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Input–output modeling of production processes for business management[1]

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Abstract

The input–ouput model is traditionally used for macroeconomic analysis. In this paper, we develop a micro-level input–output process model and demonstrate how it can be used to provide information and analytical support for making business decisions. The input–output process model provides a mathematical description of production processes and the input–output structure of a company or plant. It includes all inputs into production, and records materials flow and financial transactions among different units within a company and between the company and the outside market. We derive a set of equations to describe the basic model structure and discuss how the basic model can be extended and used by managers for structural analysis, process analysis and environmental management. © 1998 Elsevier Science B.V. All rights reserved.

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1. Historical background

Early mathematical-programming analysts (Dorfman et al., 1958; Koopmans, 1951; Manne, 1961) developed methods for firms to conduct process analyses, linear programming and game-theoretic computations. Our theoretical structure builds upon the early process-analysis work, but expands upon it by showing the explicit relationships between the input–output accounts and the processes and applying the

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IOPM to environmental management, which was not of concern to the earlier analysts. We emphasize that the IOPM is useful to managers who need to determine and trace the detailed interactions among the processes and required inputs. If they need only total profit or aggregate output information, they can make much simpler calculations than the IOPM.

Earlier, Polenske (1989) discussed how corporation (enterprise) input–output accounts can be linked to regional and national input–output accounts. Recently, Polenske (1997) showed how enterprise input–output (EIO) accounts are a necessary complement to the managerial and financial accounting systems currently used extensively by firms.[2]

Here, we update the historical material presented in those two papers first with a brief examination of some of the latest thinking on accounts in the US managerial field and then by a review of some additional information concerning EIO accounts and models.

2. Managerial accounts

Many analysts believe that the existing corporate management accounting systems, most of which were developed before 1925, are obsolete and must be revised to adapt to the changing business environment. Two of the most vocal advocates of changes in corporation management accounting systems are Kaplan and Johnson. Both have written many articles on the subject (e.g. Johnson, 1988, 1992; Kaplan, 1984, 1985, 1986), and they co-authored the prize-winning book *Relevance Lost: The Rise and Fall of Management Accounting* (Johnson and Kaplan, 1987). In the book, they maintain that the current management-accounting practices (e.g. labor, material and overhead cost accounts; cash, income and capital budgets; and various performance measures) are becoming increasingly inadequate and irrelevant in today's world of new corporate organizational forms, rapid process and product innovations, complex manufacturing processes, and global competition. Johnson (1992) argues that traditional accounting information is not useful in helping companies achieve competitive excellence. Procedures using activity-based cost-management information systems, for example, are just quick fixes that may work well for a time, but not for the long run.

Others have identified different types of problems. The managerial accounts prevalent in the 1970s and 1980s have at least six shortcomings (Dudick, 1987; Foster and Horngren, 1988; Lammert and Ehrsam, 1987; Polenske, 1989). First, they are not designed to support the operation and strategy of the firm. Second, they do not provide adequate measurements for the evaluation of division performance within the firm. Third, they do not provide complete accounting information for management decisions. Fourth, although they may be a relatively effective means of determining efficient uses of direct labor and materials, corporations now need means to assess not only the short-term direct impacts, but also the indirect impacts and long-

[2]For basic texts on managerial accounting, see Kaplan and Atkinson (1989) and on financial accounting, see Davidson et al. (1988).

term performance of the firm in the broader regional, national and global arena. Fifth, they focus managers' attention on costs rather than quality and flexibility. Sixth, the existing management accounting systems do not provide structured information for corporation planning and management decisions. Many basic planning procedures, such as sensitivity analysis, linear programming, or interdivision linkages, cannot be implemented with the data provided directly from the existing accounting system (Polenske, 1989).

3. Enterprise input–output accounts and models

An enterprise input–output (EIO) account and model can fill some of the gaps mentioned above. An EIO account shows the production processes, resources allocation and internal and external linkages of an enterprise (corporation). The account includes all inputs of production and records material flows and financial transactions among different units within a company and between the company and the outside market. Analysts in a number of countries, such as Belgium, Italy, the former Yugoslavia and the People's Republic of China (China), have developed and implemented EIO models for diverse uses within enterprises. Here, we review several of these, indicating the types of accounts and the models that they developed and implemented and the ways in which the EIO models were used.

Although we have not located corporation input–output accounts for the United States, extensive work on EIO accounts and models has been done by the Chinese and by a few analysts in the rest of the world (Filipic, 1985, 1986; LaNoce and Latorre, 1988; Li, 1991; Li and Ruikun, 1990; Limere, 1982; Pimpao, 1979; Song, 1989; Tong, 1991; Zhang et al., 1991). The Chinese were among the earliest analysts to design and implement EIO accounts and models. In 1988, our Chinese colleagues estimated that more than 50 state enterprises were employing such techniques. Today, there are undoubtedly at least several hundred enterprises using them.

In 1965, Li Bingquan and his associates from the Institute of Systems Science (ISS), Chinese Academy of Sciences, worked with the Anshan Iron and Steel Corporation (AISC) staff, in Anshan, Liaoning Province, China, to construct the first Chinese EIO set of accounts—one for 1964 in physical units. AISC now has EIO accounts for 1964, 1977, 1978, 1980, 1981, 1983, 1985 and 1987, with the tables for the latest years being compiled in both physical and value units. The staff combine data from more than 1000 types of products for each of 40 production plants. They then group these data into about 200 input–output sectors. In the recent tables, the staff have integrated physical, value, plant, and product accounts into a single table and can determine the internal and external balances for the individual plants (Zhang et al., 1991). The staff have used these accounts in various modeling exercises to help make important production decisions for the enterprise concerning inputs required, internal balances maintained, and inventories available. Because steel is one of the key enterprises within the country, the staff also use the tables to show the links between the national and enterprise economies.

Limere (1982) wrote a Ph.D. dissertation in which he analyzed the applicability of an input–output accounting system for two Belgian firms: the Bell Telephone

Manufacturing Company (BTMC) and Ford Werke. BTMC, a telecommunications company, is part of the ITT group. Limere's main objective for the 19 BTMC divisions was "to study—division-by-division—transactions in monetary terms in the company by input–output relations, and [using regression techniques] to analyze their form, stability, and reliability" (p. 107). He tested whether or not the model of the corporation was linear and the coefficients stable, using up to 60 observations for each of 16 months. Managers in the firm used these analyses to understand the interrelationships between divisions and the corresponding inventory changes. One, not surprising, finding was that those divisions that produced by order differed significantly from those that had regular production schedules in the correlation between inputs and output (p. 129).

