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Hegel on the Category of Quantity

Stephen Houlgate

In the first *Critique* Kant famously derives the categories from the logical functions of judgement.[1] Yet he also states that we cannot explain 'why we have precisely these and no other functions for judgment' (any more than we can explain 'why space and time are the sole forms of our possible intuition') (*CPR* B 146). The reason why we employ precisely these and no other *categories* thus ultimately remains inexplicable. This is not to deny that in his discussion of the 'principles of pure understanding' Kant tries to prove that specific categories, such as substance and causality, are the conditions of the objects of experience; but he never explains why we should think in terms of these categories in the first place, because he never explains why the logical functions from which they are derived are necessary.

Hegel follows Fichte, however, in wanting to exhibit the categories 'in their *necessity*', and not to leave them unexplained as Kant does.[2] To this end, he argues, the categories must be derived, not from presupposed forms of judgement, or indeed from any other presuppositions, but from what he calls the sheer 'simplicity of thinking'.[3] That is to say, the categories must be derived from the indeterminate being of thought, or—which in Hegel's view amounts to the same thing—from the indeterminate thought of *being*.[4]

Philosophy after Kant should thus not start from the idea that we judge in certain ways, or indeed that we judge at all. It should start by focussing simply on thought's indeterminate being, and it should seek to discover which categories, if any, are inherent in such being. Only in this way, Hegel believes, can we avoid making unwarranted assumptions about thought and show which categories are truly necessary.[5]

Hegel's presuppositionless derivation of the categories is, of course, carried out in his speculative logic. It is not my intention here, however, to consider the coherence or plausibility of that logic as such. I want, rather, to consider the account that emerges therein of one specific set of categories, namely those of *quantity*. In so doing, I also want to highlight certain differences and similarities between the conceptions of quantity entertained by Hegel and Kant.

In the first *Critique* Kant puts quantity at the head of his table of categories and sees no intrinsic logical connection between quantity and quality (*CPR* A 80/B 106).

By contrast, Hegel first derives the categories of quality and then shows how quality itself leads logically to quantity. In this essay I shall trace the logical development of quantity, as Hegel understands it. My account is largely expository, but it will show, I hope, that from Hegel's point of view quality and quantity are logically interconnected in ways that Kant fails to recognize.

I. From Quality to Quantity

For Hegel there is a clear difference between quality and quantity. A qualitative determination constitutes what something *is*; when this is altered, therefore, the thing becomes something else. A quantitative determination, by contrast, can be changed without changing the thing itself: 'a red, whether brighter or paler, is still red'.[6] Yet despite this difference, Hegel thinks that quantity is made necessary by quality. More specifically, it is made necessary by the qualitative category of the *one* (*das Eins*).

In the initial stages of his account of quality Hegel distinguishes between 'determinate being' (*Dasein*) and 'something' (*Etwas*). *Dasein* is rendered determinate by simple negation: it is being that is this, *not* that. Such being thus never stands alone, but always distinguishes itself from what it is not, from its negation (see *SL* 111/*LS* 105-6). The structure of *something* is subtly different: to be something is not just to be one side of a difference—this, *not* that—but to be *self-relating* being. It is to be something of one's own (*SL* 115/*LS* 110). Yet something also stands in relation to its negation. The negation of something, however, exhibits the same self-relation as something. Accordingly, it stands apart from something as the latter's *other* (*Anderes*) (*SL* 116/*LS* 111). There is thus a difference, in Hegel's view, between being the simple, determinate *negation* of X and being *other* than X: the latter consists, not just in not being X, in differing from X, but also in being something of its own and relating to itself.[7]

In the category of the *one* (*das Eins*), being determinate and being self-relating coincide completely, and in this sense quality finds its fullest expression in being one. The one has a determinate identity: it is the one, *not* anything else. It excludes everything else to such a degree, however, that there is in fact nothing outside it at all. Unlike something, therefore, the one (at least initially) has no other; it hangs alone in the *void* (*das Leere*) and in this way is purely self-relating being, or sheer being-for-self (*SL* 165/*LS* 168-9).[8]

Hegel goes on to show, however, that this void is itself another empty *one*. The one thus stands in relation to *another* one after all—and so does that one, and that one, ad infinitum. The one does not just hang alone in the void, therefore, but is one of *many* ones. Yet in this world of many ones each is the same empty one: there is no qualitative difference between them. This introduces a new

logical structure: for in the other ones around it each one encounters and relates to nothing but *itself*. This in turn means that the one *continues* beyond itself in the others.

We might think that *something* (*Etwas*) continues beyond itself, too, insofar as it relates to another that is itself something. That other, however, is qualitatively different from the first something, precisely because it is *other* than it. Every *one*, by contrast, is the very same one: each is the same empty unit. It is only with the one, therefore, that we encounter entities between which there is no qualitative difference at all. Accordingly, only at this point does the idea emerge of being that continues beyond itself; and so only now do we make the transition from quality to a new form of being.

