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SELF-REFERENCE AND THE NATURAL NUMBERS AS THE LOGIC OF DASEIN[1]

> ,,Be that as it may, Hegel now belongs to us: he cannot be all in all to us if only because his belonging to us, our possession of him, can only be of value (as the possession of a thought can only be of value) when it incites new life, new thought."
>
> Benedetto Croce[2]

1 Introduction

It is well known that self-referential statements have posed a fundamental problem for the philosophy of mathematics in connection with the semantic paradoxes of the subject. (Indeed problems associated with such statements appear in a wide range of philosophical topics.) Another fundamental problem of the philosophy of mathematics is that of justifying the concept of the natural numbers. It is generally agreed that modern mathematical philosophy, in any of its main forms of Logicism, Intuitionism or Formalism has not been able to account satisfactorily for either of these problems.

Initially, one might think that these two problems were not related to one another. The purpose of this paper is to demonstrate that in fact, the second of these problems arises out of the first and that both have their common resolution in terms of Hegel's logic of Dasein.[3] More precisely, self-referential statements as meaningful are implicitly determined by this logic, and the qualitative concept of the natural number arises out of this determination. (This qualitative concept of number is post-Hegelian, the Hegelian

concept of number being quantitative in character.) An interpretation of this logic, based on Hegel's Wissenschaft der Logik[4] is presented, together with its determination in the context of self-referential sentences, in the second section. The remainder of this introduction is devoted to preparation and motivation for this second section.

We first briefly discuss the semantic paradoxes, which give the most important examples of self-referential statements in mathematical logic. These are paradoxical statements whose paradoxicality depends on the notion of truth or of definability. A classical example of a semantic paradox is that of the Liar paradox : in its simplest (not entirely precise) form, the paradox asks:[5] A man says that he is lying. Is what he says true or not? Either assumption leads to a contradiction. Richard's paradox of 1905 is an example of a modern semantic paradox. It considers the set E of all decimals definable by a finite number of words and constructs[6] a decimal that is neither in nor out of E. Berry's paradox hinges on the paradoxical character of the least integer not nameable in fewer than nineteen syllables – but this specification of that integer involves only eighteen syllables! In Grelling's semantic paradox, an expression is called heterological if it is not truly ascribable to itself. The paradoxical problem is then the contradiction that arises when we ask if the expression heterological is itself heterological.

It will be convenient to anchor our discussion of the semantic paradoxes on one particular such paradox. (A similar discussion applies in the case of the other semantic paradoxes.) The paradox that we will use is a version of the Liar paradox which we will refer to as the card paradox. This is both precise and yet is expressed in a simple, commonsense way.[7]

We are given a card with two sides A and B. On side A is written exactly one sentence which reads: The sentence written on side B of this card is false. On side B is written exactly one sentence which reads: The sentence written on side A of this card is true.

Now if the A sentence is true, then the B sentence is false and referring to the actual B sentence, the A sentence is false. This is a contradiction. Similarly, if the A sentence is false, then it is true. Whatever truth value is assigned to the A sentence gives a contradiction. But this is impossible since (surely!) what A is asserting is either true or false. The sentences are self-referential in the sense that, for example, the A-sentence refers to the B-sentence which in turn refers back to the A-sentence.

So the A-sentence is implicitly referring to itself and similarly for the B-sentence. (All this can be put explicitly into one, obviously self-referential, statement by substituting for the B-sentence in the A-sentence to give: This sentence asserts that the sentence that this sentence is true is false.)

One would expect Hegelian logic to have much relevance to the semantic paradoxes. In fact, many of the statements that Hegel makes in his logic can be interpreted as semantic paradoxes. (This is not surprising if we agree, as the present paper claims, that the semantic paradoxes are about the Hegelian infinite.) For example, in his discussion of Ground, Hegel claims (SL 444) that ,,Essence in determining itself as ground, is determined as the non-determined; its determining is only the sublating of its being determined." This is a semantic paradox. For if Essence is determined, then it cannot be non-determined and we have a contradiction. If on the other hand, Essence is non-determined, it is not determined (as ground or anything else) and so we have again a contradiction.

However from the standpoint of Hegelian Reason (Vernunft), the difficulties associated with such a paradox only arise because we are operating from the perspective of the Understanding (Verstand). For Hegelian Reason, such a ,,paradox" is meaningful, it expresses the fluidity and infinitude of the Concept in which the hard and fast distinctions of the Understanding are broken up and the falsity of the rigid distinctions made explicit.

So given the relevance of Hegelian logic to the semantic paradoxes, the next step that needs to be decided is how such a paradox relates to the three fundamental parts into which Hegel divides his logic. These parts are Being, Essence and Concept (Sein, Wesen and Begriff). The meaning of these requires, of course, the full resources of WL to explicate. However, at the risk of greatly oversimplifying, in Being, conceptual determinations are immediate in character, expressing relations which are ,,external." The movement of Being is that of transition into otherness. Essence determinations are, by contrast, mediated in character, self-subsistent (rather than relations expressing otherness), the movement of Essence being that of reflection within it rather than transition into otherness. Essence is the first negation of the immediacy of Being, which has as a consequence that Essence has lost Being or (as Hegel puts it), the Being of Essence is der Schein, being that is a sham. Concept is the negation of this first negation so that Being is restored but as essentially mediated, with Being and Essence now

determined through each other. At the level of Concept, we have truth, the truth of Being and Essence, realized in the Idea of Concept. Furthermore, the moments of Concept, though different, are not determinations involving otherness or reflection but each gives the whole Concept in which otherness and reflection are included in sublated form.

These three major levels of Hegelian logic – Being, Essence and Concept – are readily seen to be present (even on superficial inspection) in the card paradox. The otherness and transitional aspect of Being is present in the two separate sides of the card (each with its own sentence) and the turning over of the card as we determine the meaning of the sentence on one side by referring to the other. At the level of Essence, the sentence on each side refers to that on the other which in turn refers back to the original sentence. Thus the ,,movement" of the card is its internal reflection of one side into the other. The Essence level is also apparent in the presence of the important Essence concepts of Difference (Unterschied) and Contradiction (Widerspruch) inasmuch as the sentence on one side refers to Truth while that on the other refers to Falsehood, and a contradiction seems to result. The reference in the card paradox to Truth also indicates the Concept level present in the paradox as Idea, and the intrinsic reference of each side to the other indicates the Concept feature of moments (the sides) each of which, though different, give the whole Concept.

A full investigation of the paradox in terms of these three levels is beyond the scope of this paper. Instead we will concentrate on the first of these levels, that of Being, and at that level, present the Hegelian logical development (Entwicklung) which appears initially in immediate form in the paradox. At this level essential and conceptual elements in the card paradox must be left implicit.[8]

What is important at this level is that the sentence on each side refer to that on the other. This is all that will be used in the discussion below. The card with two such other-referential sentences on its sides will be referred to as the card case. The card paradox would give an example of this but the sentences for the card case in general need not be paradoxical.

To conduct this logical development, it is natural first to identify features of the card case which will indicate which part of Hegel's treatment of Being in WL is going to be relevant. From the above discussion of Being, the two sides have to be kept separated from one another and the logical movement

between them has to be one of transition. The natural Being category that fits this is that of Something and Other. The two sides of the card are each a Something in relation to which the other side is Other. The development in the next section starts for this reason with Something and Other.

Now Something and Other is a category of the second chapter in Hegel's treatment of Being, that dealing with Dasein. This indicates that the logic of the card case at the level of Being should be that of Dasein logic.[9] Further preliminary evidence of the Dasein content of the card case at the Being level is the implicit presence in it of (what we shall call) the Finite-Finite infinite progress. Hegel (SL 136) discusses this progress briefly in the chapter on Dasein. In that progress, the inherent instability of the Finite is made explicit in its generation of a never-ending production of Finites.

