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Interpreting Peirce's abduction through the lens of mathematics

Inna Semetsky

Charles Sanders Peirce classified signs in terms of universal categories of Firstness, Secondness and Thirdness: "First is the conception of being or existing independent of anything else. Second is the conception of being relative to, the conception of reaction with, something else. Third is the conception of mediation, whereby first and second are brought into relation. In psychology Feeling is First, Sense of reaction Second, General conception Third, or mediation. Chance is First, Law is Second, the tendency to take habits is Third. Mind is First, Matter is Second, Evolution is Third" (Peirce CP 6. 7). Firstness is quality, possibility, freedom. Secondness, as a relation of the First to the Second, is of opposites, physical reality, billiard-ball forces, rigid deterministic laws, direct cause-effect or action-reaction. Thirdness establishes a relation; it is synthesis, communication, memory, and mediation in general—genuine signs have a tri-relative, dynamic, structure. Peirce described his categories in numbers which are cardinal and not simply ordinal, like sequential first, second or third. Secondness, by definition, contains one and two, so there is Firstness in Secondness, and there are three in the Third. So Thirdness always already contains the Firstness in itself.

The first pre-cognitive category, called by Peirce a quali-signification—or, a qualitative immediacy of experience—may function in a form of intuition or imagination, that is as a category traditionally left outside the boundaries of formal rationalization. The immediate Firstness—a sort of pre-modern natural attraction—was, together with the Thirdness of mediation, left out as insignificant by the "pure reason" of modernity and substituted by the dualistic sin-signification and narrow understanding of reasoning as rationality based on the conventional logic of the excluded middle. For Peirce, however, logic "is a science of the necessary laws of thought, or, better still (thought always taking place by means of signs), it is a general semeiotics, treating not merely of truth, but also of the general conditions of signs being signs" (Peirce, CP 1. 444).

Peirce posited a logical category of abduction as belonging to those "operations of the mind which are logically analogous to inference excepting only that they are unconscious and therefore uncontrollable and therefore not subject to logical criticism" (Peirce, CP 5.108); still abduction exceeds direct Cartesian intuition understood as a merely dyadic relation between the knowing mind and the known object that delivers the self-evident truths. Importantly, Peirce denounced "the Cartesian maxim" (CP 5.265) and described a vague abductive inference as having the following logical form: if A is B, and C can be signified by B, then maybe A is a sign of C. Peirce's pragmatism blends logic and psychology and allows for the presensory and preconscious—not limited to sense-data—

apprehension of reality upon which, despite its being necessarily vague, people are prepared to act. The practical bearings, as Peirce called them, of some possible pre-cognitive reasoning, are not to be ignored. Fair enough, they constitute "sensations so faint" (Peirce and Jastrow 1884, quoted in Hacking, 1990: 205) so as to bypass the level of conscious awareness. It is quite difficult to account for rational conclusions based on such vague and faint first impressions. In characteristic language, Peirce and Jastrow comment that the "insight of females as well as certain 'telepathic' phenomena may be explained in this way. Such faint sensations ought to be fully studied by the psychologist and assiduously cultivated by every man" (in Hacking 1990: 206).

We approach abduction from the viewpoint of its functional manifestations. An experiential situation is problematic, that is, it involves tension and conflict, because it encounters the otherness, or Secondness of "reaction against my will" (Peirce CP 8. 144) due to the intervention, quite beyond one's awareness of this action, of the brute facts of human experiences. It is because of "the surprising fact" (Peirce CP 5.185) embedded in experience that an inquiring mind makes a first step toward apprehending the experience by abduction, a peculiar logic of discovery, which borders on a hunch or a guess. Despite being initially preconscious and necessarily vague, the abductive inference, according to Peirce, belongs to objective logic even if it is just a hypothetical conjecture: a surprising fact is observed; but if our hypothesis of this fact was to be true, then this fact would be a matter of course; therefore there is reason to suspect that our hypothesis is true. The causal influence embedded in the semiotic process of cognition becomes indirect but mediated via inclusion of the third category that breaks down the direct dyadic cause-effect connection. The interpretation is triggered by the Firstness of abduction that, tending towards a perceptual judgement, is a hypothesis-bearing statement that asserts its conclusion only conjecturally; yet, according to Peirce (CP 5. 189), there is reason to believe that the resulting judgment, under the circumstances, is true. Peirce (CP 5. 184) was adamant that there is no sharp line of demarcation between abduction and perceptual judgement: one blends into the other along the inferential process.

In the current philosophy of science discourse, abduction is usually taken in one sense only and interpreted as an inference to the best explanation; as such abduction remains the fundamental problem of contemporary epistemology (Hintikka 1998, Magnani 2001). Abduction can be open to multileveled interpretation in both logical and psychological terms. It is a type of inference grounded in the creative, artistic, logic of discovery rather that the logic of justification. Abductive reasoning is hypothetical; the generation and selection of a hypothesis leads to a conjecture different from an inductive inference. Describing the structure of abduction, Peirce noted that "the first premise is not actually thought, though it is in the mind habitually. This, of itself, would not make the inference unconscious. But it is so because it is not recognized as an inference" (Peirce CP 8.64-65). Abduction appears to function instantaneously because this tiny temporal interval of inference remains unperceived by the conscious mind. Nonetheless, the result of such a tacit, implicit cognition becomes "fully accepted" (Peirce CP 7.37) and as such "tends to obliterate all recognition of…premises from which it was derived" (CP 7.37): the tiny inferential steps themselves

stay out of consciousness; we are not aware of them. Still, despite our remaining unconscious of the duration of abduction, it is amenable to clear insight.