Now in the sphere of quality the *limit* (*Grenze*) is where something ceases and another begins (*SL* 126-7/*LS* 122-3). With the one, however, we encounter a new form of being that continues beyond its own limit. Where one one ends and another begins, the one itself continues without interruption. In this sense, Hegel argues, the limit that differentiates one one from another is a limit to which the one as such is indifferent. Such a limit to which being has become indifferent is a *quantitative* limit. The new form of being that arises with the one is thus quantity.

A full account of the transition from quality to quantity would have to examine the relation between the 'repulsion' and 'attraction' of the many ones (see *SL* 174-8/*LS* 179-84).[9] It is clear from Hegel's text, however, that this transition is already made necessary 'in principle' (*an sich*) by the one as such (*SL* 176/*LS* 182). For our purposes, therefore, it suffices to say simply that quality gives rise to quantity via the category of the *one*.

For Hegel, in contrast to Kant, quality and quantity are thus not just two distinct categories; they are related logically because the former makes the latter necessary. Indeed, quantity is itself a distinctive kind of quality: namely, that which continues beyond any limit that is placed on it. This in turn enables a *thing* with a specific limited magnitude to go beyond that magnitude, and so to get bigger or smaller, without ceasing to be. The familiar fact that magnitudes can be increased or decreased is thus not just an unexplained given, but one that is made intelligible by the derivation of quantity from quality.

To recapitulate: quality consists in being what one is, in being oneself. Yet it is also bound to what is other than it: hence being-in-itself (*Ansichsein*) is inseparable from being other-related (*Sein-für-Anderes*) (see SL 119-20/LS 114-16).[10] The one (*das Eins*), however, is purely self-relating: it is just the one it is and so is purely for itself. Indeed, even the others to which it relates are simply further versions of itself: the one is surrounded, not by something *other* than the one, but simply by many other *ones*.

The one, therefore, as purely self-relating being, is the most developed form of quality: it is being-oneself taken to its furthest extreme. Its self-relation,

however, is profoundly ambiguous: for it is and relates to nothing but itself, yet in so doing it actually continues *beyond* itself. Insofar as it extends beyond itself, or outside itself, however, it is no longer simply *itself*, and so is no longer purely qualitative, after all. Rather, it gives rise to a new form of being: quantity. We will now consider how Hegel derives further categories from the simple idea of quantity as such.

II. Quantity as such

As we have seen, quantity is characterized by *continuity*—by the continuing of the one beyond itself in other ones (*SL* 187/*LS* 194-5). As such, quantity necessarily contains many ones or units within itself. These ones are themselves purely self-relating, and so are indifferent and external to one another: they are thus quite *discrete* units. At the same time, however, they are all identical: each is the same discrete one. In their very discreteness, therefore, they constitute a continuity: the uninterrupted continuing *of* the discrete one itself. Quantity, for Hegel, is thus 'the unity of these moments of continuity and discreteness' (*SL* 187/*LS* 195).

Hegel notes, however, that quantity also divides itself into the *distinct* forms of continuous and discrete magnitude. Where the unity of its two moments is immediate, and so directly to the fore, quantity is itself dominated by unity and continuity. It is thus explicitly continuous, uninterrupted quantity. This is how, for example, the extension of space is to be conceived. Yet where the unity of continuity and discreteness is *not* immediate, but mediated by their difference, the latter is to the fore and so quantity is itself dominated by difference and discreteness. As such, it is not explicitly continuous, but is broken up into discrete units. Such quantity is thus a discontinuous 'plurality' (*Menge*) (*SL* 200/*LS* 210). This is how we conceive of quantity when we ask *how many* Xs something contains.

Continuous and discrete quantity or magnitude are not, however, absolutely distinct. Continuous magnitude is the continuity of discrete units and so can be interrupted at any point; and the units belonging to discrete magnitude themselves form a continuous unity. Each form of quantity is thus the same unity of continuity and discreteness; but in each just *one* of the two moments is, in Hegel's word, '*posited*' as the principal characteristic (*SL* 200/*LS* 210).

Here we see an important logical difference between quantity and quality. Qualitative categories, such as something and other, pass over into one another and so prove to be inseparable: something is itself *other* than its other, and the latter is in turn *something*. Hegel notes, however, that 'in the sphere of quality, the differences […] also retain the form of immediate, qualitative being relative to one another' (*SL* 123/*LS* 119). Something may, indeed, be *other* than its other,

but being 'something' and being 'other' are nonetheless immediately different categories: each, we might say, is immediately *itself*.