This infinite progress can be seen in the card case as arising out of the indeterminacy of the sentences on the sides. When we are dealing with a ,,normal" sentence such as: Would you hand me that book over, please?, you and I are in a determinate situation. There is a book and you are present before me and the sentence is ,,realized" when you actually pick up the book and hand it to me.

By contrast, the card case gives an indeterminate situation. For example, in the context of the card paradox, to find out what the sentence on side A is really asserting, we have to determine what the claimed false sentence on side B is saying. But to understand what the claimed false B sentence is saying so that it becomes determinate, we have to refer back to A again which in turn requires for its determinacy referring to B. The endeavor to uncover the determinate content of the sentences leads to a situation where we can never stop.[10] Unlike the case of handing over the book above, we never know what either sentence really says: the information that each sentence gives to find that out (i.e. reading what is on the other side) just ,,passes the buck" back. Each is, to use a Wittgensteinian metaphor, an arrow pointing in one direction, and when we look in that direction, we are confronted with an arrow pointing in the opposite direction. It is to be compared to the situation in which a group of children is being questioned by a teacher over some impropriety and each child points to another and says:,,Not me, sir – it was him". Qua determinacy, the sentence on each side of the card only says: ask on the other side!

This is a striking illustration of the Hegelian Finite-Finite infinite progress in which the determination of any step points away from one Finite to another and is never achieved. The Finite will not stay still. In the presence of superficially stable Finite objects, the intrinsic instability of the Finite – described in detail by Hegel in Dasein – is normally not noticed. However in the context of the card case, this instability is inescapable. This is why the card case gives us the logic of Dasein, the logic of the Finite.

The Hegelian development described below shows (as an unexpected consequence) that the logic of Dasein as determined in the semantic paradoxes gives the natural numbers as a determination of the final categories, that of The Ideal (das Ideelle) and Being-for-self (Fürsichsein). The natural numbers are thus no longer, as in modern mathematics and its logic, an immediate assumption, intuitively obvious, "God-given". Rather they are a logical consequence of Dasein logic.

The philosophy of mathematics developed on Hegelian principles – part of which is given below – can reasonably be described as Hegelian logicism (in contradistinction to the Frege-Russell logicism). The former, unlike the latter, is not reductionist. It is the reflective presentation, the making explicit, of the development immanent in the on-going mathematics of the great mathematicians, not as separated off or as telling mathematicians what to do. Hegelian logic has nothing to offer the future progress of creative mathematics anymore than philosophy can tell the artist what to paint. But Hegelian philosophy provides us with the rational basis for mathematics as it exists today and the form of the infinite whose content mathematics expresses. As the Hegelian infinite, mathematics is ineluctably self-reflexive. One cannot stand back and (as it were) box it up within a formal system. (The precise expression of this is, of course, given in the incompleteness theorems of Gödel.)

It also gives an answer to the difficult question for modern mathematical logic of how applied mathematics is possible. Indeed, since the concepts of mathematics are determinations of the categories of (an extended) Hegelian logic, and these categories are also those grounding the concepts of science, we see how applied mathematics is possible. Both are expressions of the same logic.

The theorems of mathematics are the intuitive determinations of the latter's own depth as mediated in proof. Such a determination is not deep

because its proof takes many pages to write down – complicated boring tautologies can do as much – but is deep because of its content and the significance (or value) which comes from the Hegelian infinite determining itself in it. The philosophy of mathematics is then Hegelian logic, not as a fossilized collection of bizarrely related categories but as the conceptual articulation of mathematics in its living movement.

2 Self-reference and the natural numbers as the logic of Dasein

The scope of this paper does not allow for a complete treatment of all of the categories involved in Hegel's treatment of Dasein, and we shall concentrate only on those most relevant to the card case. As in our presentation of the card case earlier, the sides of the card are denoted by A and B. Each side is regarded as being given with its sentence. The discussion is divided up into subsections, each of which briefly attempts to describe part of Hegel's argument in Dasein and follows that up by exhibiting the argument's special form in the card case. The context will make clear whether we are discussing Hegel's argument in WL or its specialization in the logic of the card case. We will use Miller's translation SL of WL throughout and will use capital letters when referring to technical Hegelian categories.[11] Justification of why we start with Something and Other was given in the first section.

(a) Something and Other

The categories of Something and Other depend, of course, on what precedes them in WL, and so we give first an extremely brief, minimal account of the earlier material in WL adequate for our purposes.

In Hegel's logic, Being and Nothing vanish into each other in Becoming, the „equilibrium" of coming-to-be and ceasing-to-be (SL 106). But Becoming makes sense only if Being and Nothing are also distinguished, since otherwise, these two would coincide statically and there would be no vanishing movement (i.e. no Becoming) between Being and Nothing. But these two have been shown to vanish into one another. Hence Becoming, whose very ,,being" involves this distinction of the two, itself must vanish, not just stay there abstractly ,,fixed." This ,,vanishing of the vanishing"

is Dasein, ,,the simple oneness of Being and Nothing"(SL 109). Dasein is a sublation of the vanishing movement, a stability (since the vanishing has stopped).

As it is mediated through a sublation, Dasein has lost the completely abstract character of the earlier Being, and so is (relatively) determinate. Quality[12] is defined to be this determinate character of Dasein. The two moments of Quality reflect Being and Nothing whose sublation through Becoming was Dasein: Quality as Being is called Reality (determinate Being) while Quality as Nothing is called Negation (determinate Nothing). On the one hand, these moments are, of course, different. On the other hand, since Dasein is the ,,simple oneness" of Being and Nothing, its character Quality is also ,,simple oneness," and hence its two moments also coincide with it and therefore with each other, each moment implicitly containing the other. The sublation of Quality that makes explicit the nullity of the distinction between Reality and Negation yields the category of Something. Something is being with this distinction together with its sublation.

Something as this sublation is the mediation or transition beween the two moments of Reality and Negation, that in which the two are ,,drawn together" yet distinguished. However, as Something contains these moments in sublated form, the moments are further determined in Something. These further determined moments are called by Hegel Something and Other. (There is an unfortunate ambiguity here in that Something can either refer to the original Something or the moment. When there is the possibility of misunderstanding, the term moment will explicitly be used when the moment Something is intended.)

From the corresponding properties of Reality and Negation in Quality, each of the moments Something and Other is itself implicitly the whole Something. So Something is also a self-mediation (SL 116). (This to some extent justifies the ambiguous use of the term Something discussed in the previous paragraph.) The fact that Something and Other are more determinate forms of Reality and Negation is indicated by the fact that the Something/Other relation is the Being/non-Being relation. (This would not have made sense for the Reality/Negation pairing.)

We now discuss the determinations of Something and of the moments Something/Other in the card case. With the latter, we are immediately presented with the turning over of the card (backwards or forwards) as one

follows up the claim of the sentence on a side by referring to the other side. The unity of this turning as immediate and ,,qualitative" is Something. We can define Something as the card-in-turning from one side to the next in the quest for making determinate the other-referential sentences. As a turning, this Something is (as in Hegel's treatment of the first Something described above) a becoming, a mediation. Also as in Hegel's treatment, Something becomes determinate in the card case as the moments Something and Other. The form that these moments assume are very explicit and indeed obvious: they are just the two sides of the card (each with its sentence). Notice that the sides are completely symmetrical with respect to one another: at whichever side we start, turning gives the other side. What we have here is the logical determination that each side is the moment Something which is Other relative to the ,,other" side: when one side is facing up, the other has disappeared underneath. Each is the ,,non-Being" of the other. Further, each side generates through the following up of its sentence the turning of the card, and hence is implicitly the ,,whole" Something (the card-in-turning).

These are exactly the properties of Something and the Something/Other moment relation in general as discussed in the paragraph preceding the last.