Kihlstrom (1993) described the now-classic experiment on subliminal perception performed by Peirce and his student Jastrow, providing many references to contemporary research in experimental psychology and cognitive science on the topic of the unconscious, which is understood as "a domain of mental structures and processes which influence experience, thought, and action outside of phenomenal awareness and voluntary control" (1993: 125) whenever one mental content simply takes over other "competing" mental content. Contemporary neuroscience and neurophilosophy (Varela 1999) recognize the existence of an imperceptible temporal gap of not more than 5-10 msec in brain activity, during which a kind of unconscious processing of information is supposed to take place. This dynamics determines "the entire readiness-for-action in the next moment" (Varela 1999: 51). A multileveled relational system of signs may very well be "instantiated in the neural organization of the brain" (Jackendoff 2001: 65), encompassing both conscious and unconscious information processing. It is reasonable to conclude that it is in the midst of tiny "fast oscillations" (Varela 1999: 50) where abduction intervenes into the process eliciting implicit ethical negotiations and a "rapid dialogue invisible to introspection" (Varela 1999: 49-50). Varela refers to Walter Freeman (1975, 1999) whose research in neuroscience postulated the existence of fast resonance binding, or coordination. These "negotiations" would not "spring from reasoning, but from an *immediate coping* with what is confronting us…[It is] the situation [that] brought forth the actions from us" (Varela 1999: 5, italics in original).

Although tending towards making a judgment, the reasoning from abduction remains "subconscious [and] does not have to make separate acts of inference" (Peirce CP 5.181) each of which supposedly would have been articulated and "spelled out" in verbal propositions. Still, "if we were to subject this subconscious process to logical analysis, we should find that it terminated in what that analysis would represent as an abductive inference" (Peirce CP 5.181). If reasoning from premises to conclusion is considered to be either deductive, or inductive, or fallacious, then an abductive guess understood reductively as an inference to the best explanation, that expresses merely some likelihood, would seem to represent a fallacious kind indeed; and it is sure enough considered as such within the analytic discourse grounded in the logic of the excluded middle. However the logic of the included middle demands the presence of abduction. An implicit preconceptual "content" (not a contradiction in terms) is amenable to clear insight, therefore potentially becoming conscious.

Peirce emphasized the role of diagrammatic reasoning and asserted that "passing from one diagram to the other, the [reasoner] will be supposed to see something that is of a general nature" (Peirce, CP 5.148), hence contributing to making one's ideas clear. The purpose of such a diagrammatic mode of expression was to "depict thought's very movement, its processual character, in terms of interconnecting lines, schemes, figures, abstract mappings. In fact [Peirce] believed that all thought is sign process and hence it is capable of being presented diagrammatically" (Merrell 1995: 51). The epistemic process, for

Peirce, means denial of the Cartesian notion of arriving at propositions that simply mirror reality. The whole notion of a proposition, whose subject designates reality and whose predicate describes the essence of the said reality, is transformed by Peirce into interpretation of reality and living it out experientially: mimesis turns into semiosis.

The adductive guess as a matter of the First borders on intuition; intuitive knowledge traditionally being a synonym for immediate knowledge. Yet, there is a paradox here because, for Peirce, there is no immediate, that is unmediated, knowledge: signs, in a process of semiotic inquiry, do mediate all cognition. Such intuition differs from direct Cartesian intuition: to *in-tuit* means to learn from within, however "the parish of percepts ...[is]...out in the open" (Peirce CP 8. 144) of the experiential world. No thought is ever instantaneous because it needs an inferential process for its own interpretation. Yet the immediacy of Firstness is always presented in an instant and, as Firstness, it is experientially *had* prior to the mediative Thirdness, making inference appear to border on association and guessing. Peirce stated that cognition exists only "in the relation of my states of mind at different instants....In short, the Immediate (and therefore in itself unsusceptible of mediation—the Unanalyzable, the Inexplicable, the Unintellectual) runs in a continuous stream through our lives; it is the sum total of consciousness, whose mediation, which is the continuity of it, is brought about by a real effective force behind consciousness" (Peirce 1955: 236-237).

Signs reiterate; they become signs of signs, or representations. As Peirce (CP 5. 138) stated, "the mode of being of a representamen [...a sign] is such that it is capable of repetition" that is, of creating sensible patterns. Every sign is subject to interpretation by a series of subsequent thought-signs, and the whole triad enveloping "the relation-of-the-sign-to-its-object becomes the object of the new sign" (Sheriff, 1994: 37). In Peircean terms, it is the Thirdness of interpretation—as mediation—that creates the meaning for a sign. The following triad illustrates the process of how the interpretant of the sign in a triadic sign-object-interpretant relation becomes a sign in a subsequent triadic relation, and so forth, as shown on Figure 1:

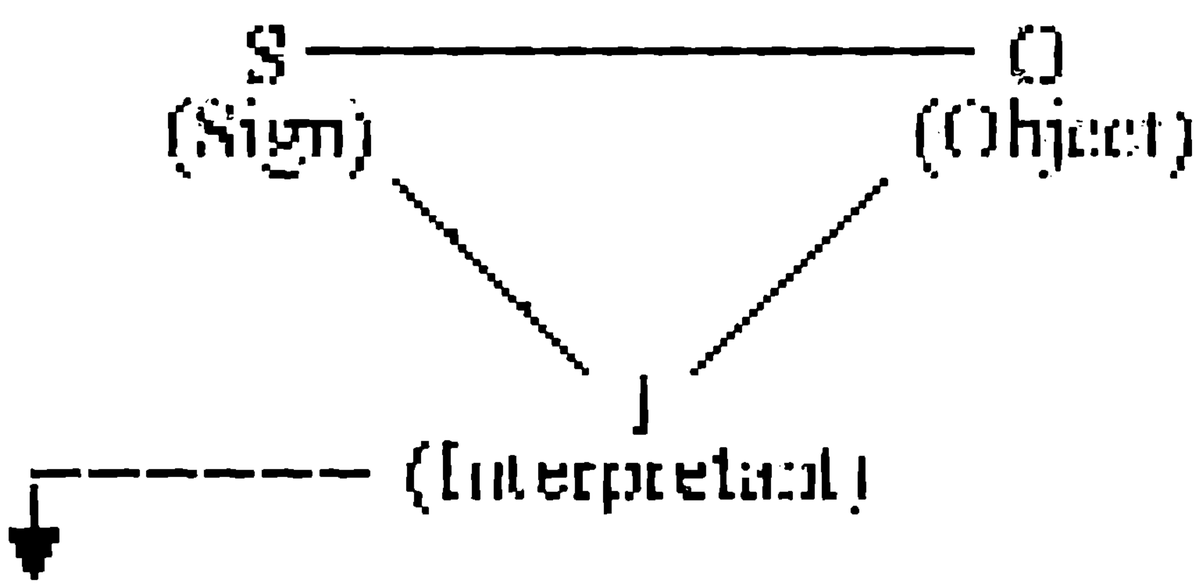


Figure 1. A triadic relation (Sheriff 1994: 35)

In ontological terms, the included middle of the interpretant delivers quite a blow to the principles of classical physics of observable world (Nicolaidis 2001) reduced to the facts of Peircean Secondness. The metaphysics of signs partakes of quantum dynamics embedded in

triadic relations, as shown in Figure 2. This dynamics defies the binary either-or logic of A versus not-A; but instead establishes a triadic relation, mediated via the included middle, T:

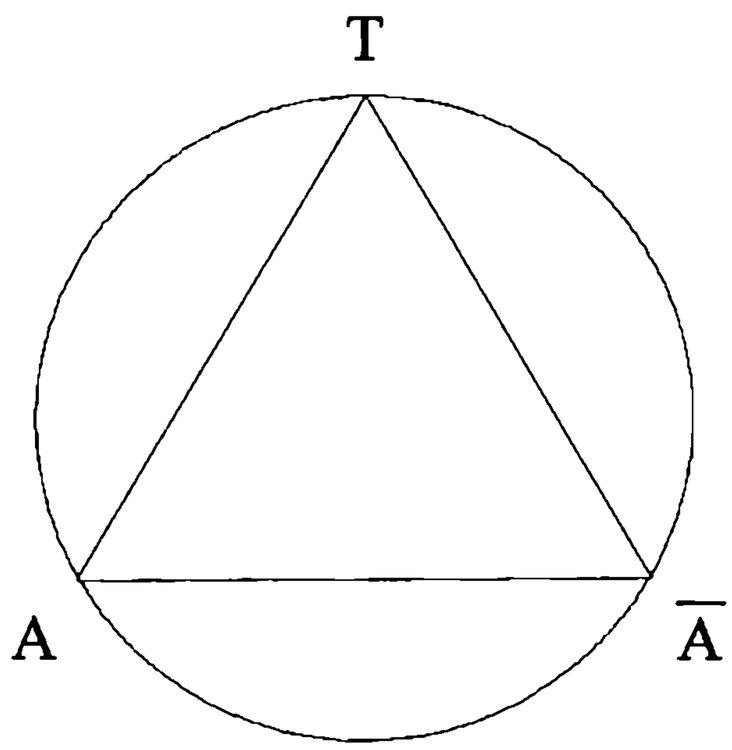


Figure 2. The triadic logic of the included middle

The third term T necessarily mediates between the dyad of A and not-A, thereby creating a semiotic bridge connecting both. A single quantum entity as a *genuine* (vs. degenerate, in Peircean terminology) sign, that is, above and over the level of Seconds, will indeed have comprised three terms in agreement with the Peircean triad (Figure 1). Sure enough, for Peirce, the whole Universe is composed of signs. As a result of multiple interrelations, signs move from one to another; they grow and engender other signs because triadic logic leads to signs always already becoming something else and something more, contributing—in the process of their growth—to human evolution and learning so that "the man-sign acquires information and comes to mean more that he did before" (Peirce 1955: 249). The value of knowledge is in its practical import, that is, the way we humans will act, think, and feel—in short, assign meaning to our own experience—as the pragmatic effect of the said knowledge. The meaning and essence of every conception depends, in a pragmatic sense, on the way the latter is applied: it "lies in the application that is to be made of it" (Peirce CP 5. 532).