In the second of his applications, Limere used both a physical and monetary table for Ford Werke, employing a limited sample of automobile parts for the physical model. He evaluated different transfer-pricing systems, which have a direct effect on profit allocation, and he studied integrated production planning, based on performance evaluations. Finally, he examined the fiscal implications of the company's production plan, because different tax rates in different countries should influence the optimal plan.

In the former Yugoslavia, Filipic (1985, 1986) and his colleagues designed EIO accounts for the self-managed economic and noneconomic Organizations of Associated Labor (OALs) within enterprises. They used the accounts and the model to examine the structure of the OALs and the interdependence between the different production systems and prices of the OALs. At the 1986 International Input–Output Association meeting in Sapporo, Japan, Filipic (1986) described the uses of the EIO accounts and the model by the Split shipbuilding industry, examining the OAL structure of purchases and sales within the enterprise. One of the prime uses was to plan and optimize the output and employment of the enterprise. According to Filipic (1985), the EIO was especially useful in making the long-term and 5-year employment plans, taking into account productivity changes. The EIO account for an individual enterprise can be expanded, he says, to cover the basic OALs for the entire national economy.

In the mid-1980s, a research group led by LaNoce and Latorre (1988) at the large Italian conglomerate IRI developed an extensive interconnected system, consisting of a data bank (i) for the company, (ii) for a 1985 IRI input–output table, (iii) for annual updates of the table and (iv) for the national market structures for activities relevant to the company. They disaggregated the company data by plant location and by primary and secondary products. In addition, they categorized each good or service used in the production of an output according to its nature, level of processing, production technique employed and product code. In 1985, they had 151 different units of homogenous production ranging from ferrous-metal foundries to air-based and land-based vehicles, food production and to provision of telecommunication and transport services and radio and television broadcasts. Of the total sales, 21% were in telecommunication services, representing 90% of the Italian total. This sector also employed 22% of the employees and absorbed 57% of the total value of investment. For the product categories in which

the IRI Group had production, the sales represented 27% of the national total and the investment was 21%.

Because of the large size and scope of IRI's production, plants are frequently transferred, broken up, or taken over by companies within and outside the IRI Group. The research team therefore assembled data at the plant level, where a plant is a combination of activities that is owned and controlled by a single entity to produce a homogenous set of goods and services, usually at a single site.

One of the research team's first uses of the IRI input–output framework was to look at the 'path' of direct and indirect inputs into IRI's motor vehicle industry, the global influence of these paths and the effect on the IRI group of an increase in the final demand for automobiles. At about the time that this research was starting, IRI sold the Alfa Romeo group, so that the research team could then study the influence on the rest of the firm from having this group as an outsider rather than insider.

Thus, some significant work has been done, primarily in China and Europe, on EIO accounts and models. In the 1970s, about 200 US companies were using the input–output technique for strategic planning, of which about 25% were making output forecasts to help determine market shares (Gols, 1976); but most, if not all, firms were relying on detailed national input–output accounts, rather than constructing their own internal accounts.

Input–output tables and models can actually be used for three separate tasks by corporations. First, they provide an excellent way to construct a consistent account of the flows from suppliers to users. Second, the account can be used to identify the structure of production (technology) and changes in it over time. These production structures can then be compared with those of other firms that produce the same product, or ones that produce different, but related, products, or ones to whom the company wishes to supply products. Third, the accounting data can be converted into an enterprise input–output model and used for a variety of important analytical tasks in the firm, such as forecasting future output and market shares, determining optimal production input structures, calculating the effect of changes in backward and forward direct and indirect requirements and assessing the energy requirements and environmental impact of alternative production techniques. In the United States and other Western countries, an important use of the general input–output technique is to determine the direct and indirect labor required for different types of processes. At least up to the present, labor estimation is a less important, often nonexistent or insignificant, part of the Chinese analyses. In this paper, we present an adaptation of the EIO accounts and models that will make it especially useful for corporation business decisions.

Most of the existing EIO models are constructed using the framework and convention of national input–output tables. The basic units of the EIO model are typically products/commodities or branches/plants within a company, which are treated in the same way as the sectors in national input–output tables. The main advantage of this approach is that it is consistent with the national income and product accounts and macroeconomic statistical conventions. The EIO tables so constructed

can be easily linked to national, regional, and industry-level input–output accounts and models (Polenske, 1989).

The national input–output accounting framework, however, is often ill-suited for business accounting and modeling. The relationships among different branches and processes of a business enterprise are different from the relationships among different sectors in the national economy. Some processes, for example, may have a large number of by-products and different plants and processes may produce the same or similar products. This presents a major problem if managers want to use plants or branches as the basic unit for EIO models. If managers build EIO models using products/commodities as the basic unit, they have difficulty interpreting the diagonal elements, which, in the national input–output table, show the self-consumption of a sector's own product. Furthermore, the national input–output framework may be too comprehensive and, at the same time, too general to be applied to the business enterprise setting. Companies typically do not collect data and compile statistics based on the model of national income and product accounts, so that there is little data readily available for EIO models.

In this paper, we develop an alternative approach to EIO modeling which remedies these problems and is oriented toward business accounting and decision-making. We build an EIO model based on production processes rather than products or branches. We also make a clear distinction between production of a commodity, which is entered into the EIO account as a positive number and consumption or use of a commodity, which is entered as a negative number. Our model is built upon the work of Zuo (1985) who used this approach to construct a new type of input–output model. We modify Zuo's formulation, derive a set of equations to describe the basic relationships of our model, and discuss how the basic model can be extended and used by managers for structural analysis, process analysis, and environmental management.

