Jean CavaillÃ¨s and the Vienna Circle RAMIREZ, Santiago Grazer Philosophische Studien; Jan 1, 1986; 27, Periodicals Archive Online pg. 155

JEAN CAVAILLÈS AND THE VIENNA CIRCLE

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I. *Introduction*

We propose "mathematical philosophy" as a new way for the philosophy of mathematics. Its sources are manifold: stoicism, Pascal, and more recently Cavaillès and Lautman. The purpose of this paper is to show the relation between Cavaillès and Lautman's works and Wittgenstein's *Tractatus* in the common ground of their criticism of the Vienna Circle.

The main sources will be the simultaneous works of Cavaillès and Lautman, "*L'école de Vienne au Congrès de Prague*" and "*Le Congrès International de Philosophie des Sciences*", published in the *Revue de Métaphysique et de Morale* (T. XLII, no. 1, 1935 and T. XLIII, no. 1, 1936). They are both related to the "*petit Congrès*" that the Vienna Circle organised immediately before the VIII International Congrès of Philosophy.

II. *Cavaillès' Wittgenstein*

Cavaillès started out his work by explaining Wittgenstein's *Tractatus* since it "played a determinant role in the birth" of the Vienna Circle.

His view of Wittgenstein claims that there are three essential points in the *Tractatus*:

1. Language is the image of the world,
2. Purely logical propositions have no content,
3. There are no propositions about propositions.

Following Cavaillès, the first thesis amounts to the following:

1.1) The real is something given in-itself (*en-soi*) and is independent of knowledge, which can only be its description.

1.2) The universe (*réel*) can be represented by one of its parts (*discourse*). This peculiar feature makes it possible to reproduce, in an image, the essential relations of the original. That is, there are

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invariant relations throughout the process of representation: these are logical articulations.

1.3) The world is a system of actual (*effective*) relations (*Was der Fall ist*) which are contingent actualizations of possible relations. The permanent element in both (the original and the image) is the object which has, in its relation to the situations in which it is involved, the same relation that words have with the phrases in which they appear. That is, objects contain the virtual possibility of participating in the possible situations (2.014).[1]

1.4) The "space" in which these relations exist is "logical space"; "logical space" does not assume the existence of the actual world but only that the world is (5.552).

1.5) Thought is projected upon language. This yields a one-to-one correspondence between the real universe and the universe of discourse (5.6). Because of this fact, we can establish common limits for the world and our language.

1.6) It is as absurd to think about a world that would escape logic as it would be to speak of something that is not in the world or about the limits of the world.

1.7) There is a difference between the *sense* of a proposition and its truth-value. Sense lies in the logical space of possibilities while truth lies in the logical space of the actual. The truth of a proposition has to be, thus, verified experimentally: there is no other source for truth (sense would demand an actualization).

2. The second thesis implies that:

2.1) there are no logical constants.

2.2) the theory of truth-tables indicates the new role of philosophy. That is, by proving logic's "radical sterility", philosophy cannot aspire to the rank of a doctrine (4.112); its only role is to clarify propositions (*loc. cit.*). Philosophy can only aspire to oppose the emergence of pseudo-problems which stem from the fact that we do not understand the logic of our language (4.003). Its results can never be formulated as propositions.

3. Propositions of philosophy (those that would propose rules for the architecture of propositions) are not propositions: logic, being what is common to the image and the original, cannot be represented.

1. Unless otherwise stated, number in parentheses are referred to Wittgenstein's *Tractatus*.

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3.1) Propositions, rather, show the logical form of reality (4.126).

3.2) Thus, the study of syntax cannot constitute a science.

3.3) The unutterable shows itself and is of no concern of philosophy (6.54).

Cavaillès sums up his view of Wittgenstein by a reminder of Carnap's rejection of Wittgenstein's impossibility of a syntax (3.2 above). Indeed, Carnap wrote, in *Logische Syntax der Sprache* that his view differed from Wittgenstein's in two points: First in the possibility of expressing a syntax:

> In opposition to this view, our construction of syntax has shown that it can be correctly formulated and that syntactical sentences do exist.*Thus syntax is exactly formulated in the same way as geometry is.*[2]

Secondly, and after quoting 6.54 and 7, he claims that

> such an interpretation of the logic of science is certainly very unsatisfactory of science and those of metaphysics.[3]

This brief description of Wittgenstein's *Tractatus* raises several issues upon which Cavaillès worked: the problem of the absolute, the problem of the role of philosophy, the problem of sense, of ethics and the criticism of the Vienna Circle. We shall deal with some of them in this paper.