In the sphere of quantity, by contrast, no category is immediately itself and immediately different from its negation. Continuity is already *in itself* the continuity of the discrete, and discreteness is *in itself* continuous discreteness. Quantitative determinations do not just pass over *into* one another, therefore, but each contains its opposite within itself. In this sense, one can say that continuity and discreteness both *continue* in their opposites. Both are thus infinite, unending, rather than finite, limited determinations.[11]

Insofar as continuity and discreteness are moments of one another, quantity clearly anticipates the logical structure of essence. In the sphere of essence, however, a determination, such as the positive, incorporates its opposite, the negative, as something *negated*. The positive thus includes the negative as *excluded* from it (see *SL* 426-7).[12] This is what Hegel means by saying that determinations of essence merely 'shine' (*scheinen*) in their opposites.[13] In quantity, by contrast, continuity is contained *affirmatively* in discreteness and vice versa. Categories of essence are thus above all the negation of their own negation, and in that sense lack any simple immediacy of their own. Quantitative categories, however, retain a relatively immediate sense, even in their lack of immediacy and independence, since each is affirmatively present as itself in the other. So although quantity does indeed anticipate the sphere of essence, it remains, with quality and measure, a moment of being.

III. Quantum

Now discrete magnitude, as we have seen, comprises a plurality of units: it is the kind of magnitude that prompts us to ask 'how many?' This plurality, however, is not just a loose, unconnected multiplicity, but forms a continuous unity. As such a unity, however, it remains characterized by *discreteness*. In Hegel's view, this means two things. On the one hand, such magnitude encompasses discrete units within itself; on the other hand, however, it is also discrete *as a unity*. That is to say, the discrete units *in* discrete magnitude form a unity that is *itself* discrete—a unity that is, as it were, broken off at either end. Discrete magnitude is not just a general kind of magnitude, therefore, but it takes the form of a single discrete unit, or '*a* magnitude' (eine *Größe*) (*SL* 201/*LS* 211).

Since this unit is broken off at either end, it is determinately different from other such units. Indeed, it is the *limit* of such units: where they stop and it begins. The single, discrete unit formed by discrete magnitude must, therefore, be a limited magnitude that excludes other such magnitudes. Accordingly, it is what Hegel calls a *quantum*. A quantum is thus nothing but 'quantity with a

determinateness or limit' (*SL* 202/*LS* 212). It is quantity, not just as continuous discreteness as such, but as *a* discrete, limited quantity.

Note that Hegel derives the quantum from discrete magnitude in particular. Discreteness, however, is to be found in both continuous and discrete magnitude. Both kinds of magnitude, therefore, must take, or be capable of taking, the form of quanta. According to Hegel, there is no quantity anywhere that is completely separate from quanta.[14]

IV. Number

The principal features of the quantum, in Hegel's view, are (a) that it is a discrete unit of quantity, a one, and (b) that it is explicitly determinate and limited. Indeed, it is both of these things together: determinate in being a one. Yet being a one is not enough by itself to render the quantum fully determinate, for in this respect all quanta are the same. Something else is required, therefore, to render a quantum fully determinate and so differentiate it from other quanta. Apart from its being a one, however, there is nothing to a quantum but the 'plurality' (*Vielheit*) of discrete units that it contains. Hegel argues, therefore, that this plurality must be what differentiates two quanta that, as ones, are completely alike.

In order to distinguish a quantum from other quanta, however, this plurality must itself be 'a determinate plurality' (*eine bestimmte Menge*) (*SL* 203/*LS* 214).[15] It must be *this* plurality and *not* that one. Such a *determinate* plurality Hegel calls an 'amount' (*Anzahl*). When a quantum is understood to contain an amount, it is understood in turn to be a *number* (*Zahl*). In contrast to the bare quantum, therefore, the number incorporates not just an undetermined 'plurality' of units, but a definite amount of them, and this amount is what differentiates one number from another. The number, however, is not reducible to the amount it contains, but it is also a continuous unity. For Hegel, therefore, '*amount* [*Anzahl*] and *unity* [*Einheit*] constitute the *moments* of number' (*SL* 203/*LS* 214). Each number is a unity—*one* number—but it is a unity with its own distinctive amount of units.[16]

Hegel notes, however, that the two moments of unity and amount, though united in number, are qualitatively distinct from one another (*SL* 206/*LS* 217). This is because the amount contains the unique determinacy of a number, whereas its unity does *not*. This in turn means that the amount, though falling within the unity of the number, *is not itself a unity*. It is, rather, an aggregate of units that are wholly discrete and external to one another (*SL* 203/*LS* 214). This sheer externality without unity is at odds with the fundamental nature of quantity in which discreteness is inseparable from continuity and unity. Nonetheless, it is

integral to number. Moreover, it is what determines the specific operations that can be performed with numbers.

Since a number contains units that are wholly external to one another, the latter do not belong together by their nature. It is a matter of indifference to those units, therefore, whether they are collected together as this amount or that, as 3 or as 4: the three units in the number 3 could just as easily be put together with a fourth to make the number 4. This indifference to the way in which they are collected in a number means in turn that units can be taken away from or added to numbers, and that we can thus *calculate* with numbers. In Hegel's view, therefore, addition and subtraction are not just contingently connected to number, but operations grounded in the logical structure of number itself (see *SL* 204-12/*LS* 215-24). Mathematics is thus firmly grounded in logic, just as Frege tried to show (albeit, of course, with a very different conception of logic).[17]

Two further things should be noted before we continue the story of quantity. First, Hegel's conception of number, as a unity containing an amount of wholly discrete units, closely matches that which was common among the ancient Greeks. Ivor Bulmer-Thomas, for example, maintains that 'the very first definition of number in the history of Greek mathematics—by Thales according to Iamblichus—was μονάδων σὐστημα, "a collection of units"', and that 'Plato's brightest pupil, Eudoxus, defined number as πλῆθος ὡρισμένον, "a determinate multitude"'.[18] It is evident, therefore, that Hegel's presupposition-less logic will not always produce radically new conceptions of categories; in some cases it may simply demonstrate, without taking it for granted from the start, that the ancients were right all along.