4. Basic input–output process model

Our corporate input–output model is based on production processes rather than commodities. We view production activity in a company as a system of interrelated production procedures that combine factor inputs to produce outputs. Table 1 shows the basic structure of the input–output process model, which consists of nine components:

Z matrix, production and intermediate consumption of main products;
Y matrix, final output of main products;
M matrix, consumption of purchased inputs;
$\mathbf{X}^{m}$ matrix, total demand for purchased inputs;
W matrix, generation and elimination of by-products and wastes;
$\mathbf{X}^{w}$ matrix, discharge of by-products and wastes;
V matrix, consumption of primary inputs;
$\mathbf{X}^{v}$ matrix, total demand for primary inputs;

Table 1
Basic structure of input–output process model

Item		Processes				Final output/total demand
		1	2	3	n	
Main products	1	$\mathbf{Z}_{11}$	$\mathbf{Z}_{12}$	$\mathbf{Z}_{13}$	$\mathbf{Z}_{1n}$	$\mathbf{Y}_1$
	2	$\mathbf{Z}_{21}$	$\mathbf{Z}_{22}$	$\mathbf{Z}_{23}$	$\mathbf{Z}_{2n}$	$\mathbf{Y}_2$
	3	$\mathbf{Z}_{31}$	$\mathbf{Z}_{32}$	$\mathbf{Z}_{33}$	$\mathbf{Z}_{3n}$	$\mathbf{Y}_3$
	n	$\mathbf{Z}_{n1}$	$\mathbf{Z}_{n2}$	$\mathbf{Z}_{n3}$	$\mathbf{Z}_{nn}$	$\mathbf{Y}_n$
Purchased inputs	1	$\mathbf{M}_{11}$	$\mathbf{M}_{12}$	$\mathbf{M}_{13}$	$\mathbf{M}_{1n}$	$\mathbf{X}_1^{\mathrm{m}}$
	2	$\mathbf{M}_{21}$	$\mathbf{M}_{22}$	$\mathbf{M}_{23}$	$\mathbf{M}_{2n}$	$\mathbf{X}_2^{\mathrm{m}}$
	3	$\mathbf{M}_{31}$	$\mathbf{M}_{32}$	$\mathbf{M}_{33}$	$\mathbf{M}_{3n}$	$\mathbf{X}_3^{\mathrm{m}}$
By-products and waste	1	$\mathbf{W}_{11}$	$\mathbf{W}_{12}$	$\mathbf{W}_{13}$	$\mathbf{W}_{1n}$	$\mathbf{X}_1^{\mathrm{w}}$
	2	$\mathbf{W}_{21}$	$\mathbf{W}_{22}$	$\mathbf{W}_{23}$	$\mathbf{W}_{2n}$	$\mathbf{X}_2^{\mathrm{w}}$
	3	$\mathbf{W}_{31}$	$\mathbf{W}_{23}$	$\mathbf{W}_{33}$	$\mathbf{W}_{3n}$	$\mathbf{X}_3^{\mathrm{w}}$
Primary inputs	1	$\mathbf{V}_{11}$	$\mathbf{V}_{12}$	$\mathbf{V}_{13}$	$\mathbf{V}_{1n}$	$\mathbf{X}_1^{\mathrm{v}}$
	2	$\mathbf{V}_{12}$	$\mathbf{V}_{22}$	$\mathbf{V}_{23}$	$\mathbf{V}_{2n}$	$\mathbf{X}_2^{\mathrm{v}}$
	3	$\mathbf{V}_{13}$	$\mathbf{V}_{23}$	$\mathbf{V}_{33}$	$\mathbf{V}_{3n}$	$\mathbf{X}_3^{\mathrm{v}}$
Process main products		$\mathbf{X}_1^{\mathrm{z}}$	$\mathbf{X}_2^{\mathrm{z}}$	$\mathbf{X}_3^{\mathrm{z}}$	$\mathbf{X}_n^{\mathrm{z}}$	

$\mathbf{X}^{\mathrm{z}}$ matrix, gross output of main products, which is the sum of elements in the columns of Z.[3]

To illustrate the model, consider a hypothetical iron and steel corporation that produces 20 000 tonnes of iron and 500 000 tonnes of steel a year, as shown in Table 2. Each row in the table describes the production (make) and consumption (use) of a commodity, input, or waste material. A positive number indicates the production of a commodity, by-product, or waste material and a negative number indicates the consumption of a commodity, by-product or input. For example, the second row in the main-products section of Table 2 shows that 474 500 tonnes of iron were produced by the corporation, of which 454 500 tonnes were used in the production of steel and the remaining 20 000 tonnes were sold to final customers or added to the company's inventory. The second row in the purchased-inputs section indicates the coal-use pattern in the company: 294 200 tonnes used for coke making and 80 700 tonnes in the blast-furnace operations. The total coal consumption was 374 900 tonnes a year.

[3]Sometimes it is not clear which good should be considered as the main product of a process. In such a case, the choice of the main product depends on the manager's information requirements and analytical needs. Changing main-product definitions will affect the mathematical structure of some of the matrices but not the model structure.

Table 2
Input–output process model: hypothetical iron and steel plant

		Coke oven	Blast furnace	Converter	Final outcome
Main products					
Coke	K tonne	213.5	−213.5	0.0	0.0
Iron	K tonne	0.0	474.5	−454.5	20.0
Steel	K tonne	0.0	0.0	500.0	500.0
Purchased inputs					
Iron ore	K tonne	0.0	−711.8	0.0	−711.8
Coal	K tonne	−294.2	−80.7	0.0	−374.9
Steel scrap	K tonne	0.0	0.0	−90.9	90.9
Limestone	K tonne	0.0	0.0	−36.4	−36.4
Others	K tonne	0.0	−730.0	−50.0	−780.0
By-products and waste					
Coke oven gas	K tonne	28.5	0.0	0.0	28.5
Blast furnace gas	K tonne	0.0	1091.5	0.0	1091.5
Converter flue gas	K tonne	0.0	0.0	318.2	318.2
Ammonia water	K tonne	47.5	0.0	0.0	47.5
Tar	K tonne	4.7	0.0	0.0	4.7
Slag	K tonne	0.0	170.8	522.7	693.6
Primary inputs					
Capital	$1000	−9490.9	−23727.3	−20000.0	−53218.2
Labor	p.year	−95.0	−300.0	−630.0	−1025.0
Main products	K tonne	21.5	474.5	500.0	

Source: compiled by the authors based on technical information about a steel production process in Japan provided by Zuo (1985, p. 82).

