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Category Theory as the Language of Consciousness

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Abstract. A form of Category theory with Grothendieck topologies is utilized to provide a preliminary mathematical formalism allowing the mathematical interpretation of the ideas of the principles of a conscious universe. Category theory is the most generalized form of mathematics and as such is shown to be the most suited tool for establishing a link between physical theory and perennial philosophies.

Keywords: Category theory, Consciousness, Functors, Noetic theory, Perennial philosophy, Sheaf theory

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1. Introduction

In a series of recent papers[19-21] Kato and Struppa have developed a mathematical formalism (essentially based on the theory of categories with Grothendieck topologies, [18] to describe and interpret a general theory of consciousness. More precisely, our efforts there were to provide a mathematical formalism which would allow us to interpret mathematically the ideas of a conscious universe, as described in the work of Nadeau and Kafatos (see [15], but also the more recent [16] and [17]).

Our efforts were, so far, at a rather primitive stage and mostly provide a language for the description of consciousness. To employ a metaphor, we felt as if we were trying to show that the theory of conics is an appropriate language to describe planetary motion, but without yet understanding Newton's fundamental laws of dynamics.

Independently and somewhat earlier, Amoroso, Draganescu, and Kafatos have developed (see [1-2], [6],[11-13]) a rigorous and coherent framework for consciousness studies, which is based on a few fundamental principles. Such principles, philosophical in nature but grounded on established physical theories, hint to the necessity of developing a mathematical tool set which will both embody and describe those principles.

This paper will show that the categorical framework proposed by Kato and Struppa is in fact

appropriate to embody the principles established by Kafatos and his collaborators. This is not surprising, since [19] was written as a consequence of several conversations between Kafatos and Struppa, and specifically of [11]. This paper will also show how the fundamental current description of physical reality find a place in this general framework, and how perennial philosophies can be interpreted in these new terms.

The plan of the paper is as follows. In section 2 we describe the fundamental principles of consciousness as they have been detailed in [12]. These are the principles which guide our model, and we will later prove that in fact our framework provides a good representation for such principles. In section 3 we provide the mathematical background for our model. Such background essentially consists of the theory of sheaves and the theory of categories. While a full description of such theories is clearly beyond the scope of this paper, we wanted to make this work self-contained, and we are therefore giving all the necessary details. Essential references for this section are Bredon [5] and Lawvere and Schanuel [25]. Section 4 is devoted to a description of the model of consciousness developed by Kato and Struppa in [19-21]. As the reader will see, the model is evolving as it is commented upon by different scientists, and therefore its current status is not necessarily the one originally described in [19]. Section 5 provides the linkage with established physical theories, and shows how our model for a conscious universe does indeed fit within currently widely accepted models for physical descriptions of the universe. Finally, the last section goes back to the issue of perennial philosophies. The links between Shaivism and modern physics have been pointed out in [14]; in this final section, we show how our new model conforms with the fundamental ideas of Shaivism, and of other Eastern philosophies.

2. Principles Of Consciousness

This section is mostly based on some principles in Kafatos [12], which have later been expanded in a work of Draganescu and Kafatos [6]. The starting point is our belief that the current science cannot completely explain not only life, mind and consciousness, but also the nature of matter and reality. We are convinced that while science has been extremely successful, a new ontological model of reality is needed to peek into the mysteries of deep and underlying reality. In particular, science has to move away from a fundamental reductionistic approach, which essentially states that the whole is composed of simpler parts yielding discreteness, and that the study of the whole can be reduced to the study of its parts. We can refer to this as to the *Simplicity principle*, which much of current science seems to espouse.

Our program, on the other hand, is based on the assumption that any model for a conscious universe (and for a theory of deep reality consistent with the most recent discoveries in physics as well as with traditional wisdom) must satisfy a few fundamental principles. The reader is referred to[1-3, 6, 11-16] for more details on these principles and their relationships with both quantum physics and more traditional points of view.

***Self-organization*:** The principle of self-organization, in the structural domain, is the result of specific formal laws that govern a domain. Gravity, for example, leads to self-organized objects like planets, starts, galaxies and cluster of galaxies. Non-linearity seems to be an essential feature for a rich range of self-organized structures. But self-organization does not restrict itself to the physical domain. Similar phenomena occur at the social level as well (as the rise and organization of Internet shows). The emergence of life in the universe is itself a process of self-organization. It is important to note that through self-organization, complex things arise and are built from simpler things. This principle, nevertheless, does not contradict the complexity principle.