Second, the numbers derived at this point in the *Logic* are the cardinal natural numbers. Fractions are not derived until we reach the idea of a direct ratio between numbers (see *SL* 245, 315-17/*LS* 265-6, 351-3); and irrational numbers cannot be conceived until we reach the idea of a quantitative infinite progress.[19] Hegel is not simply ignoring these kinds of number, therefore, but in both cases more is required to understand them than the bare idea of number as such.

V. Extensive and Intensive Quantum

Hegel has shown that quantity divides itself into discrete units of itself, or quanta, and that the latter must take the further form of numbers. Numbers, for Hegel, are thus not simply conventional distinctions, or fictions, imposed on the world by human beings, but they are essential features of the world itself. Just as the category of quantity belongs of necessity to thought, so quantity itself belongs to the fabric of being. This means that numbers in turn also belong to being: the world and the things in it are intrinsically susceptible to being counted. And yet it

remains true that quantity, and thus number as well, is that to which things are indifferent (at least to a certain degree). Thus, although it belongs to being a thing to have magnitude and to be countable, things do not come with just one number attached to them: I can increase my weight or lose my hair and still remain who I am. As we learn in the section on measure, there is a range of numbers within which a thing has to be confined in order to be what it is; within that range, however, the thing is essentially indifferent to its magnitude and number. Number, for Hegel, is thus a peculiar hybrid that is both intrinsic to being *and* a matter of indifference to, and so external to, being at one and the same time.

Hegel now shows that quanta and numbers actually come in two different forms: they must be both *extensive* and *intensive*. Let us consider first the extensive quantum.

The quantum as such, before it is a number, is simply a discrete, limited unit of quantity. Yet it is not only a unit itself, but also contains many units within it. When that internal manifold is understood to be a specific amount (*Anzahl*), the quantum is understood to be a number (*Zahl*). As a number with a specific amount the quantum is clearly distinguished from all other quanta: it is unique. Each number, however, remains a unit—remains one number—and in this respect is the same as all the others. Numbers are thus distinguished by their amounts, but they are all alike in being units that contain an amount at all. In this latter respect, Hegel maintains, all equally are *extensive quanta* (*SL* 217/*LS* 230-1).

Note that, though each number is an extensive quantum, being the latter is, logically, subtly different from being the former. The difference lies in the nature of the amount they contain. A number has a specific and thus quite distinct amount—3 *or* 4 *or* 5. To think of a number as an extensive quantum, however, is to disregard the specific amount it contains and to think of it simply as a unit containing *some* amount, just like all the others.

Hegel maintains that this idea of the extensive quantum now leads directly to that of the *intensive* quantum or degree. It has to be said that his argument at this point is especially concentrated, even by his standards, but the gist of it, as I understand it, is this.

A number, as we have seen, is a collection, or what we might call a set, of units. These units are all the same—are all one—and so constitute the unity of the number. Yet they also constitute its amount. The unity and amount of a number are thus constituted by the same set of units. As we remarked earlier, however, the two moments of number are also quite distinct from one another. Insofar as the units constitute a unity, therefore, they do *not* constitute an amount, and vice versa. This in turn means that, although the amount falls within the unity of the number, it is itself a collection of mutually external units *without unity*—units that can thus be subtracted from or added to numbers.

Recall now that an amount is by definition determinate and specific: it comprises 3, 4 or 5 units and thereby confers a unique identity on to the number concerned. Yet, as we have seen, when the number is conceived as an extensive quantum, like all the others, the amount is no longer specific but just *some* unspecified amount. An amount that is no longer specific, however, is not actually an *amount* at all, but merely an indeterminate *aggregate* (*Menge*) of units.

This point is crucial and dramatically transforms the extensive quantum: for if the units in that quantum no longer constitute an amount, they must, by virtue of their homogeneity, form a *unity*, since amount and unity are the only components of the quantum. Furthermore, in ceasing to form an amount, the units also cease being wholly *external* to one another, as they are in the number: in Hegel's words, 'the externality, which constituted the ones as a plurality, vanishes [*verschwindet*]' (SL 218/LS 232). Yet if the units within the quantum are no longer mutually external, the quantum itself can no longer be *extensive*. The unity formed by the units, therefore, must be one *without* inner extension, or a wholly 'simple unity' (*einfache Einheit*).[20] Such a quantum, without an internal sum of mutually external units, Hegel calls *intensive*. Every extensive quantum thus mutates, through its inherent logic, into an intensive quantum or degree.