III. *Wittgenstein*

If we try to establish the difference between *sense* and *truth-value* in Wittgenstein we can forward the following:

1. The comparison stated above (1.3) is not without a purpose. Wittgenstein himself insisted in the fact that words cannot be understood outside the context of non-linguistic human activities; their meaning is not an object but its use. The statement of course, is not to be found in the *Tractatus* but, later on, in his *Philosophical Investigations* which, of course, Cavaillès did not know. But the point, in a sense, was already sketched in *Tractatus* 3.3 and 3.33

2. R. Carnap, *Logische Syntax der Sprache*, 73.

3. Carnap, R., *loc. cit.* Further on, in *Philosophy and logical syntax* (1935). Carnap again quotes propositions 4.122, 6.54 and 7 and then adds: "I cannot agree with him upon his analysis of metaphysics. In the first place, it seems to me that what he is doing is contradictory..." (I, 7).

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where he stated that the meaning of a sign has no role in logical syntax. This observation has implications for Russell's theory of types (3.331) and with 3 above.[4] In order to say something about the logical properties of language we should be able to deal with a language without those properties. Such a language is impossible (3.032). But something else is possible: its use. Or, to put it in Cavaillès' terms, mathematical acts are in search of a meaning.

In other terms, mathematics, actual mathematics show something[5] and then we talk about it; first in formalized systems where meaning is excluded and then in metamathematics.

Thus, even if formal mathematics do have a logical syntax, metamathematics cannot prove about themselves that which they try to prove about formalized mathematics.

IV. *Carnap*

The Vienna Circle took Wittgenstein as its departure point but without accepting his final restrictions, i.e. the impossibility of formulating a syntax in valid propositions and the reduction of philosophy to a system of logical clarifications.[6]

So, for Carnap (according to Cavaillès), the following points would define the task of philosophy:

1) It is always possible to formulate a syntax in terms of correct propositions. That is, for Carnap the question "Is it possible to establish syntactic rules that would regulate a formal language within the language?" has an affirmative answer.[7] For such a purpose he formulates *Language I.*

4. No proposition can say anything about itself because the propositional sign cannot be contained in itself, *Tractatus*, 3.332.

5. "We *make* mathematical arguments", J. Herbrand.

6. *Tractatus*, 4.112.

7. Parts I and II of *Logische Syntax der Sprache* are consecrated 1) to build Language I (Part I) and 2) to prove that the syntax of I can be formulated in I (Part II, 18). Language I is a definite form of language: a symbol in I is *definite* "when it is either an undefined constant or a defined one in the definition chain of which no unlimited operators occur"; an expression is *definite* "when all the constants which occur in it are definite and when all the variables are limitedly bounded" (15). He claims further that "some of the tendencies which are commonly designated as *finitist* or *constructivist* find in a certain sense their realization in our definite

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2) Using Gödel's arithmetisation procedure,[8] Carnap replaces the usual logical symbols by numbers and to every numerical sequence he assigns a product of powers of prime numbers.[9] By so doing, he can consider every proposition of pure syntax as a combination of such numbers and such a proposition will be transformed into a mathematical problem. The syntactic rules have the purpose of establishing under what conditions could one attribute to a proposition the properties it would have when taken into the deductive system under consideration, such as refutable, provable, compatible (when dealing with several propositions), etc.

3) For Carnap there is a multiplicity of languages; as many as systems of rules of a determined nature.

4) Every language can be characterized, furthermore, by its definition of "consequence" (*Folge*); that is, by the conditions under which a proposition follows from a class of propositions.

5) Every scientific language has two parts:

5.a) that which consists of descriptive signs which correspond to objects and to empirical propositions.

5.b) that which consists of logical and mathematical signs.

5.a) includes those statements which, contrary to Kant, are the only possible synthetic propositions of science. It reveals the empiricist side (and the Machian inheritance) of the Vienna Circle. They are principles "borrowed" from some part of physics.

5.b) creates statements by means of formal mathematical and logical calculations.[10]

The 'unification' of 5.a) with 5.b) is considered by Lautman as the synthesis of Mach with Russell. Both parts have, also, a certain definition of *Folge.* For 5.a), we have *P – Folgebestimmungen*, for 5.b) we have *L – Folgebestimmungen*. The main problem for 5.b) is to determine when does a particular combination of signs constitute a proposition for the science in question (i.e. the science determined by 5.a)). This is the problem of *formal determination (Formenbestim-*

Language I". (16) He also states that Wittgenstein holds an "allied" opinion with intuitionism.

8. *Cf. Logische Syntax der Sprache*, 19.

9. If we have the sequence $n_i,\ldots,n_k$ we assign to it the number $p = p_i^{n_i} \ldots p_k$ where $p_i,\ldots,p_k$ are the first k prime numbers.

10. *Cf. Philosophy and logical syntax*, 5.

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mungen) and to know under which transformations we can deduce other propositions (*Umformungsbestimmungen*).

6) Carnap introduces then the *Toleranzprinzip der Syntax* which states: "*wir wollen nicht Verbote aufstellen, sondern Festsetzungen treffen*".[12] The two parts of a syntactic structure (P and L) can vary independently of each other.[13] This allows the construction of languages which respond to any physical or logical need[14], and thus:

> *In der Logik gibt es keine Moral.* Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his method clearly, and give syntactical rules instead of philosophical arguments.[15]

7) Nevertheless, there is no simple superposition of languages. Carnap introduces the concepts of "partial" languages and of "translation" which he defines as follows:

7.a) A language S is a partial language of T if every proposition of S is a proposition of T and if every *Folge*-relation in S is conserved in T.