(b) Being-for-other and Being-in-itself

The Something of (a), which is the mediation or transition between the separated moments of Something and Other, is in unity with itself, preserves itself, in the transition. This unity is made explicit in the categories of Being-for-other and Being-in-itself, the former affirming the otherness through which Something preserves itself while the latter affirms its self-relatedness in otherness. Both are distinct; but, as expressing the unity of Something, they are also implicitly identical – for example, Being-for-other establishes the moment of Other as well as Something's self-preserving in transition. But to have a transition implies the presence of the moment Something in ,,transit" to this Other and back, and hence contains conceptually the Something (with its full unity) which is (by (a)) the sublation of this transition. So Being-for-other, as well as Being-in-itself, is the full unity of Something with itself, not just ,,a part" of it. In particular, they are identical. Both kinds of Being are ,,undividedly present" (SL 120) in Something.

In the card case, the Something is ((a)) the card-in-turning between the sides. The following up of the sentence on the (up-facing) side of the card by turning the card over to refer to the other side is Being-for-other in this case. (The other side will be up-facing then.) On the other hand, Being-in-itself is the returning back to the starting position, with exactly the same side facing up as when we started (after the transition through the other side). As we saw for Being-for-self above, the turning card (Something) ,,preserves itself" qua turning by moving to (,,affirming") the other side. On the other hand, as we saw for Being-in-itself, the turning card on its return back to the starting side will coincide with its starting position and so has ,,affirmed its self-relatedness" (unity) in transiting through the other side. The (implicit) identity of the two kinds of Being is obvious in the card case, since Being-for-other applied twice gives us the return of Being-in-itself, and Being-in-itself entails having gone through the other side (Being-for-self) before the return.

(c) Determination and Constitution

We saw ((b)) that Being-for-other is implicitly identical with Something's Being-in-itself. Indeed, the argument of (b) establishing this identity can be briefly formulated in Essence terms (SL 122): Something is ,,reflected into itself out of its Being-for-other." Thus what is in-itself is Being-for-other. This identity is explicitly posited as the Determination of the in-itself in Something (by the for-other). The argument of (b) was formulated in terms of the moments, Something and Other, of Something. Determination can usefully be formulated in terms of these moments as well. It is just the positing of each moment in the other through their transition, their conceptual identity.

In the card context, the preceding gives that Determination is the explicit positing in Something (the card-in-turning) of each side in the other through the transition from one to the other. In terms of Being-for-other and Being-in-itself, the reference to the other side of the turning card, which was Being-for-other, is now determined in unity with its Being-in-itself, the return of the card back to its original position. Side A can no longer be regarded as isolated, on its own, separate from B, the ,,non-Being" of B as we had in (a), the latter side only a means through which the sentence on A is to be determined, and then discarded. It is now explicitly recognized that A and

B intrinsically involve each other. The two sides are conceptually posited as the same in the card-in-turning.

Returning to the general case, Determination is further determined by taking into account the ,,filling" that Something's Being-in-itself obtains from the reflection into it of Being-for-other. This is where the otherness comes in that was excluded from Determination as the immediate identity of the two kinds of Being. As so excluded, this filling of Something's Determination falls outside the Determination. This filling is its Constitution, and is that in the Something which becomes other while its Determination remains unchanged. Something's Constitution is what ennables the Determination to be preserved under ,,external" influences.

In the card case, the Determination of the Something (card-in-turning) has its Constitution in the external circumstances of the turning of the card through which the Determination – the conceptual identification of the two sides – is established and preserved.

Returning to the general case, the Determination and Constitution of Something are initially indifferent to one another inasmuch as the action of Constitution in relating to external influences does not seem to make any difference to the Determination – the two lead separate, independent lives. In the card case, for example, the Determination of the conceptual identity of A and B is independent of any actual turning of the card. But this apparent indifference is only immediate – it is clear from the definitions of Determination and Constitution above that each is only established through the other. Determination only preserves itself through its defending Constitution and conversely, Constitution only has meaning as constituting the Determination. They are together as a unity, both within Something.

This unity of Determination and Constitution is then made explicit and posited in Something. This is the unity of the moments of Something and Other as identical (Determination) together with the transitional, external influences through which it is preserved (Constitution). As such, the transition (alteration) between the moments of Something and Other, which that unity expresses, is now located in Something. Accordingly, the moments Something and Other now no longer relate directly as immediate Being/non-Being ((a)), but through Something, now in its developed form, which encapsulates their movement into one another. The intrinsic transition between the moments is no longer abstract as before, one external to the

other, but is through the Determination-Constitution unity in Something.[13] (This unity will be further determined in Limit below.)

In particular, in the card case, the moments (the sides) of Something (the card-in-turning) no longer relate directly to one another by the means of turning the card, which turning is then forgotten. Instead, ,,each relates itself to itself only by means of the sublation of the otherness which, in the determination, is reflected into the in-itself" (SL 125). The unity of Determination and Constitution is a determination of Something as the card-in-turning in which the sides are identified. This unity is recognized when we see the sides as saying the same thing in their other reference, their relation explicitly being in the common card-turning (Something) which implements their identity.[14]

(d) Limit

The determination or the precise specification of the above posited unity in Something through which the moments Something and Other relate to one another is called Limit. It is determined (using a spatial metaphor) as the boundary between the moments, which on the one hand determines where they ,,come together" yet on the other hand distinguishes them, gives the ,,place" where they ,,push" against each other. (So that, as we noted above, Something and Other no longer oppose each other directly but rather through the posited unity made precise in Limit.)

Limit is simultaneously the Being and non-Being of Something and Other. Something is not fully given or determined if the Limit is not specified. Dynamically, Something as determined by the Limit is ,,explosively" there in Limit. Hegel gives the (mathematical) example of a line segment generated by starting at a point on a line and moving along the line in one direction or the other. This end-point is the limit which has ,,within it" the line segment which it generates through its ,,unrest"[15] (SL 128). The end-point separates out the line segment as ,,Something" from the remainder of the line of which it is a part. This remainder is also a line segment starting at the point but going in the opposite direction, and is the ,,Other" to the original line segment's ,,Something." In the point as Limit, the two segments come together yet are distinguished.

Since Limit is the determinate form of the transition through which Something and Other relate, this determination in the card case is easy

to specify. As we saw above, Something and Other are the two sides of the card. The Limit in this case is the card ,,frozen" in the turning between the two sides, as it were a snapshot of the rotating card. As such it is the ,,unity" of the sides since the card can go from this in-between state to the state in which either side faces up. But it also separates the sides in their opposition and acts as the ,,boundary" between them. (Notice that the posited unity of the other-referential turning which was the sublated unity of Determination and Constitution in the card case has been specified further in the ,,snapshot" Limit.)

(e) Finite

Limit, the Something-Other boundary through which Something is determined, explicitly lacks this Something that it bounds (though implicitly it is dynamically the being of Something). It is the circular edge of a disk in the plane without the interior or exterior, the smile on the face of the Cheshire Cat in Alice in Wonderland with the face missing.

But since Something is only determinate in its Limit, the making determinate of the former gives Limit as explicitly ,,located" in the being of Something. Now since Limit is also the non-being of Something (or equivalently, the being of the Other ((a))), Limit as located in Something drags into the heart of Something the latter's own non-being. This is not just in a passive sense. For since Limit dynamically generated both Something and its Other, the Something as determined in Limit is now ,,posited as the contradiction of itself, through which it is directed and forced out of and beyond itself"(SL 129). As such, Something (and its Other) now coincide.

Something is this intrinsic shifting and is only determined through its negation in Limit which in turn is what it is only as negating the Something.[16] We have reached the stage of the Something as the Finite whose hour of birth is its hour of death and whose being is its ceasing-to-be (SL 129).

In the card case, Something and Other (the sides) coincide in their embracing of the Limit (the card frozen in transition ((d))) to give the Finite as the whole card as forcing itself out of itself through Limit in its rotating quest for making determinate the meaning of the sentences on the sides. Note that the original Something of (a) – the abstract card-in-turning – has become determinate in the Limit (as asserted in the second paragraph above).

