Philosophia Mathematica (III) **20** (2012), 86–110.
doi:10.1093/phimat/nkr032 Advance Access publication November 21, 2011

Husserl's Pluralistic Phenomenology of Mathematics[†]

Mirja Hartimo[*]

The paper discusses Husserl's phenomenology of mathematics in his *Formal and Transcendental Logic* (1929). In it Husserl seeks to provide *descriptive foundations* for mathematics. As sciences and mathematics are normative activities Husserl's attempt is also to *describe* the *norms* at work in these disciplines. The description shows that mathematics can be given in several different ways. The phenomenologist's task is to examine whether a given part of mathematics is genuine according to the norms that pertain to the approach in question. The paper will then examine the intuitionistic, formalistic, and structural features of Husserl's philosophy of mathematics.

Edmund Husserl, originally a mathematician, wrote about mathematics rather extensively especially early in his career. Phenomenology can even be said to have originated in Husserl's attempts to provide foundations for mathematics. Against this background it is surprising that it is not at all clear what phenomenological, or more specifically, Husserlian philosophy of mathematics is.

Indeed, Husserlian philosophy of mathematics is rather rarely addressed in the secondary literature. Moreover, the few existing views about it appear to conflict with one another: for example, Gian-Carlo Rota [1997a] has described it as realistic description, Richard Tieszen [2010] views it as 'constitutive Platonism', and Mark van Atten, having argued against both Tieszen's and Rota's views, identifies it with Brouwerian intuitionism (in his [2010] especially).

In the following, primarily on the basis of Husserl's *Formal and Transcendental Logic* (1929, hereafter *FTL*), I will argue that each of these aspects can be found in Husserl's approach that aims at description of any kind of experience. Since sciences and mathematics are essentially normative activities, Husserl's attempt is to describe also the *norms* at work in the sciences as well as in mathematics. In mathematics these norms are

[†] I have delivered earlier versions of this paper in Tampere, Paris, Turku, and Tokyo, and would like to thank the organizers of the various meetings for the invitations as well as the audiences for questions and comments. The paper has benefitted enormously from comments by Mark van Atten, Mitsuhiro Okada, Leila Haaparanta, George Heffernan, David Woodruff Smith, and Richard Tieszen. Thanks are also due to Academy of Finland for the financial support of my work.

[*] University of Helsinki, Finland. mirja.hartimo@helsinki.fi

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not uniform across the board, but different normative ideals characterize different parts of mathematics. The virtue of Husserl's approach is in enabling a synthesis of the various aspects in which mathematics is given to us.

Having discussed *FTL*, I will relate the present view to Tieszen's, Rota's, and to intuitionists' views about Husserl. While the present account is in agreement with much of what Tieszen says about mathematics, it refrains from claiming that all of mathematics is given in the same way. Different parts of mathematics have different senses to us, and they are associated with different kinds of sets of norms. Thus a part of mathematics can be thought to be given in a kind of evidence that also characterizes intuitionists' views about mathematics. A part is given in a more or less formalistic manner. A part is given structurally. The phenomenological method has to be able to accommodate these differences. Moreover, the phenomenologist's task is to examine whether the given part of mathematics is genuine according to the norms that pertain to the part in question.

When comparing the present account to Rota's view, we can point out that while Rota is very faithful to Husserl's view of phenomenology as description Rota is not particularly sensitive to the norms and ideals at work in mathematics (with possibly a brief exception in his [1991, pp. 484–485]). Consequently, Rota misses the point of Husserl's *Besinnung* in uncovering what the sciences should be like. But then again Rota's analysis shows that Husserl's view may be plagued by unjustified presuppositions, which is a genuine worry, too. This will be discussed in more detail at the end of the paper.

1. Conflicting Views

I will first discuss these above-mentioned conflicting, or at least partly conflicting, views about what Husserl's phenomenology of mathematics is all about in more detail. Let us start with Gian-Carlo Rota's view of Husserl's phenomenology:

1.1. *Rota's Descriptive Realism*

Gian-Carlo Rota [1997a] has taken from Husserl's phenomenology an ideal of realism he calls 'realistic description'. According to Rota the following rules should be followed in a realistic description:

(a) A realistic description shall bring into the open concealed features. Mathematicians do not preach what they practise. They are reluctant formally to acknowledge what they do in their daily work.

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(b) Fringe phenomena that are normally kept in the background should be given their due importance. The shop talk of mathematicians includes words like understanding, depth, kinds of proof, degrees of clarity, and sundry others. A rigorous discussion of the roles of these terms should be part of the philosophy of mathematical proof.
(c) Phenomenological realism demands that no excuses be made that may lead to dismiss any features of mathematics by labeling them as psychological, sociological, or subjective.
(d) All normative assumptions shall be weeded out. Too often, purported descriptions of mathematical proof are hidden pleas for what the author believes a mathematical proof should be. A strictly descriptive attitude is imperative, though difficult and dangerous. It may lead to unpleasant discoveries: for example, one might be led to the realization that no features whatsoever are shared by all mathematical proofs. Or else, one may be led to admit that contradictions are part of the reality of mathematics, side by side with truth. [1997a, p. 184]

Without further specifying the nature of his method Rota then goes on to examine various proofs in mathematics. His conclusion is that mathematical proofs come in different kinds, and that at least notions such as the notion of possibility, understanding, and its degrees as well as evidence have to be taken into account when describing mathematical proofs [*ibid.*, p. 195]. Elsewhere, Rota has described for example the phenomenology of mathematical beauty and the concept of mathematical truth ([Rota, 1991; 1997b]. See also [Tragesser, 1984] on phenomenology as metaphysically neutral description). Even though somewhat embryonic, Rota's view of phenomenological description provides us with an excellent starting point to Husserl's phenomenology of mathematics.

1.2. *Tieszen's Constitutive Platonism*

Richard Tieszen [2010; 2005] argues that Husserl, after 1907, holds a view that is a combination of mathematical realism and transcendental phenomenological idealism. According to Tieszen, for Husserl mathematical objects have a sense of being transcendent, but it is our experience that constitutes them as transcendent. The mathematical objects are objective and they exist independently of the mind. However, this sense of objectivity is constituted by consciousness, and the task of phenomenology is to investigate the constitution of the sense of mind-independence. Accordingly, Tieszen terms his view 'constituted mathematical realism' or 'constituted Platonism'. Thus, like Rota, Tieszen associates Husserl's phenomenology with realism. However, whereas Rota emphasizes the presuppositionless description of mathematical practice, Tieszen focuses on the

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constitution of the transcendence that belongs to the mathematical objects (see also [Tragesser, 1984]). The present view agrees with Tieszen's in that, as Husserl describes it, most of mathematics is given as something transcendent. Indeed, Tieszen's description of the givenness of the mathematical objects follows Husserl's views almost *verbatim*. However, if we take the idea of phenomenological description seriously, as described by Rota, we have to pay attention also to other kinds of givenness of mathematical objects and structures.