7.b) An L-mapping of T unto S is a correspondence between T and S that preserves L-*Folge*. A language T can be translated unto S if there is an L-mapping from T unto a partial language of S.

Thus the apparent multiplicity introduced by the "principle of tolerance" disappears if agree that all the "tolerated" languages are translatable unto a unique universal language:

11. Even if, as Carnap puts it, "*Eine Definition für den Begriff "Folge" in seinem Umfang aufzustellen, ist nicht mit einfachen Mittlen möglich; eine solche Definition ist bisher in der modernen Logik noch nicht gegeben worden...*", *Logische Syntax der Sprache*, 10.

12. *Logische Syntax der Sprache*, 17.

13. We might show a very simple example: if we take as a P-statements the proper axioms and postulates of euclidean geometry and we change them without touching the L-statements (modus ponens, excluded middle, etc.) we can obtain non-euclidean geometry. But we can also keep the P-propositions and change the L-propositions to obtain, for example, the intuitionist euclidean geometry.

14. And not only to mathematical ones, the *Toleranzprinzip* relates "to all questions of logic", *Logische Syntax der Sprache*, foreword to the english edition, p. xv.

15. *Logische Syntax der Sprache*, 17.

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> every sentence — he states in *Philosophy of logical syntax* — of any branch of scientific language can be translated into physical language without changing its content.[16]

In fact, regardless of the *Toleranzprinzip*, for Carnap there are only two possible languages: that with an expressive function ("lyrical verses, etc." and metaphysics!) and language with a "representative function". The former are uninteresting for philosophy and the latter, despite the *Toleranzprinzip* are all part of physical language which is, thus,

"an all embracing language, a universal language of science."

With these points in mind, the task of philosophy becomes clear: Philosophy is the study of scientific statements, including those that are considered as propositions belonging to the "theory of knowledge". Its role is to criticize all propositions about logical relations between scientific concepts by means of syntax. Some of these propositions will pass the test while others will be eliminated as expressions of "pseudo-problems". A condition for scientificity is thus stated: every statement must be susceptible of translation into a formalized language:

Hilbert had already stated a similar idea:

> everything that can be the object of scientific thought falls under the scope of axiomatic method *and through it, it belongs, indirectly, to mathematics.*[17]

Nevertheless, as Lautman points out, the ideas of Hilbert and Herbrand are not those of the Vienna Circle. Carnap states about Herbrand:

> Up to the present, we have differentiated between the object-language (*Objektsprache*) and the syntax-language (*Syntaxsprache*) in which the syntax of the object-language is formulated. Are these necessarily two separate languages? If the question is answered in the affirmative — as it is the case for Herbrand in connection with metamathematics — then a third language will be necessary for the formulation of the syntax language, and so on to infinity...[18]

16. *Philosophy and logical syntax*, III, 7).

17. Quoted in Cavaillès, *Methode axiomatique et formalisme*, p. 78.

18. *Logische Syntax der Sprache*, 18. It is noteworthy to quote the rest of the paragraph: "According to another opinion — Wittgenstein — there

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Carnap will hold that "it is possible to manage with one language only". Nevertheless, Carnapian syntax is not Hilbert's metamathematics since *for Hilbert (1922)*[19] metamathematics are not a part of formalized mathematics.[20]

A second difference would be the one that stems from the idea of a "universal language". For the Vienna Circle, such a language is physics. This is Neurath's thesis on physicalism:

> The consequence is that all the sciences of nature and of the spirit are nothing but partial domains of... "unified science" whose construction, by means of the establishment of links between particular sciences, and whose logical perfectionning, as well as the development of a common language, is the task of philosophy.[21]

Thus philosophy has the *negative task* of purification, i.e., the function of rejecting any proposition that cannot be transformed into an experimentally verifiable proposition. On the other hand, it has the *positive task* of building a unified science. Here, as in Wittgenstein, Neurath proposes *one* language but in a different sense. For Neurath, this unique language subsumes every other in a hierarchy which answers to the needs of the object. This is as close as the Vienna Circle gets to Comte.

Nevertheless, the above cannot produce new propositions: if we follow Wittgenstein, propositions about L-Folge are arithmetical propositions and hence tautologies. P-propositions are the laws of physics. Hence there is no "science of syntax"; the properties of propositions cannot be formulated, they are to be shown. For Wittgenstein, a proposition must always correspond to an effective situation. A valid proposition for every possible phrase must be valid, at the same time, for the totality of the universe. Yet this totality is illegitimate since it corresponds to no experience whatsoever. The Vienna Circle must abandon such a "realism": reality will impose no restrictions to symbolism; the only restrictions are those which come from symbolism itself, such as those imposed by Gödel.

exists only one language, and what we call syntax cannot be expressed at all — it can only "be shown"."