The row of the input–output process model, therefore, provides a statistical account of input–output flows in the corporation. These flows must be balanced, as follows:

$$\mathbf{Z}_{i1}+\mathbf{Z}_{i2}+\mathbf{Z}_{i3}+\ldots+\mathbf{Z}_{ij}+\ldots+\mathbf{Z}_{in}=\mathbf{Y}_i, \tag{1}$$

$$\mathbf{M}_{i1}+\mathbf{M}_{i2}+\mathbf{M}_{i3}+\ldots+\mathbf{M}_{ij}+\ldots+\mathbf{M}_{in}=\mathbf{X}_i^{\mathrm{m}}, \tag{2}$$

$$\mathbf{W}_{i1}+\mathbf{W}_{i2}+\mathbf{W}_{i3}+\ldots+\mathbf{W}_{ij}+\ldots+\mathbf{W}_{in}=\mathbf{X}_i^{\mathrm{w}}, \tag{3}$$

$$\mathbf{V}_{i1}+\mathbf{V}_{i2}+\mathbf{V}_{i3}+\ldots+\mathbf{V}_{ij}+\ldots+\mathbf{V}_{in}=\mathbf{X}_i^{\mathrm{v}}. \tag{4}$$

In matrix notation,

$$\mathbf{Z}^{*}\boldsymbol{i}=\boldsymbol{Y} \text{ main products balance,} \tag{5}$$

$$\mathbf{M}^{*}\boldsymbol{i}=\boldsymbol{X}^{\mathrm{m}} \text{ purchased inputs balance,} \tag{6}$$

$$\mathbf{W}^{*}\boldsymbol{i}=\boldsymbol{X}^{\mathrm{w}} \text{ by-products/wastes balance} \tag{7}$$

Table 3
Direct input-output coefficient matrices

	Coke	Iron	Steel
A matrix, main products			
Coke	1.000	−0.450	0.000
Iron	0.000	1.000	−0.909
Steel	0.000	0.000	1.000
B matrix, purchased inputs			
Iron ore	0.000	−1.500	0.000
Coal	−1.378	−0.170	0.000
Steel scrap	0.000	0.000	−0.182
Limestone	0.000	0.000	−0.073
Others	0.000	−1.538	−0.100
C matrix, by-products and waste			
Coke oven gas	0.133	0.000	0.000
Blast furnace gas	0.000	2.300	0.000
Converter flue gas	0.000	0.000	0.636
Ammonia water	0.222	0.000	0.000
Tar	0.022	0.000	0.000
Slag	0.000	0.360	1.045
D matrix, primary inputs			
Capital	−44.444	−50.000	−40.000
Labor	−0.445	−0.632	−1.260

$$\mathbf{V}^{*}\boldsymbol{i} = \boldsymbol{X}^{v} \text{ primary inputs balance,} \tag{8}$$

where $\boldsymbol{i}$ is a unity column vector corresponding to the dimension of each matrix it multiplies. Post-multiplication of a matrix by $\boldsymbol{i}$ generates a column vector whose elements are the sums of the elements in each row of the matrix.

Each column of Table 2 provides information on the inputs used and outputs generated in each production process. The inputs include those produced by the company (own products/inputs), those purchased externally (purchased inputs) and primary inputs (such as labor, capital, and land). We use the cokemaking (column 1) as an example. To produce 213 500 tonnes of coke, the coke-making process used 294 200 tonnes of coal and required \$9.5 trillion of capital and 95 person-years of labor inputs. The process also generated 28 500 tonnes of coke-oven gas, 47 500 tonnes of ammonia water and 4700 tonnes of tar.[4]

[4]We note that, in actual practice, a manager will have considerable difficulty in allocating the primary inputs. Also, the manager has to allocate overhead among plants. A discussion of the types of decision rules managers develop for allocating these items is beyond the scope of this paper, but changing the numbers based upon a particular manager's decision rules will not alter the theoretical structure of the model presented in this paper.

When each element in a column of Table 2 is divided by the output of the main product of the production process, we obtain Table 3, the direct or technical coefficients matrix. Each coefficient in Table 3 shows the amount of input required or the quantity of the output produced for each unit of main-product output. Each column of technical coefficients displays the input mix and output structure of a production process. It can be interpreted as a quantitative measure of production technology. The coke-oven column of Table 3, for example, indicates that in order to produce 1 tonne of coke, the company needs 1.378 tonnes of coal, $44 worth of capital input and about 0.455×10^{-3} person-years of labor input. In turn, the cokemaking process generates 0.133 tonnes of coke-oven gas, 0.222 tonnes of ammonia water and 0.022 tonnes of tar.

Mathematically, the direct input–output coefficients are obtained as

$$a_{ij} = \frac{\mathbf{Z}_{ij}}{\mathbf{X}_j}, \tag{9}$$

$$b_{ij} = \frac{\mathbf{M}_{ij}}{\mathbf{X}_j}, \tag{10}$$

$$c_{ij} = \frac{\mathbf{W}_{ij}}{\mathbf{X}_j}, \tag{11}$$

$$d_{ij} = \frac{\mathbf{V}_{ij}}{\mathbf{X}_j}, \tag{12}$$

where a_{ij} = main product i per unit of main product j; b_{ij} = purchased input i required per unit of main product j; c_{ij} = by-product or waste i generated per unit of main product j; and d_{ij} = primary input i required per unit of main product j.

In matrix notation,

$$\mathbf{A}^*\mathbf{X} = \mathbf{Y}, \tag{13}$$

$$\mathbf{B}^*\mathbf{X} = \mathbf{X}^{\mathrm{m}}, \tag{14}$$

$$\mathbf{C}^*\mathbf{X} = \mathbf{X}^{\mathrm{w}}, \tag{15}$$

$$\mathbf{D}^*\mathbf{X} = \mathbf{X}^{\mathrm{v}}, \tag{16}$$

where $\mathbf{X} = (\mathbf{X}^{\mathrm{z}})^{\mathrm{t}}$, which is the transpose of $\boldsymbol{X}^{\mathrm{z}}$.

From the direct input–output coefficients, we can calculate the total (direct and indirect) input–output coefficients, as shown in Table 4. Each element in the table describes total amount of inputs required or outputs generated, both directly and indirectly, to deliver one unit of main product to final users. The cell in the coal row and iron column, for example, indicates that in order to provide one tonne of iron to final customers, the company requires 0.79 tonnes of coal, of which 0.17 tonnes is direct consumption in the ironmaking process (see Table 3) and the other 0.62 tonnes is indirectly required because the production of coke, a major

input of the ironmaking process, requires a large quantity of coal. This way of calculating the total input requirement is similar to the engineering-process analysis (Miller and Blair, 1985).

