The Concept of the Propositional Calculus

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> At first, therefore, the Notion is only in itself, or implicitly the truth; because it is only something inner, it is equally only outer. It is at first simply an immediate and in this guise its moments have the form of immediate, fixed determinations. It appears as determinate Notion, as the sphere of the mere understanding. Because this form of immediacy is still inadequate to the nature of the Notion, for this is free, being in relation only with itself, it is an external form in which the Notion cannot count as a being-in-and-for-self, but only as something posited or subjective. The Notion in the guise of immediacy constitutes the point of view for which the Notion is a subjective thinking, a reflection external to the subject matter. This stage therefore constitutes subjectivity, or the formal Notion. Its externality is manifested in the fixed being of its determinations each of which appears independently as an isolated, qualitative something which is only externally related to its other. But the identity of the Notion, which is precisely their inner or subjective essence, sets them dialectically in movement, with the result that their separatedness vanishes and with it the separation of the Notion from the object, and there emerges as their truth the totality which is the objective Notion.
> **G. W. F. Hegel, p. 597 of** *The Science of Logic***, translated by A. V. Miller, (Humanities Press International, Inc., Atlantic Highlands, 1989).**

1 Introduction

According to Hegel[1] the logical forms of the Concept (where we are using the term *Concept* as the English translation of the fundamental Hegelian term *Begriff*[2]) are "the living spirit of what is actual; and

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[1] §162 of the *Encyclopedia Logic*, translated by T. F. Geraets, W. A. Suchting, and H. S. Harris (Hackett Publishing Company, Inc., Indianapolis, 1991).

[2] A. V. Miller, in his translation of Hegel's *The Science of Logic*, *supra*, and other Hegelian scholars have used *Notion* to translate *Begriff*. Miller's translation, which will be abbreviated to SL in the sequel, is the main Hegelian source for this paper and we will usually follow his terminology. In using *Concept* instead of *Notion* we follow most recent Hegel translations. In particular, this is used in the recent invaluable translation of the *Encyclopedia Logic* referred

what is true of the actual is only true in virtue of these forms, through them and in them".

So from the Hegelian perspective, anything of permanent and lasting value in human culture has its validity only through the Concept and can only be properly understood in terms of the logical categories of the latter.

When we try to apply this to 20th century Logic, Mathematics (and Science), we run up against against a problem of the utmost seriousness for modern Hegelianism. The importance of the Logic of Mathematics in SL is illustrated by the fact that right at the summit of the work – in the Chapter immediately preceding that on *The Absolute Idea* – is a discussion of what is effectively the logic of Euclidean Geometry in *The Idea of Cognition*. Yet there is no doubt that despite Hegel's extensive discussions about Logic and Mathematics in SL, his views have, for practical purposes, been irrelevant to the great advances in these areas in the 20th century.

There are then two possibilities. Either Hegel is simply wrong in his claim that the logical forms of the Concept are the "living spirit of the actual", the source of any truth obtaining in the world and its cultures, or that these forms really are the key to Logic and Mathematics but have remained in-itself or implicit to its practitioners. This is the claim of the present writer, and the main purpose of this paper is that of justifying this claim by exhibiting the most fundamental of the formal logical systems – the *Propositional Calculus* – as a development in Hegelian Logic.

Our main source for this development is SL. Close inspection of SL shows it to be a wonderfully integrated body of thought permeated with remarkable rigor and continual cross-reference. It is the author's belief that it is time for the extraordinary riches of SL to be explored and developed in the context of modern Logic and Mathematics.

However, in such an endeavor, it is of fundamental importance that we allow the dialectical movement of 20th century Mathematical Logic to emerge naturally out of itself. The reason why SL is relevant in the apparently alien context of modern formal logic

to above. This translation will be referred to in the sequel as EL. The *concept* (small c) of a category is the Concept as self-limiting in the category.

is that its limitations are exactly those of *unsublated immediacies* and the paper claims that the mediation of one of these (described below) is worked out in modern Mathematical Logic[3].

With hindsight, we can see, at a more general level, the inadequate immediacies of SL revealing themselves in the collapse of a vague and indeterminate absolute idealism in the early part of this century. The sublation of these inadequacies then appear inthemselves in the working out, by mathematicians such as Frege, Russell, Hilbert and Gödel, of a determinate, symbolic calculus[4]. As external, this working out is also internal or subjective, meaning being externally hooked onto the symbols. The final stage is for the hidden Conceptual activity to become for-itself or manifest, making sense of the results that have been obtained.

The *Propositional Calculus* (henceforth abbreviated to PC), whose Conceptual development is studied in this paper, and its concomitants are fundamental for 20th century thought in Mathematical Logic and the Philosophy of Science. In Mathematical Logic, it is the basic language of *Principia Mathematica* and the logic which cements together the elementary propositions to give all propositions in Wittgenstein's *Tractatus*. It is the basic formal system in

[3]Hegel was himself not completely satisfied with SL. Indeed, in the Preface to the second edition of SL, Hegel admits "the imperfection" of the treatment of the Science of Logic in the first edition, and claims "the indulgence of the reader" with respect to its treatment in the second.

[4]Hegel had no sympathy with the Leibniz program of *characteristica universalis* and its fascinating speculations for symbolic calculation. The power of calculation in Logic, Mathematics and Science was seriously underestimated by Hegel. It can and does produce profound insights in Logic, Mathematics and Science. One need only mention how the invariance of Maxwell's equations under the Lorentz transformations – which is a matter of direct calculation – points to the principle of Special Relativity with its revolutionary *qualitative* new understanding of space and time. Even the "drier" confines of symbolic logic have their concepts though the externality of the medium forces them to be "in-itself" and so subjective. In this respect, we can compare logical symbolism to the external positing of spatial figures in geometry (such as the triangle) : these have their concepts in Hegelian philosophy (SL, p. 810). An important calculus (such as the Propositional Calculus) that means something is not *just* an "uncomprehending, mechanical mode of treatment"(SL, p. 618) (though when treated unthinkingly, it can become that) : it is the Concept's determining of itself externally, leading to its spilling out of what is implicit and to new, explicitly mediated, syntheses. However it has, of course, to be freely admitted that external calculation *by itself* is a hopeless medium for representing the Concept. The same applies to the use of geometrical figures for such representation. Hegel attempted this in his Jena years, using a triangle of triangles moving through one another to express in theological terms the life of the Idea. A fascinating account of this is given by H. S. Harris in pp. 157-188 of his book: *Hegel's Development, Night Thoughts, (Jena 1801-1806)*, (Clarendon Press, Oxford, 1983). In SL, imagery of such a kind is emphatically rejected.

Hilbert's *Metamathematics*, which has led to the securing of the foundations of mathematics[5]. In the Philosophy of Science, formal systems and their models play a fundamental role[6].

If Hegelianism is going to have any chance of making sense of Mathematical Logic, it must be able to give an account of PC. The latter is the natural test case, and in such an account, many of the important issues relevant to the more advanced and interesting logical systems will be touched on, so that a proper understanding of PC has a much wider significance than might otherwise appear at first sight.

In the case of PC, we are dealing with propositions. Propositions and judgements seem at first sight very much alike, at least in form. However, Hegel distinguishes carefully between them (c.f. SL, p. 626, EL; §167), the judgement requiring that the predicate be Conceptually related to the subject as a universal while for the proposition, the relation is on the individual level and not relevant to the Concept. In an external calculus like PC, as we shall see, such difference is obliterated and the judgements form a subclass of the class of propositions. For this reason, we will usually not distinguish between propositions and judgements in the sequel. (At a deeper Conceptual level, of course, the distinction would certainly need to be made.)

However since only judgements are determined by the Concept, it is to be stressed that our treatment of the concept of PC implicitly refers only to judgements in the Hegelian sense. (At the level

[5]See, for example, *Introduction to Metatmathematics* by S. C. Kleene, (D. Van Nostrand Co. Inc., New York, 1952.)

[6]In the following comments of this footnote, we are heavily dependent on the account of modern Philosophy of Science in the exciting book: *The scientific image* by B. C. van Fraassen (Clarendon Press, Oxford, 1980). (See especially pp. 13f. and pp. 54f. of that book.) The Propositional Calculus is a paradigm for the syntactical approach in the Philosophy of Science to the language of theory in Science. In the case of Logical Positivism, a theory is a "deductive system" whose vocabulary is divided into "observational terms" and "theoretical terms" and the empirical significance of the theory is the set of sentences formulated in the subvocabulary of the theory. The distinction between theoretical and observational terms proved untenable under the criticism of philosophers of science such as G. Maxwell, W. Sellars and P. Feyerabend, which led to the rise of *Scientific Realism*, currently the dominant trend in the Philosophy of Science. Nevertheless, in Scientific Realism (as well as in the *Constructive Empiricism* of van Fraassen) the scientific theory as syntactical is still of fundamental importance though now semantically paired up with *models*. (A lucid account of this is given in Ch. 3 of van Fraassen, ibid.) The relation between a syntactical theory and its models is strikingly similar to that in the Hegelian Philosophy between the pure Concept and its reality in the Idea, though without the "necessity" of the latter.

of propositions in the Hegelian sense, the PC is just a formal calculation game with no relevance either to Mathematics or Logic.) The obviously relevant part of SL for the PC is thus clearly *The Judgement* in the *Subjective Logic*. Provisionally, we can say that the concept of PC is that of the judgement coming for-itself in external symbolism and in-itself as subjectivity in the form of an infinite urge, their unity being actual in truth-functional calculation.

We now briefly describe the fundamental unsublated immediacy in Hegel's account of Judgement : this will be that *judgement does not come for-itself.* Firstly, we recall how the Judgement arises in SL.

The entrance of Judgement into Hegelian Logic is preceded by a discussion of the three moments of the Concept : the Universal (U), the Particular (P) and the Individual (I). Each of the moments is no less the whole Concept (SL, p. 600).

The Universal U is the Concept in its simplicity, possessing within itself the richest content (SL, p. 602). The Universal as internally differentiated , *determined* in universal, self-related differences, is the second moment, that of Particularity, P. The Concept reinstated as self-identical is Individuality, I. These moments are of central importance in SL and will feature prominently in our discussion of PC. In the context of Judgement – more precisely, in the disjunctive judgement – U,P and I are helpfully thought of in the Aristotelian terms of genus, species and the individual members in which the genus is self-identical (SL, p. 646; EL, §177).