Everything is a sign: the whole universe, for Peirce, is perfused with signs; yet "nothing is a sign unless it is interpreted as a sign" (Peirce CP 2. 308) by means of triadic relations leading to each successive sign becoming an interpretant for the preceding one. What seems to be a paradoxical statement is derived from the nature of the pragmatic method itself. Abduction does seem to function instantaneously, not because there is no temporal interval of inference, but because the mind is unaware of when it begins or ends. Describing the structure of perceptual abduction, Peirce noted that "the first premise is not actually thought, though it is in the mind habitually. This, of itself would not make the inference unconscious. But it is so because it is not recognized as an inference; the conclusion is accepted without our knowing how" (Peirce, CP 8. 64-65). Abduction is indeed the "'first', present,

immediate, fresh, new, initiative, original, spontaneous, free. Only, remember that every description of it must be false to it" (Peirce CP 1. 302).

Keeping in mind the paradoxical flavour of Peirce's warning, let me ask: how to explain abduction? How to construct a model for such paradoxical inference? Mathematics helps, and a kind of Peircean diagram may indeed, as the ancient Chinese said, be worth more than ten thousand words. The diagram I propose is to be located on a complex (Gaussian) plane. The two axes are marked with imaginary numbers on the vertical axis and real numbers on the horizontal axis (Figure 3).

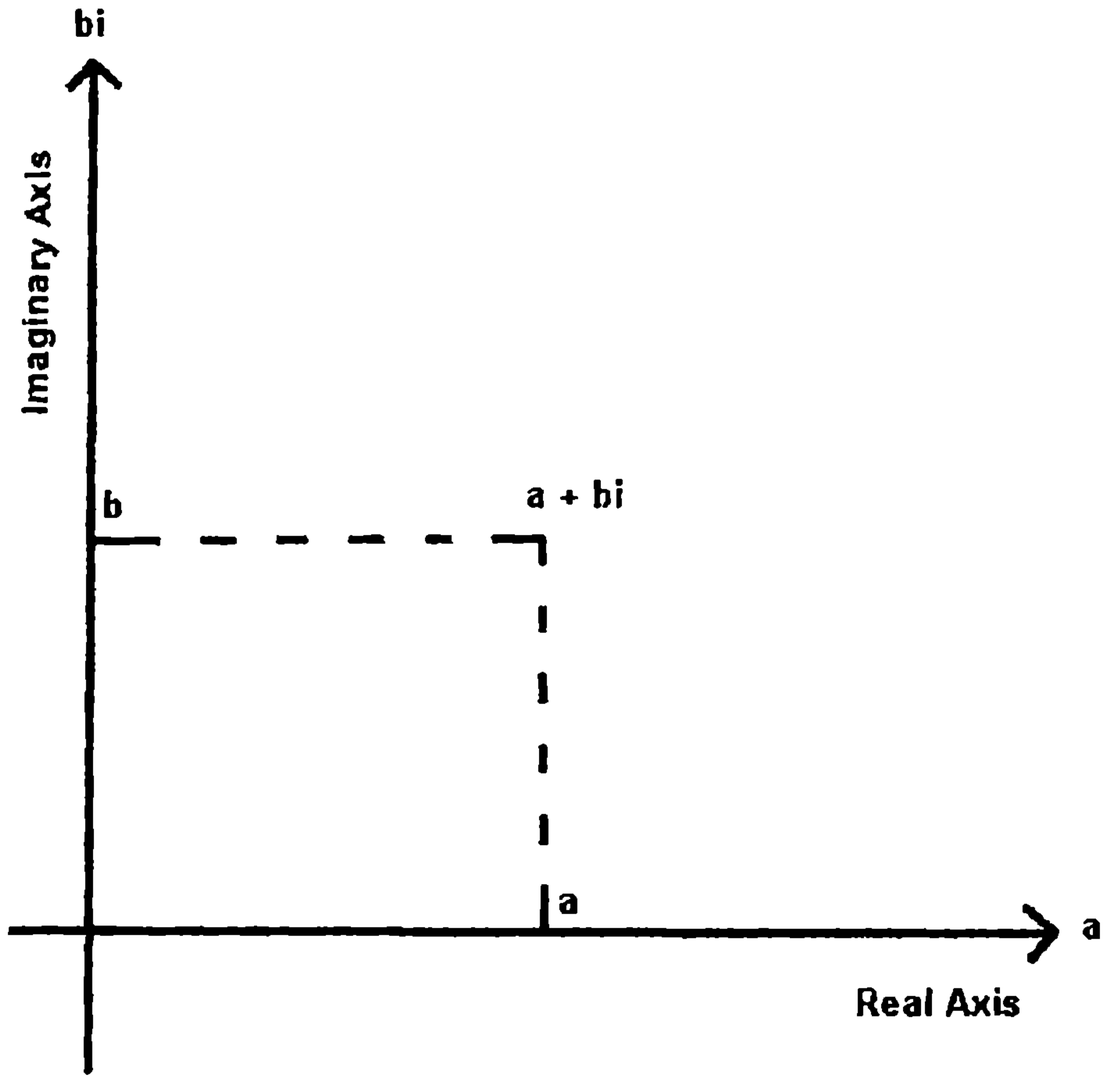


Figure 3. The complex plane

Imaginary numbers, which were dubbed magical by physicist Sir Roger Penrose (2004), do "play a fundamental role in the working of the universe" (Penrose 2004: 67). Do they perfuse the world just like Peircean signs? The imaginary number ***i*** is the square root of minus one. Descartes had a rather derogatory attitude towards imaginaries: it was he who first coined the name. There was no place for them in Newton's mechanistic philosophy either: he considered them plainly impossible. Leibniz recognized their intermediary character and positioned them at the ontological level between being and non-being. The true metaphysics of imaginary number was elusive even for Gauss. He however agreed that their geometrical representation establishes their meaning. Rational discursive reason, a, is marked along the horizontal axis by means of real numbers, in the reality of the physical world and in the form of propositional statements. But abduction's place would be on the vertical axis, it is an intuitive jump or insight, a leap in imagination so an imaginary number i seems to be a proper symbol for signifying abduction. So in this model, rational thought coexists with imagination, deduction with abduction – such complex logic being represented by means of complex numbers comprising an ordered pair a + bi on the complex plane. An

analytical representation of direction is also possible, by means of a vector, or the directed magnitude that can be considered to describe what Whitehead dubbed prehension (Whitehead 1978). The inferential processes may be represented by vectors, or arrows, which are directional forces in the physical sense. The two "add up": they converge onto a vector on a complex plane (Figure 4).