(f) Limitation and the Ought

Limit as the being of Something-as-Finite is made determinate by being ,,posited by Something as a negative which is at the same time essential" (SL 132). In this determination, the negative, bounding character of Limit (that we saw in (d)) now becomes explicit and ineluctable (essential) at the level of the Finite. Such a positing of Limit is called Limitation. But at the same time, Something as Finite can no longer remain steadfast within its Limitation; it is ,,forced out of itself" through its inherent instability and so goes ,,beyond" the Limitation. So in the determining of Limit as Limitation, ,,Something must at the same time in its own self transcend the Limit" (SL 132). This transcending negative relation of Something to its Limitation is the Ought.

The being of each of Ought and Limitation mutually implies that of the other. So reference to the Ought is meaningful only if there is Limitation which is blocking its fulfillment and conversely.

We now determine the categories of Limitation and the Ought for the card case. We saw ((e)) that in that case, the Finite was the whole card (not just separate sides of the card) in its self-generated, rotating drive for making determinate the content of the sentences on the sides. As we observed earlier, this content refuses to become determinate, and the only way that the Limit can be posited as determinate, ,,bounding" the rotation, is by stopping this rotation at a side. This stopping[17] is the Limitation of the card case. But of course, Something as Finite (in particular, containing its Limit) transcends this (arbitrary) cessation: we could still go on reading the sentences and turning the card. This is the negative relation of Something to its Limitation which is the Ought. The Ought is the driving onwards towards (illusory) determination through the turning of the card.

(g) The Finite-Finite infinite progress and the Infinite[18]

We saw above ((e)) that the Finite as Something determined as Limit was the contradiction which is forced out beyond itself. This contradiction was further determined in the Ought and the Limitation each of which mutually involved the other. To resolve the explicit contradiction, the only way possible is for it to become implicit. This is done by taking any one of the two, say the Ought, and ,,giving it its head," suppressing the Limitation. (And

indeed, in normal life, we often use the term ,,ought" in a given situation without explicit mention, or even conscious awareness, of the limitation with which it is paired.) The Ought has thus been abstracted, posited outside the Finite within which the original Ought and Limitation related unstably. The Finite and its inner conflict has been destroyed since an essential moment (the Ought) is no longer present. As Hegel says, the Finite ,,ceases to be." But implicit in this ,,outside" Ought is Limitation and when the latter is made explicit, the Limitation and Ought are once again unstably related in the Finite, another Finite. So the preceding development describes the production of the new Finite from the destruction of the old. Note that the Finite itself has been determined further since it is now seen in transition outside itself rather than being ,,self-contained" as before.

The same development occurs again with this second Finite which like the first is annihilated and produces yet another Finite, and so on ad infinitum. This is what we shall call the Finite-Finite infinite progress, a ,,bad" infinite sequence whose work in going from one Finite to the next is never done.[19] Its infinitude lies in the fact that (SL 136) ,,in ceasing to be, the Finite has not ceased to be." The badness of the infinite is strikingly indicated when one attempts to write down the sequence according to standard mathematical practice – the sequence is: Finite, Finite, Finite, $\cdots$. The tell-tale $\cdots$ (standing for ,,and so on") is effectively a giving up – to use a trenchant expression of Wittgenstein's, the dots in $\cdots$ are the dots of laziness.[20]

However, conceptually, the Finites produced are all instantiations of the same Finite and hence this infinite process is not just a ,,going on for ever." Each step in the progress is (as qualitative) identical, the same thing, viz. the Finite identifies with itself through its disappearance. The Finite identifying with itself through its self-produced disintegration is a sublation, a negation of a negation. The positing of this sublation is defined to be the Infinite. The Infinite is the sublation of a transition and so, as in the original sublation of Becoming in (a), has the form of Being.

We now identify the Finite-Finite infinite progress and the Infinite in the card case. Recall that in this case, the Finite was the whole card, the Ought was the driving towards determination of the card sentences while Limitation was the giving up of this objective after some turns. We described above how the separation of the Ought (or Limitation) from the Finite gave the disappearance of the original Finite followed by its reappearance.

In the card case, the separation of Limitation and the Ought gives the original Finite (the card) as fixed under Limitation „losing" the Ought. Since the dynamic of the Ought forcing the turning of the card has been separated out from Limitation, the card is now still. The Finite, as the card in the unstable Limitation-Ought relation, has now been replaced by the side of the stationary card facing up. When the suppressed Ought is made explicit in the side, we have that the further determined Finite referred to above is no longer the card as a whole but rather the side forcing itself out of itself in the turning.[21] In determining the form of the Finite-Finite infinite progress, the Ought as freed-up from the Finite is then realized in the side turning over. The Finite which has lost the Ought (i.e. the side at which we initially started) then disappears underneath, of course, but when explicitly determined by the new Limitation, the new Finite is the side opposite to that with which we started, the side now facing up.

The Finite-Finite infinite progress is then the endless generation of card turns, each side as Finite disappearing and generating the next as the new Finite. Since the sides are the same Finite conceptually, the sides of the card are now blank and regarded as indistinguishable.[22] In particular, the sentences have disappeared. (The remaining physical distinction between the sides will vanish later in this section in the emergence of the *-string.) The truth of the original sentences is now clear – their only content was the imperative to turn the card to the other side. The indeterminacy of the original sentences informally discussed in the Introduction has now been made completely explicit and justified in their disappearance.

The Finite which has been determined as the side now obtains an important further determination. For the Limitation for the Finite was ((f)) the stopping of the card arbitrarily at a side while the Ought was the forcing on of the turning. Now when the card is determined under Limitation as stopped at a side, what is being presented is more than just the simple side; it is the side as sublating the turns which were implemented before the stopping. So the qualitative concept of some turns is implicit in the determination of the Finite as side. When this and also the Ought (which was implicit in the fixing of Limitation) are made explicit, we have the Finite now determined in the side as the ready-to-go-on after some turns. Note particularly that the use of some indicates the qualitative and is at the level of Being.[23] Thus there is no quantitative determination involved with

,,some."

The sublation of this progress in the Infinite in the card case is just the positing of the sublated turning transition from one side to the next in which the two sides are identified qua Finite. The Infinite has two aspects, first as regards the Finite as a card side, and second, as regards the qualitative content of the Finite.

First, the Infinite is the intrinsic self-reference of a side (the Finite) as identifying with itself through the turning process, both sides being conceptually identified as indistinguishable. Of course, physically, the sides switch over when the card is turned – the important point here is that they are now actually identified, not just (subjectively) regarded as identified. In the Finite-Finite infinite progress, the sides came one after the other. Now there is no order and a side as such identifies with itself in the Infinite. Since the sides are physically not identical in the card, the card itself with its sides now falls away as inessential in the logical development and the side replaced by a simple positing.

Second, in the Infinite, the qualitative content of the Finite must also be identified with itself. Now as we saw above, this content was the some turns determined by Limitation in a side. So in the Infinite, this content must identify with itself as we turn from one side to the next. In this self-identifying, some turns becomes independent of side and is qualitatively infinite.

The preceding considerations will be clarified below when this qualitatively determined, infinite, positing will be presented in the familiar string terminology of mathematical logic. (Indeed, the Finite-Finite infinite progress and the Infinite are effectively present in abstract, unrecognized form in the formal systems of modern logic.) The discussion below could be expressed without any reference to symbols but is made clearer by using them. Thus the symbolization is strictly only a convenience, and it is important to use it (as the mathematician uses mathematical symbolization) for indicating the conceptual development, and not for its own sake. The rest of the development will then exhibit the fundamental mathematical concepts implicit in the original card case.