Accordingly, the Concept, as differentiated, is the Concept as Particularity. As such, it is the *Judgement*, the *Urteil* or *original division* (SL, p. 625). In the Judgement as abstract, the moments of I and U as separate give subject and predicate, and the copula "is" expresses their identity in the Concept.

The development of Judgement is one of specification of the determinateness of the copula in relation to the two extremes of subject and predicate. The development follows the scheme of Being, Essence and Concept, the second of these in doubled form. The copula finally becomes "pregnant with content" and passes over into the syllogism.

It is clear that this development is a dialectical dance of the

copula and the two extremes, and to this extent, since we are distinguishing components of the Judgement, the Judgement is *not* for-itself as a simple something in its own right– rather it is only as *Urteil*, a divided totality (SL p. 622). PC is the development of the positing of Judgement as for-itself, unitary in externality[7]. Thus the fundamental, unsublated immediacy in Hegel's treatment of Judgement is that the latter only remains *in-itself*, as a divided totality, and the possibility of it becoming for-itself as a unity is not considered.

This simple positing of the judgement as for-itself appears symbolically in PC as p, and clearly the "divided totality" of subject, predicate and copula in the judgement has vanished. If we follow the positing of the judgement as for-itself, we must break the forward development of SL. Indeed, by positing the judgement as simple we are effectively going *backwards* in SL to the stages of Essence and Being from that of the Concept. Indeed, if we stay at the level of the Concept, then we are in a development (EL, §161) in which the distinguished moments are "immediately posited at the same time as identical with one another and with the whole" and this *cannot* be fixed abstractly in external representation, where two distinguished things are not identical.

Nor is there any way that we could save this situation by putting in new material at the earlier stages of SL since the judgement has to have emerged *before* it can become for-itself. What is packed into the judgement is not just what has gone before sublated into immediacy but rather *new* kinds of externality that did not occur at an earlier stage flow from it. This is not the immediacy of abstract being whose content is indeterminateness (SL, p. 839). Rather, it is the Concept "pregnant with content" making itself determinate in externality and calculation. We can also express this as the Concept reverting to Being and Essence only to return back to itself as mediated.

Seen from this perspective, Logic cannot be, as Hegel seems to suggest, an accumulative *linear* progress from Being to Essence and finally to Concept, but rather an onwards development of circles. It is to be compared with the flow of a river whose bed is covered

[7]Since the Judgement thus becomes a simple, the internal distinction between judgements and propositions which Hegel makes is no longer relevant (as we commented above).

with large boulders – the river flows onwards but there are eddies curving backwards before joining in the flow again.

To motivate and give guidance to the development of PC, we will look briefly in the second section at Hegel's treatment of Judgement. The extremes of the Judgement and indeed the Judgement itself are each implicitly the others and accordingly what we have to find in Hegel's treatment is where the judgement as for-itself can emerge either directly or through Conceptual transformation of extremes. As a simple illustration, the hypothetical judgement is (in SL) : *If A is then B is.* Here A and B stand for beings (SL p. 652), or immediate determinatenesses. If the latter are replaced by the judgement as for-itself we get an immediate form of the conditional connective linking *propositions*[8] which is of great importance in PC : *if a (certain) proposition holds, then so does (another) proposition.* The Concept as relation between the two propositional positings is the connective which, in its initial immediacy, is not separated out as for-itself but is submerged between propositional symbols in the form : $p \supset q$.

The precise justification for substituting a proposition in for an extreme (such as A and B above) is the following : the extreme as a moment of the Concept is the whole Concept and therefore implicitly the whole Judgement also (as Concept in its particularity).

We will show that Hegel's argument in the *Judgement* Chapter in SL admits five forms of positing of the judgement as for-itself and these are logically developed in the third section dealing with the *Immediate Propositional Calculus*. It is to be thought of as corresponding to the Hegelian stage of *Being*. At the end of this the symbolic abstract representation of propositions (as, for example, p) falls away as representing nothing and the connectives as Concept become for-themselves, explicit, acting as truth-functions on the Essence categories of Abstract Identity and Absolute Difference (symbolically represented as T and F). This brings us to the fourth section dealing with *The positing of the connective*. It corresponds to the Hegelian stage of *Essence*.

In this section, the Concept develops its content through the spe-

[8]Recall that in the context of PC, we do not distinguish between judgements and propositions.

cific connectives. This finally sublates itself in the fifth section with the emergence of objectivity in the return of simple propositional symbols p. This is the stage of the *PC as a calculus*. Here the PC becomes explicitly what in its initial concept it was implicitly – external, conceptless calculation trapped in a bad infinity whose truth is trivial (in the form of the tautology). Nonetheless, it is to be stressed that the Conceptual content which has retreated inwards and been sublated in PC into such an apparently pointless activity reemerges in meaningfulness when PC is embedded in much more Conceptual surroundings in Mathematical Logic, such as Set Theory, Quantification and Formal Systems. These are beyond the scope of this paper.

In the Judgement's coming for-itself, the Concept, then, only *seems* to be destroying itself, fizzling out, in the external medium – however, what is really happening is that it is retreating inwardly, only to emerge later in the remarkable results of Mathematical Logic, Mathematics and indeed in modern technology, having now a precise mode of expression in which it can make explicit what previously would have had to remain implicit and inarticulate since the formal apparatus required for articulation had not been developed. This emergence is the *PC as Idea* which is discussed in the Epilogue to the paper.

For motivation, we recall that Hegelian Logic concludes by positing (SL, p. 843) Nature as the Idea in an immediate totality as the absolute unity of the pure Concept and its reality as logical Idea. In the resulting new immediacy, the Concept is disguised behind appearances and the logical forms reappear as "shapes" in the development of Science (EL, p. 58) (in the everyday sense of the term). In this sense, Science is "Applied Logic".

In a similar way, the concept of PC appears as Idea in modern life and science. The form of the Idea is that of a *Boolean algebra*, a mathematical structure associated with PC. The form of PC as Idea is that of the *Computer* whose logic (in its most basic form) is exactly that of Boolean Algebra.

The paper concludes by discussing briefly some general consequences for modern Hegelian Logic.

2 From Judgement to the Propositional Calculus

In this section, we very briefly discuss Hegel's investigations into *Judgement*[9] in SL. (We have also used the account in EL to clarify that in SL wherever appropriate.) The discussion gives the minimum that we need to start developing PC and is not comprehensive. We recall that the objective of this section is to determine where the judgement can become for-itself in Hegel's treatment of Judgement.

In the latter, judgements are classified into twelve kinds. These are further divided into four groups, each containing three kinds of judgement. The groups are ordered parallel (EL, pp. 248-249) to that of Being, Essence and Concept in SL, the second being "inwardly divided in two". (The latter is a reflection of the fact that in Essence, the "determinations are only relational" (EL, §112).)

The names and groups of the kinds of judgement are those of the Kantian *Table of Judgements*[10] though in some of the groups, the listing order is different. Hegel's group headings are all different from those of Kant.

A key difference between Kant and Hegel's treatment of judgement is that, instead of just asserting this classification as a matter of fact, as Kant does, Hegel "deduces" them through a Conceptual development starting from the Judgement as the Concept divided in its self-related differences, the Concept as Particularity (discussed in the Introduction above).

The first group heading is *Existence* or (EL) the *Qualitative* and its kinds of judgement are the Positive, Negative and Infinite. The second group heading is *Reflection* and its kinds of judgement are the Singular, Particular and Universal. The third group heading is *Necessity* and its kinds of judgement are the Categorical, Hypothetical and Disjunctive. The fourth group heading is *Concept*, and its kinds of judgement are the Assertoric, Problematic and Apodeictic.

Firstly, in the Judgement of Existence, the positive judgement is

[9] The author has found particularly helpful G. R. G. Mure's treatment of Judgement in his book : *A study of Hegel's Logic*, (Oxford University Press, London, 1950). A more recent discussion of Hegelian Logic (including Judgement) is given by E. E. Harris in: *An Interpretation of the Logic of Hegel*, (University Press of America, Lanham, MD, 1983).

[10] p. 107 of *Critique of Pure Reason* by I. Kant, translated by N. K. Smith, (Macmillan, London, 1973).

a simple, immediate positing in relation : it says I=U, subject is predicate. The positive judgement for-itself leaves only the simple positing of the judgement as such which is represented in PC by a symbol such as p. The incompatibility of I and U in the positive judgement then leads to the negative judgement which is represented by e.g. *The rose is not red*. At the same time it is implicitly assumed that the rose has a color, the question being which *particular* color is involved. Of course, the particular moment of the Concept is just P, and so the negative judgement asserts : I=P. Rewriting the judgement as a negation of a positive judgement rather than a negation of an extreme (the predicate) gives the negative judgement as *It is not true that the rose is red*. In PC, the latter falls apart into the negation and positive judgement components. These are respectively denoted by $\sim$ and p and the whole proposition is represented in PC by $\sim p$. Returning to the negative judgement, we noted above that the subject and predicate were related through an implicit universality (color in the above example) giving I=P. The incompatibility of I and P finally breaks down the negative judgement and gives the infinite judgement where there is no longer even a universal relation between subject and predicate (SL, p. 642), e.g. *Spirit is not red*. The crude externalizing of PC, in which content and the implicit are obliterated, does not distinguish between this and the earlier negative judgement *The rose is not red* and we get $\sim p$ again.