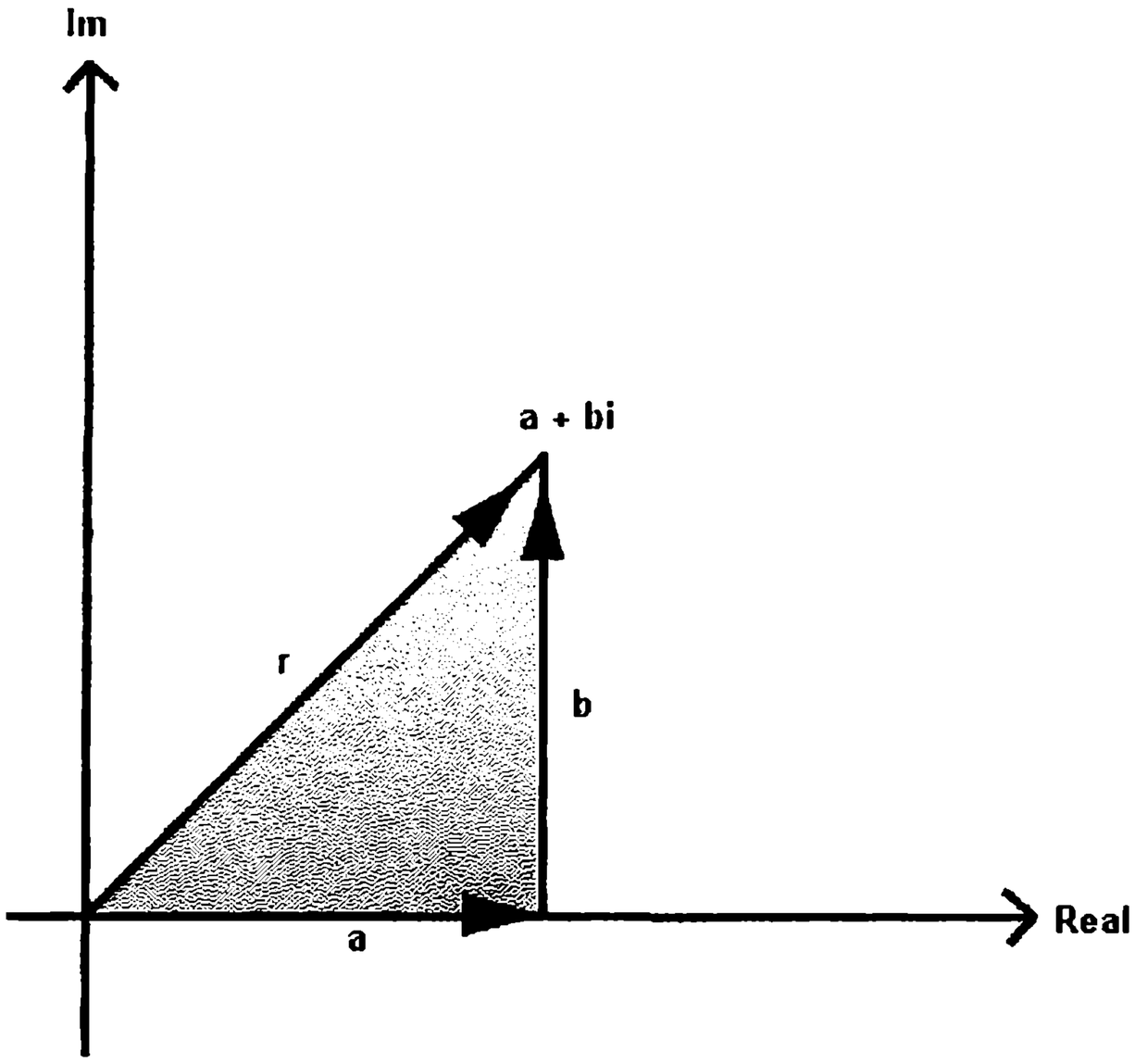


Figure 4. The resultant vector r = a + bi

A vector, by definition, has both magnitude and direction, that is, it can be described by both mathematical quality and a physical property. Together, vectors form a semiotic triangle on the complex plane. Vectors model natural entities, lines of force. In the context of philosophy of mind, Smolensky (1988, 1991) suggests a model of the connectionist network, in the framework of which the real medium of cognition is considered to be an intuitive processor, mental representations are expressed by vectors, and mental processes are described by differential equations. A vectorial diagram, therefore, represents the dynamics inherent in the structure of signs it is an organizational, relational, pattern reflecting the process-ontology of Peirce's semiosis. A higher level of complexity would have encompassed the physical (horizontal) level in itself like two nested circles, similar perhaps to the Pythagorean *tetractys* encompassing natural numbers that are inside the integers that are inside the rationals that are inside the reals, and the reals themselves being just a line among the complex numbers populating the whole plane, notwithstanding an increase in dimensions, and hence order.

