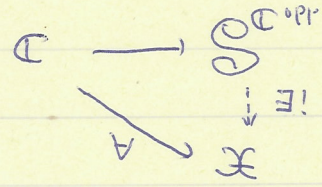
Lawvere 23 juli 66

Mainly conjectures.

$\mathcal{X}$ right complete, $\mathbb{C}$ small



So $\mathcal{S}^{\mathbb{C}^{opp}}$ is the free right complete category

Large theories in the cat of cat's. : every possible limit will be an algebraic operation.

$\mathcal{S}^{\mathbb{C}^{opp}}$ is in some sense the power set. ; so one can use it for higher order theories. Grothendieck topology is ordinary topology, but with a new power set functor.

Why power set? Think of the closed cat. of ordered set Then the analogue of $\mathcal{S}^{\mathbb{C}^{opp}}$ is

$$\mathcal{L}^{\Lambda^{opp}}$$

($\mathcal{L}$ analogue of $\mathcal{S}$) ; power set if Λ discrete.

"Triple" on ordered sets. (not really).

$$\mathbb{C} \xrightarrow{F} \mathbb{C}' \qquad \mathcal{S}^{\mathbb{C}^{opp}} \underset{\square}{\overset{adj}{\longleftarrow}} \mathcal{S}^{\mathbb{C}'^{opp}}$$

adj instead of $\square$ makes it covariant. (Corresponds to covariant power set with direct image).

$\mathcal{S} \circlearrowright P$ (Power set with direct image, is a monad).

The algebras : right complete lattices. (the morphisms preserve arbitrary sup's). Free alg's: Sets with relations.

This suggests the following : The Kleisli is just putting in more functions. Take e.g. P_o (set of finite subsets).

E.g. cat of cat

$\mathbb{C} \to \mathbb{D}$. But a generalized functor is

$$\text{Cocontinuous}\left(\mathcal{S}^{\mathbb{C}^{opp}}, \mathcal{S}^{\mathbb{D}^{opp}}\right)$$

$$\cong \mathcal{S}^{\mathbb{C} \times \mathbb{D}^{opp}}$$

Objects in here are simply pairings (sense of Cohn) $\vee$. Composition is a generalized matrix multiplication

This has been looked at in ring theory. Morita-theory: these things are just bimodules. Freyd showed the same to hold for theories. – Or take top. spaces and sheaves.

Every monad in sets had an algebraic part

The composition $P \boxtimes P_0$ carries a monad structure; it describes Rcled lattices; (distributive law $x \wedge \bigvee y_i = \bigvee x \wedge y_i$ required).

Close connection with Hausdorff^{opp} and Rcled

$$\mathcal{H}^{opp} \underset{top}{\rightleftarrows} \text{Rcled}$$

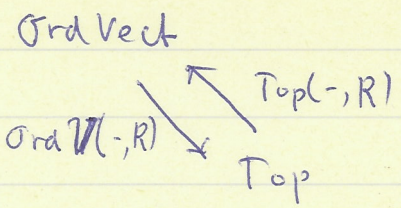
top

This was to prepare the way for a similar monad in the cat. of cat's, to get topos.

Given two cats, both relative over some common closed cat, and an object S common to both

$$\begin{array}{ccc} \mathcal{R}^{op} & \xleftarrow{\text{Cat}(-,S)} & \\ & \searrow^{R(-,S)} & \text{Cat} \end{array}$$

will automatically be adjoint. The monads should have some special properties ("analytical")



Gives the integration theory monad on Top.

The previous monad $\mathbb{R}$ is

$$\mathcal{X} \rightsquigarrow \mathcal{R}(\mathcal{S}^{\mathcal{X}}, \mathcal{S})$$

I don't know what the algebras are. If $\mathcal{X}$ has small hom sets, I have the Yoneda mapping

$$\mathcal{X}^{opp} \longrightarrow \mathcal{S}^{\mathcal{X}} \quad ;$$

restricting along this, I obtain a functor

$$\mathcal{R}(\mathcal{S}^{\mathcal{X}}, \mathcal{S}) \longrightarrow \mathcal{S}^{\mathcal{X}^{opp}}$$

The same procedure for the measure case gives

$$\mathcal{O}(C(X,R),R) \longrightarrow Meas(X,R)$$

The Riesz repr. theorem gives conditions when this is an equivalence.

Is there a Riesz theorem for

$$\mathcal{R}(\mathcal{S}^{\mathcal{X}}, \mathcal{S}) \longrightarrow \mathcal{S}^{\mathcal{X}^{opp}}$$

$$\int f\,dm \;=\; \varinjlim (-,S) \xrightarrow{n} S$$

(the canonical limit).

One can speak of the small part of a monad in the cat of cats

$$\tilde{T}_{\mathbb{R}}\mathcal{X} = \bigcup_{\substack{\mathcal{C} \to \mathcal{X} \\ \text{small}}} \mathcal{R}(\mathcal{S}^{\mathcal{C}}, \mathcal{X}) \;\subseteq T\mathcal{X}$$

$\hat{T}$ is a monad

I conjecture the alg's of over this is right complete categories (with chosen limit functors).

$$\mathrm{Dir}(\mathcal{X}) \longrightarrow \mathcal{S}^{\mathcal{X}}$$

factors ↘ ↗ conjugation (Isbell)

$$\mathcal{S}^{\mathcal{X}^{op}} \dashrightarrow \mathcal{X}$$

An algebra $\mathcal{F}$ ...

Presentations. Given a pair of small cats

$$\mathbb{R} \rightrightarrows \mathcal{S}^{\mathbb{C}^{opp}} \xrightarrow[\text{in } \mathbb{R}]{\text{coeq}} \mathcal{X}$$

Grothendieck topology

For small cat's: take the sheaves for ~~with~~ the canonical topology. - is the topple.

The topple in the cat of cat's has the following property, that

$$\mathcal{S}^{(\mathcal{S}^{\mathcal{X}^{op}})^{op}} \longrightarrow \mathcal{S}^{\mathcal{X}^{opp}}$$ then

$\mu x \to \eta_x T$; so the μ is determined by η

$$\mathcal{S}^{opp} \quad \text{with} \quad 2^{(\,)} \uparrow \quad 2^{(\,)} \downarrow \quad \mathcal{S} \circlearrowright$$

we get complete atomic Boolean alg's as alg's.