19. This will not be the case for Gödel.

20. Carnap claims that metamathematics is the "syntax of mathematical language", *Logische Syntax der Sprache*, 2.

21. Quoted by Cavaillès in *L'École de Vienne...*, p. 171.

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> We find a limit to symbolism born from symbolism itself.[22]

So the only possible role for philosophy would be

> to monitor science: to order and direct research... The only reality is the scientific edifice,... the image of the universe.[23]

Or, as Carnap himself puts it:

> Philosophy is to be replaced by the logic of science... the logic of sciences is nothing other than the logical syntax of the language of science.[24]

V. *Tarski*

Lautman and Cavaillès agree, on the other hand, with Wittgenstein's contention that logic and mathematics take care of themselves and so they rely upon the positions of the school of Warsaw and, in particular, upon that of Tarski who is burdened with the task of criticizing Carnap. According to Lautman — he quotes Tarski's conference in Prague[25] —, Tarski claims that Carnap's reasoning does not bear upon the true axioms of specific mathematical theories. Carnap's reasoning — Lautman claims to quote Tarski — is on schematic models of possible axiomatic systems which will allow to state when "a proposition can be proved", but Carnap gives no indication whatsoever as of the actual proof.

Tarski's position is not so radical. In a note to his "Concept of truth in formalized languages" he goes as far as to claim that his own definition of true sentence "still in no way provides a general criterion of truth". In the introduction to *Logic, semantics and metamathematics* (second edition), his editor writes (with Tarski's approval):

> It is one thing to know what it means to say that a sentence is true and quite another to know a criterion of truth, i.e. to have a method whereby the truth-value of a sentence may be determined.[26]

Tarski's task is to articulate a "methodology of deductive science"

22. Lautman, *op. cit.*, p. 117.
23. Cavaillès, *op. cit.*, p. 172.
24. *Logical Syntax of Language*, Foreword, p. xiii.
25. The text of the original conference is missing but it served as the base for Tarski's article "Some methodological investigations on the definability of concepts".
26. *Logic, semantics and metamathematics*, p. xx.

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as a formalizable deductive science. This would be a direct attack against Wittgenstein's claim — endorsed by Cavaillès — that any attempt to do so would lead to nonsense or to self-contradiction but Tarski also claims that his concept of truth in colloquial language[27] when applied in conjunction with the normal laws of logic, leads inevitably — as Wittgenstein showed — to confusions and contradictions.

Lautman then claims that

> the future of mathematics is in no way engaged in a syntactic research detached from the real mathematical facts. A. Tarski, following the steps of Herbrand, tries to define the metamathematical or syntactical notions in such a way that they are effectively linked with the proofs of non-contradiction, independence, etc.[28]

Furthermore, for Tarski, every definition will be purely nominal if it does not link the research of *Vollständigkeit* to an *interpretation* of the axiomatic system. Thus he attempts a formulation of a general theory of correspondance between signs and the things signified by them, i.e. a *semantic*.

Tarski points out that the "methodology of deductive sciences" (metamathematics) has already dealt with the problems of

1) axiom
2) theorem (derivable sentence)
3) rule of inference and
4) proof

which are analogous to those of

1) primitive concept
2) definable concept
3) rule of definition
4) definition

a) He then proves Theorem 1 which establishes the complete parallelism between theorem and definable concept, i.e. the problem of definability can be reduced to the problem of derivability.

b) Given two sets X and Y of sentences, Tarski defines Y as *essentially richer* than X (with respect to specific terms) if 1) every

27. "A true sentence is one which says that the state of affairs is so and so, and the state of affairs indeed is so and so", *Logic, semantics and metamathematics*, p. 155.

28. Lautman, A., *op. cit.*, pp. 118-119.

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sentence in X is a sentence in Y; 2) in the sentences of Y there are specific terms that are not contained in the sentences of X and which cannot be defined by means of terms in X alone

c) We shall say that X is *complete* if there is no Y which is essentially richer than X. Tarski shows that complete sets — against Carnap — do not exist.

d) Instead, he defines a set X of sentences as *categorical* if any two interpretations of X are isomorphic.

e) He then redefines completeness (*Vollständigkeit*) as follows:

A set of sentences X is *complete* with respect to its specific terms if it is impossible to construct a categorical set Y of sentences which is essentially richer than X with respect to its specific terms.

f) Since this definition is useless in practical terms and so he goes on to define *monotransformability.*

A set of sentences is categorical if for any two realizations of this set, there is a relation which establishes the isomorphism of both realizations. It is *monotransformable* if there is only one such relation.

g) He then proves (Theorem 4) that monotransformability implies completeness.