The infinite judgement ceases to be a judgement through the complete incompatibility of I and U. This is the *reductio ad absurdum* of the Judgement of Existence, of treating the relation between I and U in an immediate, *Being* way. Instead, we must think of these terms as *mediated*, and following the shift from Being to Essence in SL, the Judgement of Existence sublates itself in the Judgement of Reflection where I and U are now related in *Essence* terms, reflected into each other. This first appears in the singular judgement which says (SL, p. 645):*This is an essential universal*, for example (EL, p. 252) "This man is inventive". However, since Essence cannot be exhibited in externality, PC does not know the difference between a positive and a singular judgement, and both yield p. Returning to Hegel's development, the assertion *This man is inventive* implic-

itly asserts that *some* men have this property and that we have just noticed one of them. This is made explicit in the particular judgement in which I is replaced by P and "This" by "Some". The proper place for the particular judgement in Mathematical Logic is as the relation of *subset* in *Set Theory*. However, determining the Some *in finitude* gives a judgement such as *John and William are inventive*. But when treated as for-itself, e.g. as subject to doubt, the judgement is verified by checking in turn (or in reverse order) that John and William actually *are* inventive. Accordingly when treated externally, i.e. as "something" which we stand back from and "freeze", the judgement is first bifurcated into two separate singular judgements: *John is inventive*, *William is inventive*, its unity being reestablished by combining them into : *John is inventive and William is inventive*. The two singular judgements are represented in PC as p and q and the connective *and* is represented externally as $\wedge$. Thus the whole judgement is represented in PC as the *conjunction* $p \wedge q$. There is an important feature of the particular judgement which we lose when we go to the finite form just discussed. The "Some" qualifies a universal – in the case above, *men* – and implicit in the particular judgement is the fact that we are referring to men in general, *all* men. This gives the universal judgement (such as "All men are mortal") in which the indeterminacy of the "Some" achieves determinacy. The universal judgement is not representable in PC since the "all" is not in concept finite. (In Mathematical Logic, the universal judgement achieves expression at the more advanced stage of Quantification which combines with PC in the *Predicate Calculus*.)

In the universal judgement, *all men* really refers to the genus man, a "true" universality (SL, p. 648). Both subject and predicate are now as moments of the Concept the same – the universal moment U. But to have a judgement, we must (by definition) have the Concept as P, having different extremes. Hence we no longer have a judgement. This collapse of judgement is sublated in the Judgement of Necessity in which the Concept's difference is expressed in the relation of the individuals and species to their genus, the latter determining the former as their nature or their *Necessity*. The categorical judgement expresses the necessity of the genus in the

immediate individual as in *The rose is a plant*. As in the case of the singular judgement, the categorical judgement gives nothing new for PC since it cannot represent the inner necessity – the categorical judgement doesn't *say* that there is any necessary, genus connection between plant and rose. The inner necessity of the categorical judgement is expressed explicitly in the hypothetical judgement – *If A is, then B is*. As in the case of the conjunction, this separates out in PC into the two propositions : *A is* and *B is* with necessity externalized in the symbol $\supset$. The whole judgement is thus expressed in PC as the *conditional* $p \supset q$. The hypothetical judgement above can be paraphrased as saying that the being of A involves the being of B. This unity of the two beings is the universality determining both and yet distinguishing between them. As such, it is the disjunctive judgement, formulated as *A is either B or C*. Making explicit the "break" as we move from B to C in this judgement gives it in the form of a connection of propositions : *(A is B) or (A is C)*. In PC, the "or" is represented by the symbol $\vee$. The whole judgement is then expressed in PC as the disjunction $p \vee q$. In the disjunctive judgement, the subject appears as (logical) genus which is the totality or universality of its distinct specific determinations in the predicate (SL, p. 653), the genus as "the circle of its self-excluding particularization" (EL, §177). Further, in the language of EL, §177, "the either-or as well as the both-and is the genus". But recalling that the genus is the Concept as U while the species are the Concept as P, we see that the disjunctive judgement posits the unity of U and P which is the Concept reinstated as I, individuality. The determination of the third moment I is the sublation of the Judgement of Necessity into the Judgement of the Concept.

Finally, the Judgement of the Concept is concerned with the Concept as an 'ought-to-be" to which reality may or may not be adequate (SL, p. 657). The PC as external cannot represent such an ought-to-be and so we obtain no new propositional forms from the Judgement of the Concept.

The arguments described above will prove useful in the sequel. The most important of these for our purposes is the transition from the disjunctive judgement to the Judgement of the Concept in which I arises as the sublation of U and P. The movements from §3 to §4

and from §4 to §5 are both sublations from U and P to I in the appropriate PC context.

Summarizing the results of the above discussion for PC, the judgement as for-itself in externality assumes five fundamental forms : positive, negative, conjunction, conditional and disjunction. With this guide, we now proceed with the development of PC.

3 Immediate Propositional Calculus

The immediate PC develops the positing of the judgement as for-itself in symbolic self-externality. The in-itself of the judgement retreats into subjectivity while the for-itself appears in the five forms of the previous section. The discussion of the previous section, though it will prove valuable, does not give a *proof* of the five forms. Rather, these have to be developed independently from the concept of PC. As an external positing, PC is intially a reversion to the earlier stage of *Being* in SL. However in-itself, it is still (as Judgement) at the level of the Concept. Accordingly, the arguments presented here are those of the Concept establishing itself back at the level of Being and we will find adaptations of Being concepts (such as that of *The One and the Many*) combining with some of the Conceptual arguments of Judgement in the previous section. The relation between the in-itself side of the Concept and the external Being side is that of Essence, of *Schein*, Illusory Being. The presence of Schein in this section is only for us: the development here is that of *Being* with no explicit consciousness of the "two worlds" of Essence. The connective of the next section will, however, operate explicitly in an Essence context.

We note here that there is no incompatibility with the Concept operating back at the stages of Being and Essence. It is to be remembered that the Concept contains sublated in it the earlier results of Being and Essence – the Concept is at home in them albeit as severely limited and cramped. As an example of such limitation, we recall that in the previous section, Conceptually different judgements such as the positive, singular and categorical were not discriminated by PC. A fundamental limitation of the Concept operating at these earlier stages is that it loses its character as dynamic

and of being only as result. The latter though will emerge later in attentuated form as the calculation side of PC.

As we might expect, the Immediate PC sublates itself in *Essence* categories. Indeed, the sublation posits the Concept as Individuality I in the connective as for-itself. The latter completes itself as a truth-function in terms of the Essence categories of *Abstract Identity* and *Absolute Difference*, the latter two representing the remaining moments U and P respectively and corresponding to Truth (T) and Falsehood (F) in the usual presentation of PC.

(a) *The immediate positing of judgement*

The concept of the Propositional Calculus is the coming for-itself in self-externality of the judgement. Its self-externality is on the one hand its awareness of judgement in its "everyday" significance while on the other, it is a mark on a piece of paper such as p, the two being subjectively connected up in the second representing the first as for-itself. The Concept has disappeared and we are in the domain of Essence, of insubstantial determinations or *Schein* each of which is only in its other, and is not because it is in its other which is equally a non-being. The judgement as for-itself thus disappears if either side is removed, and *is* immediacy as a returning movement from one side to the other. As such, it is (according to the Hegelian definition) a *positing* (SL, p. 401).

Looked at more closely, it is the judgement whose content has collapsed into itself – p is the judgement in utter simplicity and as disconnected from any meaning, can stand for *any* judgement. When the positing itself is viewed as for-itself, all that survives is thus the intention or subjectivity – the asserting which has nothing to assert, the judgement having disappeared in the Schein of the positing[11]. The surreptitious reintroduction of meaning and

[11] The implicit awareness of the subjectivity of the judgement as for-itself in the early developers of Mathematical Logic is shown in the assertion sign $\vdash$ of Frege and the early Russell. (The latter sign still survives in formal theories as asserting that a formal sentence is a theorem.) However, Wittgenstein in 4.064 of the *Tractatus Logico-Philosophicus* trans. by D. F. Pears and B. F. McGuiness, (Routledge and Kegan Paul, London, 1974) points out that *every proposition must already have a sense: it cannot be given a sense by affirmation. Indeed its sense is just what is affirmed.* In other words, the meaningfulness of a proposition is already its affirmation and so we don't add anything to it by prefixing it by $\vdash$. In our present context, the positing of judgement as for-itself is just the subjective intention and is therefore the same

content into propositions when all that disappeared in the simple positing is the fundamental inadequacy of PC and indeed of the main mathematical philosophies of this century. This leads, in particular, for applied mathematics, to the conflation of empirical and non-empirical concepts. (See, for example, p. 58f. of *The Philosophy of Mathematics* by S. Körner, (Harper and Row, New York, 1960).)

The fact that the utterly simple symbol p is *not* what it claims to stand for – an "honest", content-filled proposition – becomes for-itself in the immediate negative positing of judgement.

(b) *The immediate negative positing of judgement*

At the level of the Concept, the negative positing of judgement reflects the judgement as not a simple identity, an abstract U. The Judgement, as we saw in the last section, is the Concept in difference and so is not an unmediated identity. This is reflected in PC as p cancelled in its immediate simplicity. The instruction to cancel is represented by $\sim$ which, at the present stage, only is as cancelling something, p, and the negation of the immediate positing of judgement as for-itself is represented in PC as $\sim p$, the immediate negative positing of judgement[12].

However, $\sim p$ stands for a proposition in its own right. It is therefore a simple positing q of judgement having otherness in-itself. As external and so at the stage of *Being*, this incompatibility of itself with itself is a self-repulsion which becomes explicit in the *immediate conjunction*.

(c) *The immediate conjunction*

As in Hegel's treatment of *The One and the Many* (SL, p. 168), the

for *every* proposition and hence is redundant.

[12]Of course at the common sense level, it appears that $\sim p$ just asserts that what the proposition (e.g. "The rose is red") represented by p claimed is wrong – the rose is actually yellow. This is fine provided we stay at the level of the subjective immediacy. However, objectively, the content of the proposition has disappeared in the representation p so that all that the latter can represent is the moment U of the Concept as abstractly simple. Hence the true meaning of the immediate negative positing of judgement is that of the Concept negating such utter simplicity in the moment of P. We will find that throughout PC, the development is powered by the Concept. This will be particularly clear at the end of this section in the emergence of the connective.

self-repulsion is "the positing of many ones" where in our context, a "one" is a propositional positing.[13] The proposition q, mediated through its contained negation, generates a "many" of propositional positings $p_1, \ldots, p_n$ (where n is some positive integer), and the latter external spread of positings, related back to the matrix q in which they have their unity, reassemble to give q as an explicitly *mediated* propositional positing, the conjunction of the p_i's :

$$p_1 \wedge \ldots \wedge p_n.$$

In the above, the indeterminacy of the integer n leads to the sublation of the conjunction in the conditional. To see this, whatever n we stop at, each of the propositions p_i, as of equal status as a positing with q, will itself generate its own propositional positings and the number grows indefinitely. Hence the conjunction is inherently a contradiction, trapped in a quantitative "bad infinite". (This, of course, is closely related to the bad infinite of the positive integers – there is no biggest positive integer – we can never get to the end of them.) In the case of *Quantum* (SL, p. 240), the sublation of the bad infinite gives a return to the qualitative in the form of the quantitative ratio (or fraction). Now a fraction is a *relating* of the numerator to the denominator through division.[14] In the case of the bad infinite of the conjunction, we also return to the qualitative as a *relating* between conjunctions of increasing length, giving, instead of a fractional number, the *immediate conditional* $p \supset q$.