Complementarity (or ***duality***): The nature of deep reality is essentially and unavoidably dual. Particle-like and wave-like behaviors are inextricably connected and must be inherent in any world description. Complementarity manifests itself in the whole/part behavior of reality, in the energetic/informational properties of deep reality (see e.g. [6] for a more detailed description of the informational nature of reality), in the wave/particle duality in the universe, in the structural/phenomenological aspects of consciousness, etc. A very interesting consequence of this complementarity principle is described in [11] and [15], through the notion of observational and theoretical horizons of knowledge. We refer to those references for more details.

Complexity: This is the opposite of the simplicity principle described before. It states that deep reality must be described in terms of complex objects which cannot be reduced to simple ones. By this we mean that while complex objects can be described in terms of simple ones, even a full description and understanding of the simple components of a complex object is not sufficient to allow a reconstruction of the nature of the complex object one started with. A different way of expressing this concept is by saying that complex objects and processes cannot be achieved by a series of simple operations. A corollary of this principle states that consciousness itself is a diffused notion whose nature cannot be explained by a reductionist approach. This principles is strongly related to the notion

of irreversibility and is the basis for the arrow of time.

Causality: The principle of casuality simply states that a certain action causes something to happen. In nature, a causal law take the form that, similarly to actions by an individual, certain processes tie causes and effects together. We can often consider the system acted upon as a closed (local) system while the disturbance causing the change is considered to be external.

Self-similarity: An analysis of both microscopic and macroscopic cosmological constants shows that the universe exhibits a surprising self-similarity. Consciousness, in particular, exhibits such a feature. The universe is a conscious entity, composed of conscious entities which, in turn, contain the entire universe itself.

Non-locality (or ***wholeness***): Non-locality, as a physical phenomenon, is currently firmly established (see e.g. [15-17]). In the framework of the standard model of quantum mechanics, a spatial non-locality produces an entanglement of elementary particles across separated regions in space. A temporal non-locality states that the path followed by a particle is not determined until a delayed experimental choice is made, and seems to indicate an entanglement of past and future. In [15], finally, a third kind of non-locality is postulated, which would imply the unified whole of space-time converging to the wholeness unity of the deep underlying reality. In this context, both spatial and temporal non-localities are now particular cases of this more global, third, kind on non-locality.

3. Mathematical Background: Sheaf Theory And Category Theory

Sheaf theory was developed mostly by Leray [9] in the forties and fifties to deal with some fundamental problems in the theory of differential equations. It then quickly spread to algebraic geometry, whose look has been fundamentally modified by the introduction of this new tool, and is now widely employed both in algebraic geometry and in algebraic analysis. The idea behind the notion of a presheaf and of a sheaf is simple enough, and was stimulated by the study of spaces of functions for which a few fundamental properties are true. The first property is that when you have two functions which coincide on sets where both are defined, then both functions are the restriction of some function defined on a larger set. The second property is that in order for a function to vanish, it is enough to analyze it locally and see whether it vanishes on a covering of an open set.

Interestingly enough (for our purposes), sheaves can be defined in two different ways which are complementary and dual to each other. For the sake of clarity we will present both of them, without showing the equivalence. For all the details on sheaf theory, we refer the reader to Bredon's treatise [5].

Let T be a topological space (we make no assumptions on this space at this point; it need not be Euclidean or even metrizable; it may or may not have order or algebraic structures). A sheaf on T is a triple (S,T,p) where S is another topological space, and $p:S \to T$ is a continuous map such that for any x in T, the fiber

$$p^{-1}(x) \tag{1}$$

is a topological abelian group, with the topology induced by S on it. Each object $p^{-1}(x)$ is said to be a "stalk" for the sheaf S. This construction shows a sheaf as a collection of localized stalks and explains the terminology "sheaf" for it. An important concept in sheaf theory (and we will soon see its role in a different construction for sheaves) is the notion of section. A section of the sheaf (S,T,p) on an open set U of T, is a continuous map $q:U \to S$, such that $p(q(x))= x$ for any x in U.

Let us point out that we have chosen to look at sheaves of abelian groups here (i.e. both stalks and sections on an open set are groups in which the operation is commutative). In actuality, one can replace groups with other algebraic structures (or even more sophisticated objects) and still have a corresponding notion of sheaves. We will come back to this issue once we have defined the notion of a category.

There is however a different way of introducing sheaves, which appears to us to be more interesting. In this different point of view, a sheaf can be seen as a complete presheaf, a representation which will show sheaves as diffused objects, rather than collection of localized stalks. A presheaf is quite simply an assignment of an abelian group $S(V)$, to every open set V in T. Namely a presheaf is a map

$$\begin{aligned} P:Top(T) &\to \text{Ab. Groups} \\ V &\to P(V). \end{aligned} \tag{2}$$

which associates to each open set V in Top (T) (by this symbol we mean the topology on T, i.e. the the set of all open sets in T) an abelian group (by Ab. Groups we mean the set of all abelian groups). In addition we require that this map satisfies a few natural conditions.

