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# The Place of C.S. Peirce in the History of Logical Theory

JAAKKO HINTIKKA

## 1 Logic as Language versus Logic as Calculus

Speaking of someone's 'place in history' is often a preface to eulogy. But we have come to study Peirce, not to praise him, and only incidentally to try to see that his good ideas are not interred with his bones or with his unpublished papers.

One of the difficulties in studying Peirce is his elusiveness. Peirce's writings are brim-full of perceptive and provocative ideas, but do they add up to a coherent overall view? Even though Peirce himself offers a general perspective on his own ideas, it is not clear how his specific results are supposed to be parts of a larger picture. This has been my problem when I have tried to offer specific observations on the details of Peirce's ideas in my early publications. How are they related to Peirce's general philosophical ideas?

In this paper, I shall pursue one lead as to how to make our ideas of Peirce clear. This way is to compare Peirce with certain other thinkers, including Frege, Wittgenstein, and Quine. Such comparisons will, I hope, illuminate both the general character of Peirce's thought, so to speak his position on the overall map of the history of philosophy, and also his specific intellectual relationships to other thinkers.

Consequently, the main purpose of this paper is to define and describe Peirce's *locus* in the landscape of certain important but specific traditions in the kind of philosophy he practised. I am not only going to offer scholarly observations concerning the details of Peirce's ideas. (Several of the detailed observations I shall discuss here have in fact been published in my earlier papers.) I shall try to show that these bits and pieces of ideas of Peirce's are in reality parts of a large jigsaw puzzle which can be put together. In other words, I am primarily endeavouring to present a general perspective on his thought. I expect that

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this perspective will help to lend direction and perhaps also a conceptual framework for more detailed studies.

The history in which I am trying to place Peirce is primarily the history of logical theory. Hence I have to discuss in some detail certain aspects of Peirce's work in logic, and to compare him with other logicians (and logically oriented philosophers). However, what I shall say has much broader implications, especially concerning the nature and the presuppositions of Peirce's pragmatism (or pragmaticism).

The main coordinate axis of the map on which I am trying to locate Peirce is provided by a distinction between two contrasting visions of language and its relations to the language user and the world. These two views have been labelled *language as the universal medium* (or *the universality of language*) and *language as calculus* (or *the model-theoretical view of language*). (See Hintikka 1988.) These terms are not self-explanatory, and have to be taken with more than one grain of salt. With a couple of interesting exceptions, the terms of this contrast are not found in the earlier literature, the main reason being that the majority of philosophers have been blissfully or banefully ignorant of the role which the contrast has played in the philosophical thought of the last hundred-odd years.

A believer in the universality of language sees language (and the conceptual system it codifies) as an indispensable intermediary between 'you' and your world (*the* world). You are virtually a prisoner of your language; you could not, for instance, step outside it and have a look at the way it is related to the world. Or, less metaphorically speaking, you cannot express in language (in 'the only language I understand,' as Wittgenstein put it) its semantic relations to the world, without committing nonsense or tautology. As either formulation shows, *semantics is ineffable* according to the doctrine of the universality of language.

As a consequence, you cannot, strictly speaking, understand another language without making it part of your own. 'A language that I don't understand is no language,' to speak with Wittgenstein again. Hence, a believer in language as the universal medium can escape linguistic relativism only by adopting a kind of linguistic solipsism. This connection between linguistic universality and linguistic relativism is one of the grains of salt needed to acquire a taste for my terminology.

Conversely, the conception of language as calculus implies that we can perform all those neat feats a universalist claims we cannot perform, for instance, to discuss the semantics of a language in that language itself. For another instance, on this view, we can vary the interpretation of our language, that is, admit other 'models' for its propositions than the actual universe. In other words (worlds?), language is like a calculus in that it can be freely assigned a

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new interpretation. Hence the term. However, a defender of this view does not hold that language (our very own home language) is like an *un*interpreted calculus, but only that it is *re*-interpretable as a calculus is. Nor does a model theorist hold that his or her language is a mere tool for purely formal (calculus-like) inferences. These observations provide a couple of further grains of the proverbial salt needed to understand the import of what I have called the idea of language as calculus.