(d) *The immediate conditional*

More precisely, the bad infinite of the conjunction is the urge to extend a conjunction in the multiplying of the Many to one of infinite length, the latter, of course, being impossible to express in the finite external medium of PC. Instead of the positing of an "infinite conjunction" *beyond*, the truth of the urge is rather in the *direction of increase* which is not away beyond the finite conjunction but is

[13] This positing of the many from the one is easily seen at level of speech in ordinary language. If I say, "Jim has taken his car in for repair.", you might well reply, "What was wrong with it." This in turn is followed by my reply, for example, "It needs new brakes." This illustrates the production of the "many" propositions from my original– the original breaks up under its own negative tension and what was implicit springs out in explicit determinacy.

[14] This is further discussed in the author's paper : *Hegel and the Philosophy of Mathematics.*

simply the relation of one conjunction being part of another conjunction. In this way the urge or direction becomes explicit and is denoted by $\supset$. For example, we would write $p \wedge q \wedge r \supset p \wedge q$ since $p \wedge q$ is *a part* of, a "subconjunction" of $p \wedge q \wedge r$.

In the final step to the immediate conjunction, the initial propositions sublate their infinite drive to infinity each returning to its "matrix" p, q with these now mediated. The difference in p which powered the production of the Many (and hence the bad infinite) now appears explicitly in the conditional symbol in $p \supset q$. More precisely, in the conditional, the denial of content in the simple symbol p is negated in its containment of another. This is the negation of the simplicity of the positing of p and hence a return to the Concept – for not only was the contracting into the symbolic simple p of the judgement the very positing of PC – and we have that explicitly contradicted in the conditional – but also the mode of the containment is inner, implicit. In the conjunction, it is true, a proposition was posited as composite $p \wedge q$. However, in that case, we could remain within the ambience of PC by first positing p and then q and both preserved their simplicity. However, the positing of the two propositions together led to the internally differentiated proposition and what has happened in the sublation to the conditional is the emergence of internal structure *in* p which was presupposed as utterly simple[15]. This contradiction is sublated in the *immediate disjunction* $p \vee q$.

(f) *The immediate disjunction*

The internally structured p cannot be represented as such in the medium of PC (since p there is posited as simple). Accordingly, it is reflected back into the judgement as Concept with which (as we noted in (a)) the PC is paired in a *Schein* relation. At the level of judgement, the internal structure of the judgement, the judgement divided in-itself, is, by definition, the Concept in its moment of

[15]This does not contradict the fact that the Propositional Calculus as a formal system presupposes a countable set of propositional symbols which are "glued" together by connectives to give the formulae of the calculus. (See, for example, Chapter VI of Kleene, *supra.*) As we shall see in §5, these original symbols remain simple and unchanged throughout. The point is of course that we are not symbolising meangingful judgements in such a system – we are merely shuffling around symbols according to explicit rules.

Particularity. Now as we saw in §2, the relation of the Concept as U to that of P for judgement is that of the genus to its species. In our case, the genus (reflecting p) divides itself into the "circle" of its (logical) species and each of these as Concept in particularity is itself the full Concept and hence as self-divided is itself judgement. The original judgement then distinguishes its species as judgements as different while yet being all of them (SL, p. 654). Finitizing the species down to two, we can project them back down in externality to p and q (not the same p and q in the conditional). The genus which is both species and yet distinguishes them then transforms into the connective symbol $\vee$ giving the disjunction $p \vee q$, p or q. This "or" is both the exclusive or (reflecting the "specific difference" of the genus, the "negative relation" of the species) as well as the "both - and" of the positive identity of the species with the genus (SL, p. 654; EL, §177).

But in the $\vee$ as a simple symbol, the Conceptual difference of the genus as differentiating and yet "completely maintaining itself" in the species (as its moment P) has been obliterated, forced into implicitness. In this respect, the dialectic that the propositional symbol p underwent above is parallel to that which $\vee$ will now undergo. In the case of p, the proposition was posited as simple but the conditional showed it to be internally differentiated and reflecting back into the Judgement level we had p as the "genus" in the form of an explicit combination of its "species" represented in PC by propositional symbols. Now in the case of $\vee$, the "specific difference" given in the exclusive "or" and the "positive identity"given in the "both - and", both present in $\vee$, must also be made explicit. Reflecting back into Judgement, we see that the connective (represented by $\vee$) is the Concept with its difference specified as the moments of the Concept as follows. Firstly, the "both - and" is U as identical in the species. As such it is U separated out as abstract. Secondly, the exclusive "or" is the species in their difference or negative unity. As such, this is the moment P as abstract in its difference. The unity of the U and P, represented in $\vee$, is (as we saw in §2) the third moment of the Concept, that of Individuality, I. Accordingly, the disjunction sublates in the *connective* as representing the moment of Individuality. (Of course, this is parallel to what happens in the

transition from the Judgement of Necessity to the Judgement of the Concept in SL.)

(f) *The connective as the Conceptual moment of Individuality*
With this development, the Concept as connective has returned to itself in I. In this return, the propositional symbols have therefore disappeared and the task of PC is that of representing only the connective-as-I in externality. This representation will give the connective as a *truth-function*.

The disappearance of propositional symbols at this stage of PC is of the greatest importance. It is normal in accounts of PC to preserve these : in terms of calculation, this is convenient. Philosophically, it is crucial that these symbols disappear. (This is partially recognized in Frege's view of propositions as names – this is discussed in a later footnote.) The connective as truth-function is not a function of propositions – it is a function of strings of the symbols T and F with no mention of propositions[16] at all. (See for example, Körner, *supra*, p. 39.)

Looked at more closely, with the emergence of the third moment of the Concept, I, out of the other two, U and P, the Concept in the limitation of PC as immediate is now complete. The Concept has to be posited as for-itself in externality in PC for that is what the concept of PC is. (Initially, such positing was of Judgement but the Concept as I is mediated Judgement.) Now in the Hegelian Philosophy, the Concept as posited immediately in externality is the Idea as Nature (SL, p. 843). Hence the positing of the Concept for PC must be regarded as the Concept as Idea at a very "primitive" level. Now the Idea as adequate Concept is Truth (SL, p. 755). Abstracted for external representation in PC, (Hegelian) Truth falls apart into two components. The first is the adequacy in which the Concept as I – the consequence of the sublation of the immediate disjunction – is identical with itself in its external other. This is the preservation of the genus in the species and therefore represents the moment U. It is represented by the symbol T. The second is the adequacy with which the moment of difference (P) in the Concept

[16]The p and q will return further on in §5 with PC as a *formal theory*, where they appear as elements of a presupposed countable set of symbols.

as I has been preserved in its sublation. This is represented by the symbol F. Both T and F represent the Concept equally well (or more accurately, equally badly) as T. As we shall see, these symbols are only in their immediacy and will be developed further in the following section.

For the present, the two symbols T and F are external representations of the moments U and P whose sublation gave the moment I.

Both equally are or are not the Concept, and the only reason that T has held onto the term *Truth* in PC is the external view of the true (in the sense of Hegel) as being the asserting of what is only positive and without any mediation[17]. Nonetheless, in the development described below, T will play a more significant role than F. This is because in external representation, the difference cannot be incorporated explicitly into the unity of I: different things remain distinct. Hence the integrity of I is obtained at the price of leaving the difference as implicit and the moment I with difference supressed just brings us back to U in abstract form.

So T represents the abstract universal, its preserving of itself as genus in the species, the "both-and" discussed above, in isolation from the exclusive "or". It is thus an expression of Identity as abstract, the *Abstract Identity* category of *Essence* (SL, p. 412).

The moment F represents the abstract particular, the Concept

[17]Hegel distinguishes between correctness and truth in the context of propositions and judgements (EL, pp. 249-250). Correctness "is only a matter of the formal agreement of our representation with its content" while truth "consists in the agreement of the object with itself, i.e. with its concept". Truth in the context of PC is usually taken to mean simply correctness. However, correctness is context dependent and indeterminate. You say "The rose is red" : I look at it and reply, "So it is." Your assertion is correct. However it is context dependent – it depends, for example, on my eyesight, on the light, on all the implicit circumstances in which the conversation is embedded and from which it derives its sense – and it is indeterminate – for example, exactly what shade of red are we talking about here, exactly what portion of space-time is occupied by "the rose". (An interesting expression of this is an early claim by Frege that the meanings of words are determined by the senses of the propositions is which they occur. (An interesting discussion of this is given on p. 117 of *A companion to Wittgenstein's Tractatus* by M. Black (Cornell University Press, Ithaca, 1966)). Frege later abandoned this view of the meaning of words in his distinction of Sense and Reference.) The truth claimed by PC is, on the contrary, absolute and not relative to context, and it is definite. As such what PC is aiming for is at bottom the Hegelian concept of truth, truth as the genuine, "absolute" truth. All "normal" propositions "contain" both moments of T and F and the attachment of one or other exclusively to propositions in the way that PC wants leads ultimately to its propositions being limited to the very special ones of formal systems in which the context determines absolutely.

as species, the exclusive "or" in isolation from the "both-and". It is thus an expression of Difference as abstract, the *Absolute Difference*[18] category of *Essence* (SL, p. 417).

Since the Concept has through the immediate disjunction returned into itself in individuality, the latter must also appear in the external positing of PC. Corresponding to the sublating of U and P in I, the Concept will appear as the connective in its uniting or sublating of the moments T and F of Abstract Identity and Absolute Difference. In this, the Concept as complete in the individual I extricates itself from its surrounding propositional symbols and becomes for-itself as the connective, thus overcoming the immediacy in which it was sunk in the earlier part the development.

Accordingly, the next stage in our discussion of PC is the positing of the connective, the development of the connective as external representation of the sublation of U and P in I.

