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Kōan-like ambiguities in John Cage's *Electronic Music for Piano*: a novel application of formal concept analysis

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In this article I present a novel application of Formal Concept Analysis (FCA) to the problem of interpreting and analysing the non-standard graphical music score *Electronic Music for Piano* (1965) by American composer John Cage. The avant-garde nature of Cage's score, presented on a single page as a series of anecdotal texts written in natural language, resists traditional approaches of music theory analysis. I utilise FCA for its suitability for delving into the work's taxonomy as well as its ability to visualise and analyse semiotic patterns and relationships within the natural language text fragments of the work. In this regard, FCA is also used as a means in which to frame what many writers have previously identified as *kōan*-like qualities in many of Cage's works. A *kōan* is generally understood as a type of unsolvable Zen Buddhist riddle filled with contradictions and ambiguities. I subsequently theorise *Electronic Music for Piano* as a type of musical *kōan*, and by applying FCA, demonstrate how various derived formal concepts, object extensions, and subset groupings of the work support this theory.

Keywords: formal concept analysis; John Cage; *Electronic Music for Piano*; *Kōan*; indeterminacy

2010 Mathematics Subject Classification: 03G10
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1. Introduction

While the utility of Formal Concept Analysis (FCA) has greatly aided research conducted within disciplines such as ontology engineering, bioinformatics, text mining, information retrieval, and machine learning (just to name a few), its usefulness as a tool for exploring and analysing underlying concepts within musical contexts has been somewhat neglected.[1] This is in spite of the fact that FCA is well suited to those traditional methodologies of musical theory and music analysis that have focused on investigations into the structural arrangement and content of the musical score – that is, musical pitch information and the patterns and relationship between musical pitches, phrasing, and the articulation of time. I present in this article a novel application of FCA to the problem of interpreting and analysing the non-standard graphical music score *Electronic Music for Piano* (1965) by American composer John Cage (1912–1992). In order to present my application of FCA to Cage's work I will first introduce the work's musical context, its innovative and non-standard graphical characteristics, and Cage's influences regarding Zen Buddhist teachings.

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[1] For counterexamples, see the work of Schlemmer and Schmidt (2010) and Motoyoshi et al. (2009).

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In particular, I discuss Cage's interest in Eastern thought and Zen as a means to interrogate what other writers have noted as the composer's use of *kōan*-like qualities in his works. A *kōan* is generally understood as a type of unsolvable riddle filled with contradictions and ambiguities. As a favoured Zen Buddhist teaching tool, a *kōan* is designed to resist any one definitive interpretation and is usually presented by a master in order to directly engage his student in the difficulties of dissolving the traditional subject–object paradigm of natural language. I will subsequently theorise *Electronic Music for Piano* as a type of musical *kōan*, and by applying FCA, will demonstrate how various derived formal concepts, object intents, and subset groupings of the work support this theory. I ultimately argue that my novel methodology greatly assists in identifying how the many ambiguities and contradictions of *Electronic Music for Piano* function as obfuscating devices that enable myriad possibilities for realisations to emerge rather than fixing the outcome to a single possibility.

2. New directions in musical notation

Though many notational innovations occurred in early music, by the middle of the twentieth century, new graphic symbols within music were rarely seen because of the prevailing hegemony of standardised musical nomenclatures inherited from the nineteenth century. In the years after the Second World War a number of composers, like Earl Brown (1926–2002), Morton Feldman (1926–1987), Roman Haubenstock-Ramati (1919–1994), and John Cage, started questioning both the role of Western music's aesthetic approaches, including its notational structures and nomenclatures, and those long-held standardised approaches used in the interpretation of music through performance. A work such as Earl Brown's *December 1952* aptly demonstrates a strikingly new type of graphic score that emerged in the post-war era. The piece consists of a series of vertical and horizontal lines of various thicknesses that are spread across the page, and for which the encoding of information regarding *how* such a score should be made into sounds and by *whom* is completely lacking and not obvious on first inspection.

Fellow American composer John Cage was equally engaged in the task of searching out new forms of musical expression through innovative notational advancements. In his monumental work, *Concert for Piano and Orchestra* (1958), Cage methodically documents and indexes the use of 84 new notational techniques in a 63-page score that can be played in part or whole by any number of musicians with or without piano. Figure 1 shows Cage's "AY" notation which is typical of the manner in which the score problemitises the traditional role of music notation as a set of codified instructions to the performer. Here, with only the presence of musical clefs (two treble and two bass), the interpreter is responsible for generating pitches, rhythms, dynamics and other sounds according to a freely-led excursion through the notation. As Heinz-Klaus Metzger and Ian Pepper have contended, in many of Cage's works

> the notations themselves refuse to generate a correlative sensuous appearance that would communicate meaning, [. . .] According to this idea, the artist is expected to determine the form, articulated to the smallest detail as a meaningful whole, whose necessity is measured by the degree to which nothing in it could be otherwise than it is, so that the precise function of each element that fulfils the whole can be grasped. (Metzger and Pepper 1997, page 54)

To produce an interpretation of a graphic score such as Brown's *December 1952* or Cage's *Concert for Piano and Orchestra* thus places a particular onus on the interpreter and their creativity. Similarly, and because of the well-established and recognised intersubjectivity between performer and notation, Cage manages to present the idea of the function of a musical score as belonging more to the realm of a meta-tool for a *co-composition*. Thus the traditional and long-established agreement between the function of performer, composer, and musical score is often

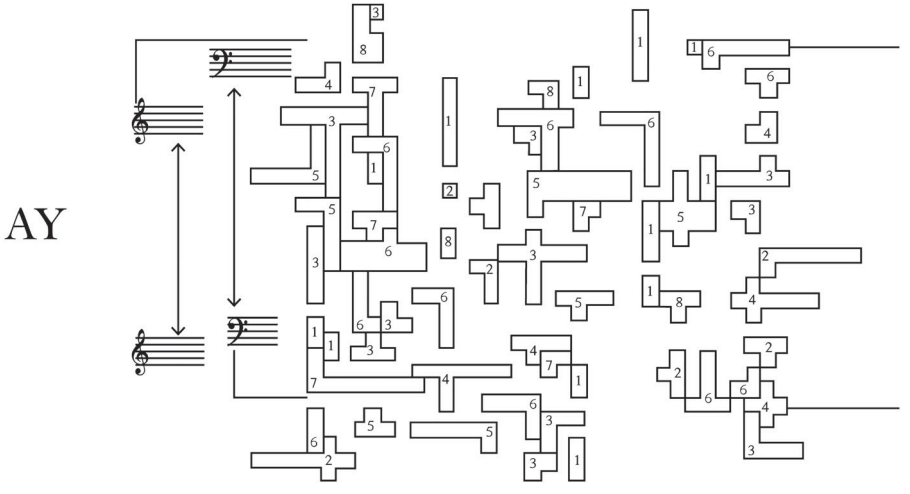


Figure 1. Notation "AY" from Cage's *Concert for Piano and Orchestra* (1958). © 1958 Henmar Press

problemitised in Cage's works by the manner in which the composer obfuscates the very function of the score to communicate musical meaning that is readily understood by both parties.

3. Cage's 1964 score *Electronic Music for Piano*

Following on from Cage's notational experiments in the *Concert for Piano and Orchestra* is perhaps one of the composer's least understood graphic works, *Electronic Music for Piano* (Figure 2). Completed in 1964 and published a year later, the work is composed of a single page of handwritten text that resembles neither clear directions nor analytic commentary. In an even more extreme fashion than his previous experiments with new graphic notations, *Electronic Music for Piano* seems wholly located at the threshold of those normative expectations regarding the function of a traditional score given its use of natural language. The work then aptly demonstrates Cage's subversion of the musical expectations extant in many of his previous graphic works, while at the same time utilising the notion of indeterminacy with respect to performance as a catalyst for enabling new performer-as-composer responsibilities to emerge.

For Cage, indeterminacy became an aesthetic ideal or a musical goal in and of itself, and was defined by the composer as simply "the ability of a piece to be performed in substantially different ways" (Pritchett 1993, page 108). Starting with works such as *Variations I* (1958), *Variations II* (1961), and *Variations III* (1962), each score becomes abstracted into collections of interchangeable points, lines, and circles (Fowler 2012). Here, the role of the performer is to *create* a score (or set of musical or written instructions) through the measurement of distances between points and lines which are then to be translated into musical information such as pitch, rhythm, and dynamics. Hence the very idea of the musical score representing a fixed entity regarding its possible sonic outcomes or its interpretation is greatly challenged (Fowler 2009).