(h) The Finite-Finite infinite progress and the Infinite for Strings

The side of the physical card is initially posited, without its qualitative content, as the simple symbol *. Any other symbol would suffice.[24] Since

above, the Finite was realized as such a side, it is now given as *.

The concept of the Finite has, as described in (g), the infinite qualititative content of ,,some turns." This content determined in the original ,,side" as * is a *-string. It is conveniently (though not necessarily) thought of in terms of an ordered sequence of *'s written from left to right on a piece of paper, the left-to-right transition from one * to the next representing the turning from one side (=*) to the next. The last * of the string is conveniently regarded as the side which was fixed under Limitation described above (and so has the qualitative content ,,some turns" of the Finite, this * determining the whole sequence of *'s as the specification of the Limitation which stops the sequence.) Examples of strings are ** and ****. It is important to stress (as we did for some turns) that such a notation is only a representation of qualitative content. This content is not restricted to strings that can be so represented – the inadequacy of the notation is illustrated by using dots for ,,and so on" (discussed in (g)) as when we say that the typical string is ***$\cdots$.

The Finite-Finite infinite progress is then that in which a string has the * (the last one of the string) which contains the qualitative content having another * adjoined (corresponding to the turning over of the card). The Infinite is the positing of the identification of string with itself through the transition of adjoining a *.

It is to be stressed that the conceptual foundation of string and the Infinite of string cannot be avoided. They are not ,,empirical" – for the actual marks made to represent different instances of * on paper will physically vary, if only under a microscope.[25]

The Infinite of the Finite-Finite infinite progress underlies the concept of what is (misleadingly) called in the Philosophy of Mathematics an incomplete infinite. The latter is a quantitative determination of the former. It is true that at the level of the Dasein Infinite as quantitatively expressed, no ,,actual" infinite set of finite strings is posited, so that in this respect there is no ,,complete infinite" in that Infinite. However the Infinite as the positing of the self-identity of string through *-ing, is qualitatively complete. (But while there is nothing ,,incomplete" about the Infinite, it is as we shall see an inadequate infinite.)

For the same reason, the related claim sometimes made – in particular, in the Intuitionism of modern logic and also in classical mathematics (for

example, by Gauss[26]) – that we avoid the recognition of the ,,complete infinite" – such as that of the set of all strings – by constructively adding *'s onto ,,existing" strings is also illusory. For in the constructive approach, the sequence of constructed strings is not intrinsically allowed to ,,stop" and so entails the implicit recognition of the ,,completed" infinite of strings which is ,,there" in the background, strings being ,,picked out" of the supposed complete infinite by flashlight, as it were. Such an implicit complete infinite is just the Infinite of Dasein in a ,,bad" form, the positing of the Infinite transformed into a stationary abstract ,,set"as arbitrarily assumed in Russell's Axiom of Infinity. The making stationary of the Infinite is the undoing of the Infinite since it has then lost the transitional movement in which it is the self-identity of the Finite, and we are back with Finite-Finite infinite progress.

So the Infinite of Hegel with which we are dealing here is not a ,,constructive" infinite. The important point is that this Infinite is not quantitatively determined at all, but, like all of the categories of Dasein, it is qualitative in character.[27]

(i) The qualitative infinite progress

Returning to Hegelian logic, the Infinite as the positing of the Finite's self-identifying through transition, as identity, has left the Finite behind. It is fixed with none of the ,,ceasing-to-be" of the Finite. But looked at more closely, the Finite cannot be ,,swept away" like this since without the Finite identifying with itself, there is no Infinite.

The relationship between the Finite and the Infinite is in fact that of a developed form of Something and Other. For as the uniting of the Finite with itself, the transition through which the uniting – the Infinite – was the result, has been negated. But the being of the Finite is just its ceasing-to-be ((e)) and it is this that the Infinite has negated. Hence the Infinite is the non-being of the Finite (SL 138) and (as noted in our discussion of Something and Other in (a)), the being/non-being relation is the Something-Other relation.

Now the Hegelian logic described earlier in (a) – (e) shows that (both) Something and Other develop into the Finite. Since the Infinite is such an Other to the Finite's Something, it follows that it itself is implicitly Finite and when this is made explicit, it is the finite Infinite.

But as we saw in (g), the Finite through the agency of Limitation and the Ought generates the Finite-Finite infinite progress which is posited as the Infinite. Since the Infinite is now the Finite, it in turn will generate the Finite-Finite infinite progress and hence the Infinite again. The process continues to give the second infinite progress, the qualitative infinite progress. This, then, consists of the original Finite, whose reuniting with itself in the first infinite progress is posited as the Infinite. This Infinite is, as we have seen, a finite Infinite and so we are back with the Finite again. This in turn generates the Finite-Finite infinite progress which produces the Infinite. This becomes Finite again, and we have the alternating sequence Infinite, Finite, Infinite, Finite,$\cdots$ repeating ad infinitum, the finiteness of the Infinite being made explicit in every Finite.

We now investigate the determination of the qualitative infinite progress in the context of *-strings. Recall that in the string context, the Finite was a *-string in transition. The Infinite was the positing of the qualitative self-identification of the string through its intrinsic transition of *-adjunction. We first have to examine how this string Infinite becomes finitized. To this end, recall that the transition from Finite to Finite in the latter's ,,ceasing-to-be" was left behind in the Infinite's positing. Now the Infinite must identify the two strings as finite, i.e. qualitatively as strings, and yet the second string is the first with an extra *. The two strings are not equal. Since the strings must be identified in the Infinite, the latter's positing must therefore undo the *-adjunction. On the other hand, since the Infinite is only through the adjoining, we have that the Infinite as identifying through external *-ing is contradictory in character. It sublates itself into an inner, implicit immediacy.[28] The positing of this immediacy is the finite Infinite. It is simply the potentiality for adjoining a * to the Finite-as-string. The movement whose sublation was the Infinite has been frozen up and the string (Finite) is opposed in a Something-Other relation to the potentiality for its augmenting (finite Infinite).

The finite Infinite will be represented by *. Indeed, the Finite-as-string is * (with the qualitative content of some *'s), and since the finite Infinite is also a Finite, it is therefore representable as *. Looked at more closely, however, as we saw above, the finite Infinite is the positing of the Essence concept of potentiality for *-adjunction. Now in the Essence terms of WL, the Finite-as-string is just the Outer in relation to this potentiality as Inner.

(Indeed, the potentiality expressed qualitatively is just the some *'s whose production it is the potential for and conversely, the concept of some *'s presupposes the potentiality for stringing together some *'s in a string.) Since (SL 524) the Inner and the Outer ,,are only one identity," their positing as Being, i.e. as Finite, together with their qualitative contents, are just the same, both representable by *.

The qualitative infinite progress for strings is then as follows. We start with the Infinite. As finitized, it is, as we have just discussed, represented by * and we are back with the Finite as * again as in (h). The development in (h) therefore applies again, giving the Finite-Finite infinite progress of *-strings. This in turn gives the Infinite of strings with which we started. We have thus gone through the sequence Infinite, Finite, Infinite, and this repeated gives the Infinite, Finite, Infinite, Finite,$\cdots$ of the qualitative infinite progress. (The Finite, Infinite, Finite part of this will reappear in sublated form in (k) below.)

(j) The Affirmative Infinite, The Ideal and Being-for-self

Returning to Hegel's treatment of the Infinite, the truth of the Qualitative infinite progress is the dynamic unity of the Finite and the Infinite, a unity in which each loses its qualitative character. Indeed, this unity is exactly what the alternation of the infinite progress is expressing. The Infinite, when its intrinsic dependence on the Finite is made explicit, becomes a Finite, so that its unity with the Finite qua Finite has been established. On the other hand, the Finite-Finite infinite progress is just asserting that the Finite with its transitoriness removed is the Infinite, thereby asserting their unity again. Both are now ,,this movement in which each returns to itself through its negation" (SL 147). Each only is in passing through its other and as such is a double negation, the first negation being that in which it disappears into the other and the second its sublatory return. The Affirmative Infinite is thus to be thought of in terms of a circle, completely present, looping back on itself with no beginning or end, rather than as a straight line which goes on ,,for ever" grasping for the illusory ∞ ,,at the end", the image of an infinite progress (SL 149).