If V_1 is an open subset of V_2, then there is a group homomorphism

$$r_{12}: S(V_2) \to S(V_1) \quad (3)$$

These homomorphisms are called “restrictions", because this is what they are in the classical examples from analysis (where sheaves are defined primarily as sheaves of functions). These homomorphisms must satisfy two conditions:

If V_1 is contained in V_2 which is contained in V_3, then R_{13} (defined because V_1 is contained in V_3) is the composition of R_{23} with R_{12}.

Finally, since each open set V_1 is contained in itself, the map R_{11} is always the identity. In this description, the elements of $S(V)$ are called sections of S over V.

It may be useful, for the reader, to fix his/her attention on a couple of important examples of presheaves. If we take T to be the real line (or the real plane), then one can easily construct the presheaf of continuous functions, by associating to each open set V the Abelian group $C(V)$ of continuous functions on V. Similarly one can construct the presheaf of bounded functions by associating to each open set V the Abelian group $B(V)$ of bounded functions on V. Note that in both cases the restriction maps are in fact the usual restrictions, namely if U is contained in V the restriction map is the map that associates to a function defined on V, the same function but defined only on U.

In order for a presheaf to be a sheaf, we need a technical property known as completeness. Intuitively this corresponds to the two following properties:

(a) objects which are locally trivial in P are also globally trivial in P;

(b) objects which locally belong to P, do actually belong to P.

These properties can be expressed in technical terms as follows:

(a) Let V be an open set in T, and let V_i be an open covering of V (i.e. V is the union of the open sets V_i). If f is a section on V, such that all its restrictions to the V_i 's vanish, then f itself vanishes.

(b) Let V be an open set in T and let V_i be an open covering of V. Suppose we have sections f_i defined on each V_i, and suppose that the restriction of f_i to the intersection of V_i and V_j coincides with the restriction of f_j to the intersection of V_i and V_j, for all indices i and j. Then there exists a section f on V whose restrictions to each coincides with f_i.

With reference to our previous examples, it is clear that continuous functions satisfy both (a) and (b) and therefore the presheaf of continuous functions is also a sheaf. On the other hand, bounded functions satisfy (a) but not (b) (just think of $f(x) = x$, which is clearly locally bounded, but not globally bounded on the entire real line). Bounded functions thus are a presheaf but not a sheaf. It is also possible to construct examples of presheaves which satisfy (b) but not (a). Such structure are called consheaves in [20], where they are studied for their role in consciousness theory. The role of “local existence" versus “global existence" is particularly important in the discussion of time in modern physics. In [23] and [24], for example, the authors argue for a solution to the problem of time based on its local existence, while rejecting its possible global existence.

When a sheaf is defined through its sections, it is possible to show that it is also a sheaf in the localized sense, by defining its stalks through the algebraic process of direct limit (see \citeB for details) by setting, for every x in T,

$$p^{-1}x = \text{ind lim } S(V) \quad (4)$$

where the inductive (direct) limit is taken over the family of open sets V containing x.

Before moving on to a description of category theory, we also wish to note that should a presheaf S be not complete (i.e. S does not satisfy at least one of the two properties (a) and (b)), then we can still build a sheaf through (1), but the sections of the sheaf one thus constructs (sheafification process) does not coincide with the sections of the original presheaf.

While sheaves and presheaves are an important concept, they are just objects of an even more fundamental concept in mathematics: categories. Category theory was created as a general framework for many different concepts in mathematics back in the forties. After a period in which it was looked upon as mostly an algebraic theory of great generality, it has now proved its fundamental importance, as the fundamental tool in algebraic analysis (see e.g. [18] for an as elementary treatment as possible).

A category C is a collection of objects, and maps between objects, such that for each map one object is specified as domain and one object is specified as

codomain. In addition, for each object in C, an identity map is defined, whose domain and codomain coincide with the objects. The maps can be composed and the composition laws satisfy the identity rule (i.e. any map composed with the identity map is left unchanged) as well as the associative law.

Examples of categories abound in mathematics. For starters, the set of all abelian groups and group homomorphisms is a category, denoted by Ab. Groups. If, instead of abelian groups, we look at any other algebraic structure, we will recognize that they also give rise to categories. The set of all vector spaces and linear transformations is a category. The set of all fields and field homomorphisms is a category, etc. Maybe the simplest example in this class is the category of sets and maps among sets.

One need not restrict one's attention to the case of algebraic structures. The set of all topological spaces and continuous maps is a category, and so is the set of all open sets in a topological space; in this case the maps are the inclusion maps between pairs of nested open sets. This category is usually referred to as Top(T), where T denotes the base topological space.