We can use the total input–output coefficient matrix to determine the total amount of inputs and outputs associated with one unit of the final output of the main product. Mathematically,

$$\mathbf{X}=\mathbf{A}^{-1}*\mathbf{Y}, \tag{17}$$

$$\mathbf{X}^{m}=\mathbf{B}*\mathbf{X}=\mathbf{B}*\mathbf{A}^{-1}*\mathbf{Y}, \tag{18}$$

$$\mathbf{X}^{w}=\mathbf{C}*\mathbf{X}=\mathbf{C}*\mathbf{A}^{-1}*\mathbf{Y}, \tag{19}$$

$$\mathbf{X}^{v}=\mathbf{D}*\mathbf{X}=\mathbf{D}*\mathbf{A}^{-1}*\mathbf{Y} \tag{20}$$

where $\mathbf{A}^{-1}$ = total main product output per unit of main product final output;[5] $\mathbf{B}*\mathbf{A}^{-1}$ = total purchased input requirements per unit of main-product final output; $\mathbf{C}*\mathbf{A}^{-1}$ = total waste generation per unit of main-product final output; and $\mathbf{D}*\mathbf{A}^{-1}$ = total primary input requirements per unit of main-product final output.[6] Eqs. (17)–(20) summarize the basic mathematical relationships of the input–output process model.

5. Model extension and application

The basic input–output process model presented in the last section provides a mathematical description of the production processes and the input–output structure of a company or plant. Just the information from the IOP account itself will provide managers with an incredible amount of important economic and production information. The account shows, for example, the level and type of purchases made to

[5] $\mathbf{A}^{-1}$ in our model is equivalent to the Leontief inverse in the conventional input–output model. It has the same property of non-negativity if the Hawkins–Simon condition must be satisfied. Diagonal elements of **A**, which represent main-product outputs, are always ones. Off-diagonal elements of **A**, representing inputs, are negative or zero. Mathematically,

$$\mathbf{A}^{*}=\mathbf{A}-\mathbf{I},$$

$$\mathbf{A}=\mathbf{I}-(-\mathbf{A}^{*}),$$

where **I** = identity matrix and $\mathbf{A}^{*}$ = off-diagonal elements matrix. The $(-\mathbf{A}^{*})$ indicates the amount of inputs required per unit of main-product outputs, which is equivalent to the direct input coefficients matrix, $\mathbf{A}_{L}$, in the conventional input–output model. In other words,

$$(-\mathbf{A}^{*})=\mathbf{A}_{L}.$$

It follows, then, that

$$\mathbf{A}=\mathbf{I}-\mathbf{A}_{L},$$

$$\mathbf{A}^{-1}=(\mathbf{I}-\mathbf{A}_{L})^{-1},$$

where, $(\mathbf{I}-\mathbf{A}_{L})^{-1}$ = Leontief inverse in the conventional input–output model.

[6] Because $\mathbf{A}^{-1}$ is generally non-negative, $-\mathbf{B}*\mathbf{A}^{-1}$, $\mathbf{C}*\mathbf{A}^{-1}$, and $-\mathbf{D}*\mathbf{A}^{-1}$ are nonnegative as well.

Table 4
Total input–output coefficient matrices

	Coke	Iron	Steel
$\mathbf{A}^{-1}$ matrix			
Coke	1.000	0.450	0.409
Iron	0.000	1.000	0.909
Steel	0.000	0.000	1.000
$\mathbf{B}^*\mathbf{A}^{-1}$ matrix			
Iron ore	0.000	−1.500	−1.364
Coal	1.378	−0.790	−0.718
Steel scrap	0.000	0.000	−0.182
Limestone	0.000	0.000	−0.073
Others	0.000	−1.538	−1.498
$\mathbf{C}^*\mathbf{A}^{-1}$ matrix			
Coke oven gas	0.133	0.060	0.055
Blast furnace gas	0.000	2.300	2.091
Converter flue gas	0.000	0.000	0.636
Ammonia water	0.222	0.100	0.091
Tar	0.022	0.010	0.009
Slag	0.000	0.360	1.373
$\mathbf{D}^*\mathbf{A}^{-1}$ matrix			
Capital	−44.444	−70.000	−103.636
Labor	−0.445	−0.832	−2.017

Source: calculated from data in Table 3.

produce each good; the interrelationships among different production processes, sales or inventories of products and the amount attributed in the accounts to labor, capital, taxes and depreciation in producing each good (Polenske, 1989). Depending upon the way the IOP account is structured, different types of analyses are possible. Managers, for example, can use it to determine:

(1) the flow of materials between different production units;
(2) input requirements for each of several product lines;
(3) the minimum use required of particularly scarce and/or expensive materials; and
(4) the amount of given materials or energy used and lost in the technological process (Polenske, 1989).

In addition, managers can use the input–output account to calculate total input coefficients, which capture both direct and indirect inputs required for making final products. The coefficients can be used to assess the total input requirements for achieving a certain sales target.

Beyond these applications, the basic IOPM can be extended in a number of directions to provide analytical and information support for many business decisions. Here we give three examples.

5.1. Structural analysis

We can extend the basic IOPM to include input costs and product prices and apply it to analyze the cost-revenue structure of a company. We do so by using the price/cost matrix to convert the physical input–output relationships into financial expenditure–revenue relationships. In fact, we can use the price and cost data to transform the entire input–output transactions table from physical units into monetary units. We can then calculate direct and total input–output coefficients in value terms and build a monetary value input–output process model, which can be used as an economic calculus for many business decisions. Managers, for example, can apply the monetary value IOPM to:

Table 5
Cost and revenue structure of the iron and steel company

Product	Unit	Quantity	Price ($/ton)	Cost or revenue ($1000)
Main products				
Coke	K tonne	0.0	30.0	0.0
Iron	K tonne	20.0	150.0	3000.0
Steel	K tonne	500.0	300.0	150 000.0
Purchased inputs				
Iron ore	K tonne	−711.8	25.0	−17795.5
Coal	K tonne	−374.9	20.0	−7497.8
Steel	K tonne	−90.9	90.0	−8181.8
Limestone	K tonne	−36.4	5.0	−181.8
Others	K tonne	−780.0	30.0	−23 400.0
By-products and wastes				
Coke	K tonne	28.5	−1.0	−28.5
Blast furnace gas	K tonne	1091.5	−1.0	−1091.5
Converter gas	K tonne	318.2	−1.0	−318.2
Ammonia water	K tonne	47.5	−1.0	−47.5
Tar	K tonne	4.7	−1.0	−4.7
Slag	K tonne	693.6	−1.0	−693.6
Primary inputs				
Capital	$1000	−53 218.2	NA	−53 218.2
Labor	p-year	−1025.0	2000.0	−2050.0
Cost, revenue and profit				
Excluding waste disposal cost				
Total cost				−112 325.0
Total revenue				153 000.0
Profit				40 674.9
Profit–cost ratio				0.362
Including waste disposal cost				
Total cost				−114 509.0
Total revenue				153 000.0
Profit				38 491.0
Profit–cost ratio				0.336

(1) trace financial flows, record expenditures and revenues, and study the cost and profit structure of the firm;
(2) conduct a market study and analyze cost and profitability associated with each of the firm's products and individual segments of the market;
(3) examine the impacts of exogenous changes, such as energy-price increases, wage hikes, and technological innovation, on the firm's economic performance; and
(4) serve as a basis for business planning and, by combining it with linear and other programming techniques, as a tool for optimization under different constraints.