Some of the further consequences of the two tenets are clear. For instance, all systematic model theory presupposes something like a belief in language as calculus, at least as applied to the specific languages (often the formal languages of a logician) whose model theory is at issue. There nevertheless is nothing incoherent in an intermediate position like Tarski's according to which we can happily develop model theories for particular formalized languages but not (just because of its universality) for our overall colloquial language.

Tarski's name reminds one of another issue that separates the calculus men from the universality boys. This issue is the definability of the central semantic concept of truth. An all-out model-theoretical philosopher would like to see the notion of truth for a language defined in this language itself, while a universalist would dismiss any such attempt as a hopeless daydream.

The thesis of the universality of language may strike you as being far more outlandish than its rival. Yet it was the dominant view in the early logical theory and the early analytic philosophy of language. The universalist position was embraced by Frege, early and middle Russell, Wittgenstein, the Vienna Circle of the early thirties, and, in a certain sense, Quine. (If you want to find an apostolic succession here, be my guest.) On the so-called continental side, the same belief in the ineffability of truth and of other semantical concepts was shared by, among others, that secret admirer of Frege, Martin Heidegger. (See Kusch 1989.) Through Heidegger's influence, the universalist tenet became a cornerstone of the hermeneutical and deconstructivist traditions.

Against such powerful (though often unacknowledged) allies, the early tradition of language as calculus might at first look like a trickle even in the history of logical theory (cf Sluga 1987). This tradition includes people such as Boole, Schröder, and Löwenheim who are scarcely household names even in philosophical families. Only later, through the work of Tarski, Gödel, and Carnap the born-again semanticist, did the model-theoretical tradition gradually gain the upper hand among logicians, though to a much lower degree among philosophers.

Moreover, at the early stages of the tradition of language (and its logic) as calculus, only a minuscule part of the totality of model-theoretical tools was brought to bear on logic, that is to say only such algebraic techniques as could facilitate the drawing of logical inferences.

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## 2 Peirce's Own Testimony

One of the main theses of this paper is that Peirce was an integral member of the model-theoretical tradition (i.e., the tradition of language as calculus). A further thesis I shall argue for is that this observation throws into a sharper relief a large number of aspects of Peirce's many-faceted thought and even opens new ways of understanding them. The import of these claims will become clearer as the paper proceeds, and so will the contrast between language as calculus and language as the universal medium.

Peirce's membership in the model-theoretical tradition is not controversial. Peirce himself identifies with perfect clarity the 'very serious purpose' of his language of graphs, by saying that 'this system is not intended to serve as a universal language for mathematicians or other reasoners like that of Peano' (4.424). This disassociates him completely from the universalists.

Peirce continues: 'Third, this system is not intended as calculus, or apparatus by which conclusions can be reached and problems solved with greater facility than by more familiar systems of expression ... But in my algebras and graphs, far from anything of that sort being attempted, the whole effort has been to dissect the operations of inference into as many distinct steps as possible' (4.424; I am grateful to Susan Haack for first drawing my attention to this passage).

In spite of the verbal disagreement, this statement shows that Peirce was dealing with interpreted logic, not merely formal inferences. In other words, Peirce was unmistakably taking a model-theoretical or 'calculus' view of his formal language. In fact, he says in 4.423 that he will only later move on to see what purely formal rules might correspond to the initial interpreted inferences: 'Part II will develop formal "rules", or permissions, by which one graph may be transformed into another without danger of passing from truth to falsity and without referring to any interpretation of the graphs ...'

## 3 Peirce versus Frege

One subject we can appreciate better from the vantage point of the contrast I have sketched is precisely what Peirce's places is in history. To begin with a relatively superficial aspect of this location task, Peirce's membership in the model-theoretical tradition will already explain something about his posthumous stature. It is usually said that the founder of contemporary logic is Frege. (Compare here van Heijenoort 1967.) Why not Peirce? Admittedly, Frege was the first to publish a treatment of the basic part of logic that is variously known as quantification theory, lower predicate calculus, or first-order logic. But Peirce and his associates developed a logic of quantifiers completely independently of Frege. What is a couple of years' publication priority among friends?