To repeat: the units in the extensive quantum no longer constitute a specific amount, but just form *some* amount or other. This means, however, that they no longer constitute an *amount* at all, since the latter is by definition specific. Consequently, the units form a unity *as opposed to* an amount. Since an amount comprises units that are external to one another, this unity must *lack* such mutual externality and so be an intensive, rather than extensive, quantum.

As we shall now see, the intensive quantum or degree has a distinctive, and somewhat contradictory, logical structure. Like all quanta, the degree is determinate and limited. We know, however, that a quantum is fully determinate only as a *number*. Yet, as we have just seen, the degree is a simple unity without an internal amount. The degree cannot, therefore, be a number in the sense in which it has been defined so far, but must be what Hegel calls a '*simple* determinacy' (*einfache Bestimmtheit*) (*SL* 219/*LS* 233).[21] Such a number, in other words, must have a definite identity but cannot be a collection of mutually external units within itself. It will thus not be a cardinal number like 3, 4 or 5, but a new kind of number altogether: the 3rd, the 4th or the 5th.

Yet there is a further complication to be considered: for numbers, by definition, are rendered determinate by a specific *amount* of units. Since the degree does not itself contain the amount that gives it its number, Hegel concludes that this amount must fall *outside* the degree and determine it from the outside (*SL* 219/*LS* 233). This does not mean that the tenth degree is surrounded by ten other degrees, but rather that the amount of degrees lying outside it determine *it* to be *the tenth*. The number ten contains ten units within itself; the tenth degree,

by contrast, has its determining ground in the amount of *other* degrees there are. This is true of every degree. Together, therefore, they form a 'scale of degrees' in which each is what it is only by means of the others (*SL* 219/*LS* 234).

The logic of quantity not only makes numbers necessary, therefore, but it also requires that they take two distinct forms. Every number must be extensive and be a cardinal number; but it must also be intensive and so be an ordinal number. The numbers 1, 2, and 3 are thus also to be understood as the first, second, and third in this series. This distinction is not merely a conventional one drawn by us, but belongs, in Hegel's view, to the very nature of numbers and quanta themselves. Quantity, for Hegel, necessarily comes both in definite amounts and in degrees.

VI. Kant and Hegel on Quantity

At this point I interrupt my account of Hegel's conception of quantity in order to compare the latter briefly with the conception of quantity put forward by Kant. The following comments are sketchy, but they will, I hope, highlight some significant similarities and differences between the two philosophers.

1) Categories, for Hegel, are to be considered by themselves: they have their own intrinsic logical structure. For Kant, by contrast, categories should be thought in relation to sensuous *intuition*. This is because they are nothing but the 'functions for judging, insofar as the manifold of a given intuition is determined with regard to them' (*CPR* B 143; see also *CPR* B128 and A 245). More precisely, categories are pure concepts through which 'the synthetic unity of the manifold in intuition' is thought (*CPR* A 79/B 105). Yet Kant also accepts that we can consider a category in abstraction from intuition, though 'one can have no insight at all into the possibility of a thing in accordance with this concept'.[22] The concept of 'magnitude' (*Größe*), for example, 'if the restriction to sensible intuition were omitted, would go thus: It is the determination through which many homogeneous elements [*vieles Gleichartige*] taken together constitute one'.[23] Kant thus conceives of the logical structure of magnitude in much the same way as Hegel, namely as a unity of homogeneous units—though he does not derive quantity from the very *quality* of being one, as Hegel does.
2) Like Hegel, Kant also distinguishes between a *quantum* and quantity or *quantitas*. Béatrice Longuenesse explains this difference as follows: '"magnitude" (= *Größe* as *quantum*) refers to an *object* immediately given in intuition'; on the other hand, '"magnitude" (*Größe* as *quantitas*) is a

predicate, referring to a quantitative determination of the *quantum'*. Only the latter, she writes, 'is strictly speaking an instance of the *category* of quantity'.[24] Longuenesse's account of Kant's distinction nicely reflects the fact that Kant understands categories, like all concepts, to be 'predicates of possible judgments' through which we understand objects to be determined in a certain way (*CPR* A 69/B 94). Hegel, however, does not assume from the start that categories are predicates of *judgements*. In his view, categories are first and foremost concepts that directly disclose ways of *being*.[25] Quantity is thus not simply a predicate but a form of being itself. A quantum in turn is not an *object* to which the *predicate* 'quantity' can be applied, but it is quantity *itself* in a determinate, limited form. To use Spinozan terminology, a quantum is a *mode* of quantity itself, rather than an object that falls under the predicate 'quantity'. This is perhaps the most important difference between the conceptions of quantity entertained by Hegel and Kant.