1.3. *Van Atten and Brouwerian Intuitionism*

The third view I would like to mention is Mark van Atten's approach. Van Atten argues against both Tieszen and Rota, and holds that L.E.J. Brouwer's notion of the construction of purely mathematical objects and Husserl's notion of their constitution coincide. Consequently, van Atten argues that '[t]ranscendental phenomenology cannot provide a foundation for a pure mathematics that would go beyond intuitionism' [2010, p. 43]. While the present paper argues that there are elements in Husserl's approach that resemble Brouwer's intuitionistic mathematics, it will contradict van Atten's latter claim arguing precisely the opposite: that transcendental phenomenology *can* provide a foundation for a pure mathematics that exceeds the intuitionistic limits. Indeed, it was Husserl's original motivation when writing *FTL* to determine the sense of formal mathematics [Husserl, 1969, p. 12]. My argument in a nutshell is that Husserl's phenomenology permits us to clarify different kinds of intentionality and different kinds of evidence and norms that go with them, including formal mathematics. Husserl's aim is not to restrict the object of investigation by any means. Obviously a lot hinges on the term 'foundations'. In the present paper it is understood to mean 'Socratic foundations', that is, rendering the subject matter examined and understood as opposed to secure and infallible.

2. Phenomenological Method

In the *Prolegomena to the Logical Investigations* (1900) Husserl characterizes the relationship between philosophy and mathematics by introducing a division of labour. According to it, mathematicians should freely construct theories and solve mathematical problems, while it is philosophers' task to think about the essence of mathematics and its relationship to the knowing subject. In 1913 Husserl described a method with which this is to be achieved as presuppositionless description of what is given to us in intuition. This means making explicit our *constitutive* activities. What is given to us turns out to be constituted by our sense-giving syntheses. In

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general, and somewhat roughly put, the *constitution* of the world around us makes the world intelligible to us. Indeed, for example, seeing things *as* something, involves constituting each of them as something. Husserl's description of the constitution evolved to analyses of passive and active syntheses in the 1920s. These analyses clarify the role of, *e.g.*, habits and our previous experiences in our constitutive activities of the world. In the 1920s, Husserl also realized that if he were to uncover the constitution of our subject-matter at hand, he would have to regard it as historically given. Each generation works within the tradition handed down by previous generations.

Note that constitution is not to be equated with construction: we do not construct the objects we perceive into existence. On the contrary, the question of their existence is 'bracketed' from the description of their constitution that aims at making explicit the way in which they are themselves given to us. The phenomenologist's task is to uncover their sense as such, that is, as mind-independent. Furthermore, the aim is to clarify the so-called eidetic structures of the experiences uncovered by a variation of individual experiences. In other words, the aim is to analyze different *types* of givenness, not this or that individual appearance. Likewise, Husserl's discussions of intersubjectivity show that the results of the phenomenological description should be valid for anyone, that is, for any normal (grown up, rational, healthy) person. Consequently phenomenology is a collaborative enterprise: we have to discuss and compare the findings of the phenomenological description to make sure that they really describe objective structures.

3. *Formal and Transcendental Logic* (1929)

Any kind of experience can be subjected to phenomenological description. Sciences and in particular mathematics are not exceptions, and if we believe Husserl, they not only could, but also *should* be phenomenologically clarified. Husserl's view is normative in demanding descriptive foundations for the sciences. Such foundations make the sciences and for example mathematics better understood and 'examined' to borrow a phrase from Socrates, which is also used by Husserl. Thus the phenomenological clarification does not *per se* aim at revision. However, sciences and mathematics differ from, say, experiences of imagination and sleep in that the sciences aim at truth, which brings normative elements into them.

According to Husserl, the giver of norms for the possibility of any genuine science is logic as the theory of science. Therefore when Husserl writes about logic as the theory of science he is describing the norms and principles of the sciences. As Husserl puts it, the task of logic is to seek out

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> the pure essential norms of science in all its essential formations, to give the sciences fundamental guidance thereby and to make possible for them genuineness in shaping their methods and in rendering an account of every step. [Husserl, 1969, p. 3]

What then is the source of normativity in logic? Husserl's answer reminds one of Aristotle: first we are to find out the essence of logic. The source of normativity is in fulfilling the essential or final sense of logic while also at the same time shaping the sense anew.

First, again not unlike Aristotle, as a preliminary guide Husserl discusses the various significations of the word *logos*. He takes these ultimately to have been speaking, thinking, and what is thought. To figure out the essence of logic Husserl then engages in *Besinnung*, sense-investigation. In Husserl's own words:

> whether sciences and logic be genuine or spurious, we do have experience of them as cultural formations given to us beforehand and bearing within themselves their meaning, their 'sense': since they are formations produced indeed by the practice of the scientists and generations of scientists who have been building them. As so produced, they have a final sense, toward which the scientists have been continually striving, at which they have been continually aiming. Standing in, or entering, a community of empathy with the scientists, we can follow and understand — and carry on 'sense-investigation'. [Husserl, 1969, pp. 8–9]

Husserl thus thinks that the scientists have always striven, in one way or other, toward fulfillment of the final sense of logic. Thus one can try to approximate this final sense by looking at the ways in which it has been striven for. However, we should not simply take the essence of logic from the traditional aims of the scientists, but while so doing, we should critically clarify it and renew its final sense. This brings an additional normative element to Husserl's approach: the description is not 'blind' but critical, and it has to be continuously renewed.