The progress asserts the Affirmative Infinite, the genuine Infinite (die wahrhafte Unendlichkeit) (and genuine Finite) as the mediation in which each of the old Finite and Infinite returns to itself through the other. As such, each

has now brought out explicitly what it was implicitly from the beginning – the Finite contains the Infinite and the Infinite the Finite. No longer is each a ,,beyond," the Finite in its Being outside itself and having no stability, and the Infinite an unattainable goal which the Finite vainly strives to achieve. The genuine infinite is then the relation of itself to itself as dynamic in which each of the Finite and Infinite is a moment.[29]

The Finite, as it is in this genuine infinite, is then (SL 150) a more developed form of the old Finite and is called by Hegel das Ideelle, The Ideal.[30] (Since the old Infinite is, as we have seen ((i)) finitized in the qualitative infinite progress, The Ideal is also the Infinite as it is in the genuine infinite.) Hegel calls The Ideal the quality of the Affirmative Infinite. To specify this quality more precisely, the (old) Finite was determined as transition outside itself ((g)). The Ideal is also a transition. It is the transition of the Affirmative Infinite within that infinite, or, as Hegel puts it (SL 150), is the ,,process of becoming" of this infinite. The Ideal can be regarded as having the two moments of Being and Negation (SL 150,158), reflecting the Being of the Infinite (no longer in external transition to the Finite in the qualitative infinite progress) which is only in its intrinsic becoming (Negation). The Ideal will play an important role in our discussion of the natural numbers in (k). A helpful example of Hegel's, illustrating the Finite as The Ideal in the context of consciousness, is discussed below.

The sublation of The Ideal as the becoming of the Affirmative Infinite is Being-for-self, Being which is infinite. This is the theme of the next Chapter of WL. Indeed, like all ,,becoming" sublations, it is a form of Being, the stabilizing of the Affirmative Infinite in its leaving and its returning in the infinite loop (to use Hegel's metaphor for the Affirmative Infinite alluded to above).[31]

Another way of expressing the above is that Being-for-self is the positing of the unity of the moments Being and Negation of The Ideal.[32] Indeed, at the level of complete chapters of WL, Being-for-self is the second negation following up on Dasein which was the first negation to abstract Being. In Being, Being and Negation were in unity together indeterminately. In the first negation, Dasein, the sphere of the Finite, Being and Negation were clearly separated and their unity was only implicit. We saw this, for example, in polarized pairings such as Something-Other and Limitation-Ought but above all in the two infinite progresses in which Negation diremptively forced

on the two progresses (in the form of the Finite-Finite or Infinite-Finite).

So Being and Negation are in a difference relation with one another in Dasein but in Being-for-self their difference is ,,posited and equalized" (SL 157), otherness being ideally present as a moment within it. An example that Hegel gives of this is Being-for-self as consciousness, in which an object of which we are conscious (,,other" to us) is present (,,equalized") in us ideally in the content of our thought. This is the object as The Ideal defined above, the Finite as it is in the Affirmative Infinite and sublated in Being-for-self.

We now examine briefly the forms that the Affirmative Infinite, The Ideal and Being-for-self assume as determined in the mathematical (string) context.

(k) The Affirmative Infinite, Being-for-self for strings, and natural numbers

Turning to the logic of strings, the Affirmative Infinite – the mediation in which each of the moments, the Finite (string) and the Infinite (the potentiality for adjoining * to the string) return to each other – is (qualitative) stringness determined as preserving itself through this *-adjunction. On the one hand, it is the complete movement of the Finite-as-string, no longer in the incomplete, unstable infinite progress of (h) but as asserting its stringness through the activity of the Infinite as the above potentiality. On the other hand, it is the complete movement of this Infinite as well, no longer abstractly apart from the string to which it is in a Something-Other relation, but establishing itself through its adjoining action on the string. As we saw in (j), this Finite (and Infinite) as it now is in the Affirmative Infinite is The Ideal. In the string context, The Ideal is the string-in-general, the Quality of stringness. As we saw in (j), the moments of The Ideal are Being and Negation. The Being aspect of The Ideal as string-in-general is the string part, while the Negation aspect is the in-general, the latter being the fluidity indicated by the refusal to be pinned down to any particular string. The Ideal will be specified further in more familiar mathematical terms once we have determined Being-for-self for strings.

As the infinite Being of stringness (the Affirmative Infinite), Being-for-self is the sublation of the string-in-general (The Ideal), the positing of the unity of its moments of Being and Negation. In more familiar mathematical terms, it is simply the qualitative determination of the natural number in im-

mediate form. This is represented in its quantitative, ,,set," form in mathematics using a symbol, usually N. For us, it is, as we have said, only qualitative in character. There is no notion yet of the quantitative – of counting or size or of a set of things.[33] The Ideal, the string-in-general, then assumes the mathematical form of the variable. It is usually represented by a symbol such as n. Thus in mathematics, one says: Let n be a natural number.

In this connection it is true that, as we saw above, The Ideal sublated in Being-for-self is the string-in-general and this is not immediately the concept of the familiar natural number. However, recall that the selection of * was arbitrary ((h)) and The Ideal is ,,the finite as it is in the true infinite" (SL 150). Accordingly, the sensuous presentation of number as strings of *'s, whose length can be arbitrarily large – infected with the bad infinite – drops off and the simple form of The Ideal in Being-for-self is reflected in external symbolism by a single symbol such as n. Number thus is an ideal form, and this ideality is reflected into the strings of *'s which gave rise to the infinite progresses in the simple form of numbers that we customarily use. Thus 1 corresponds to * and 2 to **. In terms of external symbols, there is no distinction between, for example, ** and 2 – one could in principal do arithmetic calculations in terms of either normal base 10 numbers or strings of *'s. (The latter would be much less convenient.)

However, there is an important distinction between, for example, 2 as the number two and **. The latter is immediate, conceivable without any numerical connotations. The former, however, is mediated by the ideal Finite in Being-for-self, representable in this context by n, a determination of which is 2. So 2 presupposes the variable n which in turn only is as sublated in Being-for-self in the form of N. So a number is not a separately existing thing that we intuit (Platonism), nor a ,,God-given" synthesis of moments in a priori temporal intuition (Intuitionism), nor sets of certain sets of things in the ,,world" (logicism) nor ,,marks" manipulated according to rules in a game (Formalism). Rather a number is the determination of the Finite as sublated in ideal form in the Hegelian category of Being-for-self.

We conclude this development by showing how it provides, at the level of Being, the conceptual basis for the successor function

$$n \rightarrow n', \tag{1}$$

used in the Peano axioms for the natural numbers. This is the function that takes a natural number to the next one. (So, for example, $1' = 2$ and $2' = 3$.)

Here, the natural number (The Ideal in Being-for-self as N) is determined in n, n' as difference: it has bifurcated into n and n'. Now Difference is an Essence determination. Since the development of this paper is at the Being level, such difference will not appear in it conceptually, and so n and n', even though numerically different, are conceptually identified in the natural number (as The Ideal).[34] With this conceptual identification, (1) is expressing the Affirmative Infinite as mediated through Being-for-self as N. Indeed n and n' then both stand for the Finite and the symbol $\rightarrow$ stands for the Infinite, the operation of going to the next number, corresponding to the Infinite as the potentiality for *-adjunction in string terms ((i)). As written out externally, the sequence (1) reflects part of the qualitative infinite progress which was sublated to give the Affirmative Infinite – the sequence $n \rightarrow n'$ reads: Finite, Infinite, Finite, and the identification of the extremes gives the Affirmative Infinite. The significance of the implicit mediation of Being-for-self as N lies in the fact that it is the sublated Finite, The Ideal as natural number n, n' determined by this Being, that is involved in the sequence, not the old, unstable Finite of the qualitative infinite progress.