3) It is not, however, the only difference: for, unlike Hegel, Kant understands number in terms of time. Number, for Kant, is a 'representation that summarizes the *successive* addition of one (homogeneous) unit to another' (*CPR* A 142/B 182, my emphasis). Yet if we abstract from time and consider the purely logical aspect of number, Kant's conception of the latter turns out to be quite close to Hegel's. Kant claims that the concept of number 'belongs to the category of allness', and that 'allness (totality) is nothing other than a plurality considered as a unity' (*CPR* B 111).[26] This, it seems to me, is not so different from Hegel's idea that '*amount* and *unity* constitute the *moments* of number' (*SL* 203/*LS* 214). In this respect, therefore, Hegel and Kant are by no means far apart.
4) Like Hegel, Kant also draws a distinction between extensive and intensive magnitude. This distinction has both a logical and a temporal dimension for Kant. Logically, without regard to time, 'the magnitude of something, insofar as it is represented as a plurality [*Menge*] is extensive, [and] as a unity is intensive'.[27] With regard to time, an extensive magnitude requires the successive synthesis of its manifold parts (*CPR* A 163/B 204). An intensive magnitude, however, has no parts but is a simple unity; it thus acquires the multiplicity that makes it a specific degree from the fact that 'its *emergence* and *disappearance* can be considered as composite (in time)'.[28] Whether the degree is the 3rd, 4th or 5th is determined, in other words, not by what lies within it, but by what precedes it (its 'emergence') and succeeds it (its 'disappearance'). This means that, like Hegel, Kant understands a degree to have its distinguishing multiplicity *outside* itself. Yet a difference between Kant

and Hegel remains: for, unlike Hegel, Kant does not *derive* the degree logically from extensive magnitude.

VII. The Relation of Powers

To conclude this essay, I now return to the logical development of quantity set out by Hegel in the *Logic*. We have seen quality give rise to quantity, and then seen quantity lead as far as intensive magnitude. Hegel finally goes on to show that quantity itself gives rise to quality. This is because it comes to exhibit the being-for-self or pure *self*-relation that characterises the highest form of quality, namely the one. This does not mean that quantity simply disappears: Hegel's claim is that quantity *as quantity* constitutes quality. How does this occur?

Quantity exhibits the form of quality, when one quantum, in relating to *another* quantum, actually relates to *itself* and so becomes being-for-self. This logical structure is realized, in Hegel's view, in the 'ratio' of powers (*Potenzenverhältnis*) (*SL* 321-2/*LS* 359). A brief comparison of this ratio with two others shows that quantity constitutes quality more specifically by freeing itself—at least in a certain respect—from the idea of an amount (*Anzahl*).

Each of the three 'ratios' Hegel discusses has a so-called 'exponent'. In mathematics the exponent is the superscript number that indicates how many times a number is to be multiplied by itself: so 2^3, 3^4 and so on.[29] For Hegel, however, an exponent is something different and is not just found in a ratio of powers. It is found in every kind of ratio and sets a limit to the changes that the quanta in the ratio can undergo. In the direct and inverse ratio the exponent is a fixed *amount*. So, in a direct ratio of 1:4, the exponent is the simple amount, 4. The number on the left side of the ratio can vary infinitely, but the exponent, 4, limits the changes that can be undergone by the right side. However much the left side increases, therefore, the right side must always increase so that it remains four times as big (see *SL* 315-17/*LS* 351-3).

In an inverse ratio, the exponent is once again a fixed amount, but this time it is not just a simple, given amount, but the *product* of the two quanta in the ratio. So, if one of the quanta has the value of 3 and the other 12, the exponent or product of the two is 36. It is this product, rather than either of its factors, that must remain constant to preserve the ratio. So if 3 increases to 4, 12 does not *increase* too (as in a direct ratio) but *decreases* to 9 so that the product remains 36. The two quanta in the ratio are thus not directly but inversely proportional to one another (see *SL* 317/*LS* 354).

Note, by the way, that in the first edition of the *Logic* (whose 'doctrine of being' was published in 1812) Hegel understands the exponent of the inverse ratio to be the sum, rather than the product, of the two quanta concerned.[30]

He then corrects his error in the second edition (published 1832). This is thus an interesting example of Hegel's coming to a deeper understanding of a logical structure between the two editions and revising his account accordingly.[31] Hegel takes his logic to be a priori and definitive, but this does not mean that he gets everything right from the start.

Like the direct and inverse ratios, the ratio of powers is also governed by an exponent. This exponent, however, is no longer a fixed amount or quantum, but is 'of a wholly *qualitative* nature' (*SL* 322/*LS* 359). More precisely, it consists in the quality of self-relation. In a ratio of powers, a number changes and becomes another number; in so doing, however, it must relate explicitly to itself. This occurs when the number it becomes is in fact its *own* product, that is, when the new number is just itself raised 'to a power'. The exponent of the ratio of powers is thus not a specific amount, but requires a number to be *multiplied by itself*.