In *Formal and Transcendental Logic* Husserl's *Besinnung* shows the sense of logic to be divided into three layers given in three different kinds of evidence. The layers are the grammar, the logic of non-contradiction, and the logic of truth.[1] The level of grammar formulates the rules that govern the well-formedness of the judgments. Husserl [1969, p. 137] also calls it pure theory of forms of senses. The level of non-contradiction is concerned with the meaningfulness of the judgments and the coherence of the theories. Husserl calls it 'pure analytics of non-contradiction'. It

[1] In paragraphs 14 and 15 of *FTL* Husserl distinguishes first the logic of non-contradiction as the second level of formal logic and then truth logic. He writes about

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presupposes the previous level, the pure theory of forms of senses, that is, the grammar. Questions of truth determine the next level, the logic of truth. It examines the formal laws of possible truth where truth is something aimed at in the sciences and requires an intuition of states of affairs. Or

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the logic of non-contradiction as follows:

> It is an important insight, that questions concerning consequence and inconsistency can be asked about judgments *in forma*, without involving the least inquiry into truth or falsity and therefore without ever bringing the concepts of truth and falsity, or their derivatives, into the *theme*. In view of this possibility, we distinguish a level of formal logic that we call *consequence-logic* or *logic of non-contradiction*. [1969, p. 54]

Having made these distinctions in the beginning of paragraph 16 Husserl writes that they are not enough:

> There is need of more penetrating substantiations, which explicate the correspondingly differentiated evidences. . . . [1969, p. 56]

Husserl then goes on to discuss distinctness, which is distinguished from clarity in §16b:

> *Two evidences* become separated here. First, the evidence wherein the *judgment itself, qua judgment*, becomes itself given — the judgment that, as itself given, is called also a distinct judgment, taken from the actual and proper judgment-performing. Second, *the evidence* wherein *that* becomes itself given *which the judger wants to attain 'by way of' his judgment* — the judger, that is, as wanting to *cognize,* which is the way logic always conceives him. To judge explicitly is not *per se* to judge with '*clarity*': judging with 'clarity' has at once *clarity of the affairs*, in the performance of the judgment-steps, and *clarity of the predicatively formed affair-complex* in the whole judging. [1969, pp. 60–61]

Evidence of clarity is then divided into clarity in the having of something itself and clarity of anticipation. In paragraph 17, Husserl relates distinctness to the pure analytics, that is the logic of non-contradiction, concluding that 'the purely analytic logician has the essentical genus, distinct judgment, with its sphere of possible judgments, as his province' [*ibid*., p. 63]. The next paragraph continues to address the question:

> While remaining *entirely within* this province, what can we state about possible distinct judgments *in forma*, after the antecedent logical discipline, the theory of pure forms, … has constructed the multiplicity of possible forms and placed it at our disposal?

While answering this question Husserl says for example the following:

> '*Non-contradiction*' therefore signifies the possibility that the judger can judge distinct judgments *within the unity* of a judgment performable with distinctness. [*ibid*., p. 64].

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as Husserl puts it [1969, p. 127], truth requires verification by means of evidential having of the states of affairs themselves. Whereas the ultimate logic of the sciences consists of all the levels including the last one, formal mathematics and purely formal logic, what he also calls 'pure analytics', belong to the first and second levels. The distinction between the second and the third level is something novel in *FTL*, and by means of it Husserl is able to distinguish formal mathematics from logic considered broadly as a theory of science. Husserl explains that

> the problem that was guiding me originally in determining the sense of, and isolating, a pure logic of 'non-contradiction' was a problem of evidence: namely, the problem of *the evidence of the sciences making up formal mathematics.* It struck me that the evidence of truths

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Next Husserl points out that

> In these researches, then, we must never go outside the proper essence of judgments or judgment-forms, never go beyond distinct evidence. But we go beyond this apriori sphere, as soon as we ask *questions concerning truth* or as soon as, with regard to the objects taken at first only as distinct judgments, we ask questions concerning their adequation to the affairs themselves: in short, as soon as we bring the concept of truth into our theme. [*ibid*., p. 65]

As Husserl earlier related clarity to having affairs themselves or expectation of them, it seems obvious enough that logic of truth is related to clarity. Nevertheless, in §16c Husserl identifies evidence of clarity with evident judging, which is the term Husserl occasionally uses when discussing the logic of truth. In paragraphs 21 and 22 Husserl discusses the evidence related to the grammar, 'the third evidence', and the broadest conception of judgments.

Further quotes can be found throughout the second part of *FTL*. In paragraph 70 Husserl summarizes his earlier discussion, explaining his three-fold stratification of logic. He writes

> Without exception these investigations were *directed phenomenologically to the subjective*; they concerned the contrasting of three different focusings in judging, with the interchanging of which the direction of actual and possible identification — the directedness to something objective — becomes altered, and the pointing out of three different evidences, three correspondingly different modes of empty expectant intention and of fulfillment, and three different concepts of the judgment, which become originally separated accordingly. (p. 178)

For a meticulous exposition of the three levels of logic, corresponding three notions of judgment, and the three different conceptions of evidence, see [Heffernan, 1989] and also [Lohmar, 2000, pp. 40–63]. In general, Lohmar's book offers us what Robert Sokolowski has described as 'an authoritative and illuminating commentary on Husserl's *Formal and Transcencental Logic*, the first comprehensive study since Suzanne Bachelard's work, which appeared in French in 1957 and English in 1968'. [2002, p. 233]. For a clear summary of Husserl's argument in the *FTL* in English, see [Tieszen, 2004, pp. 283–303].

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> comprised in formal mathematics (and also of truths comprised in syllogistics) is entirely different from that of other *a priori* truths, in that the former do not need any intuition of objects or of states of affairs as concrete examples, even though they do relate to these, albeit with the universality of empty forms. ([Husserl, 1969, p. 12], translation modified so that *Sachverhalt* has been translated as a state of affairs).

Indeed, the evidence characteristic of formal mathematics is evidence of *distinctness* [Husserl, 1969, pp. 62–63]. He further explains that 'mathematics of mathematicians' belongs to the level of non-contradiction. According to him formal mathematics is '*pure analytics of non-contradiction*, in which the concept of truth remains outside the theme' [Husserl, 1969, pp. 11, 54, 142–143]; see also [Husserl, 2001a, p. 150]:

> In these researches [concerning pure analytics], then, we must never go outside the proper essence of judgments or judgment-forms, never go beyond distinct evidence. But we go beyond this apriori sphere, as soon as we ask questions concerning truth or as soon as, with regard to the objects taken at first only as distinct judgments, we ask questions concerning their adequation to the affairs themselves: in short, as soon as we bring the concept of truth into our theme. [Husserl, 1969, p. 65]

Contrary to the pure analytics, logic of truth is about possible truth, 'as possibly standing in a relationship of adequation to the corresponding judgments that give the supposed affairs themselves' [*ibid*., p. 65]. The evidence in question is termed evidence of clarity [*ibid*., pp. 60–61]. The logic of truth rules out 'material counter-sense' and other 'untruths'; it is the actually applicable part of mathematics and logic.

Incidentally, Husserl's distinction between consequence-logic and truth-logic is curiously similar to Wittgenstein's *Tractatus* [1961, §6.2321]. Accordingly in an appendix to *FTL* Oskar Becker, himself an intuitionist, further specifies the connection of Husserl's doctrine to the *Tractatus*, explaining that as any contradiction excludes from the start all questions of adequation, so does any tautology. Tautologies are *self-distinct* and thus cannot be true in Husserl's sense [Husserl, 1969, pp. 339–340].