For Lautman, this is a great step, since

> A. Tarski... gives back to formalism its linguistic nature which serves to express a reality, he also tries to furnish this formalism with a definition of truth that guarantees the correspondance between the results of calculus and the real.[29]

VI. *Herbrand*

As to the continuous mentions to Herbrand, mainly those given by Lautman, the main point is to show that Herbrand had already stated the problem of *Formenbestimmungen* and *Unformungsbestimmungen* that Carnap posed in 1935.

This is indeed true. As a matter of fact, Herbrand proposed the study of signs by stating two types of rules:

a) rules that indicate which combinations of signs are propositions and

b) rules that indicate the ways in which we are allowed to form new combinations of signs which will become propositions.

29. Lautman, *op. cit.*, p. 120.

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He further proposes the study of the "laws of reasoning" which are, again, of two sorts:

a) Those that indicate true propositions and

b) those that allow the deduction of new propositions

which will be regarded as true, from propositions already known to be true.[32]

Nevertheless, Herbrand states two cautions: first of all, it is clear that any proposition (combination of signs) is a translation of a proposition of ordinary language (and thus, a "true" proposition must translate something) but, on the other hand he "wonders whether there are propositions that a system of signs will not be able to translate, and proofs that are not captured by these rules".[33]

To the second part of his worries ("uncaptured proofs") Gödel will answer that, indeed, there are such proofs (Gödel's second theorem. The first one has to do with "propositions" that could not be *said* in a formal language).

Now, if as Herbrand claims, this system of signs is equivalent to the totality of mathematics, there could be a set of "propositions" that would, in no way, be mathematically stated. These would consist another reality that would not only be unfoundable in mathematics but that could not even be formulated in them.

The second point of importance, as enhanced by Lautman, of Herbrand's work is that of the necessary distinction between mathematics and metamathematics. The mathematical sense is used "when we *make* a mathematical argument"; the metamathematical sense is used when "we *talk about* mathematical arguments".[34] This will be closely connected to Tarski's reintroduction of semantics..

I have spoken of re-introduction because semantics as a part of mathematics does not stem from the failure of Hilbert's program: as far back as 1900, Padoa had raised — without finding too much echo — the issue of semantics.[35]

As for Cavaillès, his references to Herbrand are more scarce and most of them are very technical. Yet, he links Herbrand to the idea of model and to the reintroduction of semantics.

32. Cf. Herbrand, *Logical writings.* It must be noted that for Herbrand, the concepts of true and provable stand for the same idea, p. 48.

33. *Loc. cit.*

34. *Op. cit.*, p. 51.

35. Cf. Ramírez, S., *L'inconnu # 5.*

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Herbrand has used a new method (based on Loewenheim) to prove non-contradiction for type I formalizations. For Loewenheim, the progressive construction of a domain constitutes the proof of non-contradiction and he proves that this is always possible. Nevertheless, Loewenheim's theorem is proved in a semi-formalized system. When trying to do the same in a "true formalism", Herbrand proves the converse: if a proposition is contradictory it cannot be satisfied in a field. This, again, resembles very much Wittgenstein's 4.462 where he claims that a contradiction allows no *Sachlage.* Furthermore, identities, according to Herbrand, are "true no matter what atomic propositions they are formed from".[36] This fills the other part of 4.462.[37]

VII. *Kant*

In 1941, Cavaillès taught at *La Sorbonne.* We have been able to find two sets of notes of his course.

The first meeting dealt with the problem of the relative and the absolute. Cavaillès stated that the purpose of science is to reach the absolute. The examination of Hegel's position takes to analyze the works of Descartes and Kant.

One year later, he wrote *Sur la logique et la théorie de la science.* There he claimed that, for Kant, the absolute was consciousness which, for Cavaillès, relates to Port-Royale:

> logic — he quotes Kant — is knowledge by itself of understanding and reason following form.[38]

After a very long consideration, Cavaillès concludes:

> In a philosophy of consciousness, logic is either transcendental or non-existing.[39]

It is at this point, very close, again, to Wittgenstein 6.13,[40] that we must recall Kant's differentiation between *transcendental* and *transcendent.*

1) *Transcendent* principles are defined, in opposition to immanent

36. Herbrand, *Logical writings*, p. 207.
37. Tautologies, said Wittgenstein, allow every *Sachlage.*
38. Cavaillès, *Sur la logique et la théorie de la science*, p. 2.
39. *Ibid.*, p. 10.
40. *Die Logik ist transcendental.*

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ones, as "those which profess to pass beyond these limits (the limits of possible experience)";[41] furthermore "there can never by any adequate empirical employment of these principles".[42] That which is *transcendent* occupies itself "with an object of experience, but only in that aspect in which it ceases to be an object of experience".[43] Whenever something "is posited in that which lies entirely outside the sensible world... the ideas become transcendent... they detach themselves completely from experience, and make for themselves objects for which experience supplies no material".[44]

2) *Transcendental* refers to an employment or to a misemployment of categories "which is merely an error of the faculty of judgement when it is not duly curbed by criticism, and therefore does not pay sufficient attention to the bounds of the territory within which alone free play is allowed to pure understanding. I mean *actual* principles which incite us to tear down all these boundary-fences and to seize possession of an entirely new domain which recognises no limits of demarcation".[45]

Transcendental assertions, thus, "lay claim to insight into what is beyond the field of all possible experiences".[46]

3) Transcendental ideas "have their own good, proper and therefore immanent use";[47] it is not the idea but its employment which can be liable of a transcendental or an immanent use.