Electronic Music for Piano develops this precedent, utilising fragments of natural language rather than exploring new types of graphic nomenclatures or symbols. The work is written in Cage's own handwriting and was notated on hotel stationary ("Hotell Malmen, Stockholm /Sweden"). The composition features texts describing actions as well as numerous other passages with crossed out words, edited punctuation, or somewhat obscured descriptions. But the use of natural language was not new for Cage given the composer's earlier 1962 composition

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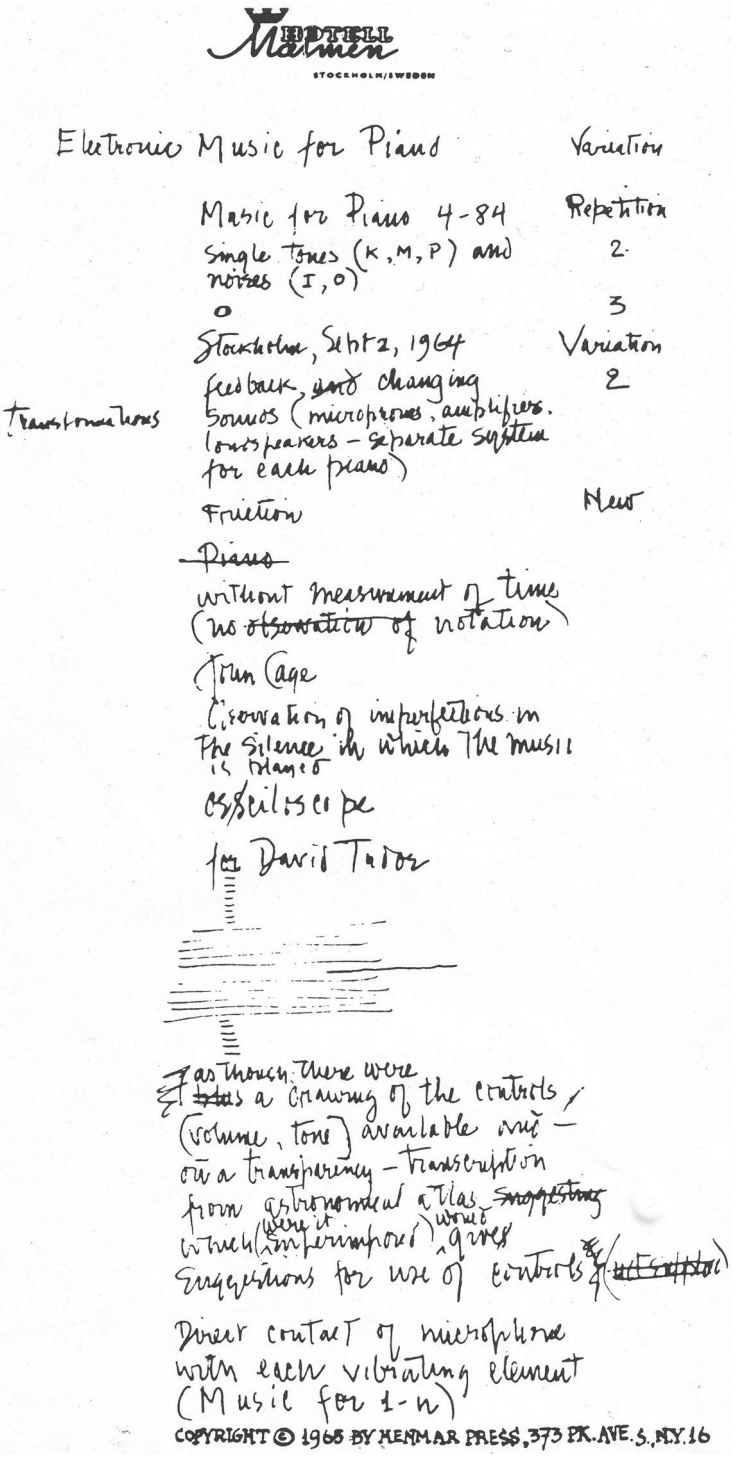


Figure 2. Complete score of Cage's *Electronic Music for Piano*, published in 1965. © 1965 Henmar Press

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0′00″ (*4′33″* No. 2). Here the simple direction of the published score "in a situation provided with maximum amplification, perform a disciplined action," was by its second performance qualified by the composer with the retainer that "the performer should allow any interruptions of the action, the action should fulfil an obligation to others, the same action should not be used in more than one performance, and should not be the performance of a musical composition" (Pritchett 1993, page 138).

In the later *Electronic Music for Piano* though, the use of natural language is not presented in such a clear fashion as it is in *0′00″*. As such, much of the content of the single page of *Electronic Music for Piano* successfully obfuscates the intent of the work as a set of discrete instructions for performance. Cage's name appears as well as the dedication "for David Tudor", a lifelong interpreter of Cage's electronic and piano works who produced a version of the piece in a concert on September 10, 1964 at Fylkingen, Stockholm. The score also has the time stamp "Stockholm, Sept 2, 1964" indicating its presumed initial function as a prescriptive device for performance that sits prior to the act of realisation.

For composer and performer Chris Burns (2004) and pianist Steffen Schleiermacher (1998), the process of realising their versions of *Electronic Music for Piano* are rather straightforward in that they rely on systematic foreground clues in the score concerning instrumentation, electronics, and thematic material. David Tudor's extensive technical knowledge of electronic systems for generating live electronic music suggested for Burns the fundamental conceptual position a realisation should take:

> Cage's notations include the phrases "feedback" and "for David Tudor." During the 1960s and 70s, Tudor gradually reoriented his career from the performance of avant-garde works for piano to the creation of live electronic music, with electronic feedback systems as the defining component of his work. [. . .] In appreciation of Cage and Tudor's work in the domain of feedback, we designed this realisation around an acoustic feedback network, implemented in software. (Burns 2004, page 193)

In approaching the interpretation of *Electronic Music for Piano* as a merger between live amplified sounds and a feedback system for spatialisation, and transformation, Burns relies on *Music for Piano 4-84* as source material. His realisation situates a pianist performing selections from *Music for Piano 4-84* being fed into a software system that variously applies indeterminate amounts of ring modulation, amplification, and spatialisation to the signal via randomised computation.

Similarly, for Schleiermacher, the mention of *Music for Piano 4-84* and the text fragment "Single tones (K, M, P) and noises (I, O)" are also strong evidence for a piano-led realisation. Cage refers here not only to previous piano works, but the performance practices used within those previous works. The notation K refers to sounds produced on the keyboard, M refers to muted notes, P is for pizzicato, I for noises occurring inside the piano, and O for those noises created on the exterior case. Like Burns, Schleiermacher makes use of those occasional references in the score to amplification and the use of astronomical charts. By using star charts printed on transparencies, Schleiermacher devised a system in which to assign durations to sections of the work, which later became overlapped through multi-tracked recording.

Both Burns and Schleiermacher approach *Electronic Music for Piano* as sitting comfortably within the normative paradigm of score as mediator for a musical performance. In each instance, though Cage provides no instructions on the appropriate methodology for realisation, both interpretations rely on the ostensive surface (or foreground) evidence to suggest, or perhaps necessitate, a performative response. But as both writers also agree, the nature of the score means that a realisation is a function of the interpreter-led realisation methodology, and questions of authenticity become relevant only in regard to the conceptual framework constructed by the interpretation. What other paradigms then might account for the lack of specific directions within *Electronic Music for Piano*, while at the same time allowing for a theory to explain the work's ambiguities?

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3.1. *Zen Buddhist influences*

Perhaps one of the most well-reported facts about the musical aesthetics of Cage was his interest in the concepts of Zen Buddhism and how these might serve to radically disrupt those inherited nineteenth century musical aesthetics he found within contemporary American musical composition in the postwar period. By constructing a complex and highly disciplined framework for musical composition based on Eastern philosophies, Cage sought to embrace the traditional aversion towards the unintentional within music. Cage thus attempted to negate value judgments in his music by rejecting the idea of the "constraint" of pre-meditation within the compositional process.