To sum up, formal mathematics and formal logic are given in a different way from the logic of truth. Whereas mathematics and formal logic seek evidence of distinctness, the logic of truth requires the possibility of intuition of objects or of states of affairs and another kind of evidence, evidence of clarity. Accordingly the purely formal logic, which is *a priori*, pure analytics, excludes the questions of truth. As an example Husserl discusses a theory of multiplicities (presumably he is not only talking about

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set theory, but the third task of logic of the *Prolegomena*, that is, a theory of axiomatic theories along the lines of Riemann, Cantor, Lie, Grassmann, and Hamilton) that the questions of truth are excluded from it:

> Let us now note that *the theory of multiplicities has no compelling reason* to include in any manner within its theme *questions about the possible truth* of its theory-forms or, correlatively, questions about *the possible actuality* (the possible true being) *of any single multiplicities* subsumed under its formal ideas of a multiplicity. Equivalently, the *mathematician as such* need not be at all concerned with the fact that there actually are multiplicities in concrete 'actuality' (for example: such a thing as a mathematically cognizable Nature; or a realm, such as that of spatial formations, which can perhaps be apprehended as a *Euclidean* multiplicity); nor indeed need he be at all concerned with the fact that there *can* be something of the sort, that something of the sort with some material content or other is thinkable. *Therefore he does not need to presuppose possible multiplicities*, in the sense of multiplicities that might *exist concretely*; and — as a 'pure' mathematician — he can frame his concepts in such a manner that their extension does not at all involve the assumption of such possibilities [Husserl, 1969, pp. 138–139].

From this passage one may also infer what Husserl had in mind with the truth-logic: the actually applicable part of mathematics, presumably Galilean physics (*cf.* also [Husserl, 1969, p. 292], Euclidean geometry, and actually perceivable concrete multiplicities.

Husserl continues to point out that formal mathematics does not even *need* to refer to truth, that '*formal mathematics, reduced to the above described purity, has its own legitimacy* and that, for mathematics, there is in any case no necessity to go beyond that purity' [1969, pp. 140–141]. In this manner the proper sense of 'formal mathematics' becomes clarified [Husserl, 1969, p. 141]. No possible intuition of any physical things or states of affairs is presupposed. Consequently, 'pure mathematics of non-contradiction, in its detachment from logic as theory of science, does not deserve to be called a formal ontology. It is an ontology of pure judgments as senses and, more particularly, an ontology of the forms belonging to non-contradictory — and, in that sense, possible — senses: possible in distinct evidence' [Husserl, 1969, p. 144].

4. Transcendental Logic and the Evidences

In addition to *Besinnung* of logic and the sciences, Husserl's quest is to clarify the outcome by means of transcendental logic. Transcendental logic describes the evidences and norms related to the various *strata* of logic. The transcendental logic ultimately enables us to discriminate between the

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genuine and the spurious. As already mentioned above, Husserl holds that the different levels of logic are given with different kinds of evidences: in particular, the logic of non-contradiction is characterized by what Husserl calls 'evidence of distinctness' [1969, pp. 62–63] whereas the logic of truth is additionally characterized by the 'evidence of clarity'. Husserl further divides the evidence of clarity into clarity of anticipation and clarity of having of something itself. The latter is the same as evident judging, judging that 'gives its meant state of affairs itself' [1969, p. 61] (translation modified so that *Sachverhalt* is translated as a 'state of affairs'), the former is clear in the sense of 'an intuitional anticipation, yet to be confirmed by the having of the state of affairs itself' [*ibid.*]

Evidence in general means giving of something itself [Husserl, 1969, p. 157]. Husserl writes that:

> Thanks to evidence the life of consciousness has *an all-pervasive teleological structure*, a pointedness towards 'reason' and even a pervasive tendency toward it — that is: toward the discovery of correctness (and, at the same time, toward the lasting acquisition of correctness) and toward the cancelling of incorrectnesses (thereby ending their acceptance as acquired possessions). [1969, p. 160]

In other words, we want to get things right; we want to know, as Aristotle puts it in the opening line of *Metaphysics*, and in order to do so, we seek evidence not only in the sciences but in all our conscious life. In the sciences, however, this takes place systematically and critically. Moreover, Husserl points out that the evidence is not infallible, the 'possibility of deception is inherent in the evidence of experience and does not annul either its fundamental character or its effect; though becoming evidentially aware of ⟨actual⟩ deception "annuls" the deceptive experience or evidence itself' [1969, p. 156]. A new experience can cancel the previous believing.

Generally the objectivities are given as the numerically identical objects that can be experienced repeatedly. They are given as transcendent objects, as objects themselves. Thus their identity surpasses our experiences of them [*ibid*., pp. 162–165]. '*Experience is the primal instituting of the being-for-us of objects as having their objective sense*. Obviously that holds good equally in the case of irreal objects, whether their character is the ideality of the specific, or the ideality of a judgment, or that of a symphony, or that of an irreal object of some other kind' [*ibid*., p. 164]. Husserl could not be clearer about the transcendence of the objects. He proceeds to explain the evidential givenness of something itself as a process of constitution, 'a process whereby the object of experience arises' [*ibid*., p. 165]. The transcendence that belongs to all objects, whether ideal or real, is constituted by us, and thus brings an ideal element to all experiences of objects themselves. 'In it consists the "transcendence" belonging

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to all species of objectivities over against the consciousness of them'[*ibid*., p. 165). Tieszen's characterization of Husserl as constituted Platonist is affirmed by Husserl almost word for word.

The task of transcendental logic is not only to describe but also to reflect on different kinds of evidences. The 'evidence — evidence of every sort — should be reflectively considered, reshaped, analyzed, purified, and improved; and that afterwards it can be, and ought to be, taken as an exemplary pattern, a norm' [*ibid*., p. 176]. The reflected and refined evidences thus yield norms pertaining to each level. In a way, Husserl appears to think that we carry out *Besinnung*, sense-investigation, on each level of logic and mathematics to find out what is the essence or *telos* of the level in question. After this we may examine whether our object of investigation is genuine or spurious:

> The formations and universal forms (formations belonging to a higher level), which are 'given' in the activity and are, at first, all that is 'given', must now be 'clarified' reflectively in order that, by clearing up the intentionality that aims at and actualizes its objective sense originaliter, we may rightly apprehend and delimit this sense and secure its identity against all the shiftings and disguisements that may occur when it is aimed at and produced naïvely. … That is to say, we *examine the evidence* awakened by our reflection, *we ask it what it was aiming at and what it acquired*; and, in the evidence belonging to a higher level, we identify and fix, or we trace, the possible variations owing to vacillations of theme that had previously gone unnoticed, and distinguish the corresponding aimings and actualizations, — in other words, the shifting processes of forming concepts that pertain to logic. [*ibid*., pp. 176–177]

In other words, the transcendentally clarified and purified evidence peculiar to each level of logic provides that level with the norms of what the logic of that level should be like.