"Human reason, according to Kant, has a natural tendency to transgress these limits".[48]

Transcendental logic is built, thus upon an expectation,[49] it has the role of "directing understanding towards a certain goal... This point is indeed a mere idea, *a focus imaginarius*..."[50]

This same idea is forwarded in the *Prolegomena*[51] where Kant puts forth the question:

41. *Critique of Pure Reason*, A 296, B 352.
42. *Ibid.*, A 308, B 365.
43. *Ibid.*, B 427.
44. *Ibid.*, A 565, B 593.
45. *Ibid.*, A 296, B 352.
46. *Ibid.*, A 425, B 453.
47. *Ibid.*, A 643, B 671.
48. *Ibid.*, A 642, B 670.
49. *Ibid.*, A 57, B 81.
50. *Ibid.*, A 644, B 672.
51. *Prolegomena*, A 165f.

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> who could be satisfied with the new experimental knowledge in all cosmological problems about the duration and magnitude of the world, of freedom or of natural necessity since, no matter how we proceed, every given answer brings up a new one which demands a new answer, thus showing the insufficiency, for the tranquility of reason of all the physical modes of explanation?... who does not see in general contingency and dependence of everything that can be thought and accepted according to the principles of experience, the impossibility of remaining in them and does not feel himself compelled, in spite of every prohibition, to get lost in transcendental ideas...?

4) Transcendental philosophy "treats only of the understanding and of reason, in a system of concepts and principles which relate to objects in general but take no account of objects that may be given (*Ontologia*)". Thus the problem of the relation of logic to ontology arises.

We could sum up by stating that there is indeed a world (*Was der Fall ist*), its limits are the limits of language (5.6) but, nevertheless there is something else: that which can be shown (6.522). This effort to go beyond the limits of language as something actual, this movement towards a "mystical" *focus imaginarius* is transcendental: it is logic, it is ethics and it is aesthetics and, inasmuch mathematics are a logical method, they, too, are transcendental.

About this subject, there is a very strange coincidence between Cavaillès and Wittgenstein: For Wittgenstein, it is well known, ethics can be either absolute or relative. For him, every proposition is a proposition about facts and it can be formulated in such a way that it looses every appearance of being an absolute value-judgement. Conversely, no statement about facts could imply an absolute value-judgement. Nevertheless, when speaking of absolute value, we remember a situation. For Wittgenstein, the best description would be conveyed when he says: "I marvel about the existence of the world".

Cavaillès, upon ethics, wrote a very early article (1928) which is not collected in any of his bibliographies: it appeared in *Cahiers de Foi et Vie*, no 3, Cahier B, 1er février 1928, under the title "Education moral et laïcité".

Morality, for Cavaillès is defined as a peculiar sort of contact with the absolute — from which morality surges — and the contingent — which morality conditions. Morality, he claims, lies "*au-dessus*" (above) the phenomenic plane. Furthermore, morality is:

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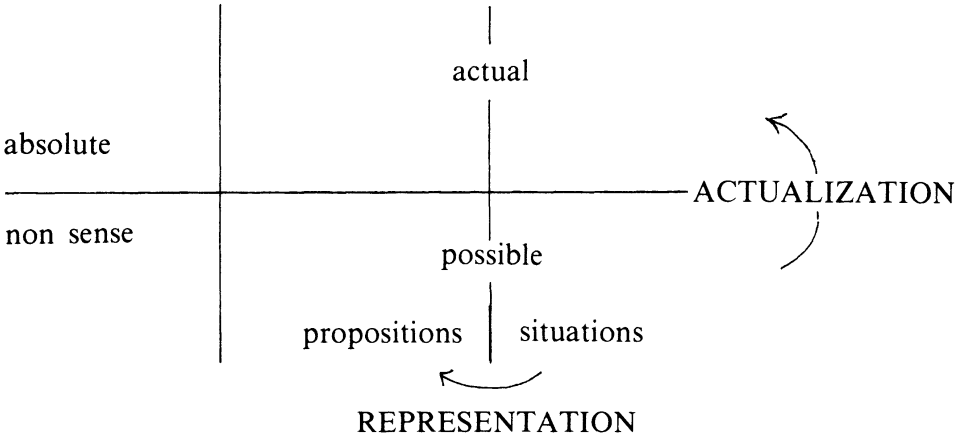
> impossible to be represented in concepts or uttered in word. It is a movement where an intuition leaves the trajectory by describing the tangent, allowing, thus, a consciousness, but an obscure consciousness or at least a flash of consciousness which is dissolved in analysis but that could be remembered when revived in a sort of game, but never by means of signs or as an intelligible fact of signs or as an intelligible fact...[52]

I shall not go further into the similarities between Wittgenstein and Cavaillès on ethics — namely on its transcendental character. The above serves only to show that both have a very similar idea about it.

