On Naturalness in Logic

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Introduction

Logic and Natural sciences: how are they connected?

- Logical positivism answer : logic as a linguistic organon
  - The connexion is methodological (cf. "Logic and the methodology of sciences")
  - Does not depend on the particular "natural science" under consideration
- Logicist answer:
  - Make the question transit through mathematics:
  - Mathematics and Natural sciences: how are they connected?
  - Descriptive / Predictive power of mathematics as a discourse

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Logic and Natural sciences: how are they connected?

Each view is problematic:

- if Logic is a linguistic organon: how to avoid a dogmatic justification of the rules
- if Logic/Mathematics are descriptive: how to reduce the epistemological weakness of metaphysical realism

Note anyway that none of those alternatives give even a beginning of explanation on the *"Unreasonable effectiveness of mathematics in the natural sciences"* (E. Wigner, 1960)

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Is Logic really about language ?

Both views (Logic as an organon; Logic/mathematics as a description tool) share a same conception : logic is about language.

Hence, if logic is not about language (or is only indirectly about language), the link between logic and natural sciences (nature) can be theorized in a completely different way.

The talk's pedagogical method

- a journey in the +/- recent history of logic (from the Boole-Peirce-Frege turn to the contemporary dialogue between Logic and Computing theory)

- following the "naturalness in Logic" theme and its metamorphosis
- as we will see the words "natural" and "nature" are very polysemic ("it is natural" often means "it is convenient", the "nature" of something means "the essence of that thing", the "natural science" refers to the world as it is. . .)

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Formal logic and artificial languages

In logic, **artificiality** has been early claimed.

It is a methodological means which :

- avoids ambiguities of our "natural" (historical) languages
- simplifies them by making them more regular and uniform
- maybe leads artificial languages even going beyond the natural language

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Artificiality denounced

- *Artificiality* in Logic has been also early denounced.
- The formalism denounced as a *purely artificial* game, with no "touchstone", e.g.
  - Descartes intuitionistic view,
  - Poincaré critical view in particular against impredicative reasoning,
  - Brouwer's Irony about Peano etc.

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Naturalness claimed as an evidence for formalism

Against the risk of artificiliality, an anchorage into "Natural reasoning" has been early claimed.

- Boole's algebrization of logic, *"The laws of thought"*
- Hilbert's proof theory : *"The fundamental idea of my proof theory is none other than to describe the activity of our understanding, to make a protocol of the rules according to which our thinking actually proceeds."*

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The "Imitation" research programme

Lukasiewics's program (a methodological proposition). To ensure the reliability of formal logic : reflect the practice, mimic the way mathematicians structure their proofs.

- Jaskowski's *Suppositional logic*
- Simultaneously : Gentzen's *Natural deduction*

With which particular sense, the word *Natural* is used here ?

- *Natural* (in *Natural Deduction*) it is mainly synonymous of "fitting our practice".
- it only vaguely, implicitely refers to the deducer's membership to nature: (s)he is an cognitive animal endowed with the *natural reasoning* that the evolution gave him/her.

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Is Natural Deduction however so natural even is this weak sense?

- **External evidence against naturalness:** The "format" of proofs does not fit the "*protocol of the rules according to which our thinking actually proceeds*". Typically : Multiple conclusions Natural Deduction (during the deductive process, we move from the current conclusion to other ones).
- **Internal evidence against naturalness (pathologies):** *"In the natural calculus, the special situation of the negation is an embarrassing exception"* (Gentzen, *Second consistency proof*). This purpose echoes :
  - the particular status of the double negation rule (that Gentzen adds to reach classical logic) which is neither an introduction rule, nor an elimination rule.
  - and, overall, the obstacles encountered when trying to generalize (in presence of the double negation rule) the proof that natural deduction proofs can be transformed into analytic ones (proofs normalization).

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Toward a rectification of naturalness

- *"[. . .] our new concept of sequent [. . .] deviates slightly from what is 'natural' and its introduction is primarily justified by the great formal advantages of the representation of inference that we will present"*
  (G. Gentzen, *Second consistency proof*)
- *"The special situation of the negation [. . .] disappears as by magic in sequent calculus"* (ibid.)
- However, both systems (the natural one and the unnatural, magic one) have the same level of "reliability":
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"The major contributions to the advancement of mathematical knowledge of nature are precisely based on the method of idealizing data to simplify their study",
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Idealizing nature

The critique of naturalness reveals a deeper, better structured, idealized nature (Gentzen, *Second consistency proof*):