In terms of the basic stages in Hegelian Logic, the move from the Immediate PC to the positing of the connective is parallel to that from Being (in its immediacy) to the two-fold world of Essence, the truth-table as the quantitative "appearance" of the connective. (As we noted earlier, this is also the case in the Hegelian treatment of Judgement, the two-fold character of Essence appearing in the Judgements of Reflection and Necessity.)

4 The positing of the connective

The moments of U and P have become for-themselves in T and F, and thus fall apart in external representation. The dynamic sublating of U and P in I is, however, reflected at the external level by the

[18] It is in this sense that we have to understand the strange claim of Frege that all true propositions are *names* of the truth value T. The truth of this lies in the insight that "correct" judgements, abstracted in the propositional form p are all equally empty and all that survives is what they have in common, a simple "something" without content, the universal moment of the Concept as Abstract Identity which is just T. Frege's version of Logicism is "realist" (c.f. Körner, *supra*, Ch. 2). On the other hand, the "nominalist" version of Logicism as developed by Russell is dominated by Absolute Difference : thus propositions divide up into "elementary" propositions which are logically "fitted" together by connectives to give the original. The trouble with this is that any recognizable judgement, as potential Concept, must always be *other* referent and hence not elementary. The "elementary" propositions of formal systems which are just the elements of the presupposed countable set (discussed in an earlier footnote) are only symbols, not propositions at all.

operation of the connective as a *truth function*. The truth-function is dynamic – it *does* something[19]. It sends members of a finite set, initially just the two-point set $\{T, F\}$, into the two point set $\{T, F\}$. (We use the term "set" here for convenience – strictly, it is finite set in its immediacy.) More generally, we will have to consider the connective as defined on sets of the form $\{T, F\}^n$ whose elements are strings of *T*'s and *F*'s of length n. The connective mediates Abstract Identity and Absolute Difference by acting on *all* of the strings of length n. At the same time, the qualitative abstractness of T and F is quantitatively determined by the connective as function by its function values.

In this section the connective will develop itself through the sequence of connectives that were present in immediate form in §3. The first two of these are the positing of Abstract Identity and Absolute Difference at the connective level, and sublate in the positing of the set of pairs of *T*'s and *F*'s on which the remaining three connectives operate. In the latter, the connective as Concept acts on the columns as external "species", and hence as a primitive "genus" must reestablish its universality U as identity (T) in working through the species elements. Hence the concepts of these three connectives determine their truth-tables in terms of T. The section culminates with the making explicit of this genus-species relation at the connective level, and U and P sublate in I for a second time, this time in the connective as *free*, the positing of the *arbitrary connective* which expresses itself in the PC as a logical calculus in §5.

[19]The concept of function is of importance in Hegel's treatment of Mathematics and in particular, of the Differential Calculus (SL, pp. 240-313). The qualitative character of the function is shown in the shape of its graph and in its defining instructions while its quantitative side is shown in the calculation of its values and in its graph as the explicit set of ordered pairs $(x, f(x))$. (The graph as quantitative will play an important role in our discussion below.) According to Hegel (SL, p. 251), the problem of incommensurability "leads to the infinite which mathematics has introduced with such functions, in general, with the *functions of variable magnitudes*. This infinite is the *genuine mathematical* qualitative infinite ...." (The author discusses this in more detail in his paper "Hegel and the Philosophy of Mathematics".) We may say that as the genuine mathematical infinite, the function is the best representation of the Concept in Mathematics. It is thus not surprising that, in our situation, the Concept as I, represented externally, appears qualitatively as a function acting quantitatively on representatives of the two other moments of the Concept. Of course, the truth-function, as acting on a finite set, is far less adequate than the "genuinely infinite" real-valued function of Mathematics: the latter is at a much more advanced level.

(a) *The immediate positing of the connective*

The development of the positing of the connective is that of the representation of the sublation of U and P in I. The latter first appears in the shape of the holding together of U and P in the connective while keeping them apart in their external difference. This is the immediate positing of the connective: the connective is the *identity function* $\mathcal{I}$.[20] This leaves T and F exactly as they were before : as a function, it is given by : $\mathcal{I}(T) = T$ and $\mathcal{I}(F) = F$. This is, however, an advance inasmuch as the two are now taken together and unified as the domain of the identity function which embraces both together. It is the weakest possible way for T and F to relate together as representing the sublation to I. Its action as mechanical is presented in the counterpart to the arithmetic multiplication table – the *truth table*.

Usually, truth-tables are presented in terms of propositional symbols, but as we have seen above, such "aids" are philosophically misleading at this stage since propositional symbols disappeared in the emergence of the connective as for-itself. The truth-table for $\mathcal{I}$ is given below :

T	T
F	F

This way of tabulating the connective as a truth-function is to be interpreted as follows. On any given row, the function value of the string of T's and F's to the left of the double vertical line is the truth-value on that row to the right of the double vertical line. For example, the first row of the truth-table for the conjunction in (c) below just says that $\wedge(T, T) = T$.

The Concept as the Essence category of Abstract Identity has thus reappeared at the function level as $\mathcal{I}$: everything stays identical with itself under the connective. As such T and F remain completely indifferent to one another, are "kept apart". However, the function as Concept's moment I needs to incoporate within itself the negativity which is at present only down in the domain of $\mathcal{I}$ in the polar opposites T and F. The $\mathcal{I}$ function does not do this and does not mediate the opposition. Indeed, as Hegel comments

[20] The identity truth-function usually does not feature in normal treatments of the PC, but is philosophically necessary as the connective coming for-itself in simple immediacy.

on SL, p. 615, in the Concept "identity has developed into universality, difference into particularity". Since these moments U and P are (with I) the same in the Concept, this identity must also show in their representatives T and F in the PC. The identity is initially achieved in the connective as *negation*.

(b) *The negation connective*

The negation connective $\sim$, as representing the Conceptual moment of I, establishes the unity of Abstract Identity and Absolute Difference, the projections in Essence of the Conceptual moments U and P, by sending T to F and F to T. The truth-table for $\sim$ is given below :

T	F
F	T

This is more clearly seen in terms of $\sim$ as function in its external, quantitative form, i.e. in terms of its *graph*. The graph of a function f in Mathematics is, as we saw above, just the set of pairs $(x, f(x))$ as x ranges over the x's for which $f(x)$ is defined. Viewed from this perspective, $\sim$ pairs together as a unity T with F and F with T. As such, it is at the connective level an external representation of Absolute Difference. According to Hegel, Absolute Difference is "self-related, therefore simple difference" (SL, p. 417). The self-related difference in our case lies in T being paired with its "opposite" F which relates back to T as *not* T. (In this respect, our treatment of Absolute Difference at the connective level is entirely parallel to Hegel's treatment (ibid) of the Absolute Difference of A and *not-A* with T in place of A and the simple "not" that constitutes it as $\sim$.)

Accordingly, the moments of Abstract Identity and Absolute Difference, originally posited in the truth-values T and F, have now raised themselves to the level of the connective, the former represented by $\mathcal{I}$ and the latter by $\sim$.

Recalling that U and P sublate in the moment I, we now have to see how the connectives $\mathcal{I}$ and $\sim$, as representing U and P, sublate their positings in a connective.

Since there are no other truth-functions of one variable, the Con-

cept as I cannot realise itself directly as a connective. Instead, what is implicit in the two connectives $\mathcal{I}$ and $\sim$ must first become explicit. What is implicit is that each connective, regarded as a graph (as described above) effectively identifies x with $f(x)$. The identification is expressed in the ordered pair $(x, f(x))$. The ordered pairs which we obtain from $\mathcal{I}$ are (T, T) and (F, F), while those obtained from $\sim$ are (T, F) and (F, T). Thus the two connectives as combined in their graphs immediately sublate themselves in the finite set $\{(T, T), (T, F), (F, T), (F, F)\}$. Thus the finite set is the immediate sublation of Abstract Identity and Absolute Difference, the "union" of the two connectives as graphs, corresponding to the combining of T and F in a set which we saw in the immediate positing of the connective. It is not, of course, a connective, and it remains to be seen how complete sublation gives a connective (the conjunction).

The moment I as externalized appears as the set of four pairs. As such it is a "loose" amalgamation of its elements, an abstract universality whose difference is explicitly exhibited in the separate elements. This "looseness" shows, in particular, in the *indifference* to the order in which the four pairs can be written down to constitute the set. We can, for example, write down the set as $\{(F, T), (F, F), (T, T), (T, F)\}$ rather than as that of the preceding paragraph.

As such, the individual as this set is (SL, p. 619) the "false path" of abstraction, rather than the "individuality to which the universal in the determinateness itself descends". Such a descent must move down to the level of the pairs as the determinateness of the set's universality.

At the level of the pairs, an importance observation is that *the simple positings of Abstract Identity in* T *and of Absolute Difference in* F *have now become mediated*. For example, a T at the first position is now differentiated from an identical T at the second position. Thus the simplicity of the initial positing has been mediated through the order relation[21]. Truth as mediated comes to explicit

[21] A similar mediation occurs in Hegel's treatment of Arithmetic in which the number as "indifferent, inert" is "actuated from without and so brought into a relation" (SL, p. 205). The relations in Arithmetic are the arithmetic processes such as addition and multiplication. However in our context, the relation is that of pairing, or more generally, forming (finite)

recognition at the connective level in the *conjunction*.

(c) *The conjunction connective*
Since the Individual is the "immediate identity of the negative with itself" whose soul is abstraction (SL, p. 621), its positing is immediately one of abstract identity and hence T. The moment I as the sublating of U ($\mathcal{I}$) and P ($\sim$) is then the recognition of mediated truth alone as true. In other words, the conjunction asserts truth only when the two Tś are together – any pair which is tainted by an F is rejected or posited as false.

The truth-table for the conjunction $\wedge$ is thus :

T	T	T
T	F	F
F	T	F
F	F	F

It is instructive to compare the argument presented above with that used in the last section to obtain the immediate conjunction from the immediate negative positing of judgement. The latter positing, as incompatible with itself, like the One in SL, repulsed itself from itself into a Many of posited judgements which recombined in finitude to give the positing of the immediate conjunction. In the argument above, the simple moments T and F reproduce themselves combinatorially in pairs. Instead of the immediate positing of the conjunction, we have posited a connective which asserts truth (in the sense of T) only in the presence of double, mediated truth. The immediate conjunction is an immediate, qualitative asserting, unmediated by "doubt", while the conjunction connective asserts truth uniquely from a range of four explicit possibilities, the other three being asserted as false (in the sense of F). This illustrates how the present section mediates the former.