Table 5 shows hypothetical product prices and input costs for our iron and steel corporation. Assume that the company does not sell any of the by-products and does not have any waste-disposal costs. Then, the total cost of production is the sum of all the input costs, which is equal to the quantity of inputs used times the input prices. Mathematically,

$$TC = \mathbf{P}^{m} * \mathbf{X}^{m} + \mathbf{P}^{v} * \mathbf{X}^{v} = \$112 \text{ million}, \tag{21}$$

where TC = total cost; $\mathbf{P}^{m}$ = prices of purchased material inputs; and $\mathbf{P}^{v}$ = prices of primary inputs. The total revenue of the company is equal to the sales of its products,

$$TR = \mathbf{P}^{y} * \mathbf{Y} = \$153 \text{ million}, \tag{22}$$

where TR = total revenue; and $\mathbf{P}^{y}$ = product-price matrix.

The excess of total revenue over total cost is the profit of the company, which amounts to about \$41 million dollars, with a profit-to-cost ratio of 36.2%. Mathematically, we can calculate the gross profit of a company using the following equation:

$$\text{Profit} = TR - TC \tag{23}$$

$$= \mathbf{P}^{y} * \mathbf{Y} + \mathbf{P}^{m} \mathbf{X}^{m} + \mathbf{P}^{v} \mathbf{X}^{v},$$

$$= [\mathbf{P}^{y} | \mathbf{P}^{m} | \mathbf{P}^{v}] * \frac{\mathbf{Y}}{\frac{\mathbf{X}^{m}}{\mathbf{X}^{v}}} \tag{24}$$

$$= [\mathbf{P}^{y} | \mathbf{P}^{m} | \mathbf{P}^{v}] * \frac{\mathbf{AX}}{\frac{\mathbf{BX}}{\mathbf{DX}}}$$

$$= [\mathbf{P}^{y} | \mathbf{P}^{m} | \mathbf{P}^{v}] * \frac{\mathbf{A}}{\frac{\mathbf{B} * \mathbf{X}}{\mathbf{D}}},$$

$$= [\mathbf{P}^{y} | \mathbf{P}^{m} | \mathbf{P}^{v}] * \frac{\mathbf{A}}{\frac{\mathbf{B} * \mathbf{A}^{-1} * \mathbf{Y}}{\mathbf{D}}}, \tag{25}$$

where

$$[\mathbf{P}^{y}|\mathbf{P}^{m}|\mathbf{P}^{v}]^{*}\frac{\mathbf{A}}{\frac{\mathbf{B}}{\mathbf{D}}} = \text{profit per unit of gross output of main products.}$$

This equation answers the question: "if total production of main products—coke, iron, or steel—increases by 1 tonne, how will the profit change?"

$$[\mathbf{P}^{y}|\mathbf{P}^{m}|\mathbf{P}^{v}]^{*}\frac{\mathbf{A}}{\frac{\mathbf{B}^{*}\mathbf{A}^{-1}}{\mathbf{D}}} = \text{profit per unit of final main product output.}$$

This equation is used to determine the profit associated with one unit of main product sold to final customers. Managers can use Eqs. (24) and (25) to examine the economic impact of changes in product prices ($\mathbf{P}^{y}$), input prices ($\mathbf{P}^{m}$, $\mathbf{P}^{v}$) and input–output structure ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$) on the cost, revenue, and profitability of a company. They can also use the equations to analyze the cost-structure and revenue sources and to identify ways to boost the profitability.

5.2. *Process analysis*

The input–output process model also provides a quantitative description of production technology employed by a company and reflects engineering characteristics of production processes. It is a valuable tool for researchers in conducting an industrial-process analyses (Manne and Markowitz, 1961) and activity analysis of production and allocation (Koopmans, 1951). Central to both the process and activity analyses is the concept of an activity, which is defined as a process for transforming inputs of production into outputs of goods and services (Manne, 1961). From the direct or technical coefficient matrix (Table 3), managers can easily derive a material-flow chart for each process, as is shown in Fig. 1. In the steelmaking process, for example, the material inputs of 0.909 tonnes of iron, 0.182 tonnes of steel scrap, 0.073 tonnes of limestone and 0.1 tonnes of other materials generates 1 tonne of steel, 0.636 tonnes of converter gas and 1.045 tonnes of slag. The individual processes in Fig. 1 can be combined and linked to show the entire operation of the iron and steel production activity, as is shown in Fig. 2. Just as in an input–output model, the output from one sector may be an input into the sector itself or into other sectors; if so, some feedback loops can be included in Fig. 1.