Heavily influenced by the teachings of Zen scholar Daisetz Suzuki (who introduced Zen to the US in the 1950s and 1960s) as well as the use of chance and aleatoric methods via the Chinese oracle *I-Ching*, Cage's compositional *modus operandi* sought to reject the notion of definitive authorship of a work of art and thus question what the meaning of music is, and how such meaning is interpreted through musical sounds. In fact, in his assessment of one of Cage's final performances of the large spoken word piece *Muoyce* (based on chance-manipulated excerpts of James Joyce's classic stream-of-conscience novel *Finnigan's Wake*) at Stanford University in 1992, Charles Junkerman suggests that Cage appropriated the nature of the Zen *kōan* as a device for obfuscation:

> I think Cage planned his reading as a kind of Zen *kōan* for the audience, an activity that would challenge the widespread conviction (especially in the University) that "understanding" is our goal and that logic and the intellect are the means of attaining it. (Junkerman 1994, page 51)

Daisetz Suzuki describes the function of the *kōan* within the *Rinzai* sect of Japanese Zen Buddhism as a disciplined methodology for discovering the truth through meditation in which a story or question is given from master to student. The story often contains "enigmatic and often shocking spiritual expressions based on dialogical encounters" (Foulk 2000, page 3) that are intended to awaken a student to their fundamental nature:

> the searching mind is vexed to the extreme as its fruitless strivings go on, but when it is brought to an apex it breaks or explodes and the whole structure of consciousness assumes an entirely different aspect. (Suzuki 1974, page 61)

Cage's introduction to Zen through Suzuki, and his creation of various compositional devices that sought to distance the ego and place value on non-intentionality are well documented. Junkerman's suggestion of the *kōan* metaphor to describe the performance of *Muoyce* is an intriguing example. It is a rare example of an actual Zen paradigm becoming a direct reference for musical composition and is similar to Cage's musical translation of the dry rock garden at the Kyoto temple *Ryōanji* (Fowler 2009). But for Austin Clarkson the *kōan* is similarly a means in which to frame Cage's embracing of indeterminacy:

> Jung explained the ability of the *kōan* to produce *satori* [enlightenment] as a breakdown of the rational defences of the ego-system so that contents from the unconscious can flow in and activate the self, which for Jung was the key to spiritual and mental health and to the successful outcomes of the individuation process. Jung's comments on the *kōan* and *satori* illuminate Cage's description of the music of indeterminacy, his call for dangerous, sudden listening, and his belief that a piece of music should be like a work of Nature. Cage's compositions provide musicians a *kōan*-like guide to a breakthrough that brings musical enlightenment. (Clarkson 2001, page 91)

What then are the implications for positioning *Electronic Music for Piano* as a type of *kōan*? Both Clarkson and Junkerman suggest the usefulness of theorising Cage's work as *kōan*-like without suggesting how such a text might be analysed or examined in a more objective fashion for *kōan*-like qualities. Cage's own spoken word performances and his numerous commentaries about the way in which Zen ideas influence his performance practice provide a wealth of evidence to suggest such a theory. As an example, take his seemingly irrational *kōan*-like approach

to answering questions after a spoken word performance of *Lecture on Nothing* at the Artist's Club, in New York City in 1958. Cage recalls that, "I gave 5 prepared answers regardless of the questions. This was a reflection of my engagement in Zen" (Cage 1959). Importantly, Cage's introduction to the ritual of *kōan* practice likely arose from Suzuki's North American lectures and later publications on Zen and Japanese culture. Both Griffith T. Foulk (Foulk 2000) and Kazue Yamada (Yamada 2004), identify a common transcendental element in the *kōan* and its importance as a device used in master–student teaching situations and for reflective meditation. But perhaps Barry Stephenson's structuralist definition provides the most useful introduction to the paradigm. He places the *kōan* in terms of an identified duality in which meaning and function arise from two distinct contexts:

> [*k*]*ōan* may be understood as psycholinguistic riddles capable of generating a certain type of experience or as a literary genre with a complex history embodying centuries of religious and philosophic discourse. (Stephenson 2005, page 476)

These two distinctions are important when considering how a *kōan* is to be analysed and especially how the realisation process in *Electronic Music for Piano* might be activated through two divergent paradigms. Stephenson's acknowledgement of the dual nature of the *kōan* is further discussed by Victor Sōgen Hori, who identifies two methods of approach in which the *kōan* is understood either as an *upāaya*, or instrument, "that deliberately poses a problem unsolvable by the rational mind in order to drive the mind beyond the limits of rationality and intellectual cognition" (Hori 2003, page 8), or through Hee-Jin Kim's realisation model in which "one experiences the *kōan* not as an object standing before the mind that investigates it, but as the seeking mind itself" (Hori 2003, page 9). To consider the instrumental approach to reading a *kōan* means negating normative expectation of an inherent singular underlying meaning present. Such an approach focuses on the linguistic constructs used and how they force contradictions in meaning and produce seemingly unsolvable conflicts.

If the interpretation of *Electronic Music for Piano* taken by Burns and Schleiermacher represents a realisation model of *kōan* analysis – in which the text is transmediated, that is, performed and made real through example – what accounts of the work might interrogate it as an instrumental paradigm? The realisation model of Kim is focused on transmediating the text, "by making real, that is, by actually becoming an example of the nonduality of subject and object [for which] the practitioner also realises, i.e., cognitively understands, the *kōan*. The realisation of understanding depends on the realisation of making actual" (Hori 2003, page 9). Conversely, to interrogate *Electronic Music for Piano* within Hori's paradigm of regarding a *kōan* as an "instrument of irrationality" implies an examination of the semiotic and structural qualities and those connective devices that occur within the text so as to discover the work's essential ambiguities and non-dualities.

The utility of FCA represents then an objective means in which to formally investigate the text passages of *Electronic Music for Piano*, and thus theorise the work as a musical *kōan*. Through the use of FCA, I will demonstrate how a primary demarcation of the work between two superconcepts, $\mathfrak{c}_{\text{perf}}$ and $\mathfrak{c}_{\text{comp}}$, and their extents, and the subset groupings of objects within the taxonomy support this theory. Approaching the analysis of the work in such a fashion allows the many ambiguities and contradictions of the text of *Electronic Music for Piano* to be brought into a clearer light regarding their function as intentional devices that enable myriad possibilities for realisations to emerge rather than fixing their outcome on a single possibility – an established favouritism regarding Cage and indeterminacy. Ultimately, the application of FCA will also allow for a means in which to qualify Hori's instrumental paradigm of *kōan* analysis through understanding the work as an instrument for irrationality and thus reveal how deeper-level structural traits within the piece can be codified to produce a definitive picture of its semantic structure.

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3.2. *Spatial partitioning and theme areas*

An important aspect of the approach of FCA lies in its mathematical exploration of incidence relations, *I*, between a set of objects, *G* (from the German word *Gegenstände*), and a set of attributes, *M* (*Merkmale*), that are first recorded in a cross table or formal context, $\mathbb{K}$, and then visualised via the graphical representation of a Hasse diagram (Wolff 1994). Although the founder of FCA, Rudolph Wille, instigated the approach in the 1980s as a means to restructure lattice theory, Tobias Schlemmer and Moreno Andreatta have argued that "music is a major inspirational field for applying formal concept analysis, [although] this approach did not establish itself as a paradigm in mathematical music theory" (Schlemmer and Andreatta 2013, page 189). In the case of *Electronic Music for Piano*, FCA will be applied in order to theorise the natural language texts of the work as the universe under consideration. But in order then to construct a viable context, $\mathbb{K}$, suitable objects and corresponding attributes that describe those objects must be identified from the materials within the score.

Examining then more closely the structure of the score and its text fragments and the actions or processes to which they refer reveals an emerging spatial scheme of commonalities (see Figure 3). As shown in the mappings of text fragments or *text objects* to spatial structure in Table 1, the work contains five larger spatial groups (A, B, C, D, E). These are distributed across the one-page score, yet are also inter-dispersed via what appears to be the use of multiple elements within each. Table 1 is ordered according to the original spatial distribution of text objects found in the score (that is, as each text fragment occurs going from top to bottom and left to right), but also includes an object ID for each text object.[2]

Most pertinent for a reading of the work through Hori's instrumental paradigm are the theme areas of the spatial groups B and C. The theme area of spatial group B refers to actions or objects suggestive of thematic materials or common performance practices previously used by Cage, or allusions to potential performances or realisations of the work. The earlier composition *Music for Piano 4-84* is mentioned as well as sound-making devices such as the piano, contact microphone, and an oscilloscope. Additionally, Cage describes the use of live electronics (microphones, amplifiers) and the compositional method he had previously used in the 1962 work *Atlas Eclipticalis* (that is, generating material through superimposed star charts onto manuscript paper).

The theme of spatial group C sits as a foil to B and describes elements that suggest the construction of musical form or the thinking through of structures for a musical composition. Spatial group D refers to the site of composition and its date, while the theme of spatial group E lists personnel associated with the work. The relationship between B and C describes the normative expectations of a musical framework in which production (the act of composition) and re-production (the act of performance) produce a musical event, though within each spatial group it is apparent that elements could equally be deemed as relating to *both* Cage's performance practices as well as his favoured compositional methods. Of note too is the lack of directions Cage gives regarding how many of the text objects are to be used in any given realisation of the piece. As with many of Cage's graphic works, such as the *Concert for Piano and Orchestra* or *Winter Music* (1957), a single page may consist of numerous markings and notations, though the performer is free to choose exactly how many and which of the notations to realise into sound. It remains unclear if this is also a possibility for a realisation of *Electronic Music for Piano*.

[2] Given that the score is to be freely interpreted by the performer, the order presented in Table 1 is arbitrary. The object IDs are used later as a means by which to distinguish parts of the score more easily in the line diagram of the concept lattice presented in Figure 4, but given the nature of FCA, any such ordering of text objects will yield the same result.