The diagram is constructed on the basis of projective geometry, employing a perspectival composition, which uses the technique of parallel projectors emanating from an imaginary object and intersecting a plane of projection at right angles (co-planar) to create

images. In the context of Peirce's semiotics, René Thom (1985) presents a case of projected shadow as an example of structural isomorphism produced by interaction or coupling. It is light illuminating the original and casting the shadow as its image that itself performs the function of interaction. The formation of images is a manifestation of the universal dynamics that "allows the appearance of forms…charged with meaning" (Thom 1985: 280). In Whitehead's geometry, the very idea of the point is the point as irreducibly complex. Whitehead posited one world both without and within; this is a single world, yet multileveled or complex, which won't consist of substantial things (as for Descartes) but of relational signs and processes (as for Peirce).

The complex point symbolizes the closure of the triangle on itself, in agreement with a genuine Peircean triadic self-referential sign; yet strictly speaking this point is akin to the vanishing point on a perspectival composition, as a vertex completing what Penrose (2004) would have called an impossible triangle. It is at this point where the "physical universe ceases to be merely physical" (Deely 2001: 621)—that is, it becomes irreducible to its description in terms of classical physics—because this is where "[t]he realm of brute force and physical interaction as such…becomes caught up in the semiotic web, and the universe becomes perfused with signs" (Deely 2001: 621). Abduction, as the magnitude along the vertical axis, creates the depth in the understanding that amounts to a sign's intelligibility because of non-local "contact with some sort of Platonic world" (Penrose, Shimony, Cartwright, and Hawking 1997: 125). Epistemologically, then, abduction reaches out to the realm of Platonic ideas or Peirce's generals hence getting out of the constraints and limitations of the physical world available to sense-perception.

The length a (in our three-dimensional classical world) is just a shadow or projection of the diagonal line onto the horizontal axis, not unlike a Platonic copy as the image or shadow on the walls of the proverbial Cave. The phenomenal realm of the real numbers along the horizontal axis appears to be just a projection, a shadow cast by complex entities constituting the world of Ideas which are genuinely real in the Platonic sense, even if appearing unanalyzable, as Peirce pointed out, and virtual. The parallelogram of forces amounts to the resultant vector r as the geometrical addition of vectors and that, sure enough, represents the mediated indirect connection in accord with the rules of projective geometry when two coplanar lines intersect. The resultant vector as a vectorial sum $r = a + bi$ represents new knowledge or rather knowledge different from the preceding, infamous, prior knowledge, because abduction contributes to explicating that what was yet tacit and implicit therefore enabling a jump onto the succeeding level on a complex plane. Peirce indeed distinguished between what he called an *ampliative* and *explicative* forms of reasoning, suggesting that the former aims at increasing knowledge while the latter, by contrast, is capable of making hidden or implicit knowledge explicit, of making manifest what is perhaps latent. True, the addition vector as a whole is not the sum total of its parts, because as a resultant, it is not a sum in the arithmetical sense, it is in-between, necessarily having a partial and fractal quality but pointing nevertheless in the direction (Figure 4) determined by our collateral experience (real axis) together with the flight of abduction (imaginary axis). Abduction, which may be psychologically interpreted as representing the depth of insight, the strength of intuition, or

the power of imagination creates a magnitude along the vertical axis. The abductive suggestion "comes to us as a flash. It is an act of *insight*" (Peirce CP 5. 181) which is fallible but still has a mysterious power "of guessing right" (Peirce, CP 6. 530) even while being preconscious and not rationally controllable.

Without the Firstness of abduction, all knowledge would remain pretty sequential, because signs would stay at the level of Secondness, perhaps growing in magnitude solely because of their arithmetical progression along the horizontal line but not being able to change direction geometrically. It is merely some prior knowledge that would be amplified, but a tacit and preconscious implicit "knowledge" would lack any possibility of explication so as to enable a new knowledge—represented now as a vector on a complex plane, pointed to by the end of the arrow—to come into being, to enter cognition. It is Thirdness as mediation that enables the coming into being of new objects of knowledge as newly created concepts and meanings. A novel hypothesis might – almost literally, as we can see in Figures 3 and 4—bring a new direction into the line(s) of thinking, and the semiotic categories of Firstness and Thirdness, the two categories outside of formal logic, functioning only on the margins the latter, are capable of constructing the totally new level of knowledge at the higher level of organization. If we imagine positioning ourselves alongside the resultant line –that is the midst of genuine signs—two perspectives may emerge: "Viewing a thing from the outside, considering its relations of action and reaction with other things, it appears as matter. Viewing it from the inside, looking at its immediate character as feeling, it appears as consciousness" (Peirce CP 6.268); yet it is on the complex plane, exceeding the realm of real numbers, that matter blends with mind and psychophysical reality of signs showcases its full presence! Peirce's semiotic categories of Firstness, Secondness and Thirdness are indeed the "conceptions of complexity" (Peirce CP 1.526).