Sheaf and presheaf homomorphisms can be defined, and then both sheaves and presheaves form categories. The usual notation for these categories is Sh(T) and Presh (T), where again T denotes the base topological space.

Maps between categories can be defined as well, and they are called functors. A functor is a map

$$F: A \to B, \tag{5}$$

between two categories which associate to each object in A an object in B, and to each map in A a map in B, with the proviso that if $f: A_1 \to A_2$ is a map between objects in A, then its image $F(f)$ under the functor F is a map $F(f): F(A_1) \to F(A_2)$ between the corresponding objects in B**.** Moreover, a functor maintains composition of maps, and maps identity maps to identity maps.

The simplest example of a functor is the so called forgetful functor. Given an algebraic category A (say, the category of abelian groups and group homomorphisms) and the category S of sets, the forgetful functor associates to each abelian group the set of its elements and to each group homomorphisms the corresponding set map. In other words, this functor effectively "forgets" the algebraic structure of the groups, and only looks at them as sets.

Another example of a functor is a presheaf. In fact, a presheaf associates to each object in Top(T), an object in the category of abelian groups, and to each inclusion in Top(T) (i.e. a map in Top(T)), a group homomorphism (the careful reader will note that the direction of this second map is actually the reverse of the inclusion; we say that this is a controvariant functor when this happens).

Finally, let us point out that while our original definition of a presheaf (and sheaf) was with values in the category of abelian groups, one may in fact replace that category with any other category, and still have a presheaf. For our analysis of consciousness, as a matter of fact, we will consider presheaves with values in an (a priori) infinite product of categories (one can show that products of categories can be defined and are still categories). Another interesting use of category theory occurs in connection with the study of holographic principles in quantum spacetime (see [26]).

4. A Category Theory Model For Consciousness

In this section we build on our two previous sections and we summarize the ideas expressed in [19-20], in order to show explicitly how they fit the fundamental principles described in section 2.

In our framework we have a topological space T, which we think of as a "generalized time", and conscious entities are described as presheaves (or consheaves, see [20]) from T to a product of categories over a (possibly uncountable) index set J.

A few words on T. The time which we, as conscious beings, experience, is a linear, uni-dimensional space, i.e. the real line R. This allows us to experience notions such as the "time arrow", past, present, and future, as well as birth and death. From our point of view T is a general topological space (a priori not necessarily Euclidean, nor even locally Euclidean). We only ask that there is an embedding

$$i: R \to T \tag{6}$$

so that conscious entities "live" and are "diffused" over this generalized time T. At the same time, any conscious entity can be restricted to I®) and such a restriction is what we perceive as the "life" of the conscious entity in "our" linear time. Note, in this regard, that the restriction of a (pre)sheaf to a closed subset of T (in this case the image of R) is still a (pre)sheaf on the closed subset. Note also that, in general, the original (pre)sheaf cannot be reconstructed only from its restriction. There are, in sheaf theory, notions of specialized sheaves which can be reconstructed from their restrictions. When sections on an open set can be uniquely extended to global sections, for example, we say that the sheaf is flabby. In our theory, therefore, we will have some entities which are flabby, but not all entities need be. A similar notion can be given for

reconstructions from closed sets (soft sheaves).

We also note that this model allows for multiple times, through different embeddings of R into T. This model is therefore consistent with the many-worlds interpretation of quantum mechanics; in particular, and consistently with the many-worlds interpretation (see [36]), our model does not require, neither does expect, a collapse of the wave function to generate different worlds. In our model, the several worlds exist simultaneously. One may mention here that as time is connected to the notion of change, so our generalized time must necessarily lead to a notion of generalized change.

A few words are given here on how the fundamental principles indicated in section 2 can be mapped onto our theory.

Simplicity emerges when we define a sheaf as simply a union of its fibers or stalks and, therefore, we look at them as composed of simple parts. On the other hand, presheaves also demonstrates why complexity cannot be reduced to simplicity in our model. In fact, we see how a presheaf allows for the creation of *complexity* (interpreted as non-triviality) to arise from trivial objects. This appears to be in agreement with the fact that conscious beings are composed of atoms which may not be conscious entities in itself.

Self-organization is a very clear principle which is embedded in our description of the universe. The structures we look at are self-organized at several different levels. At the lowest level we have *elements,* which come together to form, for example, *groups* or other algebraic structures. These algebraic structures come together to form *categories*, and one can then push further to create *cluster of categories*, and so on.