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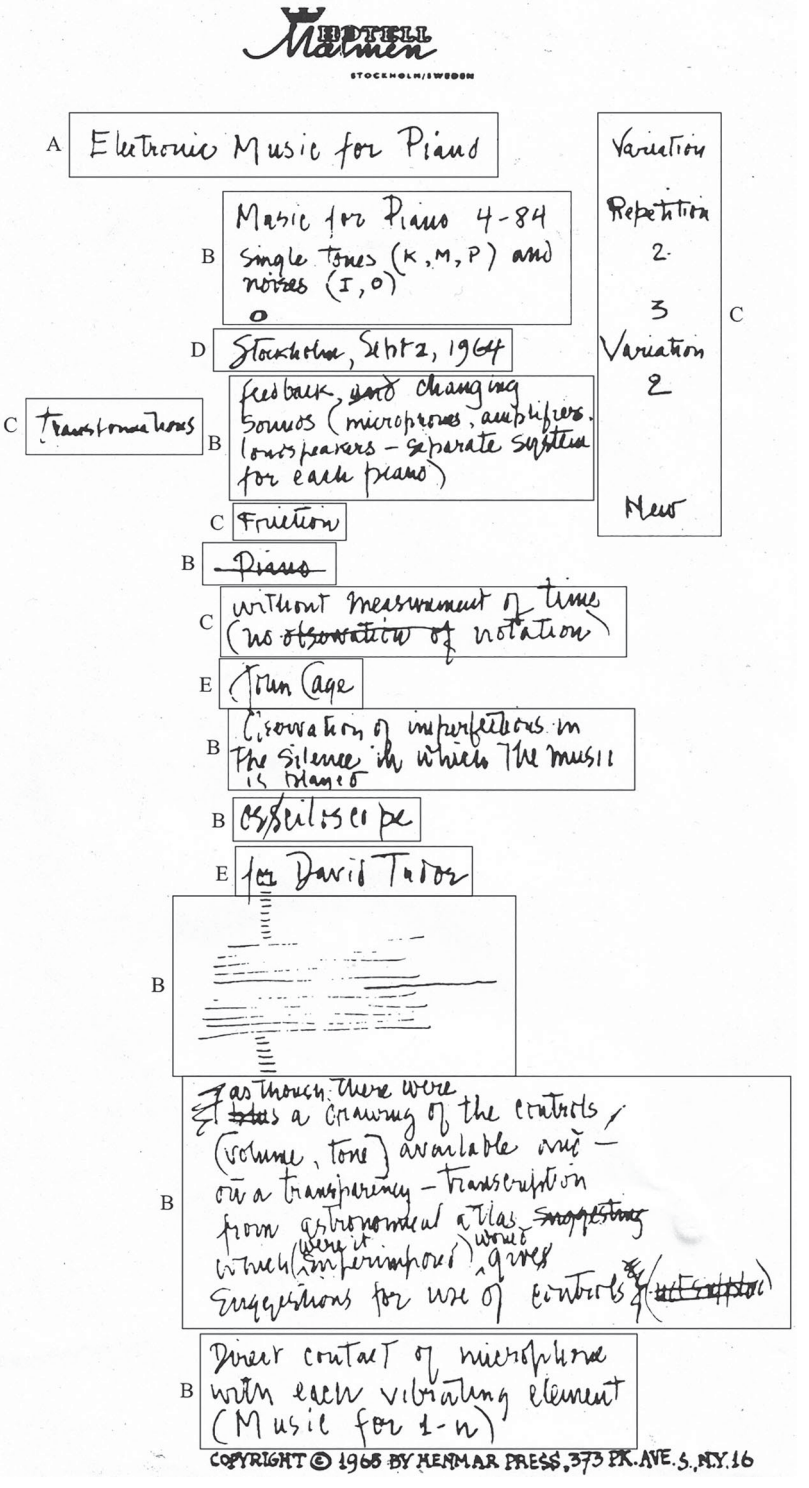


Figure 3. Spatial deconstruction of inherent text objects within *Electronic Music for Piano* in the vein of Cage's previous notational structure "AY" from the *Concert for Piano and Orchestra* (see Figure 1). The grouping follows five suggested theme areas: A (title), B (performance practice and thematic materials), C (structural or formal elements), D (site), E (personnel).

Table 1. Spatial theme areas of *Electronic Music for Piano*. The crossed-out words and crossed-out letter "s" appear as in the original work in Figure 2 and its spatial deconstruction in Figure 3.

Text object	Spatial group	Object ID
Electronic Music for Piano	A	1
Variation Repetition 2. 3 Variation 2 New	C	2
Music for Piano 4-84 Single tones (K, M, P) and Noises (I, O) *whole note*	B	3
Stockholm, Sept 2, 1964	D	4
transformations	C	5
Feedback ~~and~~ changing sounds (microphones, amplifiers, loudspeakers – separate system for each piano)	B	6
Fruition	C	7
~~Piano~~	B	8
without measurement of time (no ~~observation~~ of notation)	C	9
John Cage	E	10
Observation of imperfections in the silence in which the music is played	B	11
oscilloscope	B	12
for David Tudor	E	13
empty music staff with ledger lines	B	14
as though there were a drawing of the controls / (volume, tone) available and – on a transparency – transcription from astronomical atlas ~~suggesting~~ which (were it superimposed) would give~~s~~ suggestions for use of controls	B	15
Direct contact of microphone with each vibrating element (Music for $1 - n$)	B	16

4. An application of formal concept analysis

As Bernhard Ganter and Rudolf Wille suggest, "the basic notions of Formal Concept Analysis are those of a *formal context* and a *formal concept*" (Ganter and Wille 1999, page 17). Though the approach of FCA is concerned with the discovery and analysis of mathematical relations and mathematical constructs, Wille argues that "a high percentage of established mathematizations concerning the real world can therefore be moved up to the support of rational communication about the real world" (Wille 2010, page 29).

Each of the sixteen text objects identified in Figure 3 can be described by a series of attributes, m, that denote conditions inherent in one or many of the spatial groups of A–E. Consequently, a generation of a cross table or *formal context* of *Electronic Music for Piano* is given in Table 2. Attributes are nominated so as to generally describe the inherent musical intentions or characteristics of all the texts contained within the spatial groups A–E in an effort to reveal connections between objects. In Table 3, the attributes of $\mathbb{K}$, that is the set $M = \{$re:composition, re:performance, instrument, acoustic, thematic materials, observation, electronic$\}$ are further qualified. Each of the text objects from the derived spatial groups identified in Figure 3 are thus considered as an object, that is $g \in G$, whose attributes, $m \in M$, can be described by particular binary relations.

The generated line diagram (or Hasse diagram) $\underline{\mathfrak{B}}(\mathbb{K})$, shown in Figure 4, is particularly useful given its ability to reveal deeper formal relationships between the text objects and attributes of *Electronic Music for Piano*.[3] As Rudolph Wille suggests "the labelled line diagram of a concept lattice is logically considered as a hierarchical network linking nodes with object names to nodes with attribute names and thereby establishing conceptual meanings" (Wille 2005, page 29). As

[3] The line diagram found in Figure 4 together with the concept lattice of *Electronic Music for Piano* were generated using the software ConExp (Concept Explorer, https://sourceforge.net/projects/conexp/). For a discussion of this software, see Yevtushenko (2000). The notation for the line diagram $\underline{\mathfrak{B}}(\mathbb{K})$ includes an underline, whereas the notation for the formal concept lattice $\mathfrak{B}(\mathbb{K})$ does not include the underline.