In terms of contemporary information theory, a message's expression of its meaning, its worth, or value would be defined as logical depth. The shaded area on the Figure 4 is equivalent to such logical depth (cf. Hoffmeyer 1993). Hoffmeyer labels logical depth "a semiotic freedom" (1993: 66). In Peircean terms, freedom at the level of action, that is, as the category of ethics, is Firstness that manifests in the logical form of creative abduction. Logic is not understood by Peirce as just an invention of logicians but is a *ratio* that always already exists in human *praxis*. And it is the logic of the included middle that by its very definition creates a mathematical proportion, that is, *ratio*. Ethics (action) and logic (knowledge) complement each other because it is specifically triadic semiotics, based on the logic of the included middle, which is defined as an ethics of thinking that, for Peirce, is inseparable from human conduct, that is, ethics understood as the logic of action. For Peirce there is an analogical relation between "ethical reason [and] experimental logic" (Peirce CP 5.430): the right or proportionist action reciprocates with the right reason. Meaningful knowledge is necessarily an embodied knowledge. The complex plane as a whole contains what Peirce would have called an admixture as the weighted sum of real and imaginary components, a and bi. Peirce's categories of Firstness, Secondness and Thirdness demand the admixture of mind-dependent and mind-independent relations comprising "dream and

reality, possibility and actuality" (Deely 2001: 645) that can solve the problem of intelligibility only when functioning together as a complex whole.

And the right thinking must include a faint sensation of abduction. As though envisioning contemporary arguments for emotional intelligence (Freedman 2007), Peirce emphasized the feeling-tone of abduction saying that every abductive inference involves a particular emotion: "the various sounds made by the instruments in the orchestra strike upon the ear, and the result is a peculiar musical emotion...This emotion is essentially the same thing as a hypothetic inference" (Peirce CP 2. 643). The nonlinear—triangular—quasi-causal circuit demands accessing what Peirce dubbed a quasi-mind in nature so as to ensure the sign's ultimate relation to itself as a condition for the sign's full intelligibility. Structured by sign-relations, human experience is an expression of a deeper semiotic process. Because every sign conveys the general nature of thought, and Thirdness is ultimately a mode of being of intelligence or reason, generality does come about from a quasi-mind "located", in our model, on the complex plane. Peirce referred to such quasi-mind, in quite a Platonic manner, as a repository of Ideas or significant forms. Signs are capable of exchanging information; they are this invisible information transmitting something of the thought's general nature. The logic of the included middle transforms these generals into significant meanings at the level of human experience, which cannot be reduced to either "merely a physical [or] even merely a psychical dose of energy" (Peirce in Deely 2001: 629).

Aristotle, when attempting to solve Meno's *aporia* presented in the famous Platonic dialogue, denounced Plato's doctrine because of the absurdity of "having the starting-points" (Olshewsky 2001: 141) for knowledge and instead proposed that the first truths "should be arrived at by induction from experience" (Olshewsky 2001: 142). The gulf between immediate intuition and inferential induction appears to be unbridgeable and the problem unsolvable unless we grant that abduction performs both functions: psychological immediacy and logical mediation constitute a "mediated immediacy" (Peirce CP. 5. 181). At the psychological level, abduction as an unconscious inference (yet having the necessary logical form) is intuition, which blends into the *Nous* of the intellectual knowledge. In fact, the generality of *Nous* is what motivates and informs abduction (Oatley and Johnson-Laird 1987) because when "[s]tripped of its thirdness, the sign slips back into the dyadic order of mere actual existence" (Deely 1990: 35). The triadic quality is a must: it is only through "a derivative way" which takes the form of projection (in our model) that signs do express the deeper, virtual, semiotic reality. It is abduction that insightfully grasps the moral meanings implicit in the quasi-mind, the *Nous*, as *primum cognitum.* Therefore "a *transcendental relative*" (Deely 2001: 619) is always already *immanent* in human perception. A genuine sign has to comprise all three Peircean categories so as to enable reasoning (Thirdness) in a right, ethical, way, that is, analytically (Secondness), but also abductively (Firstness). Only functioning together would they form a genuine sign representing what Peirce called synthetic consciousness and enrich experience with meaning that as such guarantees "*uberty* or richness of thought" (Deely 2001: 627).

The mathematics of analytic geometry on the complex plane paradoxically confirms Plato's insight regarding what he dubbed *mathematicals* as the necessary intermediaries

between the universal Ideas and sensible particulars. And the information about the universe, which is always already perfused with signs, must come to us in a fully Platonic manner, both as intelligible (virtual) and the sensible (actual). All objects of experience as signs contain potentialities as deep virtual meanings, even if they are not yet actualized or made explicit. As Peirce said, "All thinking is dialogic in form. Your self of one instant appeals to your deeper self for his assent" (Peirce CP 6.338). A dyadic relation alone would not lead to the creation of meanings: a sign, "to actualize its potency, must be compelled by its object" (Peirce CP 5.554) via the abductive leap from the unconscious towards being integrated into consciousness. The diagonal line casts its shadow on the horizontal axis, appearing as if from nowhere—via projection—because it exists at a level of complexity exceeding the realm of real numbers. In the case of a dyad, the logic of identity as the excluded middle indeed excludes, by its very principles, the Thirdness of mediation expressed as a resultant vector or a diagonal line in the parallelogram of forces that thereby creates a closed area as the operation of integration. The fact that the triangle in Fig. 4 closes on itself makes the total cognitive structure genuinely self-referential and self-reflective, even as we encounter the different levels of knowledge with complex numbers encompassing both the reals and imaginaries. As Peirce pointed out, "what is growth? Not mere increase" (Peirce, CP 1.174) as it would have been in plain arithmetic. An operation of simple addition would maintain the linearity of the process subscribing to the logic of the excluded middle. But a triadic relation as a prerogative of genuine signs is embedded into the nonlinear dynamics of semiosis in accord with the logic of the included middle, the included Third.