If indeed conscious entities are described as presheaves on T, their *complementarity* nature is naturally embedded in their very way in which presheaves and sheaves are constructed. When sheaves are looked upon from the point of view of the triple (S,T,p), they exhibit a localized nature, while when they are described through the notion of complete presheaves, they assume a diffuse character. Interestingly enough, while both descriptions are adequate ones, on each given theorem (which we may regard as an experiment) we will most likely use only one or the other representation, in accordance with the principle of complementarity.

Causality can be expressed through maps between different sheaves. The irreversibility which is in embedded in the notion of (local) causality can be expressed by maps which don't admit an inverse.

We also want to show how \it self-similarity is modeled within our sheaf theoretical model. Recall that sheaves on a topological space T, $\mathrm{Sh}(T)$, form a category. Therefore a conscious entity P is a sheaf with values in a product C of categories. If one of these categories is a categories of sheaves, we have that within that same conscious entity P, and we find again a conscious entity of the form of a sheaf on a (new) topological space X. The process repeats ad infinitum, thus providing a clear evidence for self-similarity in this model.

Finally, *Non-locality* is a typical property of an element of a sheaf (or a presheaf). Since T is our generalized time (or spacetime) we see that an element of a sheaf is globally defined on such a spacetime, but its local expressions are indeed linked by properties which are non-local. The beauty of our model is that the interpretation of T allows us to have a unified way to consider both space and time non-locality. In fact, it allows us to consider what [15] defines as non-locality of the third kind (within spacetime).

5. Category Theory and the Physical World

Recent debates [3-4] on the global non-existence of time have drawn lot of attention in the scientific community. Hartle [10] considered the equation

$$H\psi = 0 \tag{7}$$

where, ψ is the state of the universe belonging to a suitable Hilbert Space H where H is the total Hamiltonian of the universe defined on the Hilbert space H. This equation implies that there is no global time for the universe, as the state of the universe is an eigenstate for the total Hamiltonian H and, therefore, does not change. However, this does not imply the non-existence of local time either [24]. Kauffman and Smolin [22] drew two conclusions about the nature of time:

1. Time is essentially local.
2. Time is to be identified with the evolution of the system itself (which we can call the local system).

We emphasize that Kauffman and Smolin did not challenge the assumption that the absence of global time implies the absence of local time.

In this paper we have introduced a concept called “generalized time" in the global or topological sense. This is not be confused with the usual property of time i.e., local time (where there is ordering like past, present and future). In our case no ordering is considered for the set (like partial ordering). This can be understood easily if we consider the physical observables since they are closely connected to the issue of time. An operator corresponding to the physical observable must locate the information it is to measure by reference to the physical configuration of

the system. This is not necessary in the case of global or topological information about the field like the gravitational field. For example, general relativity is a local field theory, with local degrees of freedom and as we are local observers, it is necessary to construct operators that describe local measurements. So if one is not interested in global or topological information, we can define local time with the ordering properties.

Now if we consider the concept of "generalized time" in the global sense, we can think of a change say "generalized change". Again this change can not be interpreted in the spirit of Bohr's original operational interpretation of quantum mechanics. Because at the level of pure consciousness, it is not possible to construct the state of consciousness using local measurements or operational procedure. It is not at all clear how to define the information for this global system devoid of any local measurement procedure. This may be possible if one can define an operational procedure defined for the entire global system. We can have some insights from "perennial philosophies" which will be discussed in the next section.

Category theory was developed as a general framework for many fundamental concepts like sheaves, presheaves etc. in algebraic analysis. Initially sheaves, presheaves and functors thought as purely mathematical objects. Now with the advancement of quantum cosmology it is believed that these abstract mathematical objects may play a significant role in physics.

Markopoulou and Smolin [26] proposed a formulation of the holographic principle, suitable for a background independent quantum theory of cosmology. Here, a relationship between the flow of quantum information and the causal structure of a quantum spacetime is discussed. The concept of elementary screens is defined here as set of events at which the observables of a holographic cosmological theory may be measured. These screens are spacelike-2 surfaces on which relevant degrees of freedom of the theory exist. Then they define a class of quantum spacetimes consisting of causal networks of such screens. This is known as screen networks. Also, in a partially ordered set of screens, in which two screens are related, $s \leq t$, when one of the future components of s precedes one of the past components t. There can be at most one edge (covering relation) from s to t. This means that, if s is in the immediate past of t, t can see one side of s. Then one can turn a screen network into a network of elementary quantum mechanical systems. In doing so it is assumed that quantum information propagates without change between screens and undergoes non-trivial evolution only when going through a screen. This can be done by assigning a Hilbert space to every edge of the screen network, and two (unitary) evolution operators to each screen. Then we can define the *edge screen network* (*ES*) as the partially ordered set whose elements are edge-sets, sets of space-like separated edges *a,b,...* in the screen network *S*. A *quantum screen network* (*QS*) can be defined as a *Functor* from the edge screen network *ES* to the category of Hilbert spaces. Hence, *QS* is the *Functor*.

$$QS: ES \rightarrow \text{Hilb}. \tag{8}$$

such that for every edge-set a in *ES* there is a finite-dimensional Hilbert space $H(a)$ in *QS*. Based on these concepts a discrete background independent holographic theory can be formulated where it is possible to build up a model of transmission of quantum information. In fact, an area of screen is defined as a measure of its capacity as a channel of flow of quantum information. However, this framework is its infancy and needs to be further developed. This may turn out to be a special case of our framework developed here.