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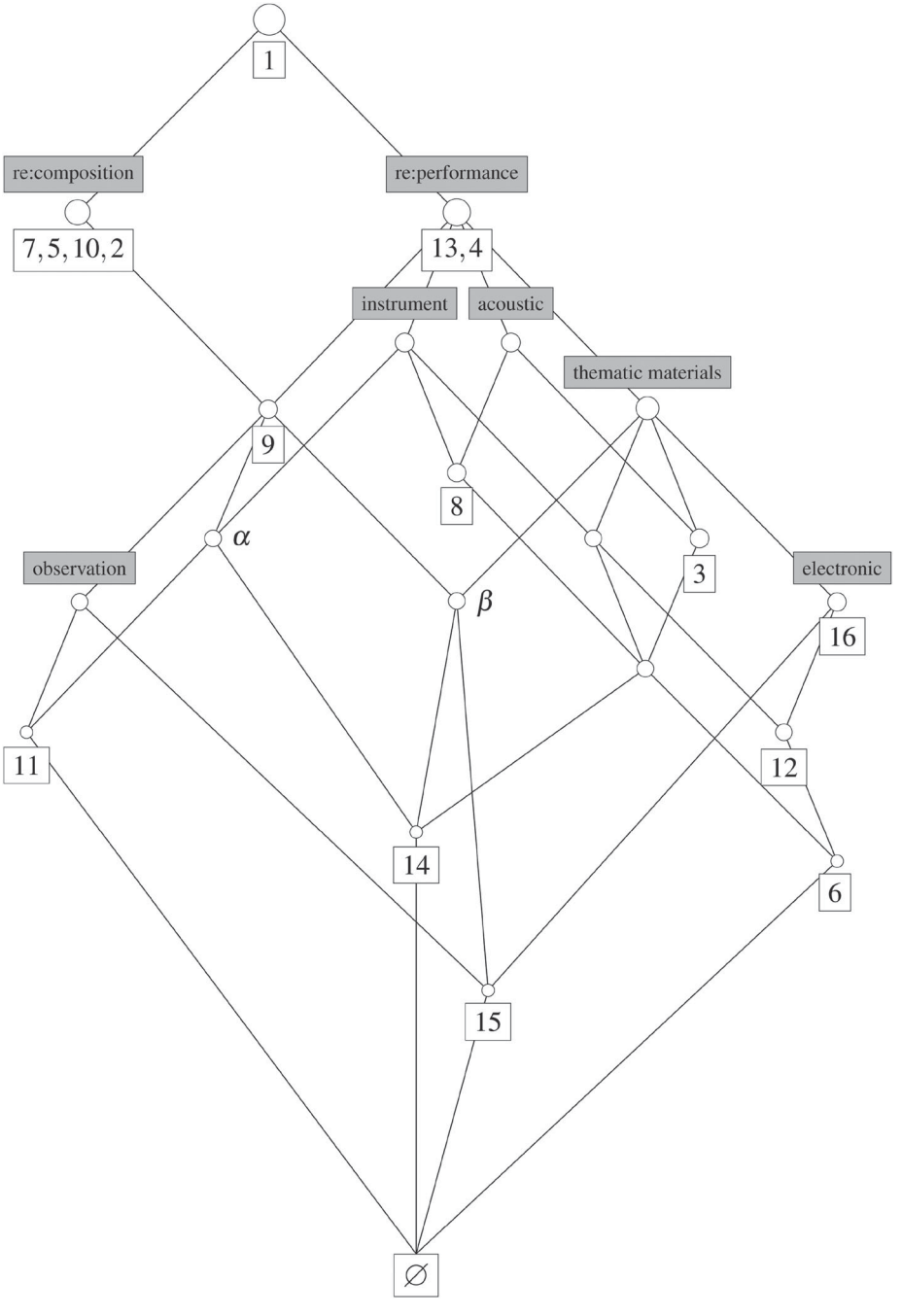


Figure 4. The Hasse diagram $\underline{\mathfrak{B}}(\mathbb{K})$ for *Electronic Music for Piano*. Attributes *m* sit above nodes, while objects *g* are indicated below them. The objects are as in Table 1: 1 = *Electronic Music for Piano*, 7 = Fruition, 5 = Transformations, 10 = John Cage, 2 = Variations Repetition 2. 3 Variation 2 New, 13 = for David Tudor, 4 = Stockholm, Sept 2, 1964, 9 = without measurement of time (no ~~observation~~ of notation), 8 = ~~Piano~~, 3 = *Music for Piano 4-84* Single tones (K, M, P) and Noises (I, O) *whole note*, 16 = Direct contact of microphone with each vibrating element (Music for $1-n$), 11 = Observation of imperfections in the silence in which the music is played, 12 = oscilloscope, 14 = *empty music staff with ledger lines*, 6 = Feedback ~~and~~ changing sounds (microphones, amplifiers, loudspeakers – separate system for each piano), 15 = as though there were a drawing of the controls / (volume, tone) available and – on a transparency – transcription from astronomical atlas ~~suggesting~~ which (were it superimposed) would gives suggestions for use of controls.

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Table 2. The formal context $\mathbb{K}$ of *Electronic Music for Piano*.

	re:composition	re:performance	instrument	acoustic	thematic materials	observation	electronic
Electronic Music for Piano							
Variation Repetition 2. 3 Variation 2 New	×						
Music for Piano 4-84 Single tones (K, M, P) and noises (I, O) *whole note*		×		×	×		
Stockholm, Sept 2, 1964		×					
transformations	×						
Feedback ~~and~~ changing sounds (microphones, amplifiers, loudspeakers – separate system for each piano)		×	×	×	×		×
Fruition	×						
~~Piano~~		×	×	×			
without measurement of time (no ~~observation~~ of notation)	×	×					
John Cage	×						
Observation of imperfections in the silence in which the music is played	×	×	×			×	
oscilloscope		×	×		×		×
for David Tudor		×					
empty music staff with ledger lines	×	×	×	×	×		
as though there were a drawing of the controls / (volume, tone) available and – on a transparency – transcription from astronomical atlas ~~suggesting~~ which (were it superimposed) would gives suggestions for use of controls	×	×			×	×	×
Direct contact of microphone with each vibrating element (Music for $1-n$)		×			×		×

Table 3. Definitions of $m \in M$ in the formal context $\mathbb{K}$ of *Electronic Music for Piano*.

$m \in M$	Spatial group	Definition of m
re:composition	B	Refers to signifiers or remarks that are most often associated with the construction or formulation of musical structure: either through enumeration, a formal description or goal-oriented suggestions.
re:performance	C, D, E	Actions or objects suggestive of previously used performance practices of Cage. The earlier composition *Music for Piano 4-84* is mentioned as well as sound making devices such as the piano, contact microphone and an oscilloscope. Additionally, Cage describes the use of live electronics (microphones, amplifiers) and the compositional method he had previously used in the work *Atlas Eclipticalis* (that is, generating material through superimposed star charts onto manuscript paper).
instrument	A, B	Objects or significations of sound producing devices
acoustic	B	Primarily referring to the creation or production of sound through mechanical means
thematic materials	B	Includes instruments, previous works, sound generating devices, methods for the generation of thematic materials through electronic or other means
observation	B, C	The act of observation either as a process within a compositional context or within a performance context (such as occurs in the work *0′00″*)
electronic	B	Sounds or thematic materials produced through digital or electronic means

found in Figure 4, the line diagram consists of circles (nodes) in which labels above nodes indicate attributes (shaded in grey) and labels below indicate objects. Thus at the top of the diagram can be found only the object "1" $= \{$*Electronic Music for Piano* $\}$, given that it has no incidence relation for any $m \in M$. This can also be seen in the cross table of the context $\mathbb{K}$ (Table 2). The definition of a formal context is usually given as a triple $\mathbb{K} = (G, M, I)$ in which G and M are the complete sets of objects and attributes, while I is a binary relation between G and M such that $I \subseteq G \times M$. More generally, $(g, m) \in I$ is read as follows: the object g is in a relation I to the attribute m if the object g has the attribute m, or, equivalently, if the attribute m applies to the object g (see Burmeister 2000).

What becomes interesting and revealing in the formal context $\mathbb{K}$ of *Electronic Music for Piano* are the formal concepts that arise and how they enable the philosophical understanding of a concept, $\mathfrak{c}_n$, as a unit of two parts, the extent $\text{ext}(\mathfrak{c}_n)$ and intent $\text{int}(\mathfrak{c}_n)$, such that all concepts of $\mathbb{K}$ are described as the concept lattice $\mathfrak{B}(G, M, I)$. As Karl Wolff notes, "the extension covers all objects belonging to this concept and the intension comprises all attributes valid for all those objects" (Wolff 1994, page 2). This can be easily grasped through the line diagram that visualises the concept lattice. Here, the edges between nodes indicate their extension or intension. The extent forms all the individual objects under consideration that fall under an attribute and can be traced via an edge, while the intent is all attributes above a given attribute that can also be traced via a common edge. As such, a label on the top of the uppermost node of $\underline{\mathfrak{B}}(\mathbb{K})$ would indicate an attribute all objects have, while the label at the bottom of the lowest node represents the object all attributes describe. In the case of *Electronic Music for Piano*, only the text object "1" sits at the bottom of the uppermost node, since no attribute in $\mathbb{K}$ describes it, while at the bottom of $\underline{\mathfrak{B}}(\mathbb{K})$ we have the empty set since no object is described by all attributes. Thus as Johannes Gadner and Doris Ohnesorge suggest:

> [t]he meaning of a concept derives not only from specific words but also from the context in which it is used. Concepts do not stand alone; they are embedded in complex networks that are joined together by syntactic and grammatical rules. (Gadner and Ohnesorge 2004, pages 70 and 71)

But in relation to FCA, a formal concept, $\mathfrak{c}_n$, is defined very specifically as a pair (A, B), where A is a maximal set of individual objects (the extent), and B is a maximal set of shared properties or attributes (the intent). A derivation operator for defining this pair through assigning arbitrary