Note that in exhibiting the quality of self-relation, the number remains related to *another number*: 2 relates to 4, 3 to 9, and so on. In the relation of a number to its square, therefore, the *quality* of self-relation coincides completely with the *quantitative* relation between two different numbers. Indeed, the identity between quality and quantity is even deeper than this: for precisely by relating to *itself* in the power to which it has been raised, and so exhibiting quality, the quantum continues *beyond* itself and so shows itself to be quantitative (see *SL* 322/*LS* 359).

VIII. Conclusion

Hegel's account of quantity is complex and detailed, and it is easy to get lost in its complexity. The logical development of quantity, however, has a relatively simple overall structure. Quantity is first made necessary by quality in the form of being-for-self and the one (*das Eins*): it is the continuity of the one *outside* itself in other ones. In the ratio of powers, however, quantity returns to being purely *for itself*, and so to being qualitative, in being quantitative. As John Burbidge points out, therefore, there is a 'double transition' from quality to quantity and then from quantity back to quality—a transition that Hegel describes as being 'of great importance throughout the whole compass of scientific method' and which we encounter at several points in the *Logic*.[32]

Note, however, that the ratio of powers does not just return us to quality by itself (which would lead once again to quantity and so give rise to an endless cycle). It confronts us, rather, with the immediate identity of quantity *and* quality, of being-outside-oneself *and* being-for-self: a quantum, in becoming another *quantum*, thereby constitutes self-relating, *qualitative* being. This identity of quantity and quality implicitly contains a further category that opens a new sphere of being altogether.

A quantum in the ratio of powers becomes another *quantum*: 2 becomes 4, 3 becomes 9, and so on. It is also *qualitative* insofar as it relates to itself and its own product in becoming another. If, however, a quantum is to be fully and explicitly qualitative in being quantitative, it must become *wholly* self-relating or for itself. This in turn means that it must cease—for the moment, at least—being related to another quantum. It can thus no longer raise itself to a higher power, but must become what Hegel calls a *measure* (*Maß*). A measure is still a quantum; but it is one that relates solely to itself—and so stands alone with 'its own determinacy'—and precisely in so doing constitutes the *quality* of a thing (*SL* 333/*LS* 371).[33]

A thing, as we know, is indifferent to its quantum or magnitude as such: it can alter its magnitude—and so become bigger or smaller—without ceasing to be what it is.[34] A thing's measure, by contrast, is the magnitude to which it cannot be indifferent. This is because that measure is the magnitude (or range of magnitudes) that *gives the thing its quality* and so makes it what it is. The sphere of measure is thus a third sphere of being that is distinct from quality and quantity by being the fusion of both.

Discussion of measure, however, will have to wait for another occasion.[35] To end, I will simply restate the main thesis of this essay: namely, that Hegel sees an intimate *logical* connection between quality and quantity that Kant fails to recognize. What grounds this difference between them, in my view, is this: Kant proceeds on the basis of what are for Hegel unwarranted assumptions, including the assumption that quality and quantity are logically (if not epistemically) separate categories. Hegel, by contrast, derives categories *without presuppositions* from the sheer 'simplicity of thinking'.[36] He is thus able to discover a logical connection between quality and quantity that Kant is unable to see. Note, by the way, that Hegel does not discern this connection because he is somehow predisposed to find it. He discerns it—though Kantians find this claim incredible—because he takes *less* for granted about thought (and being) than Kant does. This, in my view, is the true secret of Hegel.

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Notes

[1] Immanuel Kant, *Critique of Pure Reason*, (hereafter *CPR*), A 70, 80/B 95, 106. Further references are given in the main text.

[2] G. W. F. Hegel, *The Encyclopaedia Logic (with the Zusätze)*, 84 [§42 Remark].

[3] Hegel, *The Encyclopaedia Logic*, 124 [§78 Remark].

[4] See Stephen Houlgate, 'Hegel's Logic', in *The Cambridge Companion to Hegel and Nineteenth-Century Philosophy*, 120–1.

[5] On Hegel's concern to develop a presuppositionless account of the categories, see Stephen Houlgate, *The Opening of Hegel's Logic. From Being to Infinity*, 9–53.

[6] G. W. F. Hegel, *Science of Logic*, (hereafter *SL*), 185–6; G. W. F. Hegel, *Wissenschaft der Logik. Die Lehre vom Sein (1832)* (hereafter *LS*), 192–3. Further references are given in the main text. Note that I have occasionally altered Miller's translation.

[7] On the difference between determinate being and something, see Houlgate, *The Opening of Hegel's Logic*, 312–21.

[8] I have compressed Hegel's argument at this point for the purposes of this essay (which is principally about quantity, not the one as such). In fact, the absence of anything else outside the one does not lead directly to the presence of the void, as I have implied here. A full account of the void will, however, have to wait for another occasion.

[9] For a helpful account of Hegel on repulsion and attraction, see Richard D. Winfield, *Hegel's Science of Logic. A Critical Rethinking in Thirty Lectures*, 123–31.

[10] See also Houlgate, *The Opening of Hegel's Logic*, 331–47.

[11] On the difference between the 'bad' and the 'true' infinite, see *SL* 137-50/*LS* 135–51, and Houlgate, *The Opening of Hegel's Logic*, 414–32.