Let us now discuss the various foundational principles discussed in the context of consciousness as they apply to the physical world.

1. *Simplicity Principle*: The universe is remarkably simple at fundamental levels. Symmetry in all its aspects governs nature, both physical and human. A few symmetries are needed to describe the behavior of fundamental particles which are the building blocks of universe. So it may be said that the universe is remarkably simple at fundamental levels, because a few elegant symmetries are all that seem to be needed to describe fundamental particles. For a structural universe, mathematics is the language that describes it, and Quantum Mechanics and the General Theory of Relativity are elegantly simple. Complexity in the structural universe manifests at higher and higher levels due to self organization.

2. *Self Organization*: Out of the simplicity at the fundamental levels, self organization emerges. Particles combine to form nuclei, nuclei combine to form heavier nuclei, nuclei and electrons combine to form atoms, atoms combine to form molecules. Increasing levels of organization at the molecular level yield molecules which are responsible for life. The propensity of matter to move towards self-organization requires non-linear dynamics at a certain level. In the structural domain, self-organization is produced due to the specific formal laws which are governing this domain or sub-domains of it. Gravity leads to self-organized objects like planets, stars, cluster of galaxies. Self-organization is acting in structures with cellular automata, complex adaptive systems, artificial life systems, deterministic chaos processes, all these forming the newest domains of the structural science.

3. *Complementarity*: Niels Bohr proposed the principle of complementarity in quantum mechanics as a logical framework in the conscious construction of reality. Then he tried to apply it to other domains, like biology and psychology. In fact, Bohr maintained that the most fundamental complementarity is the relationship between object and subject. In Quantum Mechanics, the complementarity manifests in the wave/particle duality in the universe. The microparticles like electrons, photons, neutrons etc. behave like particles or waves (as interference phenomena)as mutually exclusive aspects. However, both the aspects can be measured simultaneously but not with infinite precision. There will be blurring or unsharpness due to the inherent indeterminacy of quantum principle. In this sense one can think of simultaneous existence of these complementary aspects as description of reality.

4. *Complexity*: Complexity generally refers to the study of large-scale systems with many interacting components. What makes these systems complex, aside from the bare constituents, is that the most interesting ones exhibit behavior on scales above the level of constituent components. For example, in a superconducting metal, where electrical resistance vanishes, it does not make sense to ask what the resistance of a given electron is. Rather, the superconducting effect arises from a large number of electrons interacting with the atoms in the crystal lattice of metal. The functioning of the human brain. or even any of its subsystems, like the visual cortex, is a property of the neurons and their circuits operating together. Thus, the functioning of complex systems often reflects cooperative behavior and the emergence of structure. Complexity as a character of physical (or natural) processes has two distinct and almost opposite meanings.

Kolmogorov made an attempt to give an algorithmic foundation to notions of randomness and probability and Shannon studied the communication channels via his notion of information. In both the cases, complexity is synonymous with disorder and the lack of structure. The more random a process is, the more complex it is. An ideal gas, with molecules bouncing around, is complex according to the notion of Kolmogorov and Shannon. Here, the sense of "complexity" implies the degrees of *Complication*. Again, complexity refers to how the structured, intricate, hierarchical a natural process is. In this sense, complexity is an indicator of how many layers of order or how many internal symmetries are embedded in a process. The human brain is complex this sense due to high degree of structure in its neural architecture. The distinction between these two meanings can be revealed by answering a simple question: Is it complex or merely complicated?

In 1970's and early 1980's there is a steady progress of complex systems arising of the studies of chaos and nonlinear dynamics. During the latter period main focus was how the randomness appeared from simple systems. During these investigations, the complementary question arises, how does order emerge in large, complicated systems? Initially we had complication arising from simplicity, then we had simplicity emerging from complication. Complex nature is the interplay of just these sorts of tensions.

5. *Causality*: This principle is encountered in all physical science, including classical and quantum physics. It is closely associated with the passage of time and the finite speed of light: An effect cannot occur before its cause. In classical physics, causality is always assumed to be related to local events. In quantum physics, however, certain correlations seemingly violate the principle of local causes (see principle 7 below).