5. Distinctness, that is, the Givenness of Formal Mathematics

Husserl next continues clearing up the givenness of mathematics in the above mentioned three strata. Transcendental examination of pure mathematics shows that it presupposes idealization of ideal identities, reiteration, 'and so forth', and the law of non-contradiction. These have to be in place, constituted, before we can construct mathematical proofs and theories. Husserl spends the most time on elaborating the first presupposition of the ideal unity of the objectivities in question. In other words pure mathematics presupposes that its objects and formations are unchanging and permanent. They have an identity. We may speak of the same judgment, 'which becomes itself-given in evidence as the same — the same

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that was at first a confused meaning or opinion and then became distinct' [Husserl, 1969, pp. 184–185]. The same judgment exists for us at all times:

> Logic relates, not to what is given only in active evidence, but to the *abiding* formations that have been primally instituted in active evidence and *can be reactivated and identified again and again*; it relates to them as objectivities which are henceforth *at hand*, with which, taking hold of them again, one can operate in thinking, and which as the same, one can further shape categorially into more and more new formations. At each level they have their manner of evident identifiability; at each they can be made distinct, can be united in evidently consistent or evidently inconsistent complexes; out of them, by cancellation of inconsistencies or by suitable transformation, purely consistent complexes can be produced. [Husserl, 1969, pp. 185–186]

The objects of mathematics are constituted as transcendent to the current living evidence in which they are actually given. Moreover, the possibility of expressing verbally the constituted objectivities is also presupposed. According to Husserl 'verbal expression, …, is an essential presupposition for intersubjective thinking and for an intersubjectivity of the theory accepted as ideally existing' [*ibid*., p. 188]. Thus intersubjectivity and speech are continuously presupposed throughout his discussion.

Another idealizing presupposition in pure mathematics is re-iteration, 'the fundamental form, "and so forth"', that is the thought that one can form infinite series. Yet another presupposition is the law of non-contradiction, which in its subjective formulation is: 'Of two judgments that (immediately or mediately) contradict one another, only one can be accepted by any judger whatever in a proper or distinct unitary judging' [*ibid*., p. 190]. When regarded objectively it is a proposition about ideal mathematical 'existence' and coexistence as distinct [*ibid*., p. 190]. Husserl ends his discussion of the evidences related to pure mathematics by repeating that the evidence related to the grammar (theory of forms) and the evidence related to mathematical analytics together form the foundations for analytics, that is mathematics and formal logic:

> All these evidences, with the essential structures belonging to them, must be explicated as functioning together in the subjective and hidden 'methods' of intentionally constituting the various ideal unities and connexions that join the theory of forms with consequence-theory to make up the unity of mathematical analytics. All the subjective structures have an *Apriori* pertaining to their function. All of them must be brought out; and on the basis of a clear self-understanding, this *Apriori* must be consciously fashioned, to

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> become the originarily clear method for a radically legitimized theory of forms and a full analytics legitimately grounded in such a theory — an analytics for which there can be no paradoxes and the legitimate applicational sense of which must be beyond question. [Husserl, 1969, p. 191]

Note that Husserl discusses here all mathematics, not only the intuitionistically given part of it, contradicting van Atten's claim that '[t]ranscendental phenomenology cannot provide a foundation for a pure mathematics that would go beyond intuitionism' [2010, p. 43]. However, van Atten also thinks that the evidence that characterizes intuitionism is not distinctness as discussed here but the kind that pertains to the logic of truth and to formal ontology. I will discuss the logic of truth below.

6. Evidences that Pertain to the Logic of Truth

So much about the idealizing presuppositions of pure mathematics. When the considerations of truth are added to it, mathematics acquires its 'logical function' as a theory of science. 'Pure mathematical analytics then becomes, as we have said, what is properly an analytic theory of science or the equivalent of such a theory, a "formal ontology"' [Husserl, 1969, p. 191]. The evidence related to the principles of logic such as the law of contradiction relates to the 'evidential creating of the concepts of truth and falsity'. This means, that the judgment can be confronted with the 'affairs themselves', so that the states of affairs themselves fulfill the judgment or else show it to be false. Likewise for example purely analytic consequence, subjectively formulated as 'The possibility of distinct evidence of that analytic antecedent judgment necessarily entails the possibility of such evidence of the consequent judgment' [*ibid*., p. 195] turns into *modus ponens* of the form 'when the syntactical (categorial) actions involved in judging the antecedent are performed on the basis of originality of "the affairs themselves" (on the basis of "experience"), the same possibility of *material evidence* must exist also for the actions involved in judging the consequent' [*ibid*., p. 195].

What is notable here is that the truth requires the possibility of material evidence, and the logical laws become laws of possible material truth. As a 'transitional link' between pure-logic of non-contradiction and the truth-logic, Husserl discusses a reduction of pure analytics to ultimate something-meanings [*ibid*., p. 202]. The analytic reduction, '*by following up the meanings*' [*ibid*., p. 203], shows that analytics is based on ultimate something-meanings.

> For *mathesis universalis*, as formal mathematics, these ultimates have no particular interest. Quite the contrary for *truth-logic*: because

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> ultimate substrate-objects are *individuals*, about which very much can be said in formal truth, and *back to which all truth ultimately relates*. . . . In analytic logic one can go so far, and only so far, as to say that, in the sense, there must be certain sense-elements as the ultimate core-stuffs in all syntactical forms, and that one is brought back to judgment-complexes of ultimate judgments having 'individual' substrates. *Analytically one can assert nothing about the possibility or the essential structure of individuals*. [*ibid*., pp. 203–204]

Husserl next explains how there is a corresponding reduction of truths:

> . . . of the truths belonging to a higher level to those belonging on the *lowest level*, that is: to truths that relate directly to their matters and material spheres, or . . . that relate directly *to individual objects* in their object spheres — individual objects, objects that therefore contain within themselves no judgment-syntaxes and that, in their experienceable factual being, are *prior to all judging*. [*ibid*., p. 204]

This shows, according to Husserl, that

> *every conceivable judgment ultimately . . . has relation to individual objects* (in an extremely broad sense, real objects), and therefore has *relation to a real universe*, a 'world' or a *world-province*, 'for which it holds good'. [*ibid*., p. 204]

Thus the truth-logic is, as Husserl puts it elsewhere, 'logic of the world' [Husserl, 1969, p. 291; 1973, p. 40]; it is ultimately the logic of our experience of the mundane worldly entities [Husserl, 1969, pp. 201, 243–244]. When logic is related to the worldly entities by means of categorial intuition it obtains its function as formal ontology (*cf*. [Lohmar, 2000, p. 111]). Husserl also writes that even though the numbers and multiplicities are formal-analytic universalities, their *sense* involves 'possible application to arbitrarily selectable objects with material content' [1969, p. 205].