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$A \subseteq G$ and $B \subseteq M$ conditions is used under the following mappings (Burmeister 2005, page 115):

$$A \longmapsto A^{\uparrow} = \{m \in M, \text{ such that } gIm, \text{ for all } g \in G\},$$
$$B \longmapsto B^{\downarrow} = \{g \in G \text{ such that } gIm, \text{ for all } m \in M\}.$$

Thus, as Ganter and Wille suggest "the extent A and the intent B of a concept (A, B) are closely connected by the relation I. Each of the two parts determines the other and thereby the concept" (Ganter and Wille 1999, page 18). This close connection is further identified in the fact that each part of a formal concept can be derived from the other because $A = B^{\downarrow}$ and $B = A^{\uparrow}$. As already discussed, in $\underline{\mathfrak{B}}(\mathbb{K})$ we find that the uppermost node, the concept described as $(G, G^{\uparrow})$, contains object "1" and attribute $\varnothing$, given that there is no m that describes all G. But Ganter, Stumme, and Wille (2005) also note that from the derivation operators further hierarchies emerge. The subconcept/superconcept relation is of particular interest to the emergent structure found in $\underline{\mathfrak{B}}(\mathbb{K})$ and is expressed as

$$(A_1, B_1) \leq (A_2, B_2) : \Longleftrightarrow A_1 \subseteq A_2 \iff B_1 \supseteq B_2.$$

Concepts that sit higher up within the line diagram, and for which lower-level concepts are contained within their extension, are considered superconcepts and are notated as (A_2, B_2). Superconcepts may include lower level attributes, and thus corresponding subconcepts, (A_1, B_1), which then accounts for the notion of an inheritance of attributes or class inclusion (Priss 2008, page 120). This characteristic of $\underline{\mathfrak{B}}(\mathbb{K})$ enables a clearer picture to emerge of attribute implications, in which

$$X \to Y \iff X \subseteq Y \quad \text{and} \quad X, Y \subseteq M.$$

Here, X implies Y if and only if X lies in the extension of Y and both X and Y are a subset of the set of attributes M. What becomes apparent from the line diagram of the concept lattice in Figure 4 are the two main factors that are evident in the concept lattice according to the attributes {re:composition} and {re:performance}: a structure already suggested in the spatial analysis and theme derivation described previously. These two attributes account for all objects within the concept lattice, and not surprisingly are suggestive of the interpenetrating nature between the traditional roles of composer and performer in *Electronic Music for Piano*.

That $\underline{\mathfrak{B}}(\mathbb{K})$ contains the two wide-reaching and important concepts, $\mathfrak{c}_{\text{perf}}$ and $\mathfrak{c}_{\text{comp}}$, must also be examined in light of the two attributes, {re:composition} and {re:performance}, and the nature of their extents. These two concepts can be formally expressed as

$$\mathfrak{c}_{\text{perf}} := \Big(\{13, 4, 9, 8, 3, 16, 11, 12, 14, 6, 15\}, \{\text{re:performance}\}\Big), \tag{1}$$

and

$$\mathfrak{c}_{\text{comp}} := \Big(\{7, 5, 10, 2, 9, 11, 14, 15\}, \{\text{re:composition}\}\Big). \tag{2}$$

The connection between the concepts found in (1) and (2) is via the intersection of a number of common objects. This connection is also highlighted by the shared subconcept $\mathfrak{c}_{\text{obs}}$, its particular attribute {observation}, and therefore the manner in which the semiotic understanding of $\{\text{observation}\}^{\downarrow}$ should be approached in light of Cage's aesthetic and the formal context from which it arises. Within the intersection between the extension of the two superconcepts

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of $\mathfrak{c}_{\text{perf}}$ and $\mathfrak{c}_{\text{comp}}$ is the following concept generated from the node labelled with the attribute {observation}:

$$\mathfrak{c}_{\text{obs}} := \Big(\{11, 15\}, \{\text{observation, re:composition, re:performance}\}\Big). \tag{3}$$

The concept $\mathfrak{c}_{\text{obs}}$ is a subconcept of both $\mathfrak{c}_{\text{comp}}$ and $\mathfrak{c}_{\text{perf}}$ and contains two objects, "11" and "15", that are similarly found within the extent of the aforementioned superconcepts. Both objects "11" and "15" are a proper subset of the larger intersecting set of objects that will be formally defined here as $\mathscr{L}$. The significance of $\mathscr{L}$ lies in its membership within the extension of both factors of the lattice: that is of {re: performance} and {re:composition}. These attributes are based on two divergent semantic areas that are both contrary, yet complementary, to each other in light of Cage's aesthetics of performance and composition:

$$\begin{aligned} \text{Let } \mathscr{L} :=& \text{ext}(\mathfrak{c}_{\text{comp}}) \cap \text{ext}(\mathfrak{c}_{\text{perf}}) \\ =& \{9, 11, 14, 15\}. \end{aligned} \tag{4}$$

From the definition of (3) we see that the condition of $\mathfrak{c}_{\text{obs}} \leq \mathfrak{c}_{\text{comp}}$ and $\mathfrak{c}_{\text{obs}} \leq \mathfrak{c}_{\text{perf}}$ is derived from the fact that the intent of $\mathfrak{c}_{\text{obs}}$ contains the attributes {re:composition} and {re:perforrmance}. This then indicates that {observation}→{re:composition} and {observation}→ {re:performance}. This is evident too in the objects of $\mathscr{L}$, which are varyingly concerned with time and observation (objects "9" and "11") as well as apparent performance directions or notations (objects "15" and "14").

Also of note is the already observed fact that $\{\text{observation}\}^{\downarrow} = \{11, 15\}$ is a subset of $\mathscr{L}$ and that the object "15" finds itself squarely between what the terms *performance* and *composition* mean in Cagean aesthetics. Recall the seemingly normally "notated" yet empty score of *4′33″* (1954), in which a performance involves the observation of non-intentional sounds. There is thus a seeming ambiguity, or at least a conflict in the meaning of $\mathfrak{c}_{\text{obs}}$, given Cage's rejection of traditional nineteenth century musical aesthetics. The question arises as to whether the objects within the extent of $\mathfrak{c}_{\text{obs}}$, as well as those in $\mathscr{L}$, are texts that are to be understood in the score as *prescriptive* statements and therefore presumedly as performance directions, or as *descriptive* statements that report on a previous performance as a model for what the performer might do in their interpretation. Here, a classic and *kōan*-like binary opposition is created by Cage regarding the exact interpretation that should be taken by the performer. There is a case then for a reading in which $g \in \text{ext}(\mathfrak{c}_{\text{obs}})$ might function simply as evocations regarding a previous realisation or interpretation of the piece, which can be traced to a concurrent work of Cage, namely his 1965 graphic work *Variations V*. Here the score is a simple construction of 37 remarks concerning an earlier multi-media performance that James Pritchett nominates as an example of musical scoring *a posteriori* (Pritchett 1993, page 144). What further anchors a non-duality or obfuscation in meaning of those important linking text objects of $\{\text{ext}(\mathfrak{c}_{\text{comp}}) \cap \text{ext}(\mathfrak{c}_{\text{perf}})\}$ found in $\mathscr{L}$ are the two left-most empty nodes in the line diagram. Labelled as α and β (see Figure 4), they represent a link between two factors: the opposing attribute features of {re:composition} and {re:performance}. The concepts generated from these empty nodes have the following definitions:

$$\mathfrak{c}_{\alpha} := \Big(\{11, 14\}, \{\text{instrument, re:performance, re:composition}\}\Big), \tag{5}$$

$$\mathfrak{c}_{\beta} := \Big(\{14, 15\}, \{\text{thematic materials, re:performance, re:composition}\}\Big). \tag{6}$$

These two concepts and their intent and extent play a pivotal role in development of the theory of *Electronic Music for Piano* as a type of *kōan*. The full notations (natural language text fragments)

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of their object extents suggest a dichotomy between what Cage has deemed *non-action* to that of *action*:

$$\mathrm{ext}(\mathfrak{c}_\alpha) := \left\{\begin{matrix}\text{Observation of imperfections in the silence in which the}\\ \text{music is played, } \textit{empty music staff with ledger lines}\end{matrix}\right\},$$

$$\mathrm{ext}(\mathfrak{c}_\beta) := \left\{\begin{matrix}\text{as though there were a drawing of the controls /}\\ \text{(volume, tone) available and – on a transparency –}\\ \text{transcription from astronomical atlas } \sout{\text{suggesting}} \text{ which}\\ \text{(were it superimposed) would give}\sout{\text{s}} \text{ suggestions for use}\\ \text{of controls, } \textit{empty music staff with ledger lines}\end{matrix}\right\}.$$

Given that $\mathfrak{c}_{\mathrm{comp}}$ and $\mathfrak{c}_{\mathrm{perf}}$ are superconcepts of $\mathfrak{c}_\alpha$ and $\mathfrak{c}_\beta$, that is, $(\mathfrak{c}_{\mathrm{comp}}, \mathfrak{c}_{\mathrm{perf}}) \geq (\mathfrak{c}_\alpha, \mathfrak{c}_\beta)$, and the empty staff notation contained in object "14" is within $\{\mathrm{ext}(\mathfrak{c}_\alpha) \cup \mathrm{ext}(\mathfrak{c}_\beta)\}$, the concepts of $\mathfrak{c}_\alpha$ and $\mathfrak{c}_\beta$ reinforce a type of binary opposition that Cage favoured, especially within his borrowed Buddhist notion of challenging binary relationships between subject and object or action and non-action. As already discussed, the latter binary pair was most famously explored (or perhaps exploited) by Cage in his earlier work *4′33″*, in which the score directs the performer(s) to sit in silence for a chance-determined amount of time. Here, the sounds and thematic materials of the work are produced through the non-action of the performer and the "actions" of the audience and the environmental context.