VIII. *Transcendence*

Cavaillès claimed that logic, if it is, it is transcendental. Wittgenstein claimed exactly the same notion (6.13). We know, now, that such transcendentality of logic means that logic can be used immanently to establish limits or it can be used transcendentally to point towards a *transcendent* absolute.

We can "map" Cavaillès' idea of Wittgenstein's *Tractatus* as follows:



Nevertheless, it is always *possible* to state propositions *without sense.* These cannot be actualized nor they can represent any possible situation. Thus they "lead" to something which is beyond the limits of the world, i.e. to an absolute: *non-sense is the transcendental use of language.*

This would be close to Kant's transcendental use of categories, i.e.

52. *Cahiers de Foi et Vie*, no 3, 1928, p. 19.

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transcendental dialectics and it is in this sense that Cavaillès' last claim would make sense:

> The generating need is not that of an activity but that of a dialectics.[53]

On the other hand, beyond Cavaillès' knowledge, it is the sense of Wittgenstein's remarks:

> The comparison with alchemy seems natural. We might speak of a kind of alchemy in mathematics.[54]

As G. Zambrana has shown, alchemy is, indeed, a way of attaining the sacred, the supernatural, the transcendent.

Thus, it seems that there is a similarity between Cavaillès and Wittgenstein inasmuch as both claim a similarity between ethics and mathematics: neither can be referred to a meaning or to a fact; both are a transgression of the limits of experience:

> But why should not mathematics, instead of "teaching us facts" create the forms of what we call facts?[55]

By the above, mathematical concepts become transcendental ideas which are used in a transcendental way, pointing towards the unutterable absolute that lies beyond language and experience. Consequently, language and experience (and their common ground as well) cannot serve as foundations for mathematics. Whatever such foundation might be is proposed by Cavaillès and Wittgenstein. For Cavaillès, it would be:

> a central intuition impossible to describe...[56]

and for Wittgenstein,

> It might be said: what you see does of course look more like a gleaming Fata Morgana; but look at it from *another quarter* and you can see the solid body...[57]

The problem, of course, is this other quarter. It seems that this other quarter must be a point of view that allows us to *look* at mathematical construction (in the Kantian sense) as an act searching for a sense. It would be the inverse of "actualization", i.e. instead of

53. Cavaillès, *Sur la logique et la théorie de la science*, p. 78.
54. Wittgenstein, *Remarks on the foundation of mathematics*, V, 16.
55. *Ibid.*, VII, 18.
56. Cavaillès, *Philosophie mathématique*, p. 29.
57. Wittgenstein, *op. cit.*, V, 16.

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searching the truth of whatever makes sense, we should search for *the sense of what is true.* This would be, to a certain point, Husserl's *search of origins which is not the search for the foundations.*

IX. *Plato*

As for the existence of mathematical objects and the place of mathematical experience, one must assume, as all philosophy has, the duality between a "sensible world in itself" and thought: we should return to Plato and state the existence of an intelligible system of reference that would serve as a guarantee for empirical consciousness:

> There is, rather, the recognition of the impossibility of remaining within the system of objects effectively constructed (and the) statement of a complex notion of mathematical existence instead of an indication that would lead to a solution.[58]

Cavaillès, as Wittgenstein, will not allow himself to be trapped in this duality and he states that this duality does not allow actualisation but that it is the unforseenability of the synthesis which defines existence. The statement is close to Wittgenstein's "paradox":

> Such is the paradox: an experience, a fact, seems to have a supernatural value.[59]

But then, contrary to Wittgenstein, for Cavaillès, the *thematic* field is not outside the world and the application of mathematics to "reality" has no bearing upon the problems of foundation:

> The young child playing with his abacus is already a mathematician and he can only do mathematics..."[60]

Yet, in accordance with Wittgenstein, we must abandon a priori. But at the same time the limits become unclear and we are led to the confusion between existence (i.e. empirical perception) and non-contradiction. For Cavaillès, existence is the true sense of non-contradiction and not the other way around.

Thus, mathematics never quits the world: non-contradiction masks an effective procedure, in particular, that of the introduction of the infinite.

58. Cavaillès, *Méthode axiomatique et formalisme*, p. 176.
59. Wittgenstein, *Conference on ethics.*
60. Cavaillès, *Méthode axiomatique et formalisme*, p. 180.

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X. *Cavaillès and Lautman*

At this point, a nuance must be introduced: the difference between Cavaillès and Lautman.

On february 4, 1939, the French Philosophical Society organized a meeting in which both delivered papers about their recent works.