In the development of the immediate conjunction to the immediate conditional, the conjunction was caught in a bad infinite as the result of its urge to extend indefinitely under the self-dividing of its propositional symbols.

strings of T's and F's.

In the context of the conjunction connective, there is also a bad infinite. The recognition of mediated truth in the two variable conjunction necessarily leads to this bad infinite. Indeed, to posit this recognition absolutely, we must not stop at the 2-variable conjunction – the general n-variable conjunction Λ_n $(n = 3, 4, \ldots)$ also recognizes truth only when it acts on a string of n T's, and the mediation is greater in this case since there the number of variables (and rows) involved is greater. The bad infinite unfolds as we let $n \to \infty$.

In the case of the immediate conjunction, the sublation of the bad infinite in the immediate conditional came about through the positing of the relation between a conjunction and a part of it. Similarly, in the present situation, where there are no propositional symbols, the sublation of the bad infinite of truth mediations in the Λ_n's is given by the form of the containing relation between a Λ_n and a Λ_m where $n > m$. Λ_m is, as it were, a "part" of Λ_n. More precisely, when Λ_n asserts truth, all n-variables must be T, and hence the first m of these variables that Λ_m acts on are T as well. So the truth of Λ_n forces the truth of Λ_m. The posited form of the containing, forcing relation between Λ_n and Λ_m – which is independent of the particular choice of n and m – is the concept of the 2-variable *conditional connective* $\supset$.

Since this positing is no longer in the PC as immediate, it has to be expressed as mediated, i.e. in a quantitative, "truth table" form. (This quantitative mediation is not the truth-table of the conditional which we shall derive below. Rather it is the mediation *directly* expressing the conditional's positing and hence involves the Λ_n's.) This is the quantitative determination that expresses the forcing relation between Λ_n and Λ_m through the connective $\supset$. More precisely, $\supset (\Lambda_n, \Lambda_m)$ is unconditionally true since the relation by definition holds between Λ_n and Λ_m. In the medium of PC connectives, unconditionally true means "identically" true in its truth-values, i.e. is a *tautology*. Spelling this out in detail, for any n-tuple of truth values, compute Λ_n's value on the n-tuple to get a truth-value V_1. Then compute Λ_m on the first m entries of the n-tuple to get a truth-value V_2. Then $\supset (V_1, V_2)$ is *always* T (no matter which of the 2^n possible selections of the n-tuple we started with).

(d) *The conditional connective*
The truth-table for the conditional $\supset$ is given by :

T	T	T
T	F	F
F	T	T
F	F	T

The top two rows of the table express explicitly the concept of the conditional connective as described above – when the first variable is T then the conditional is true if the second one is, and is false otherwise. (In the latter case, the forcing relation does *not* hold and so we obtain an F truth-value.)

The second two rows are determined by considering the tautology $\supset(\Lambda_n, \Lambda_m)$. We will consider the case where $n = 2$ and $m = 1$ (though the general case will give the same result). Then Λ_2 is the usual conjunction Λ while $\Lambda_1 = \mathcal{I}$.

Now $\wedge(T, F) = F$ and $\mathcal{I}(T) = T$, so that

$$\supset(F, T) = \supset(\wedge(T, F), \mathcal{I}(T)) = T$$

giving the third row above. The last row follows similarly by considering (F, T) in place of (T, F). (The reader may find the above easier to follow by using propositional symbols and calculating the truth-table of the tautology $(p \wedge q) \supset p$. As we have commented above, this is not philosophically justifiable at the present stage since such symbols have disappeared, but is correct in the PC as a formal system with a presupposed set of elementary "propositional" symbols. In fact, the above tautology can be taken as one of the postulates for PC as a formal system (p. 82 of Kleene, *supra*).)

The truth-table of the conditional is just the making explicit of the position of Philo in his famous debate with his master Diodorus. According to Sextus Empiricus[22] *Philo says that a sound conditional is one that does not begin with a truth and end with a falsehood.* So we only get the conditional giving an F value when the first variable is T and the second is F. This is often regarded as disturbing since it seems to say that any false statement implies a true, and

[22] p. 128 of *The Development of Logic* by W. and M. Kneale, (Clarendon Press, Oxford, 1963).

that flies in the face of common sense[23]. However, the problems with ordinary language don't apply since the content of propositions has long gone – there aren't even any p's left as representatives of "real" propositions or judgements. The problems disappear when we realise that in PC, we are not talking about content but form, and about the Concept, not contingent matters of fact[24].

When we look more closely at the conditional connective, we see that the variables T and F differ in their Conceptual significance in accordance with the column to which they belong. The columns are invested with the significance of the *order relation* which has come for-itself in the externality of the left column dominating the right through the conditional connective.

But in itself, in its externality, a T in the first column does not differ from one in the second. The Concept as the conditional connective is not Necessity as attached externally to one column: instead, the Necessity of the Concept works through both columns equally, just as in the case of cause and effect, neither precedes the other but each presupposes the other (SL, p. 566).

The making explicit of the equal significance of the two columns is achieved in the disjunction connective.

(e) *The disjunction connective*

The truth-table for the disjunction connective is :

[23] For example, consider the "counterfactual" : *If chocolate had not been eaten by anybody, then the second world war would not have happened.* A discussion of the problems associated with the Philonian conditional appears in the book : *Formal, Transcendental, and Dialectical Thinking* by E. E. Harris, (SUNY, Albany, 1987).

[24] Of course, in PC as a *formal* language, the truth-table for the conditional is not a problem since the presupposed propositional symbols are just marks with no meaning and all other "propositions" are built up from these using connectives. For such a language, Wittgenstein's assertion (p. 18 of the *Tractatus*, *supra*) is true : *A proposition can determine only one place in logical space: nevertheless the whole of logical space must already be given by it.* The exact formulation of this is given in Metamathematics. It is simply a theorem in Metamathematics that given PC as a formal system, then there is a $\{0,1\}-$ (or, if you like, a $\{T,F\}-$) valued function on the "proposition letter formulae" giving truth-tables in the usual sense. Using these, it is another theorem that this yields Consistency, i.e. that one cannot derive, for some p, both p and $\sim p$ within the system. The latter is the precise formulation of what Wittgenstein meant in the preceding quote. An excellent treatment of the Metamathematics of PC is given in Chapter 6 of Kleene, *supra*. In the world of Mathematics, practising mathematicians do not think in terms of a false statement mysteriously "implying" the truth and the falsity of everything else: rather, as above, they think in terms of Consistency, or, what is philosophically equivalent, Coherence.

T	T	T
T	F	T
F	T	T
F	F	F

In more detail, the Essence category of Necessity operative as containment in the conditional sublates itself in the Concept as genus realising itself in the relating of the columns as species. The columns in themselves are, of course, identical but are differentiated in the order relation. Each as a unity of *T*'s and *F*'s is the potential Concept or genus. As "species" each is distinguished by position but at the same time, they are treated equally by the genus. This is the genus as its "circle" of species (which we discussed in its immediate form in the *immediate disjunction* of the last section) realized insofar as is possible in the present medium in respect of the species as the specific difference of the genus while each is the whole genus. As establishing its self-identity in its action, the disjunction as genus searching for-itself as self-identical (T) in the species finds itself as such in the self-identity (T) of (at least) one of the column "species". In such a determination, the species as proto-genus is giving back to the genus its own self-identity. On the other hand, the genus remains divided (F), does not find itself, if both columns are false. This is the content of its truth-table.

The disjunction as representing the genus in its circle of species thus collapses since it does not always return to itself in self-identity U. The moment P, which is also the whole Concept, is not properly included in the disjunction, with the latter's one-sided obsession with truth as abstract self-identity representing U.

There are two immediate consequences of this. Firstly, the disjunction as not representing the Concept adequately, has to sublate itself in a connective in which the one-sided emphasis on T is overcome. Secondly, it is not enough that the column as "species" be only a proto-genus – there has to be a sublation in which the column actually achieves the unitary status, the wholeness of the genus, not just a falling apart into separate T and F.

Both these sublations are consequences of the sublation of the disjunction, which, as in the case of the sublation of the syllogistic disjunction (SL, p. 702) is the transition to *Objectivity*. We now

look at this more closely.

The Hegelian sublation of the disjunction in the moment I is initially that of the positing of an *objective totality of simple objects* p. In this positing, the moments of T as U and F as P, which at the end of §3 were sublated in the connective as I dynamically, are now sublated in the objects p which represent the moment I. Indeed, in the former sublation, the connective only *related* the T's and F's to give T's and F's but left them in themselves still indifferently apart. Now they are sublated in symbolic objects p. The symbol p has the potentiality to be T and F, the moment I containing within it these as sublated moments. Note that externally, p stands only for an abstract symbol: there is no connotation of it standing for a "proposition" – the latter disappeared at the end of §3[25]. Note also that the multiplicity of such objects p is the preservation in the sublation of the indefinite number of variables that can occur in a connective[26].

At the same time, the one-sided emphasis in the disjunction on the moment T is sublated in the *arbitrary connective* which can assume both truth-values in a completely "free"[27] way. In this way, a false truth-value is now an expression of the moment P in the connective, the latter being as intrinsic to the connective as U is. The connective now acts on the propositional symbols p by realising their potentiality as columns of T and F.

For the arbitrary n-variable connective Λ, then, anything goes, with arbitrary choices for truth-values and the number of variables

[25] Of course, of particular importance, is the case where the p's stand for claims in a mathematical proof but this is not essential. PC is now purely syntactical and a variety of interpretations is possible. The network example of §6 is one possibility. We should also recall (in an earlier footnote) that the PC as a calculus can be, and nowadays usually is, presented axiomatically in terms not involving "truth-values". Indeed, in Metamathematics, it is common to list certain sentences as axioms with *modus ponens* as the only rule of inference (Kleene, *supra*, Ch. 6). The "truth-value" results are then theorems of the PC.