We have included this very brief discussion of the successor function to indicate how the logical development of self-referential sentences grounds (unexpectedly) basic concepts of the natural numbers and the mathematical logic associated with them. Of course, the foundations of mathematics requires a far more extensive development (including categories of Essence and Concept and indeed new, non-Hegelian categories) which is beyond the scope of this paper.

Appendix: The Translation of Dasein

The issue of translating into English Hegel's logical term Dasein – especially in the context of its first introduction in WL – is a thorny one. J.H. Stirling[35] translates the term literally as there-being while in their translation EL of the Encyclopedia Logic, Geraets, Suchting and Harris, on a majority decision, translate it as being-there or thereness. These neologisms, however, have the spatial connotation of place. Spatial considerations are quantitative in character (as Hegel says at the beginning of his philosophy of nature) and indeed, in WL, are considered by Hegel in Quantity – for example in his treatment of the Kantian antinomies. But Quantity comes after Quality in WL and Dasein is in the earlier section on Quality. Place, with its spatial and hence quantitative, significance, should not be part of Dasein.

Geraets and Harris[36], in their discussion of criticism by J. H. Wilkinson, agree that Dasein should not be connected with a representation of space, and argue against the spatial connotations of being-there and thereness, pointing out that ,,there" does not necessarily carry a spatial connection, as evinced by non-spatial terms such as ,,there-by", ,,there-fore" and ,,there-of". However, there is no hyphenation in any of these terms in normal English usage, and it seems clear that the meaning of each of these non-spatial terms is not obtained by externally combining the meaning of ,,there" with that of the other word. In the case of a neologism such as being-there, there is, in contrast, no alternative but to take the meaning of being and of there separately and combine them meaningfully somehow. Since the primary significance of ,,there" is surely spatial, these hyphenated terms ought to be also.

J. Burbidge[37] has rendered Dasein as a being. A problem with this rendering is that a being implies one being among others, and so entails the one and the many. Yet Hegel's treatment of the one and the many comes after Dasein in Fürsichsein (Being-for-self).

Translating Dasein by existence or life is unsatisfactory since these concepts occur much later in WL than Dasein. In response to an objection by Harris and Geraets to his suggestion that Dasein should be translated by existence, J. H. Wilkinson[38] argues that a distinction should be made in English between existence (Dasein) as ,,connected with" the finite and Existence (Existenz) as what emerges out of Ground. However, this distinc-

tion is not valid since this Existence that has emerged out of Ground, for Hegel, applies every bit as much to a finite (though mediated) Dasein such as a book[39] as it does to God.

It is important to stress that the meaning of Dasein deepens, becomes more concrete, as it is continually further determined through the accumulating sublations of an advancing Hegelian logic, so that, for example, at the level of Existenz, Dasein, instead of being simply qualitative, as it was in Chapter 2 of WL, is to be thought of in terms of a presupposed Grund and (as Geraets and Harris point out) is used to refer even to the existence of God.

Unfortunately, this deepening is not clear from the ,,contextual" translation SL which does not render Dasein consistently. However, in SL, the popular translation of Dasein as determinate being is often used. This is open to the Geraets-Harris criticism[40] that Dasein is the most completely indeterminate form of finite being. Further, the being that we surely have in mind when using the expression determinate being is positive being, while[41] Dasein is the simple oneness of being and nothing with being given no preference. It seems, therefore, better to reserve, as we did in (a) of the second section, the expression determinate being for Hegel's later Dasein category, Reality, the determinate character (Quality) of Dasein as being.

It seems to me that even in German, Hegel's choice of the term Dasein in Chapter 2 of WL was not good. A vague, ,,qualitative" term like the English something, as when, in informal conversation, we are pressed for details of which we are not sure and say that ,,it was something like that", seems closer, but something is itself an (early) category of Dasein. Maybe neither German nor English has a word for what Hegel means, a word that has the difficult job of ranging in meaning from the almost inconceivably indefinite through to existing things and beyond, and in this paper, I decided to leave Dasein untranslated in the hope of building up its early meaning by extrapolating backwards from the more familiar concepts that Hegel develops out of it rather than opt for an English word which would be misleading.

Notes

[1]The author is grateful to H. S. Harris and M. Hoffheimer for their advice and encouragement. He is particularly indebted to Dr. Hoffheimer for his detailed criticism of the paper and numerous valuable suggestions.

[2]B. Croce: An Unknown Page from the Life of Hegel. In: Philosophy, Poetry, History: An Anthology of essays. London 1966. 191.

[3]The logic of self-reference discussed below is only a determination of the logic of Dasein – for that logic applies to a much wider range of thought than just the mathematical. A discussion of various English translations of Dasein, and the reasons for staying with the original German term in this paper, are given in the Appendix.

[4]Hegel's Wissenschaft der Logik will be abbreviated to WL in the sequel. A. V. Miller's translation of WL, Hegel's Science of Logic. Atlantic Highlands, N.J. 1989, will be abbreviated to SL. The translation by T. F. Geraets, W. A. Suchting and H. S. Harris of Hegel's Encyklopädie der philosophischen Wissenschaften – The Encyclopedia Logic. Indianapolis 1991. – will be abbreviated to EL.

[5]W. and M. Kneale: The Development of Logic. Oxford 1962. 114.

[6]The construction is similar to that used by Cantor in his proof that the set of real numbers is not denumerable.

[7]The card paradox is taken from W. S. Hatcher: Foundations of Mathematics. Philadelphia 1968. 113. The preceding book will be referred to in the sequel as FM.

[8]However, following the practice of Hegel, it will at times be helpful to use Essence/Concept categories to describe more clearly the development.

[9]More confirmation that Dasein is the part of Hegel's treatment of Being relevant to the semantic paradoxes is given by the fact that it is there that Hegel deals first with the infinite as qualitative. This infinite (in its ,,genuine" form) is self-referential. Russell argued against locating the problem of the semantic paradoxes in the infinite: for example, the liar paradox does not seem to involve an infinite set. However, the self-referential property which he regarded as characterizing all of the paradoxes is (as we shall see) the (qualitative) infinite in the sense of Hegel, to be thought of in terms of a circle bending back on itself. A good example for Hegel of the infinite as self-referential is that of self-consciousness which is ,,the nearest example of the presence of infinity" (SL 158).

[10]As we will see below, the category of Limitation in the card case is in fact the arbitrary stopping. But this is unstable and the Ought with which it is coupled forces on the infinite progress.

[11]It is important to distinguish between the Hegelian and everyday use of a term, the latter being a vague, imprecise version of the former. For example, the Hegelian finite is much more logically developed than the ,,everyday" finite since it explicitly embraces, seething within it, conceptual contents such as becoming, something, constitution, limit and the ought that are left blurred over in the everyday use. Further, the conceptual contents of Hegelian terms determine the strictness of the logical development. Reserving, for example, finite for the ordinary finite as defined in a dictionary and Finite for the logically structured, Hegelian finite is perhaps the least objectionable way to make this distinction.

[12]A fundamental theme of Hegel's treatment of Dasein (and therefore also of this paper) is the importance of the qualitative. In 20th century mathematical logic, this emphasis is also clear in Russell's no class theory, which (in its simplest form) can be regarded as removing substantiality from a class of things and assigning it to the property or attribute that describes it, the supposed justification being that coherent properties are immune from paradox. (See, for example, R. M. Sainsbury: Russell. London 1979. 293 ff.) The qualitative character of property is discussed in SL 487, where quality is shown to be property in an immediate, non-reflected form. It is of interest that Russell's theory had already been anticipated by Hegel in his treatment of Existence (SL 491) in WL. There, Hegel shows how property as a relation between things becomes itself self-subsistent while the things become unessential.