Husserl discusses at length the presuppositions and evidences related to the truth-logic. I will here mention only one: the truth-logic has a fundamental presupposition that every judgment in itself can be decided, even though, for us, most of the judgments that are somehow possible can never be evidently decided in fact [1969, pp. 197–198]. This is a fundamental belief, constituted by us, that characterizes scientific activity. It also shows our constituted scientific realism.

Since truth-logic seeks material evidence for the fulfillment of its claims, mathematics within truth-logic has to be something applicable. Husserl's example in *FTL* is from geometry: Riemannian geometry in general is characterized by the evidence of distinctness; within it the Euclidean

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manifold can gain evidence of clarity. Mathematics within truth-logic appears to consist of acts founded on perception of individuals. Thus for example intuiting the conjunction of *A* and *B* is perceiving the *A* and *B* as a whole in a unified form.

Husserl's discussion of evidence related to truth-logic summarizes many of his earlier results: preliminarily it consists of rather simple acts of counting, adding, and collecting, and of what follows from these by means of *modus ponens* thus recalling his early investigations into authentically given parts of mathematics in the *Philosophy of Arithmetic* (1891). Not surprisingly Husserl also refers to the *Philosophy of Arithmetic* in *FTL*, saying that the first part of the *Philosophy of Arithmetic* amounted to a phenomenologico-constitutional investigation [1969, pp. 86–87]. Husserl also incorporates his doctrine of categorial intuition and truth from the *Logical Investigations* into it. The applicability of truth-logic is a reversed way of explaining that it has been obtained from intuition of concrete objects and states of affairs by free variation. Thus Husserl works his doctrine of eidetic intuition into the truth-logic. In *FTL* Husserl in addition takes into account the whole tradition from Plato onwards and further specifies the idealizations that take place in the constitution of the theoretical world. Due to these idealizations, empirically applicable mathematics applies not to the everyday-life world but to an idealized world of exact measures and ideally straight lines, *etc*. [1969, p. 292].

7. Phenomenology of Mathematics

7.1. *Mathesis Universalis*

How then does Husserl's view stand in relation to the views mentioned at the beginning? Where does he stand within the field of philosophy of mathematics? As we have seen, the basic tenet of phenomenological description is antirevisionist. In the *Prolegomena* (1900), Husserl thought that mathematicians are seeking a theory of theories in which individual axiomatic theories can be compared and ordered. He lists as partial realizations of this ideal Riemann's theory of manifolds, Grassmann's extension theory, Lie's theory of transformation groups, Hamilton's quaternions, and Cantor's set theory. Also in his later writings Husserl thinks that mathematics has developed freely, even though guided by the Euclidean ideal (which he had discussed in more detail in his *Definitheit* double lecture in Göttingen in 1901) [Husserl, 1969, pp. 94–97]. In his characterizations of formal mathematics and logic Husserl is much indebted to the algebraic tradition starting with Leibniz and Vieta and continued by Boole and Schröder [Husserl, 1969, pp. 73–75]. Furthermore, a characteristic of modern mathematics, according to him, is that the system-forms themselves can be subjected to mathematical treatment. Because of this he holds Riemann to be

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the 'founder' of modern mathematics [Husserl, 1969, p. 93]. Husserl still has in mind the idea of an all-embracing task: 'to strive toward a highest theory, which would comprise all possible forms of theories…' [Husserl, 1969, p. 98]. This is the theory of manifolds mentioned above.

In *FTL* Husserl is more sensitive to the normative issues and for example to the fact that there should be no paradoxes in mathematics. The philosophers' task is to clarify the sense of formal mathematics within the sense of logic, and then reflect on the evidences that pertain to it. Husserl's basic assumption is not to restrict mathematics to any part of it, but to examine whether it is genuine in terms of the evidence that belongs to its sense. This demands an account of the constitution of the kind of mathematics in question. Husserl explicitly says that the ideal elements transcend our experience, and thus we have to examine the constitution of them as independently existing objects. As already noted above, Tieszen's constitutive platonism is an accurate description of phenomenology of *mathesis universalis*.

More specifically, given Husserl's indebtedness to the algebraic tradition from early on, I think his conception of 'mathematics of mathematicians' should be labeled 'constitutive structuralism'. For example, in the *Logical Investigations* Husserl expressed in detail views that are unambiguously structuralist. According to him, mathematics is about abstract structures. Even the concrete sciences derive their 'theoretical stock' from such abstract sciences. Even in concrete sciences when 'our purely theoretical interest sets the tone, the single individual and the empirical connection do not count intrinsically, or they count only as a methodological point of passage in the construction of a general theory' [2001b, p. 148]. A theoretical interest, to Husserl, means seeing things structured in a certain way. The task of logic is to uncover the pure *a priori* form, the structure, that is common to all theoretical systems.

Structuralist leanings can be found also in *Formal and Transcendental Logic* where Husserl discusses the importance of the algebraic approach to the development of *mathesis universalis* and explains how formal mathematics is formal in the sense of having as fundamental concepts 'derivative formations of anything-whatever' [1969, p. 77]. Moreover, Husserl's structuralism explains the distinction between 'mathematics of mathematicians' and the logic of truth: mathematics of mathematicians is characterized by distinctness that arises from the coherence of the theories involved, not from intuition of concrete objects. For formal mathematics '*there can be no cognitional considerations other than those of "non-contradiction"*, of immediate or mediate *analytic consequence or inconsistency*, which manifestly include all questions of *mathematical "existence"* ', Husserl writes [1969, p. 140]. Thus the only access we have to the mathematical objects is in terms of the structure that defines them. This is also the reason why pure mathematics of contradiction cannot be called a formal

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ontology. Instead, '[i]t is an ontology of pure judgments *as senses* and, more particularly, an ontology of the *forms* belonging to non-contradictory — and, in that sense, possible — senses: possible in distinct evidence' [*ibid*., p. 144].

Thus I think there are grounds to call Husserl a 'constitutive structuralist' and in line with Tieszen, perhaps even a 'constitutive *ante rem* structuralist' about mathematics. The task of phenomenology is then to uncover the way in which the mind-independence of mathematical entities is constituted.