The semiotic triangle closes the Platonic gap between the sensible and the intelligible. Because the growth of reason consists "in embodiment, that is, in manifestation" (Peirce CP 1. 615), in this semiotic, communicative, and informative process (and the Latin *informare* literally means giving material form or body), the sensible world becomes intelligible while affording a degree of sensibility to the intelligible world. The reflective knowledge was characterized in the early Greek scheme as διάνοια or proper thinking (as a step away from mere opinion) which enables us to realize that sensible things are semiotic representations of those universals "residing" in the intelligible realm. Logic as semiotics demands that a premise is to be generated first, or created, which is indeed the task of the leap of abduction and without which no proper deductive reasoning from premise to conclusion would be possible because there won't be any premise in question. Plato's famous division, first presented as the *Line* in his *Republic IV* (509e-511e), was an indication of his indeed envisioning the multiplicity of levels of knowledge even if he did not have all the mathematical tools, such as the geometry on the complex plane, available to us today, to sufficiently describe them.

The complex plane always already is; it is just that we are unable to access it without the flight of abduction. As Peirce said, "In the beginning – infinitely remote – there was a chaos of unpersonalized feeling" (Peirce CP 6.33); such gut feeling as abduction is the first, virtual, "starting point" in the cosmic growth as well. Peirce does posit abduction as indispensable for growth in intelligibility; abduction borders on paradoxical "instinctive reason" (Peirce CP 6.475); still it conforms to, and is informed by, the greater "continuum of

feeling" (Peirce, CP 6.152) as living, semiotic, cosmic reality. Without abduction as Firstness and mediation as Thirdness, we appear to be doomed to confine our knowledge of the real to the realm of Platonic shadows, to illusions as appearing on the walls of the infamous Cave. Perhaps Plato himself, when calling upon giving up a sole reliance on sense-data but instead using the power of reason to examine the intelligible world, anticipated the logical necessity of what Peirce will have later called abduction.

References

Deely, J. 1999. Postmodernism and the perfusion of signs. In: E. Taborsky (ed.), *Semiosis, evolution, energy: Toward a reconceptualization of the sign*, 7-13. Aachen: Shaker Verlag.

Deely, J. 2001. *Four ages of understanding: The first postmodern survey of philosophy from ancient times to the turn of the twenty-first century*. Toronto: University of Toronto Press.

Freeman, W. 1975. *Mass action in the nervous system*. New York: Academic Press.

Freeman, W. 1999. *How brains make up their minds*. London: Weidenfield & Nicholson.

Hacking, I. 1990. *The taming of chance.* Cambridge: Cambridge University Press.

Hintikka, J. 1998. What is abduction? The fundamental problem of contemporary epistemology. *Transactions of Charles S. Peirce Society* 34: 503-533.

Hoffmeyer, J. 1993. *Signs of meaning in the universe*, trans. by Barbara J. Haveland. Bloomington: Indiana University Press.

Jackendoff, R. 2001. Language in the ecology of mind. In: P. Cobley (ed.), *The Routledge companion to semiotics and linguistics*, 52-65. London: Routledge.

Kihlstrom, J. F. 1993. The rediscovery of the unconscious. In: H. J. Morowitz and J. L. Singer (eds.), *The mind, the brain, and complex adaptive systems*, Vol. 22, 123-143. Santa Fe Institute, Studies in the Sciences of Complexity: Addison-Wesley.

Magnani, L. 2001. *Abduction, reason, and science: Processes of discovery and explanation.* Dordrecht: Kluwer Academic/Plenum Publishers.

Merrell, F. 1995. *Peirce's semiotics now: A primer.* Toronto: Canadian Scholars' Press.

Nicolaidis, A. 2001. The metaphysics of reason, available at http://diacentro.physics.auth.gr/rtalks/pdfs/nikolaidis01.doc

Oatley, K. and Johnson-Laird, P. N. 1987. Towards a cognitive theory of emotions. *Cognition and Emotions* 1: 20-50.

Olshewsky, T. 2001. Grasping the staring-points. In: K. Boudouris (ed.), *Greek philosophy and epistemology,* Volume 1. 141-155. Athens: Ionia.

Peirce, C. S. 1860-1911. *Collected papers of Charles Sanders Peirce*, C. Hawthorn and P. Weiss (eds.). Cambridge: Harvard University Press.

Peirce, C. S. 1955) *Philosophical writings of Peirce,* J. Buchler (ed.). New York: Dover Publications

Penrose, R. 2004. *The road to reality: A complete guide to the laws of the universe*. London: Jonathan Cape.

Penrose, R., Shimony, A., Cartwright, N., and Hawking, S. 1997. *The large, the small, and the human mind*. Cambridge: Cambridge University Press.

Sheriff, J. 1994. *Charles Peirce's guess at the riddle.* Bloomington: Indiana University Press.

Thom, R. 1985. From the icon to the symbol. In: R. E. Innis (ed.), *Semiotics: An introductory anthology*, 275-291. Bloomington: Indiana University Press.

Varela, F. J. 1999. *Ethical know-how: Action, wisdom and cognition*. Stanford: Stanford University Press.

Whitehead, A. N. 1978. *Process and reality: An essay in cosmology*. New York: Free Press.