[13]A good example of this relating of the Something-Other moments through the unity of Something is provided by the sons Shaun and Shem in James Joyce's Finnegans Wake. The unity in this case is their father HCE in whom their opposition is reconciled. As Michael Hoffheimer has pointed out to me, in Finnegans Wake, Joyce, who obviously identifies with Shem, also identifies with his father and even with his brother. (See the discussion of this in Bernard Benstock: Joyce-again's Wake. Washington 1965. 232ff.) In Hegelian terms, such identifications are to be thought of, not in the Being terms of Something and its moments but in the more advanced terms of the Concept, each of whose moments is the whole Concept.

[14]This identity of sides and turnings through each other prefigures the closed ,,loop" of the Affirmative Infinite, to be discussed in (j), in which each of the Finite and the Infinite returns to itself through the other.

[15]This argument is used in the derivation of the line from the point in Hegel's Philosophy of Nature.

[16]We have here a precursor of Schein in Essence, the movement of nothing to nothing (SL 400).

[17]Of course, that is exactly what we would normally do when we realized the vicious circle that we were trapped in when trying to resolve the card case – we would burst out laughing and go and do something else! However, this action is a giving up, and the resolution of such a self-referential problem can only be obtained by staying strictly with it and allowing it to develop itself logically.

[18]SL 136.

[19]There are, of course, many other infinite progresses in philosophical thought, not just in Hegelian logic. There is, for example, the quantitative one produced by halving as in Zeno's first argument about motion (Aristotle: Physics 239b). The Finite-Finite infinite progress is qualitative in character, and in form is simplest possible since the terms of that progress are all the same.

[20]This is quoted by G. E. Moore in Wittgenstein's Lectures in 1930-33. In: Mind 64 (1955), 3. The ,,laziness" in the use of $\cdots$ for Wittgenstein was in connection with the replacement of the formula $(x).fx$ (i.e. for all x, fx holds) by a logical product $fa.fb.fc.\cdots$ in the case where the x's range over a finite alphabet, and we use the $\cdots$ when we cannot be bothered (are too lazy!) to finish the enumeration explicitly. In the context of this paper, the ,,laziness" in the use of $\cdots$ is different though related. It concerns the use of $\cdots$ in the infinite progress (sequence) through which we stay content with the ,,idea" of the progress suggested by giving a few initial terms followed by the $\cdots$, thus evading the serious issue of the bad infinitude involved. Such evasion is inadmissable for the philosophical understanding of mathematics. The paper argues that the true meaning of the $\cdots$ is to be found in the Hegelian infinite.

[21]The moment Something was also a side but in unmediated form. So the further determined Finite is a return to this Something but as mediated, the moments of Limitation and Ought having been made explicit.

[22]In the positing of Determination and Constitution ((c)), the moment sides were posited as identical only ideally in the Something, but remained externally separate as moments. This separation was overcome in the Finite

in (e) (for the card case) in which the two sides identified immediately in the whole card. But in this case, they disappeared as only implicit in the whole card. Now, however, the sides have both separated out and then explicitly identified in the infinite progress.

[23]The term ,,some" at the more advanced Concept level is used in Hegel's treatment of the Particular Judgement (SL 645). But in the context of the present paper, ,,some" is immediate relative to this latter ,,some."

[24]The use of * here follows the practice in formal systems (such as the Statement Calculus) of denoting statement letters by $x^{***\cdots}$ (cf. Hatcher in FM 144 ff) except that we have dropped the inessential x. In that context it is used as a device for constructing an infinitely countable number of distinct statement letters from a finite alphabet.

[25]Hatcher, in FM 17, comments that the ,,semantically precocious" reader may notice that when we speak of a sign such a *, we do not really mean ,,the particular blob of ink on the particular part of this page." He suggests that the sign is really the ,,equivalence class of ink blobs under the relation sameness of shape." However, there is nothing precocious about such a view of signs – it is simply a confusion. For a start, it would imply that when a small child recognizes a sign, he has implicitly used infinite set theory and checked an equivalence relation! The real problem is that the proposed equivalence relation is not properly defined for mathematical use. In fact, the proposed relation is ultimately purely subjective – two blobs are equivalent if they look the same to us. (Compare this to a genuine equivalence relation such as that which defines two integers to be equivalent if their difference is divisible by 2.) The truth is that such a sign (or what we have called a symbol) is more than its physical appearance on the paper – it expresses something, it is a positing and so is given as conceptual in character.

[26]Gauss's horror of the infinite as something consummated is described in D. M. Burton: The History of Mathematics. Boston 1985. 593.

[27]The importance of the qualitative in questions involving the Infinite is illustrated by Hegel's qualitative solution to the infinite in the Differential Calculus (SL 274f). This is sometimes dismissed derisively (for example, by Russell). But it is to be observed that the modern ,,solution" to the defining of a derivative in the Calculus uses new concepts relative to Hegel's time, such as that of limit and the real line as a set (rather than as conceived

geometrically), and these involve the qualitative in the sense of Hegel.

[28]This is to be compared to Hegel's treatment of the Essence category of Contradiction: Contradiction resolves itself and ,,withdraws into Ground" (SL 434).

[29]The two infinite progress arguments, in which the reuniting of the Finite with itself sublates to give the Infinite and the dynamic uniting of the Finite with the Infinite sublates to give the Affirmative Infinite, illustrate a fundamental principle of Hegelian logic. For example near the end of Essence, the ,,bad infinite" of cause and effect is sublated in a causality which presupposes itself (SL, 564) and even at the advanced logical level of the Concept, the Idea itself is the sublation of the ,,infinite progress of mediation" (SL 749) of ends and means.

[30]Miller, in SL, translates the term das Ideelle as ideal being. However, as a moment in the genuine infinite, it does not have the immediacy of being but rather is intrinsically mediated. Further, as we shall see, for Hegel, the term is to be thought of in terms of becoming rather than of being, and anyway the being of the Affirmative Infinite is that of Being-for-self which is different from (though it includes in sublated form) das Ideelle. For these reasons, it is better to remove any reference to being in translating the term, and we have rendered it as The Ideal.

[31]It is the positing of the coincidence of beginning and end as asserted in the Bhagavad Gita and the poetry of T. S. Eliot. It is also the linking of the first and last sentence-fragments in Finnegans Wake. (This latter observation is due to Michael Hoffheimer.)

[32]Being-for-self, the unity of the moments of Being and Negation of The Ideal posited as the Quality of the Affirmative Infinite, is to be compared with the Something of (a) which expressed the identity of the moments Reality and Negation in the sublation of Quality.

[33]This notion of Being-for-self is effectively what is being expressed externally in the fundamental formal system **S** for elementary arithmetic, the formalization of Peano's axioms (eg. Hatcher in FM 70). (The successor function that appears in these axioms is briefly discussed below.) A merit of this system for mathematical logic is precisely its limited set theoretic commitment.

[34]Of course, the essential difference that is being expressed in (1) is of fundamental importance for the full conceptual content of the successor

function, and indeed the Dasein determination of the function, discussed above, is far from doing justice to this content. We note that the conceptual identification of n, n' above is, however, present implicitly in the embarrassing problem for logicism that for some n, n could conceivably be the same as n', a possibility that Russell sought to avert by his Axiom of Infinity.

[35] The Secret of Hegel. London 1865.

[36] On Translating Hegel's Encyclopedia Logic: A Response. In: Owl of Minerva 26 (1994), 95-97.

[37] J. Burbidge: Hegel's conception of logic. In: F. C. Beiser: The Cambridge Companion to Hegel. Cambridge 1993.

[38] J. H. Wilkinson: On Translating Sache in Hegel's Texts: A Response to a Response. Owl of Minerva 27 (1996), 211-230.

[39] SL 490.

[40] EL xx.

[41] SL 109.

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