The present approach is thus mainly in agreement with Tieszen's views when looking at what Husserl called 'mathematics of mathematicians'. However, what I want to add to Tieszen's approach is that the descriptiveness of the phenomenological method admits that different parts of mathematics may be given in different kinds of evidence. It is a fact that mathematics is given in a variety of ways, and the truly descriptive approach has to be able to accommodate the possible differences in the givenness of various parts. I will next consider Husserl's logic of truth.

7.2. *The Logic of Truth and the Clear and Distinct Experiences*

Husserl's logic of truth suggests that some parts of mathematics can be given in evidence of distinctness and clarity. This is the part of mathematics that we can directly experience to be true. The logic of truth takes into account the contents of judgments, so as to rule out the material countersense. It is the applicable part of mathematics and it can be obtained by free variation of intuition of concrete objects or states of affairs. Thus it is about the real world. However, as mentioned above, in *FTL* Husserl further specifies that the theories that belong to truth-logic are idealized, so that for example geometry is about ideal straight lines, circles, and so forth, instead of actual and possible appearances. Exact geometry as well as exact Galilean physics are accordingly regulative ideas, norms [Husserl, 1969, pp. 243, 292].

Husserl's logic of truth, however, has been identified with Brouwer's intuitionism. Mark van Atten's fundamental point about Brouwerian intuitionism and Husserl's logic of truth resides in the concept of evidence at work in both of them. Indeed the immediacy of the experience of truth in Husserl's account of truth is similar to what the intuitionists aim at. However, if we want to see intuitionistic features in Husserl's logic of truth, I think it is more fecund to compare it to Per Martin-Löf's intuitionistic type theory rather than to Brouwer's views. One reason for this is that to Brower mathematics is essentially a languageless activity while Husserl presupposes verbal expression. Moreover, Martin-Löf's views are easily applicable to the Husserlian framework mainly because in both the fundamental form of judgment is the old *S* is *P* form. The

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fundamental form of judgment shows how the judgments are ultimately about individuals. Moreover, the type-theoretical universe is sorted as it is for Husserl. Martin-Löf's intuitionistic type theory thus appears to be close to what Husserl refers to as a 'transitional link' between truth-logic and consequence-logic, the analytic reduction to the judgments about individuals.

Martin-Löf's analysis of truth is also easy to relate to what Husserl is doing. Whereas on Husserl's analysis a true judgment requires having the states of affairs themselves, on Martin-Löf's analysis the truth-maker is the proof. In place of Husserl's view of truth as an experience of agreement between affairs themselves and the proposition, Martin-Löf discusses truth in terms of a proof and a proposition. Husserl's having affairs themselves then translates to having an actual proof for a proposition.

As I said above, in *FTL* Husserl is primarily thinking of logic as a theory of science, and thus it has to be applicable to the real world, albeit idealized as explained above. Husserl's examples are Euclidean geometry and Galilean physics. However, by means of intuitionistic type theory one could demarcate a logic of 'formal' truth, to which analogous evidence applies as to Husserl's worldly logic of truth. Besides, independently of the question whether Husserl himself had such a theory in mind or not, his phenomenological method should allow examining and clarifying it. We should notice also that within mathematics there are different types of evidence at work, and an intuitionistically given part of mathematics is given in a different way from say, set theory, and its 'higher flights' as Quine would put it. Therefore, while Husserl does not seem to be talking about intuitionistic mathematics, I think it is fully legitimate and, indeed, an interesting project to discuss intuitionistic mathematics and the evidence that pertains to it from the phenomenological point of view.[2]

[2] One may raise a question about what to do in case of conflicting results in, say, classical and intuitionistic real analysis. The situation can be analyzed in two ways: either the two results are given in distinctness with formal mathematical sense, but within different formal mathematical systems with different sets of axioms and inference rules. This is entirely possible on a basis of the way in which Husserl defines the level of non-contradiction. It is also possible that the two systems are given with different senses, so that one is given purely mathematically and the other is taken as true, that is, as holding in the world. This is also perfectly possible. Husserl's writings about Copernican and pre-Copernican views suggest that within different attitudes we may hold contradictory claims while phenomenologists' primary task is to uncover their respective senses (*cf.* [Husserl, 1981, pp. 222–233]). The virtue of the Husserlian point of view is that with it we may explicate the kinds of norms and evidences demanded in the two approaches so that the situation is better understood. In this respect, Husserl's approach is not unlike that of Wittgenstein. However whereas for Wittgenstein there may be contradictions in a mathematical system (see for example his [1978, p. 119]), for Husserl a formal mathematical system cannot be contradictory: non-contradictoriness is a defining feature of a system to be formal mathematical.

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7.3. *Husserl and the Games-Meanings*

In addition to the above mentioned 'mathematics of mathematicians' that 'has its own legitimacy', and the logic of truth that is an ultimate norm for the sciences, Husserl distinguishes what he calls games-meanings, *Spielbedeutungen*, that given signs have by virtue of the rules of a game of which they are part. The rules give the signs games-meanings. Such symbolic thinking has a new intentional act, and thus symbolic thinking should be distinguished from arithmetical thought:

> We do not therefore operate with *meaningless signs* in fields of symbolic arithmetical thought and calculation. For mere signs, in the sense of *physical* signs bereft of all meaning, do duty for the same signs alive with arithmetical meaning: it is rather that signs taken in a certain *operational or games-sense* do duty for the same signs in full *arithmetical meaningfulness*. [Husserl, 2001b, p. 210]

These games-meanings capture yet another way with which we may relate to mathematics. Indeed, much of mathematics is given as games-meanings, *i.e.*, as purely combinatorial results of calculations, without any deeper sense to us. Husserl's descriptive approach can accommodate this too, and it is interesting how he thereby distinguishes arithmetical and purely operational meanings, in other words, between structuralist and formalist attitudes. In his view, the difference is that the formalist is focused only on signs and 'blind' operations on them, whereas the structuralist's attitude involves understanding what the signs refer to, even if the references are to purely formal objects implicitly defined by the axiomatic system.