But the presence of these dualities within $\mathscr{L}$ only seems to accentuate the ambiguity and *kōan*-like quality in the text objects of *Electronic Music for Piano*. Unlike *4′33″*, where the performance directions are explicit and prescriptive, meaning in *Electronic Music for Piano* is continually challenged so as to confuse the function of the score. The work then outwardly appears as neither a direct document of a previous performance (as is the case of *Variations V*), nor a score of discrete quantised directions for a realisation.

As is clearly seen in the line diagram of Figure 4, the nearest upper intersection of the two unnamed nodes that generate the concepts of $\mathfrak{c}_\alpha$ and $\mathfrak{c}_\beta$ is at the unnamed node whose object is "9", that is, $\{9\}^\uparrow = \{\text{re:composition, re:performance}\}$. The object "9", {without measurement of time (no ~~observation~~ of notation)} is also located at the node of the first intersection of the extents of the two main superconcepts of $\underline{\mathfrak{B}}(\mathbb{K})$ and is therefore significant given that it contains only two attributes in its intension, whereas the other objects of $\mathscr{L}$ have four additional attributes:

$$\begin{aligned}\{9\}^\uparrow &= \{\text{re:composition, re:performance}\},\\ \mathscr{L}^\uparrow &= \left\{\begin{matrix}\text{thematic materials, electronic, instrument,}\\ \text{re:composition, re:performance, observation}\end{matrix}\right\}.\end{aligned} \tag{7}$$

The semiotic structure of text object "9" {without measurement of time (no ~~observation~~ of notation)} can thus be considered as more general regarding its attributes, of which there are two, as opposed to the attributes that describe the objects within $\mathscr{L}$, of which there are six. That the two attributes of object "9" are the main attributes of the concept lattice, {re:performance} and {re:composition}, suggests a potential complication in the musical interpretation of the contents of the object.

There is also a significance in the subconcept of $\mathfrak{c}_{\mathrm{them}}$, from the node labelled with the attribute {thematic materials}, which is formally defined as

$$\mathfrak{c}_{\mathrm{them}} := \Big(\{3, 16, 12, 14, 6, 15\}, \{\text{thematic materials, re:performance}\}\Big). \tag{8}$$

Regarding the extent of $\mathfrak{c}_{\mathrm{them}}$, it can be determined through the line diagram that there exists only one object, "14" {*empty music staff with ledger lines*}, in the intersection between $\mathrm{ext}(\mathfrak{c}_\alpha)$

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and ext($\mathfrak{c}_\beta$), which is also within the extent of $\mathfrak{c}_{\text{them}}$. Furthermore, object "14" is not an element of the extension of the concept from the attribute node {electronic}, that is, ext($\mathfrak{c}_{\text{elec}}$), and given $\mathfrak{c}_{\text{elec}} \leq \mathfrak{c}_{\text{them}}$, there exists only one object in $\mathscr{L}$ that is *also* an object in the extent of $\mathfrak{c}_{\text{elec}}$: namely, object "15."

There is a significance then in the two objects "14" and "15" and how they may assist in interpreting *Electronic Music for Piano* given the structure of the concept lattice. The object "14" sits prominently within the line diagram at the very centre, lower down in the meeting between the two superconcepts defined in (1) and (2). The nature of the object's musical contents and its attribute descriptors also present a significant conundrum as to how it should be interpreted. If "14" can be said to be located at a crucial juncture of the line diagram, the situation of "15" presents us with further complications and obfuscations.

From the attribute {electronic} and the implication {electronic}→{thematic materials, re:performance} comes the concept definition of

$$\mathfrak{c}_{\text{elec}} := \Big(\{16, 12, 6, 15\}, \{\text{electronic, thematic materials, re:performance}\}\Big). \tag{9}$$

It is important to note here the position of object "15" within the line diagram as the only intersecting object in $\mathscr{L}$ for which its intention includes the two attributes of {electronic} and {thematic materials}:

$$\exists! g \in \big\{\{\text{observation}\}^{\downarrow} \cap \{\text{electronic}\}^{\downarrow}\big\}. \tag{10}$$

The two attributes of {observation} and {electronic} then share a predictably similar semiotic duality as to their superconcept attributes of {re:performance} and {re:composition}. While the attribute of "observation" has the implications of {observation}→{re:performance, re:composition}, the latter produces {electronic}→{re:performance}. While the concept derived from the attribute of {observation} may be *both* a performance instruction or compositional process, the concept derived from the attribute {electronic} is a function of the attributes that suggest a performative process only. Further to these relations is the hinging of object "14" within the concept lattice and the way in which the entire interpretative agenda of a realisation of *Electronic Music for Piano* seems to hang on the meaning of the empty staff. As an element of both the two main branches of the line diagram, that is, from the superconcepts of (1) and (2), the meaning of object "14" presents a type of non-resolvable duality. Is the empty staff to be interpreted as non-action in the same manner as its partner element in $\mathscr{L}$, object "11" = {Observation of imperfections in the silence in which the music is played}, and similarly as a function of those attributes in the intension of $\mathfrak{c}_\alpha$, that is, {instrument, re:performance, re:composition}?

Such a framing of object "14" within the context of "non-action" suggests developing a similar approach to the one used in realising *4′33″*, in which observation or simply listening becomes the response to the notation of the empty staff. Indeed Cage's first completed manuscript version of *4′33″* consisted of only ruled manuscript lines without clefs and simple chance-derived time references for the duration of each page (Gann 2010). But when object "14" is read within the context of $\mathfrak{c}_\beta$, where the notion of "action" arises via the attributes of its superconcepts $\mathfrak{c}_{\text{them}}$ and $\mathfrak{c}_{\text{perf}}$, then along with the other element of the subset, object "15", the attributes of int($\mathfrak{c}_\beta$) = {thematic materials, re:composition, re:performance}, suggest that the empty staff notation of object "14" is to be populated in some fashion and performed (perhaps in a similar fashion to the directives of *0′00″*). The dual nature of object "14" becomes a major point for interpretation of the work – whether the composition itself is simply about the observations of things and musical processes, or whether it is a directive for the generation of sounds, or specifically about the making or performing of those sounds.

FCA is an invaluable tool then for displaying the duality of meanings within *Electronic Music for Piano* and the ambiguous nature of the combination of statements it contains through exploring formal mathematical relations. As Peter Eklund and Rudolf Wille argue, such an approach

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"underlines that appropriate mathematizations of semantic structures and their relationships are very important for understanding and handling those structures" (Eklund and Wille 2007, page 26). Indeed the obfuscating nature of the text objects of *Electronic Music for Piano*, so pervading across the work, is also reflected in its very title. Lending itself as a perfect précis of a *kōan*, the question raised is whether "Electronic Music" is the object intended for a performance with a "Piano", or "Electronic Music for Piano" is the object to be performed – and not necessarily by a pianist. This ambiguity runs right through the text objects, and FCA highlights those crucial connective text objects as a subset of the extent of the superconcepts of $\mathfrak{c}_\alpha$ and $\mathfrak{c}_\beta$. As Eklund and Wille note:

> [f]or creating *adequate mathematizations*, elaborated semantic structures on the philosophical level may function as a useful bridge to reach well abstracted mathematical structures from given concrete structures. (Eklund and Wille 2007, page 26)

Within the formal context of *Electronic Music for Piano*, such mathematisations have revealed numerous non-trivial relations. At the intersection of the two main factors of $\underline{\mathfrak{B}}(\mathbb{K})$ are those more ambiguous objects of $\mathscr{L}$. A closer look at $\mathscr{L}$ reveals one further striking subset, formally defined here as τ, whose member objects sit in the union between $\mathrm{ext}(\mathfrak{c}_\alpha)$ and $\mathrm{ext}(\mathfrak{c}_\beta)$:

$$\tau := \mathrm{ext}(\mathfrak{c}_\alpha) \cup \mathrm{ext}(\mathfrak{c}_\beta) \tag{11}$$

$$= \{11, 14, 15\}.$$

By examining more closely the subset τ we find a linking connection between objects via their intentions and what is expressed in the line diagram. The intensions of each element of τ as corresponding attribute set memberships are summarised as

$$\{11\}^\uparrow := \{\text{observation, instrument, re:performance, re:composition}\},$$

$$\{14\}^\uparrow := \{\text{thematic materials, instrument, acoustic, re:performance, re:composition}\},$$

$$\{15\}^\uparrow := \{\text{electronic, observation, thematic materials, re:performance, re:composition}\}.$$

The special case of object "14" (*empty music staff with ledger lines*) is highlighted here again. In the intent of object "14" are the attributes {thematic materials, re:performance, re:composition, instrument, acoustic}, which indicate that object "14" is an element of both $\mathfrak{c}_\alpha$ and $\mathfrak{c}_\beta$. In contrast then to the greater and more direct or obvious statements in the objects $\{8, 3, 16, 12, 6\}$ and their attributes {acoustic, thematic materials, electronic, instrument, re:performance}, here in τ, object "11" as well as object "14" anchor the concepts $\mathfrak{c}_\alpha$ and $\mathfrak{c}_\beta$, while object "15" falls within the extent of the concept $\mathfrak{c}_{\mathrm{elec}}$.