- The architectonic of Sequent calculus organizes rules into three groups corresponding to three separated dynamic roles (identity, logical, structural).
- Pathologies vanish : *"The special situation of the negation [. . .] disappears"*,
- Homogeneous treatment of dual connectives recovered : *"All the connectives symbols are deeply equal in this system [. . .]"*.
- Symmetries of classical logic (duality "laws") now realized not simply at the level of theorems, but *at the level of proofs* themselves: *"There is a complete symmetry between* $\wedge$ *and* $\vee$ *and between* $\forall$ *and* $\exists$

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The abandon of naturalness is not only a 'methodological' choice or an 'aesthetic' position: a new target

- Naturalness theme was a way toward the defense of reliability (gives an "external" evidence).
- The reliability is partly inherited by the new "artificial" systems (proofs are now "artificial", but they reach the same theorems than the "natural" ones),
- But, anyway, reliability is no more the priority for Gentzen (cf. his laconic interpolated clause in his panoramic *State of Researches on the foundations of mathematics*: *"It is not always necessary worrying about the safety"*, Gentzen, 1938)
- New priority : an "internal" criterion for reliability, namely consistency
- New target : analyticity of proofs (which entails consistency)
- **New focus : the process** of proofs' analytization.

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Analytization of proofs under focus

- If we take into account only the 'logical rules', then the passage from Natural Deduction to Sequent Calculus *"returns to abandon the natural succession of propositions within a proof [...], to create instead an artificial order[...], the order of the growth of the complexity of propositions."* (Gentzen, "Second proof")
- The alteration of *naturalness* of proofs aims at a better understanding of the *analytization process*: more regular, simple, homogeneous and structured.

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- The alteration of *naturalness* of proofs aims at a better understanding of the *analytization process*: more regular, simple, homogeneous and structured.

To conclude about the Naturalness theme in Gentzen

The naturalness of "Natural deduction" refers to a tentative imitation of an human practice

- In on hand:
  - it hardly induces lighting on the links between logic/mathematics and nature (but maybe through an implicit reference to the deducer as produced by evolution and belonging to the physical/biological world).
  - in particular, it induces no direct, precise explanation of the *"unreasonable effectiveness of mathematics in the natural sciences"*
- In the other hand, the abandon of the naturalness and the explicit revendication of a mathematical artificiality, put at the center of mathematical attention :
  - analytic proofs (sub-formula property)
  - the composition of proofs (the "cut" rule, is not really a rule, but a composition operation)
  - the process of proofs analytization (normalization, cut-elimination)

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The computational turn in logic : from proofs to processes

- The Brouwer-Heyting-Kolmogorov programme: proofs are algorithms
- The Proof-Programs paradigm (Curry-Howard isomorphism, +/-1969)
  - Proofs in Natural deduction are Programs in Lambda-Calculus
  - Normalization is a computational process
  - It started with natural deduction for intuitionistic minimal logic
  - Today, it is extended to (almost) all parts of classical logic, set theory included
- The Formulas-as-Type paradigm :
  - The external view : Logic selects some programs (those tamed by the the type discipline)
  - A programming language "selects" a logic : formulas **are** sets of algorithms ("types"); logical rules are operations on sets of algorithms.

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From “Logic is about language” to “Logic is about process”

- Processes come first (programs constitute a “protologic”). Logical languages are so to speak a by product

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A few questions about computational processes and programs

What is a *computational* process ?

- Can any a priori non computational process be turned to (or seen as) a computational one ?
- Typical question of "Natural computing" (use natural processes to compute)

The come back of language:

- *Computational* processes kept by means of Programming languages
- A programming language is **an axiomatic for actions**
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Naturalness revisited

For programming languages, the "naturalness" theme again splits into two meanings :

- The weak meaning (cf. the meaning of "natural" in "natural deduction"): all programming language being artificial, "natural" then means "convenient". It thus depends on the goal:
  - Ergonomically : understandable by the human programers
  - Mathematically : structuration, architectonic, compactness
- The strong meaning : Do the programs meet nature ? And then how they do ?

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Processes and Natural sciences

- The usual ambiguity of the epistemological status of axioms (description versus convention) extends to programming languages.
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How programming languages meet nature ?

- When Programming languages are descriptive axiomatics:
  - Means that the link between computation and nature is "imitation" of the structure of some informational natural processes.
  - Typically, the so called 'Natural computation" : bio-computing (intra-cellular interaction processes), DNA computing etc.
- When Programming languages are conventional axiomatics:
  - They generates a given space of computational processes
  - Conventionalism implies that some limits are required:
    - Internal limits : computational consistency (the dynamic does not collapse everything), expressivity (representation properties)
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