As a process-analysis tool, IOPM can be used to help managers:

(1) Trace the flows of energy and materials, calculate direct and indirect input requirements for each production process and estimate the demand for energy, materials, capital, labor and other inputs under alternative production targets.
(2) Identify bottlenecks of production activities and coordinate capacity expansion of related production processes. Managers, for example, can apply the IOP model to determine how much the iron-production capacity has to increase in

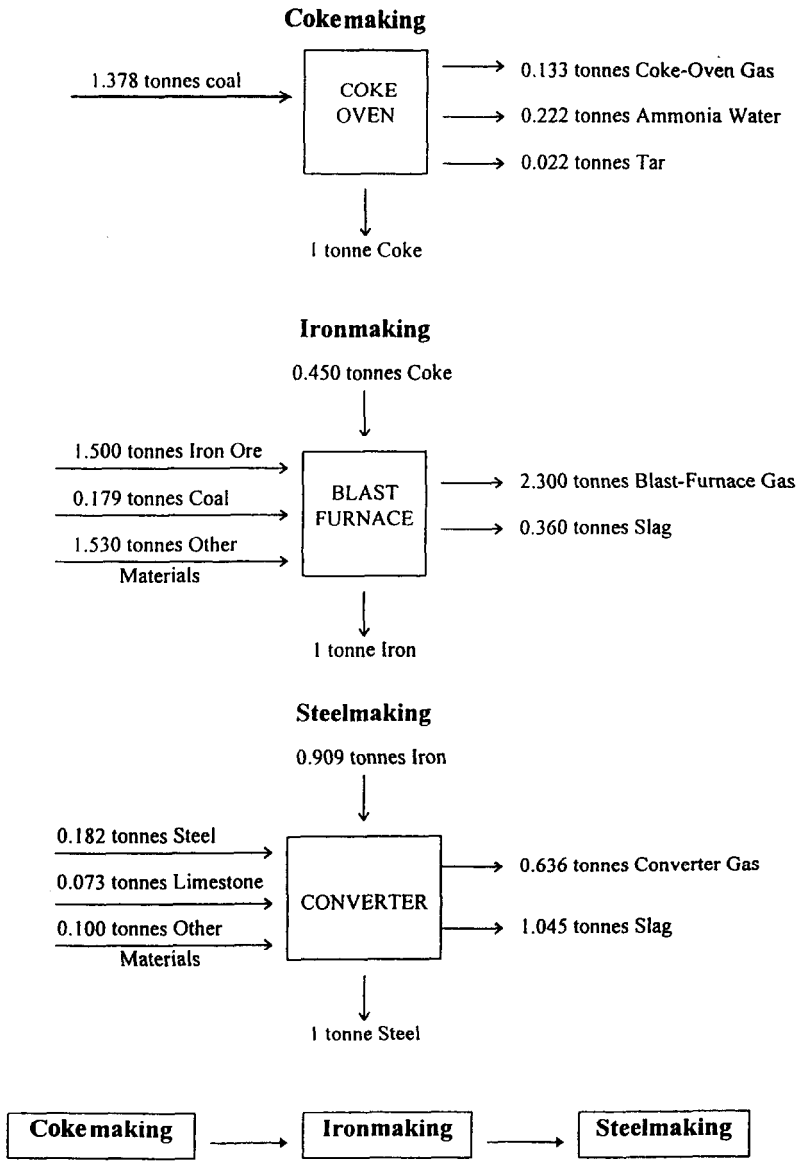


Fig. 1. Material-flow chart for three main product processes.

order to meet an additional demand for pig iron from the expansion of steel-production facilities.

(3) Evaluate alternative technologies or processes for a production line. Managers, for example, can assess the impact of introducing the electric-arc furnace, which will change the input–output coefficients for the steel production process, on input requirements, production costs, and profitability. They can also analyze

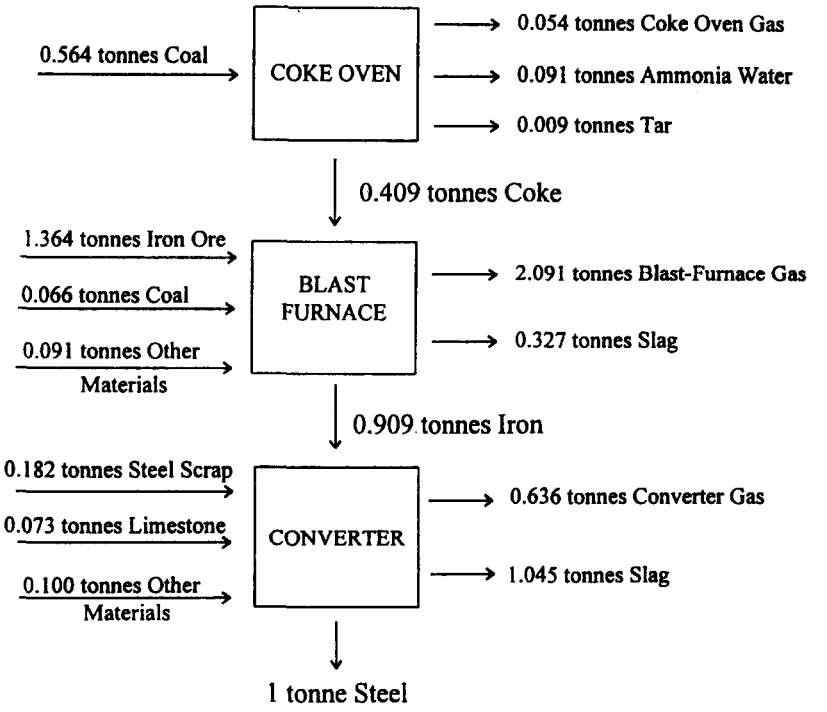


Fig. 2. Input–output structure of steel production.

the effects of other changes, such as energy-efficiency improvements and ore-quality changes, on the company. Based on the input–output data for alternative processes for making the same product, managers can use mathematical-programming techniques to determine the extent to which, and the circumstances under which, one or another process should be employed.

Because of the correspondence between material flows and input–output coefficients, managers can construct the IOPM from the bottom up based on engineering-process information under the assumptions of proportionality. Proportionality means that the input–output relations are fixed and do not vary with production scales. If an activity can be operated at its base level, it can also be operated at any level that is a multiple of the base level, with all inputs and outputs varying proportionately. Fig. 2 provides a listing of the inputs and outputs required to produce 1 tonne of crude steel. The proportionality assumption says that if we build a plant with the production capacity of 100 000 tonnes of steel, all the inputs will be multiplied by 100 000.[7]

The IOPM, then, can serve as a bridge between the industrial-engineering process model and the economic production-function analysis. Traditionally, economists see

[7]It is possible, however, to obtain numerical solutions that adjust for increasing or decreasing returns (Manne, 1961).

the enumeration of all possible physical input–output relationships, which is the basis for the specification of the production function, as being the responsibility of plant engineers. Engineers, however, are usually busy solving technical problems and optimizing plant operations and do not view the examination of all possible efficient allocations of inputs and outputs as being within the domain of their work. One important contribution of process analysis is to "explore some of the territory that lies between the domain of the economist and the industrial engineer to exploit the latter's detailed knowledge of production processes in order to provide the economist with a better characterization of the set of efficient production possibilities" (Manne, 1961, p. 420).