6. *Self Similarity*: A geometrical resemblance or physical correspondence between the parts of a system and a system as a whole is often encountered in the physical world. In a holographic image, each part of the image represents information about the whole so that a significant portion of that image may be lost with only insignificant loss of the information provided by the whole. In fractal geometry, the same principle appears. We have already mentioned that Smolin et al used the concept of Category of Hilbert spaces, Functor etc. to develop a holographic theory of quantum cosmology. So the self similarity prevails in all scales in the Universe.

7. *Non-locality*: The concept of non-locality raises lot of debates after the publication of the famous paper by Einstein-Podolsky and Rosen in quantum mechanics. The two particle like photons or electrons are said to be in non-local correlation if this correlation can't be explained by any local hidden variable theory. This non-locality is a property of the quantum system. Recent experiments clearly indicate [35] that this type of non-local correlation does not depend on the distance scale. This, however, does not mean that we have only spatial or temporal correlation. It also refers to the spatio-temporal non-locality [15]. The large-scale homogeneity of the universe might have natural explanation if we consider Bell-type of correlation in the early universe [29]. For example, the cosmic microwave background radiation seems to be $3^{\circ}K$ all over the causally disconnected regions of the universe. We can perhaps explain it by considering a kind of non-local correlations between the frequencies of the background photons [30].

6. Perennial Philosophies And Category Theory

Perennial philosophies [33] concern themselves with the nature of consciousness, the individual's place in the universe and the connection between individual and universal consciousness. Here we are concentrating on some statements that are more of a philosophical rather than a predominant religious tone, although we have to emphasize that the link between philosophy and religion is, often, very tight. Our purpose is not to do a comparative analysis of different perennial philosophies as they relate to consciousness. This would be a vast task and beyond the scope of this work [14]. Rather, our purpose is to examine the ties between perennial philosophy and our proposed framework of category theory as the language of consciousness. The foundational principles are applicable to perennial philosophies and as such, they link consciousness to its formulation in terms of category theory.

We are concentrating here on statements found in, perhaps, the most ancient Indian philosophical system, Shaivism and its more recent specific form as developed in Kashmir, [7-8, 27, 31-32]. Kashmir Shaivism is a philosophical system developed by sages such as Abhivanagupta, Kshemaraja, Vasugupta and others in ancient Kashmir between the eight and thirteenth centuries A.D. It holds a three-fold monistic view of ultimate reality, which is termed *Paramashiva*, which translates to Supreme Shiva or Supreme Consciousness and it is in harmony with the broad principles, although not necessarily the details, of other major Eastern perennial systems such a Vedanta [28]. Shaivism holds that the ultimate, underlying principle is Universal Consciousness and is also the ultimate source of the mind. As we will see below, the dynamic Universal Consciousness gives rise to an increasingly complex but rhythmical, flowing universe or, more accurately, infinite universes. The creative process itself is a vibration (spanda) of Ultimate Reality and as such, Shaivism is also the doctrine of vibration \citeD. In terms of the foundational principles formulated above, the same perennial principles map to the mathematical/physical principles in the most direct way. One should not consider these principles, as somehow, forming a hierarchy or sequence. Rather, if anything, they apply simultaneously at all levels with or without any specific manifestation.

According to Shaivism [14], consciousness enjoys and is characterized by a five-fold act which constitutes the dance of Shiva Nataraj: Creation (or emanation) identified with the deity Brahma; Sustenance (or preservation) identified with the deity Vishnu; Reabsorption (or destruction) identified with the deity Shiva or Maheshwara; these first deities constitute the fundamental triad of Shaivism; there are, however, two other most important acts in Universal Consciousness: Concealment (the property of consciousness that it forgets its own Ultimate Reality, its origin from a universal level): and Revelation (or grace, through which consciousness or the individual self realizes its own Ultimate Self or remembers its origins). As we will see below, these five acts find a correspondence in category theory.