[12] See also G. W. F. Hegel, *Wissenschaft der Logik. Die Lehre vom Wesen (1813)*, 44–6.

[13] See Hegel, *The Encyclopaedia Logic*, 306 [§240], and G. W. F. Hegel, *Enzyklopädie der philosophischen Wissenschaften im Grundrisse (1830). Erster Teil: Die Wissenschaft der Logik. Mit den mündlichen Zusätzen*, 391.

[14] A discrete, limited quantum does not, however, bring the continuity of quantity to a definitive end; rather, quantity continues beyond any given quantum in a series of further quanta ad infinitum (see *SL* 188, 224-5/*LS* 196, 239–40).

[15] Miller has 'specific aggregate'.

[16] The one by itself, as a form of quality, is thus not a number. The number one is a unity that contains one unit—rather than, say, two or three units—as its amount (and that displays the other features that belong to number, such as being both extensive and intensive).

[17] A quite outstanding discussion of Hegel's conception of mathematics, and especially his understanding and defence of differential calculus, is to be found in Michael Wolff, 'Hegel und Cauchy. Eine Untersuchung zur Philosophie und Geschichte der Mathematik' in *Hegels Philosophie der Natur. Beziehungen zwischen empirischer und spekulativer Naturerkenntnis*, 197–263.

[18] Ivor Bulmer-Thomas, 'Plato's Theory of Number', 384.

[19] See *SL* 249/*LS* 270: 'We may further remark that the existence of infinite series which cannot be summed is an external and contingent circumstance with respect to the form of the series as such. They contain a higher kind of infinity than do those which can be summed, namely an incommensurability, or the impossibility of representing the quantitative ratio contained in them as a quantum, *even in the form of a fraction*' (my emphasis).

[20] Miller has 'simple oneness'.

[21] Miller has '*unitary* determinateness'.
[22] Immanuel Kant, *Notes and Fragments*, 377 [6338a].
[23] Kant, *Notes and Fragments*, 377 [6338a]; Immanuel Kant, *Gesammelte Schriften*, 18: 659.
[24] Béatrice Longuenesse, *Kant and the Capacity to Judge*, 266, my emphasis on 'object' and 'predicate'.
[25] This is not to deny that categories and concepts will turn out later in Hegel's *Logic* to be inseparable from judgement (see *SL* 623ff). Yet Hegel's initial understanding of the categories leads him to a very different conception of judgement from Kant's. Since categories, for Hegel, are principally ways of disclosing being, this will also be true of judgement. Judgement will thus not just be the structure of our thought *about* being (as it is for Kant), but it will be a structure exhibited *by* being itself: as Hegel puts it, 'all things are a judgement' (*alle Dinge sind ein Urteil*) (Hegel, *The Encyclopaedia Logic*, 246 [§167]; *Enzyklopädie der philosophischen Wissenschaften im Grundrisse (1830)*, 318).
[26] See Immanuel Kant, *Kritik der reinen Vernunft*, B 111: 'der Begriff einer *Zahl* (die zur Kategorie der Allheit gehört)'.
[27] Kant, *Gesammelte Schriften*, 18: 241 [5587], my translation.
[28] Kant, *Gesammelte Schriften*, 18: 241 [5588], my emphasis. See also Longuenesse, *Kant and the Capacity to Judge*, 313–14, and Paul Guyer, *Kant and the Claims of Knowledge*, 198.
[29] See John W. Burbidge, *The Logic of Hegel's Logic*, 50.
[30] See G. W. F. Hegel, *Wissenschaft der Logik. Das Sein (1812)*, 218, 221.
[31] In the second edition Hegel undertakes a similar revision of the sections on determinate being and something; see Houlgate, *The Opening of Hegel's Logic*, 320–1.
[32] See *SL* 323/*LS* 361, and Burbidge, *The Logic of Hegel's Logic*, 51. Double transitions are found in the dialectics of being and nothing, something and other, determination and constitution, the finite and the infinite, extensive and intensive magnitude, and in many other places in the *Logic*, but they are not found everywhere. The transitions, for example, from becoming to determinate being, from limit to finitude, and from the direct ratio to the indirect ratio and then to the ratio of powers, are simple, one-way transitions.
[33] This does not mean that such a quantum cannot change into *another* quantum. The quality of the thing concerned, however, is tied to the specific determinacy of *that* quantum (or to a specific range of quanta); so if the latter changes, the quality and the thing necessarily change as well (and may, indeed, be destroyed) (see *SL* 334-5/*LS* 372-3).
[34] See *SL* 186/*LS* 193, and Winfield, *Hegel's Science of Logic*, 135.
[35] For a fine discussion of measure in the *Logic*, see John W. Burbidge, *Real Process: How Logic and Chemistry Combine in Hegel's Philosophy of Nature*, 27-73. See also G. W. F. Hegel, *La théorie de la mesure.*
[36] Hegel, *The Encyclopaedia Logic*, 124 [§78 Remark].

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