[26] As commented above, it is instructive to compare the emerging of objects p above and the transition in SL from the syllogistic disjunction to the world of objects encompassed by Mechanism (SL, pp. 701-711). Hegel comment (SL, p. 710) that "objectivity is an immediacy whose moments, by virtue of the totality of all the moments, exist in a self-subsistent indifference as objects outside one another, and in their relationship possess the subjective unity of the Concept only as an inner or an outer unity. This is mechanism." We will see the "mechanism" for PC in the form of the computer in §6.

[27] Of course, this is not free in the full Hegelian sense since the latter in the Concept presupposes Necessity (EL, §158). We will see later how the Necessity earlier in the conditional will reemerge in the tautology.

involved. Finally, the sublation of the columns of T and F into simple "propositional" symbols such as p now applies to the columns of truth-values for the connective to give it as acting on an n-tuple $p_1, \ldots, p_n$ of such symbols. The resultant truth-table – usually much more complicated than $\{T, F\}$ – is then sublated to give another constructed "proposition" $\Lambda(p_1, \ldots, p_n)$, the latter notation documenting the new proposition's dependence on the p_i. The propositional symbol has now returned to itself mediated through the connective. The connective can then apply to n-tuples of mediated propositional symbols in the same way as it did for the simple p_i's.

The dialectic of PC has thus returned as mediated to its starting point. In the Immediate Propositional Calculus, we started with the immediate judgements of ordinary life, only to find that their positing as simple led to their disappearance and the emergence of the connective as for-itself. Now the sublation of truth-table columns has led to the positing of simple propositional symbols which the connectives build up into complex ones, and we are back with a set of "judgements" on which the PC. These are genuinely built up out of "atomic" propositions" and give a logically sound theory. But of course they are nothing like the propositions that we started out with since they have no content and are purely syntactical.

With the emergence of the "totality" of all propositional symbols or *sentences* on which the arbitrary connective can act, the positing of the connective has sublated itself in *PC as a calculus*, PC as the formal system studied in Metamathematics in the work of mathematicians such as Bernays, Post, Hilbert and Ackermann.

5 PC as a calculus

The arbitrary connective differs from that of the posited connective of §4 inasmuch as the latter was not mediated – indeed, the development of §4 was exactly that of the species of connective or alternatively expressed, the development of the moment P mediating the connective. With the arbitrary connective, the moment P is now present in the sense described above that T and F are now equally respected. However, at the level of the connective as such, the arbitrary connective as the moment I only contains the Concep-

tual moment of U in abstract form, since it is just the positing of any function of n-variables into T and F without division into kinds. The abstraction is that of the set of such functions as totality.

The moment of particularity P, however, is implicitly present in the arbitrary connective. The moment P, as expressed in terms of the species of connective discussed in the preceding section, comes to explicitness in the theorem[28] that the arbitrary connective *is built up from the species of connectives* $\mathcal{I}$, $\wedge, \supset$ *and* $\vee$. For example, the 3-variable connective given in the "sentence" $(p \wedge q) \vee r$ is built up from the two other connectives $\wedge$ and $\vee$.

At the same time, the totality of propositional sentences is also mediated and hence has its moment P. The totality of sentences as the moment U is mediated through the action of the connectives which presents a sentence as built up out of the presupposed simple sentences p as exemplified by the example in the previous paragraph. Thus the sentences are mediated by its simple "species" – the p's - and the species of connective which "glue" the simples together.

In this totality, the three moments U, P and I of the Concept now coincide. The moment U is the totality, not just as a blank collection, but as the form of sentence. The latter is explicitly given as P, a circle of species at two levels – firstly the presupposed symbols as species, which secondly unify with the species of connective to give the sentence. Finally, the moments U and P sublate in the moment I, the individual sentence which implicitly is every sentence (and hence U) through the logical structure in which alone it has any meaning[29] We are now therefore in the sphere of the Concept (in limited form, of course).

But the Concept is only as result and PC as presented above is static, the identification of the three moments being only inner and not in the external working out. The latter is the determining of sentences in terms of their moments of truth-values. This is truth-table calculation and there was no mention of this in the previous paragraph discussing the identity of the three moments of the Concept in PC.

Indeed, the simple propositional symbol was the sublation of

[28] Mendelson, *supra*, p. 24

[29] The identity of I with U is essentially the same point that Wittgenstein makes in (3.42) of the *Tractatus* discussed in Footnote 22 above.

$\{T, F\}$. The *complete* determination of the sentence is not only its expression as I but is also its determination of the other two moments in terms of T and F. This is PC as a Calculus[30] and is essentially the truth-table computation for connectives as given in terms of the species of connectives.

Hegel points out (SL, p.216) that occupation with calculation is an "unthinking, mechanical one". However, it is a different matter with the great theorems of Number theory (such as the Prime Number Theorem). These involve the Concept (c.f. SL, pp. 792-794). In the case of PC, we also have that calculation for its own sake is "unthinking and mechanical". However, the sublation from that of *The positing of the connective* to the *PC as calculus* was a movement from *Essence* to the *Concept* and as such has, like the Judgement of the Concept (SL, p. 657) an "ought-to-be to which reality may or may not be adequate". The ought-to-be is (in Aristotelian language) *that for the sake of which* the calculations are performed. The sentences producing those special connectives for the sake of which calculation is performed are the *tautologies*, the "theorems" of PC.

This is the Concept as determinate in the following manner. The presupposed symbols are indeterminate, sublating T and F but inadequately as just the potentiality for splitting up into T and F. The symbol p is only a promise of their unity, but in actual fact it is only as a name for holding them together externally, a name for the pair $\{T, F\}$[31] The determinacy of the Concept in PC cannot

[30] In this respect, it is to be compared with Arithmetic discussed by Hegel in SL, pp. 202-212. In that case, the "species" of calculation are the basic arithmetic operations such as addition and multiplication. Calculation in the latter case uses *multiplication tables*. In our case of *logical* calculation, the "species" are the connectives and instead of multiplication tables, we use the *truth-tables* which were constructed in §3.

[31] In the context of Wittgenstein's *Tractatus*, if the presupposed propositions p are the elementary propositions – those which represent possible atomic facts – and we know their truth-values, then the world as the totality of facts articulated as sentences in PC is determined. (For a discussion of the truth-values of elementary propositions in the *Tractatus*, see Ch. XL of *A companion to Wittgenstein's Tractatus* by M. Black, *supra*.) An obvious problem is that of determining when an elementary proposition is false, and this is aggravated by not knowing what needs to be checked since there are no known elementary propositions. However, as we have seen above, the "elementary proposition" p is not in principle determined as true or false: rather it is the naming of the sublation I of U and P, the latter two abstractly represented by T and F. It is the positing of a proposed sublation which cannot by reason of the abstract externality, be carried out. In this respect, it is only an external reflection of the true sublation of U and P as I in the Concept.

therefore be achieved in the presupposed propositional symbols and is only to be found if at all in composite propositional sentences G.

Such a G as determined must be independent of the particular truth-values arbitrarily attached to the presupposed symbols since otherwise it will depend on arbitrary choice and so be indeterminate. The truth-table of G must then be constant and there are only two possibilities : the constant is either T (G is a tautology) or F (G is a contradiction). Now the determination that is being sought in PC is, as we have seen throughout this paper, that reflecting the sublating of U and P in I. But as we commented earlier, I as represented externally as a simple symbol coincides with U since the moment P as difference cannot survive the simplicity of the symbolism. Hence the determination of I in PC must coincide with that of U, Abstract Identity or T.

Hence the complete determinacy of the Concept in PC is the *tautology*, the theorems of PC as a formal system, that for the sake of which we do PC calculations. The PC as consummated in the tautology is the unmediated positing of the Concept as blank T, abstract individuality as Abstract Identity. With this, the Concept sinks without trace in the bad infinity of the mechanical checking of truth-tables and the establishing of tautologies, contentless truth-functions.

Looking back to the beginning of our discussion of PC, we recall that we started with judgements or propositions in their usual non-technical sense. When we substitute these in place of propositional sentences in a tautology we obtain propositions which are silly and useless, which have no content – propositions such as: *Either the sun will shine tomorrow or it will not.* It may seem strange that the development above has resulted in propositions which have no content, which say nothing (in a complicated way, of course). What we are seeing here is the whole enterprise of PC as the *Judgement posited as for-itself in externality* reaching its climax. As we commented earlier, such a positing of Judgement is the destruction of content since the internal structure of the Judgement which gave it its meaning has been abandoned.

Nevertheless, it has to be stressed that the dialectic that has been developed here has not evaporated – it has merely retreated

inward, waiting for a context (such as Number Theory) for the submerged Conceptuality to emerge semantically in Idea. The meaning provided by the context lets loose the sleeping Concept, as content flows back in and PC lives. In this respect, it is to be compared with arithmetical calculation in Number theory as discussed above. As calculation for calculation's sake, operations such as multiplication and long division are mechanical and often those who have been best at performing them have in other matters, shown a low level of thinking skills. It becomes an entirely different matter when the Arithmetic operations go to work in the context of the great theorems of Number Theory.

The emergence of the concept of PC in Set Theory, other richer formal systems (such as the Predicate Calculus) and indeed in modern Mathematics can not be gone into here[32]. Instead, in the Epilogue, we will briefly consider PC as Idea in the sense of Science and present day life.

6 Epilogue

At the end of SL, the Idea posits itself as "absolute unity of the pure Concept and its reality, and thus contracting itself into the immediacy of being, is the totality in this form – nature."(SL, p. 843). As we commented in the Introduction, when Science (in the usual sense of the word) is developed, the immediacy of being is mediated and the logical forms of the Concept reappear in the former's "shapes". So all the categorial development which was submerged in the immediacy comes back, not in pure form, but rather as Idea in the rich reality of Scientific development.