The five acts, which are ever-flowing steps in the universal creative unfoldment of consciousness, taken in pairs constitute complementary aspects of the creative process (principle of complementarity). Viewing consciousness in terms of the five-fold act, provides a view that everything in the universe is fundamentally the same, i.e., consciousness is remarkably simple (principle of simplicity). In fact, Shaivism states that Paramashiva, possessing the three-fold nature of Being, Consciousness, and Universal Bliss (Sat-Chit-Ananda), gives rise to the universe through the five-fold act. In other words, the universe at its fundamental level is remarkably simple, it is nothing other than Consciousness itself. At the deepest levels, Paramashiva integrates the most fundamental complementary relationship, that between subject and object, in fact it integrates all relationships and oppositions. As such, we see that even though it may not be possible to understand the universe intellectually, due to its vast complexity, it can, nevertheless be experienced at the deepest levels as an unifying, underlying, Consciousness of Being. By an infinite combination of these acts, infinite universes and objects arise (principle of complexity). In other words, the creative process gives rise to a vast system of complexity, which, taken in its complementary aspect of simplicity, derives from self organization (principle of self organization). The complex universe, possessing self-organizing relations between its parts, is greater than just the sum of its parts. Yet, at a fundamental level it is remarkably simple, it is Universal Consciousness itself. The five-fold act applies at all derived steps of the creative process and through the projection of consciousness into the object-subject relationship (principle of self-similarity). Self-similarity at the physical levels is nothing but a manifestation of the self-similarity of consciousness that applies at all levels. Undifferentiated consciousness in its basic nature as Sat-Chit-Ananda and its five-fold act extends “everywhere", beyond space, beyond time, with no beginning or end, except as viewed from a limited (concealed or local differentiation) point of view (principle of non-locality). The five acts can also be considered to provide a causal engagement that ultimately reduces to the fundamental complementarity between subject (Consciousness) and object (the Universe): It

appears that the creation of something causes the destruction of something else, etc. (principle of causality). The five acts are the way that self-projection gives rise to self-organization, the organized parts being created, sustained, and ultimately, destroyed, thus appearing as a series of causes.

To recapitulate, the five-fold act reveals an underlying dynamic, or ever-flowing (spanda) Universal Conscious-ness (Paramshiva), assuming complementary aspects or roles (static: Shiva, dynamic: Shakti) and projections (expansions and contractions) [8, 27, 31]. Universal Consciousness provides a self-similarity which is inexorably tied to a profound simplicity, the ultimate levels are exceedingly simple, yet from these levels an exceedingly high complexity arises in a series of self-similar projections of Universal Consciousness, giving rise to many universes and levels of creative process[14]. The five-fold act of Universal Consciousness providing Self-organization, or in Shaivite terms Self-projection, is not static, it is ever-flowing and dynamic. Ultimate Consciousness (Paramashiva) in its Being or static aspect (Shiva) is, also or simultaneously, dynamic, creative and possessing all powers (Shakti).

Kafatos and Kafatou [14] considered the application of the five-fold act to quantum processes as derived from quantum electrodynamics, the dance, say, of a proton which becomes a myriad of particle-loops (creation, sustenance and destruction), losing its identity if seen in finite spatio-temporal ranges (concealment) but, in reality, never losing its true identity as a proton (revelation). These actions naturally arise in category theory.

In our categorical framework, Consciousness (or Paramashiva) is identified with the category C of all presheaves from the fixed topological space T to the given infinite product of categories. The will of Paramashiva consists in the choice of a single presheaf P to be considered. When such a choice occurs, the will of Paramashiva creates P and at the same time, in a typical complementarity behavior, destroys all the non- P (or the entities not chosen). It is at this stage, as in Shaivism, that the distinction between object and subject is first created. Sustenance of the presheaf P is identified with its “life" over an open set U in T; correspondingly (and dually), P chooses non-living or non- U. When P is considered as an individual functor, this corresponds to concealment. P, as an individual functor, is oblivious to its real origin (concealment). But reabsorption and grace (revelation) arise when the functor is seen as an integral part of the universal category C. Note how in this interpretation each action has a dual or complementary one. Note also that each presheaf, as a functor, takes values in a product of categories. Each category represents a different state of consciousness. The choice of a specific category is to be interpreted as the action of the conscious entity, while the functor itself is knowledge.

7. Conclusions

In conclusion, in the present work we have established the link between category theory, physical theory and perennial philosophies as they all related to the issue of consciousness: Category theory as a generalized language of mathematics has been shown to be widely applicable to physical theories and as such it is not just a powerful mathematical language, it is also perhaps the foundation of physical theories. Since consciousness may be difficult, if not impossible, to formulate in terms of analytical methods, a powerful, “pre-analytical" mathematical language may be appropriate for consciousness, as it is for physics. The “glue" that allows us to perceive the connections in a most direct way is a set of foundational principles. These appear to apply at all levels, pointing perhaps to a dynamic, ever-flowing, five-fold creative process of consciousness as being of most fundamental significance for the universe. In Indian perennial philosophy schools of thought, “Sankhya" is believed to be one of the oldest schools. The meaning of the name “Sankhya" has been the subject of considerable speculations. According to one school, the very meaning of the word”Sankhya" indicates either wisdom in general or that knowledge which consists in enumerating different categories. Another school holds that not only the term signifies the enumeration of categories but also signifies discrimination or choice for matter and pure consciousness. This is very much similar to the concept of choice which we have associated with category theory.

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