After the questions were posed, Cavaillès concluded:

> I would not be far from Lautman except for the word "real" that bothers me; one must try to distinguish if one means the sensible real — in which case I would have to disagree — or if one means the "effective" real of beings — in which case I do agree with him... Lautman drifts away... personally, I would abhor to place anything else that would dominate the effective thinking of a mathematician. I find the demands in the problems themselves. That is, maybe, what he calls "dialectics"... but to transform it into a philosophical position would seem to me impossible.[61]

To this, Lautman responds:

> Lastly I will address Cavaillès. The precise point of our disagreement is not the nature of mathematical experience but its sense and consequences. That such an experience is the *sine qua non* condition of mathematical thought goes without saying, but I think that we must find in the experience something else and more than experience, one must grasp, beyond the temporal circumstances of discovery, an ideal reality which alone can give sense and value to mathematical experience. I conceive this ideal reality as independent of any spiritual activity which appears only when the creation of effective mathematics is at stake...[62]

The above relates to Lautman's concluding remark to his paper:

> We see now which must be the task of mathematical philosophy, and even of the philosophy of sciences in general. We have to construct the *Theory of Ideas* which demands three types of research: those that lead to what Husserl called descriptive eidetics, i.e. the description of the ideal structures embodied in mathematics and whose richness is endless... (secondly) to establish, as Plato did, a hierarchy of Ideas and a theory of their genesis from one another. There is left, lastly, the task of rewriting the Timeus, i.e. to show, within ideas, the reason of their application to the sense-Universe.[63]

This position, disregarding the reference to Plato can be related to

61. *Bulletin de la Société Française de Philosophie*, t. XL, 1946, p. 36.
62. *Op. cit.*, p. 39.
63. *Op. cit.*, p. 17.

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Carnap's as stated in *Logische Syntax der Sprache*: while stating the difference between Frege and formalism, he poses the problem of the foundations of mathematics in the following terms:

Formalism, as developed by Hilbert, wants to construct mathematics and logic together in a common calculus. Frege's formalism asks for a logical foundation of mathematics "giving an account of the meaning of mathematical symbols and sentences". For Carnap,

> the formalist point of view is right in holding that the construction of the system can be carried out by a pure formalism, that is to say, without reference to the meaning of symbols.[64]

This would be Lautman's "first task".

Nevertheless, Carnap goes on: this construction is not enough. It does not contain all relevant sentences and in particular it does not contain "those sentences which are concerned with the application of mathematics".[65] Such a problem would be tackled by the "inclusion of mathematics in the total language".[66] Such an inclusion would be Lautman's second task.

Lastly mathematical structure would be needing a P-structure which would permit calculation with empirical objects and empirical magnitudes (sense-Universe): "A structure of this kind fulfils, simultaneously, the demands both of formalism and logicism".[67] It would, at the same time accomplish Lautman's third task.

Cavaillès, whose opposition to Carnap could be eventually explained in terms of one single impacting phrase of the latter, namely "*In der Logik gibt es keine Moral*", answered Lautman's program in the following terms:

> I do not seek to define mathematics, but through mathematics I try to know (*savoir*) what does it mean to know (*connaitre*) and to think; it is a modest reprisal of the problem posed by Kant. Mathematical knowledge is central to know what knowledge is.[68]

To my understanding, this implies — to put it in Lautman's terms — the rewriting of *Theetetus*. He goes on to say, in a Wittgensteinian mood:

64. Carnap, *Logische Syntax der Sprache*, 84.
65. *Loc. cit.*
66. *Loc. cit.*
67. *Loc. cit.*
68. *Bull. Soc. Fr. Phil.*, p. 34.

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what do you call real world? I am not an idealist, *je crois à ce qui est vécu*.[69]

XI. *Conclusions*

Without considering the technical details of Wittgenstein's proposal, we can now conclude the following:

1) For Carnap and in a certain sense for Lautman, philosophy of mathematics is a logic of mathematics, a syntax of the language of mathematics (for Lautman it would be a dialectics of mathematics) which would rely upon two structures: a formal L-structure developed by Hilbert, and a P-structure developed by Frege. The concept of application of mathematics yields the connection between both structures; the *Toleranzprinzip* allows for the apparition of the unexpected. By analogy, Lautman's theory of ideas would correspond to the L-structure and the sense-Universe would account for P-structure. In any case, P-structure is immanent.

2) For Wittgenstein and for Cavaillès, P-structure is *transcendent.* Thus mathematics is transcendental and is, hence, closely linked to ethics. L-structure is to be related, also, to a transcendental reason and not to an objective logic.

3) Hence we can claim first, that the difference between Carnap and Wittgenstein and between Cavaillès and Lautman has to do with the difference between immanent and transcendental points of view.

4) In either case both positions lead us to the problem of meaning: either to an empirical conception of significance as is held by Carnap, Lautman and, possibly, Engels or to a conception of mathematics that appeals to no philosophical idea of reality. This would be Tarski's contribution, the necessary continuation of this paper.

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69. *Ibid.*, p. 34.

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