The nature of the lattice's structure highlights the differences between objects in terms of their perceived meanings (in light of their attribute intentions) regarding descriptive or prescriptive activities, action or non-action, composition or performance. Cage manages to construct text objects that are suggestive of being read for different appropriations, procedures and actions, thus guaranteeing myriad interpretative actions to arise from the interpreter(s). The longest of these objects, "15", implies the control of sound sources as well as referrals to common compositional procedures previously encountered in the composer's works concerning transparencies and star charts. Object "11" is even more problematic. Is this a reference to an action involving composition, or the production of the composition during, for, or *as* a performance? All are possible and probable; it is only those decided combinations or disregarded sections of the score that will most greatly affect the shape of the realisation. Indeed as Charles G. Zug suggests, the very nature and function of the *kōan* is to provide a multifaceted ground such that "when the student has achieved the classic answer to a particular *kōan*, the master will give him another one [answer] according to his particular needs" (Zug 1967, page 86).

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5. On reflection

If FCA has revealed that *Electronic Music for Piano*, at the formal ontological level, is a work that seeks to skew an obvious or clear meaning regarding its descriptive or prescriptive qualities, consider also that I have generalised those loaded meanings and captured temporalities of Cage's struck out words and apparent edits (see Figure 1). I contend that Cage's inclusion of struck out words within the formal objects {6, 8, 9, 15} of *Electronic Music for Piano* does not provide a definitive negation of the words or actions in question, but rather disrupts their meaning. As the main attributes of {re:performance} and {re:composition} have revealed, there are myriad ways in which to interpret the formal objects given Cage's non-standard performance practices, predilection for novel thematic materials, and unique compositional processes. For example, when considering object "8" = {Piano}, I took the word ~~Piano~~, as found in the score, as equivalent to "Piano." This raises the question of whether ~~Piano~~ refers to the absence of a piano, that is, its absence from the work, or its changed or transformed status in the work, or any number of other ways of interpreting such a graphic representation of an ostensive revision.

This reduction is perhaps part of the mechanism within the work that presents the greatest doubt for a definitive action to take place (within the sphere of production or re-production), and fulfils Yamada's assertion that "*kōan*s bring us into a land abounding in contradictions of ideas and concepts" (Yamada 2004, page 33), and what Hori describes as the problem of duality:

> [*k*]ōan after *kōan* explores the theme of non[-]duality. Hakuin's well-known *kōan*, 'Two hands clap and there is a sound, what is the sound of one hand?' is clearly about two and one. The *kōan* asks, ['']you know what duality is, now what is non-duality?['] In 'What is your original face before your mother and father were born?' the 'father and mother' alludes to duality. This is obvious to someone versed in the Chinese tradition, where so much philosophical thought is presented in the imagery of paired opposites. The phrase 'your original face' alludes to the original non[-]duality. (Hori 2003, pages 289 and 290)

Focusing then on the line diagram $\underline{\mathfrak{B}}(\mathbb{K})$ of *Electronic Music for Piano* as representing a means for applying Hori's instrumental model of *kōan* analysis has highlighted the underlying ambiguities that Cage constructs within the work. But its relationship to the earlier composition *0′00″* is an important precedent that provides a strong aesthetic context to the manner in which Cage approached both composition and performance. As Pritchett has noted, the minimal instructions of *0′00″* mean that there is no real score to speak of, nor any concept of the appropriate sound world to be produced,

> [i]nstead, there exists a totally subjective situation, in which the performer acts in a deliberate and personal fashion. The action[s] of a performer in *0′00″* are not arranged according to any elaborate plan or process, but are arrived at simply and directly. In its clear presentation of a musical world quite different from that of the 1950s, *0′00″* represents that rarity in music history, a clear line of stylistic demarcation. (Pritchett 1993, page 140)

Like *0′00″*, the score of *Electronic Music for Piano* questions the normative expectations of the role of the composer and performer by bringing into focus the duality between actions of composition and acts of performance. The four critical objects {9, 11, 14, 15} mediate and ultimately dissolve the demarcation between production and re-production: the act of composition becomes performance, and the act of performance becomes composition. But unlike the clear directive of *0′00″* in which Cage indicates that "under circumstances of maximum amplification one performs a disciplined action," *Electronic Music for Piano* is ambiguous in regard to action(s), outside of those that may simply be the act of contemplating the score's contradictions. Again, that Tudor's first performance becomes the only direct composer-led manifestation of the work only highlights the fact that Cage left no explanations of the prevailing circumstances within the single page score, nor his intentions for future performances by other people – a point he made of resolving for numerous other graphic or indeterminate works, however obscure the instructions. In the second edition of *0′00″*, Cage was insistent about the amendments to the single line

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direction, stipulating that the action to be performed by "an everyday action" rather than the performance of a musical work. A further amendment declared that other activities may or may not interrupt the performance, and that each performance of the work should not resemble a previous interpretation – a wish that would seem also relevant to *Electronic Music for Piano*.

As a foil perhaps to the *a posteriori* scoring of *Variations V*, *Electronic Music for Piano* can be perhaps readily identified in the same manner to which William Fetterman sees the ambiguous nature of the *Variations V* score:

> [t]he score is thus a practical and aesthetic document for disciplined improvisation and collaboration rather than being (as is usual) a prescriptive notation indicative of future performance. It is also not a description of the actual performance, but is evocative and formal in its use of language. (Fetterman 1996, page 130)

Many of Cage and Tudor's electro-acoustic performances throughout the 1960s involved a number of situations in which the act of performance was conceived through the channels of improvisation such that notation and composition came to co-exist within the moment of creation and the unfolding of the musical object. In an interview from 1984, Tudor discusses the nature of this simultaneity and the outgrowth of this approach into his own work:

> [u]nless there is a formal structure the performance is like performing the possibilities which are in front of you. And so what's in front of you becomes the composition. And then going on from that, you choose among the possibilities what you want to appear, and then it's the task to make those things appear. So if they don't appear at one time, they might [. . .] then you try another time. The initial choice is, how much variety you wish there to be. So you try to make it happen or you try to arrange for it to happen. That's what I do. (Fullermann 1984)

This then is perhaps the most serviceable way in which to frame a theory of *Electronic Music for Piano* in terms of Hori's instrumental approach to analysing a *kōan*. FCA has revealed those ambiguous object relations that describe and prescribe a methodology or musical process, yet simultaneously negate any explicitness as to the score's true function or the shape or structure of a realisation. In this sense the score works like both a *modus operandi* led by the superconcept $\mathfrak{c}_{\text{perf}}$ and manifesto from the superconcept $\mathfrak{c}_{\text{comp}}$ for which no musical object or outcome is singularly obvious.

But this lack of specificity of the score also ostensibly validates the seemingly myriad possibilities for musical or non-musical outcomes to arise. Like the rumination of the Zen student seeking to grasp the true meaning of a *kōan* – whose language works to counter rational explanations of the story or provide a clear manner in which to interpret the meaning – *Electronic Music for Piano* can be considered as sitting within the normative expectations of a musical framework at the same time as sitting outside of such a framework of expectation. Whether to approach then those crucial objects of $\mathscr{L}$ in direct terms as Burns and Schleiermacher have via the paradigm of Kim's realisation model, or to analyse their ontological structure through Hori's instrumental model as I have, seems not to limit those myriad possible outcomes seemingly pregnant in Cage's score. What perhaps becomes most relevant then is the importance that lies in the act of considering how the conundrum of the empty music staff of object "14" might be resolved, and whether its meaning can be (or should be) resolved, acquitted, or codified through an embracing of active or passive means.

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