5.3. Environmental management

Environmental management is becoming a key component of the business strategy in many companies. In a survey of 41 firms, most of which were Fortune 500 companies, Lent and Wells (1994) found that the environment is becoming a central strategic issue and is being managed as an arena of competition rather than as a compliance-driven function. More and more managers recognize that beyond the costs of environmental regulation lie economic opportunities and that environmentally sound practices can contribute not only to low emissions and compliance costs, but also to higher productivity (Willig, 1994).

One major obstacle to effective corporate environmental management is the absence of cost/material accounting systems that include information on environmental performance (Wells et al., 1994). This makes it difficult for managers to examine alternative pollution-control measures and to evaluate the costs and returns of environmental-quality improvements. Furthermore, because the conventional corporate-accounting systems record little information on environmental performance, managers are not rewarded/penalized for good/poor environmental quality. Therefore, they have no incentive to minimize environmental pollution.

The IOPM can fill some of the gap. From the material-balance perspective, we can view a production activity as physical, chemical and/or biological transformations, through the use of capital equipment and inputs of human and nonhuman energy, of one or more raw materials into one or more desired outputs (Kneese et al., 1970; Kneese and Bower, 1979; Duchin, 1990). No production process, however, can have 100% conversion of raw materials to useful outputs because of the second law of thermodynamics. Thus, in addition to useful outputs, there are nonproductive outputs—waste materials, unwanted by-products, or residuals—generated in the production process, which are quantified by the $\mathbf{W}$ matrix and $\mathbf{X}^{w}$ matrix in the IOPM. The unwanted outputs consist of:

(1) nonproduct materials formed in the production process;
(2) raw materials not used or not transformed in the production process;
(3) nonused or nondesired energy outputs from the production process (Kneese and Bower, 1979).

Those unwanted outputs eventually have to return to the environment and are a root cause of many of our environmental problems.

Even if there are no government environmental regulations, the company may still have to pay the waste-disposal costs. A steel producer, for example, must find a way to shift ammonia water, tar, or slag away from the production site and put them elsewhere. To illustrate, let us assume that the disposal cost for all types of waste is \$1 per tonne for our hypothetical iron and steel corporation (see Table 5).

The total cost of waste disposal, then, is:

$$C^{w} = \mathbf{P}^{w} * \mathbf{X}^{w} \tag{26}$$

where $\mathbf{P}^{w}$ is the matrix of waste-disposal costs.The waste-disposal expense increases the total cost of production and reduces the profit of the company.

$$\text{Profit} = \mathbf{P}^{y}\mathbf{Y} + P^{m}\mathbf{X}^{m} + P^{v}\mathbf{X}^{v} - \mathbf{P}^{w}\mathbf{X}^{w} \tag{27}$$

$$= [\mathbf{P}^{y} | \mathbf{P}^{m} | \mathbf{P}^{v} | - \mathbf{P}^{w}] \begin{matrix} \mathbf{Y} \\ \hline \mathbf{X}^{m} \\ \hline \mathbf{X}^{v} \\ \hline \mathbf{X}^{w} \end{matrix}$$

In this case, the profit of iron and steel production declines to \$38 million and the profit/cost ratio drops to 33.6% (Table 5). It makes economic sense for a company, therefore, to reduce waste and recover or reuse some portion of it. The extent to which it is economic to practice waste reduction and materials recovery at a particular plant depends on the relative costs of waste reduction versus disposal and the relative costs of recovered materials versus new materials.

Managers can apply the IOPM to examine the impacts of environmental regulation on the company and to evaluate alternative compliance strategies. If the government charges pollution taxes, $\mathbf{P}^{w}$ will go up, which will increase the waste-disposal cost and reduce profit. The company can pay the pollution tax and/or spend money to abate pollution, depending on the marginal costs of pollution control relative to pollution-tax bills. If the government sets a quantity limit on pollution, the company must reduce the waste and the amount of pollutants discharged into the environment.

The amount of waste generated, $\mathbf{X}^{w}$, is determined through the following equation:

$$\mathbf{X}^{w} = w * i = C * X = C * A^{-1} * Y. \tag{28}$$

$\mathbf{X}^{w}$ can be reduced by lowering the output level, $\mathbf{Y}$, reducing pollution intensity, $\mathbf{C}$, or revising the production technology, A. The company, for example, can introduce end-of-pipe equipment to treat flue gas before it is released into the environment. Better yet, it can utilize blast-furnace gas, coke-oven gas, and converter gas to generate heat or electricity, which can then be used in the company or sold to outside customers. In both cases, the basic IOPM can be extended to include additional waste-management processes, pollution control or waste-utilization process. Managers can use the extended models to quantify the costs of waste disposal, identify factors that influence waste/pollutant generation, and evaluate alternative

options to comply with environmental regulations. Because the IOPM also shows the interrelationships among different types of pollutants, they will be able to consider all pollutants simultaneously to determine the optimal strategies for company-wide, integrated pollution control and waste management.

6. Conclusion

We have introduced a process-based, micro-level input–output model and demonstrated how it can be used to measure a company's performance and provide information to support business decisions. Our modeling approach has at least three advantages over the traditional enterprise input–output model that is based on the macroeconomic national input–output table framework. First, it focuses on production processes and their underlying input–output relationships. This enables us to build a bridge between economic/financial analysis and technology/engineering analysis. It also allows us to account for by-products or multiple products, which have created a major difficulty for enterprise input–output modeling because the conventional approach assumes that each firm produces only one homogenous (or similar) product.

Second, the model is easy to implement. The model structure is simple, straightforward, and accessible to managers and business analysts. Almost all the data required for the model construction are now readily available in current business accounts and statistics and from production-process engineering literature. The model can be operated in any spreadsheet environment, such as Lotus 1-2-3 and Excel, which again are widely available in most corporations.

Finally, the model is flexible and can be extended or modified in other ways to meet the analytical needs of the company. Managers, for example, may expand the model to include detailed information on product sales, such as locations of major customers and average prices for each group of them, and to account for the skill and wage levels of labor and the vintages of capital equipment. On the other hand, they may decide to simplify the model structure by grouping inputs or products together or treating some processes exogenously. We believe that the corporate input–output model should be task-oriented and as simple as possible, given the level of detail dictated by the needs of users. It is not the purpose of modeling to elaborate the obvious in a complex language. Managers will not use the input–output model unless it is user-friendly and helps them find practical solutions to business problems.

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