Contrary to his discussion on formal mathematics Husserl's wording about the formalistic games is not entirely descriptive; he suggests that we should avoid relating to mathematics as if it were mere deductive games on symbols. Instead he holds that we should keep in mind the objects about which we are talking. He writes,

> [W]e must not define merely in terms of signs and calculational operations — for example: 'It shall be allowed to manipulate the given signs in such a manner that the sign $b + a$ can always be substituted for $a + b$'. Rather we must say: 'There shall obtain among the *objects* belonging to the multiplicity (conceived at first as only empty Somethings, 'Objects of thinking') a certain *combination-form* with the *law*-form $a + b = b + a$' — where *equality* has precisely the sense of actual equality, such as belongs to the categorial logical forms. [1969, p. 100]

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Thus, contrary to his descriptivist attitude, Husserl appears to think that blind calculations are not desirable, but they should be interpreted to be about something. The sense of formal mathematics is not to consist in mere calculations. In other words, the formalistic approach towards mathematics is not in Husserl's view 'genuine'. Nevertheless, today much of mathematics is given with the aid of computers in such a formalistic manner. Husserl's description of the givenness of mathematics as formalistic games is perhaps even more pertinent today than during Husserl's lifetime.

7.4. *Intermediate Conclusion*

In conclusion, in Husserl's texts one can identify at least three ways in which mathematics is given: most of it, 'mathematics of mathematicians', is given as purely formal, non-contradictory theories that are characterized by the evidence of distinctness. Second, some of it coincides with the logic of truth, that is, refers to the part of mathematics and logic that is actually applicable to the world. Third, some of mathematics is given as mere calculation on symbols, as empty formalistic games. In addition, as I only hinted above, Husserl also discusses the evidence related to the broadest conception of judgment and the level of grammar. The level of grammar and the evidence related to it are presupposed in all the other levels of mathematics; thus it characterizes a layer of norms embedded in all of mathematics. Statements in mathematics have to be grammatically correct.

What this shows is that when describing mathematics with a phenomenological method, we need not assume that mathematics is a monolithic whole, but different kinds of ways of givenness and sets of norms may pertain to it. Thus mathematics is not only given as constitutive platonism but also in other ways and with other sets of norms for correctness. Nothing precludes our distinguishing even more sets of norms related to it than what are discussed above or than what Husserl ever thought of.

Indeed, the differences between philosophies of mathematics are typically differences in the norms considered to be right for mathematics. In other words, the difference between these approaches is in the normative ideal of what mathematics should be (a somewhat similar claim can be found in [Tragesser, 1973, p. 293]). Phenomenology can distinguish and differentiate among these approaches. Because of that, it clarifies the different kinds of understanding involved in mathematics.

8. Practice of Mathematics

In the final part of this paper I will turn to Rota's view of the phenomenology of mathematical practice. Rota's approach anticipates the recent, increasingly popular, trend in philosophy of mathematics, in which

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epistemology of mathematics is extended to include the practice of doing mathematics. In it the general idea is that by focusing on the practice of mathematics one may acquire a much richer view of the epistemology of mathematics compared to single-minded focusing on the Benacerrafian problem of access to abstract objects. Studies in this tradition aim to reflect on different aspects of mathematical practice such as an epistemic role that visualization may have in mathematics, what kind of explanation is sought for in mathematics, how the ideal of purity of methods plays a role in mathematical practice, *etc*. The shift in focus is analogous to the trend in the philosophy of natural sciences where both historical and contemporary case studies have broadened philosophers' interest beyond the traditional epistemological problems. (*Cf.* [Mancosu, 2008].)

From early on, Husserl is interested in both the objects of our intentionality as well as the acts in which the objects are constituted. Accordingly he holds that the elucidation of the theory of the science of logic has to be two-fold: 'subjectively directed toward the activity of knowing and, on the other hand, objectively directed toward theory' [2001a, p. 29]. Phenomenological investigations address both the object as well as the acts, or as Husserl later calls them 'the accomplishments of the consciousness', in which the theory becomes objective. Thus phenomenology of mathematics should be at least as much about mathematical practice as about abstract structures. This would amount to a description of what Husserl calls active syntheses. Strictly speaking proving a theorem is, according to Husserl, active synthesis; the proved theorem however sinks into passivity. While proving something our mathematical knowledge resides passively in the background, but available to us, so that we may for example decide to use a strategy of proof that we know has already been used somewhere else.

However, Husserl held that we should also describe the normative ideals at work in mathematics. Rota does not seem to pay much attention to the normative nature of mathematical activity and expresses views rather to the contrary. Consequently, according to Rota, the strictly descriptive attitude may lead to unpleasant discoveries:

> … for example, one might be led to the realization that no features whatsoever are shared by all mathematical proofs. Or else, one may be led to admit that contradictions are part of the reality of mathematics, side by side with truth. [Rota, 1997a, p. 184]

This is not what Husserl would think. Husserl thinks that mathematics is guided by the normative ideal of non-contradiction. This is the reason that he thinks that in the phenomenologically clarified theory there should be no paradoxes. While mathematical practice may be contradictory and guided by all kinds of psychological motives, this is not how Husserl thinks that mathematicians think the situation should be. Rota

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thus dismisses this normative ideal in his description of mathematical practice.

Then again, Heidegger notoriously criticized Husserl claiming that it is impossible to approach anything without presuppositions. Indeed, Gian-Carlo Rota's practice-oriented description can be viewed as revealing Husserl's prejudice against the axiomatic ideal of theories. There is no final answer to this dilemma. Whether the normative ideals are indeed normative ideals or whether they are unjustified prejudices is a question that has to be continuously reflected upon in the process of renewing our view of the essence of mathematics and logic.

9. Conclusion

Husserl's description of logic as a theory of science, described above, divides logic into three levels. In so doing, Husserl distinguishes among three different kinds of normativity: the first level describes what is right from the point of view of grammar, the second what is correct and incorrect in terms of coherence of theories, and finally the third level adds the consideration of truth to the logic. The same holds of mathematics. Instead of attempting to reduce mathematics to anything more primary, Husserl clarifies and distinguishes different kinds of senses or evidences and thus different sets of norms, with which different parts of mathematics are given. Thus Husserl distinguishes between (1) structuralist 'mathematics of mathematicians', (2) empirically applicable parts of mathematics, which have an account of evidence analogous to the intuitionist's conception of evidence. In addition, from his texts we can discern also (3) the blind following of the rules of the formalistic or purely computational approach. There are no reasons to think that these three kinds of understanding of mathematics are the only ones of interest. The descriptive phenomenological method could show us further ways in which mathematics is given. The aim in phenomenology is to describe mathematics, and the norms in it, in their factual diversity. The phenomenological clarification means examination of how mathematical knowledge in question is constituted, and whether the basic concepts of the approach in question fulfil the normative demands that it has set to itself.

Therefore:

> *[O]nly a science clarified and justified transcendentally (in the phenomenological sense) can be an ultimate science; only a transcendentally-phenomenologically clarified world can be an ultimately understood world; only a transcendental logic can be an ultimate theory of science, an ultimate, deepest, and most universal, theory of the principles and norms of all the sciences.* [Husserl, 1969, p. 16]

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