At a somewhat humbler level, PC also is posited as Idea in the everyday world of the 20th century and is of the greatest importance. Arising out of the study of formal systems in the work of mathematicians such as Hilbert and Turing, the concept of PC as mechanical, machine-like manipulation has literally assumed the form

[32]We will only say that it entails the view of Mathematics as the Idea of Logic in its externality. In particular, in terms of the three main 20th century schools on the nature of Mathematics, Mathematics is not Logic (the logicist position) nor the study of disguised empirical statements in the context of formal systems (the formalist position) nor languageless construction in temporal intuition (the intuitionist position).

of machine-like immediacy – the *computer*. The basis of the logic of computers is just *Boolean algebra*. This belongs to the mathematical development of PC in which one associates with PC in a precise mathematical way an algebra, the *Lindenbaum algebra* (Mendelson, *supra*, pp. 43-44). This can be intuitively thought of in terms of the set $\mathcal{P}(X)$ of all subsets of a given set X with $\sim$, $\wedge$, $\supset$ and $\vee$ corresponding in $\mathcal{P}(X)$ to complementation in X, intersection, containment and union respectively. This Mathematical form of the PC as Idea has its embodiment in "Nature" in the circuitry of the *modern Computer*.

A simple illustration of this is given by an electrical circuit containing only on/off switches. Then T corresponds to "on" and F corresponds to "off". The switches correspond to the variables p in PC. So for example, if there are only two switches p and q independent of one another, then the conjunction $p \wedge q$ being true just means that current flows through the circuit. (More complicated, but more interesting illustrations are given in Mendelson, *supra*, pp. 21-23.)

The profound impact of computing in the modern world is arguably comparable to that of the French Revolution of the 18th century, so shocking and exhilirating to Hegel's generation, and is surely as significant in the development of culture as the geographical conditions whose importance Hegel appreciated for the life of a nation. It is a clear indication of the Conceptual profundity of Mathematical Logic that it provides the Logic of such a revolutionary "machine", a profundity which it is easy to lose sight of in the technicalities of the subject as treated for its own sake by its practitioners and in the antipathy of those who regard it as a "mere" machine, beneath the level of serious Conceptuality. It is a delightful twist in the Hegelian cunning of Reason that in the late 20th century, the bringing together of the world, the breaking up of the isolationism of the nations and the lives of their members, has been achieved, not so much by any of the grander concepts of Hegelian Spirit (such as Wars) but by the development of a completely external technology, a paradigm of which is the computer and its concomitants[33].

[33] Alvin M. Weinberg, former director of the Oak Ridge National Laboratory, has commented (c.f. p. 32 of *Peirce's Philosophy of Science* by N. Rescher, (University of Notre Dame

The truly important Conceptual changes to come in the exciting period in which we now live cannot be anticipated. But it is in them that we may see the teleology of the Hegelian Reason, which lives in its opposite, self-alienated in externality, only to emerge in a transformed world order "pregnant" with the Concept, in which the latent Conceptuality operating in the development of external and apparently "mindless" calculation breaks open and serves the ends of the Concept.

In this respect, our approach to Formal Logic is substantially different to that of E. E. Harris in his interesting book *supra*. Harris claims (pp.42-45, ibid) (with Russell and the early Wittgenstein in mind) that the metaphysical presupposition of Mathematical Logic is that the world is made up of atomic facts and that these do not take into account "internal relations". Hence we require a different kind of Logic. He then considers Transcendental Logic which fails because of its "subjectivism" (p. 117, ibid) and finally settles for Dialectical Logic, the Logic of system (p. 131, ibid). The difference between such a logic and the Hegelian one (which we have attempted to follow here) is that for Hegel, categorical limitation is set free to show itself through *its own* dialectic rather than being externally judged in terms such as "internal relations", "holism" and examples from the advancement of Science, however valuable and suggestive these may be. It is not enough to see Formal Logic as having "a legitimate and important province and application" (p. 44, ibid). True Logic in the Hegelian sense will include Formal Logic in all its precision and will not pigeon-hole it in its "proper place" (a euphemism for saying that we do not need to take it seriously in philosophical thinking or allow it's truths to permeate the more Conceptual categories of Logic) while, at the same time, not allowing it's will to closing up in itself (for the "professionals" only) to blind us to its intrinsic pushing outside itself in sublation towards the Conceptuality beyond. The Concept will certainly burst through the straightjacket of Formal Logic but this will come from the Concept

Press, Notre Dame, 1978)): *When history looks at the 20th century, she will see science and technology as its theme; she will find in the monuments of Big Science – the huge rockets, the high-energy accelerators, the high-flux reactors – symbols of our time just as surely as she finds in the Notre Dame a symbol of the Middle Ages.* From the standpoint of the late 20th century, it seems more arguable to take the existence of a Personal Computer on (almost) everyone's desk as a symbol of the period.

within the subject itself, and not from an external perspective with its inevitable presuppositions.

We have tried to show in this paper that fundamental categories of SL are expressing themselves with Hegelian necessity in the apparently technical and external developments of Formal Logic. It is true, of course, as Harris points out, that Formal Logic as understood by its founders, is far from adequate. As we stressed above, from a present day perspective, it is hard to believe that philosophers could regard the abstract symbolism of the PC as at all adequate to "real life" propositions whose content they had shrivelled away into nothing. Yet the Concept is operating in the great work of these founders, and the later philosophy of one of them – Wittgenstein – can be regarded as an "unscientific" return to the Hegelian Concept[34]. That the central objectives of these founders were not, for the most part, achieved, is not a surprise from the point of view of the cunning of Reason which achieves its own ends rather than those of the active participants.

In a similar vein, we also have to disagree with the claim made by Charles Taylor in his fine book[35] that for Hegel "a purely formal logic is a chimaera". Taylor supports this view by referring to Hegel's treatment of Truth and points out that "Truth requires two terms". However, Hegel's treatment of Truth in SL only comes after hundreds of pages dealing with Being, Essence and most of Concept. Formal Logic belongs to this earlier stage. (And indeed, there seems not much difference in formality between Formal Logic and the Differential Calculus to which Hegel devotes many pages of SL.) Certainly, Formal Logic is far from the last word in Logic (in

[34] The conclusion of Wittgenstein's later philosophy agrees with Hegel's (c.f. EL, p. 302) in that the sublation of the philosophical "cramps" brought about by supposed linguistic deception does not lead to "metaphysical" theories of the world but rather to the realisation that everything is in order as it is and an acceptance of the "forms of life" in which language is meaningful. It is unscientific in the sense that the "cramps" as Conceptual difference – the moment P above – are abstracted into simplicity, in the Wittgensteinian "sublation" instead of being preserved as in a true sublation. In the Wittgensteinian approach, the immediacy that returns is essentially the unthinking, unmediated immediacy of the "ordinary person" who gets on with his life without troubling his head about abstruse philosophical issues. In Wittgenstein's famous "fly" metaphor, the only distinction is the abstract one of the pure negativity of the wariness of the fly who has escaped from the bottle in contrast with the fly who has always known freedom – but the world they fly about in is otherwise the same for both – the two flies have identical attitudes (except when there are bottles around)!

[35] *Hegel*, (Cambridge University Press, Cambridge, 1975), p. 300

the Hegelian sense) but, as commented above, if we take seriously the Hegelian claim that a category contains what has been previously sublated and only is as result, the content of the Idea of the True can only be enriched by the sublating of the earlier immediacies in SL, one of which has been mediated in PC, a branch of Formal Logic. So Formal Logic will not operate explicitly at the very advanced level of Truth[36] and its concomitant, the Idea. We have also indicated that Taylor's requirement that (Hegelian) "Truth requires two terms" is actually satisfied by PC, the second term being the Computer.

The true significance of Formal Logic can only be appreciated in terms of the Hegelian vision. It then reveals itself as having significance only in the Concept and as a part of SL. While Hegelian Logic makes sense of PC, the latter in turn rewards Hegelian Logic by mediating an important unmediated immediacy of SL. Properly construed, PC flows with and enriches the Hegelian Logic, bringing a level of precision previously undreamed of while at the same time transcending itself in the sublation of its external rigidity to the deeper dimensions of Conceptual thought. Hegelian Logic does not in any way depreciate the great advances of Formal Logic. Indeed, Gödel's incompleteness theorem is one of the most striking illustrations of the Hegelian bad infinity – and its *precision* is what makes it stand out as superior when compared with the many examples that Hegel gives of the bad infinite. (In this respect it is wise to remember Hegel's comment (EL, p. 128) that the foremost requirement of Philosophy is *that each thought should be grasped in its full precision and that nothing should remain vague and indeterminate.*)

For this reason, too, we should dissent from the approach to reading Hegel's *Science of Logic* suggested by J. N. Findlay[37]:

> ... that the line of Hegel's long argument comes out better in the shorter work *(i.e. EL)*, and that many of the intricacies of the longer work are merely an interesting but dispensable, and at times artificially stirred up, foam upon its basic waves.

[36] What is called Truth in PC, i.e. T, is as we have seen, Abstract Identity, an Essence category.

[37] p.vii of the Foreword to *Hegel's Logic*, translated by W. Wallace, (Oxford University Press, London, 1975).

The danger with this approach to SL is that of leaving one with a vague blurred view of Logic without the necessary rigor that Hegel stressed for any *scientific* development of the subject. The opposing view is supported here – that the *details* of Hegelian Logic are its truth and the "basic waves" are mere initial immediacies which need to be developed in detail before they seriously mean anything. This is essentially the same point as Hegel makes in his famous comment (EL, p. 304) comparing the utterance of religious statements by a child with the utterance of the same statements by an old man "for whom they carry the significance of his whole life".

Above we have tried to show how using the *detail* of arguments in SL, we can enter the apparently hostile territory of Mathematical Logic and exhibit the Hegelian categories, normally hidden behind the scenes, which give it its life and meaning. The PC is arguably the most Conceptless of all the main parts of modern Mathematical Logic – the author plans to consider the much richer parts dealing with Set Theory, Quantification, the more adequate Formal Systems and indeed Mathematics proper elsewhere. (They are richer than PC since they involve the genuine infinite, whereas PC is finite. The latter, of course, is exactly why it is in essence the logic of computers: almost by its definition, the machine is ineluctably finite, performing only finite sequences of definite "moves". The Turing machine is the classic illustration of this (Kleene, *supra*, Ch. XIII).)

Finally, as we commented in the Introduction, in the discussion of PC in this paper, we have been compelled to go *back* from the Concept in SL to Being and Essence since incomplete sublation required new but earlier categories. The author believes that in general, such reversals in the order of flow in SL are necessary if the riches of SL are to be made available to present day thought. In the words of T. S. Eliot in *Dry Salvages* :

> And the way up is the way down,
> and the way forward is the way back.

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