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It is a historical fact that Peirce not only independently discovered the importance of quantifiers and developed a notation for them, but that he also discussed their nature in a most perceptive way. In particular, Peirce showed much keener awareness than Frege of the nature of dependent (nested) quantifiers and their importance for logical reasoning. For instance, in 4.483 Peirce says that 'when a proposition contains a number of *anys* and *somes*, it is a delicate matter to alter the form of statement while preserving the exact meaning.' These insights of Peirce's are not trifling; related ideas have since played a crucial role among other developments in Hilbert's metamathematical thought and in game-theoretical semantics. I shall soon return to this subject in greater detail.

Unlike Frege, Peirce also outlined or foreshadowed several subsequent developments in logic, especially in modal logics. In general, the bag of logical tools we find in Peirce's writings is much richer than what we find in Frege.

Peirce's failure to have much of his best work in logic published explains some of the modesty of his later reputation. But Frege, too, was little known in his own day. Hence Peirce's failure to publish does not fully explain why his reputation should have languished in comparison with Frege's.

The fate of Peirce's logical work is to a considerable extent due to the fact that he happened to be a member of a less dominant tradition than Frege. Both Frege and his discoverer, the early Russell, were firmly in the universalist tradition, and so in a way is W.V. Quine, who has probably more than anybody else helped to turn Frege into the major saint in the calendar of modern logicians that he is by now, in the majority view (cf Hintikka 1990). Admittedly, from the vantage point of the universalist tradition, Frege's claims to fame are exceedingly well-founded. For instance, he created the conception of a formal system of logic which has remained paradigmatic for a century (cf van Heijenoort 1967).

Frege's universalist stance helped to make his work look impressive. Frege presented first-order logic as the central part of a relatively simple universal language which was calculated to capture faithfully the world of our concepts. (This is the point of Frege's calling his formal language a *Begriffsschrift*.) For Peirce, the self-same quantification theory was only one kind of logical system among many. Characteristically, Frege did not see any need of explaining or analysing its ingredients, such as quantifiers, any further. He probably would have agreed with one of his latter-day followers who has said that if he does not understand first-order logic, he does not understand anything at all in logic. In contrast, far from taking any set of accustomed logical principles for granted, Peirce was constantly trying to give them a deeper foundation or extending their range.

> Now to say that the graphical procedure is more analytical than another is to say that it demonstrates what the other virtually assumes without proof. (NEM4: 319)

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> The one universally valid method is that of mathematical demonstration; and this is the only one which is commonly avoided by logicians as fallacious. (NEM4:21.)

As a consequence of this constant presence of interpretational ideas in his work on the formal rules of logic, Peirce did not feel the need of making a hard-and-fast distinction between, on the one hand, the formal axioms of logic and its formal inference rules, and, on the other hand, its derived truths. They all had to be justified by means of semantic considerations, which could be evoked at any stage of the proceedings. Thus, to the detriment of his later reputation, Peirce, unlike Frege, never came to develop explicitly the idea of a formal system (axiomatization) of logic.

## 4 Peirce as a Modal Logician

Several aspects of these remarks need to be explained further. First, Peirce's pioneering work in modal logic reflects deep philosophical assumptions which are closely related to his model-theoretical conception of logic. A universalist like Frege could never have developed a modal logic. Why not? The answer is obvious. If language (our actual language or any alternative that can do an appreciable part of the same job it does) cannot be re-interpreted, it can be used only to speak of one and the same world, what is to say, our actual world. No serious alternatives are possible, and hence the notions of possibility and necessity lose their natural Leibnizian sense. Consequently, modal notions have either to be abandoned as meaningless, as Quine urges, or to be suitably re-interpreted, as in Russell's proposal to define necessity as universality. In particular, for a universalist logical truths are not truths about all possible worlds. Rather, as Russell once put it, for a universalist thinker like himself logic deals with the actual world quite as much as zoology, albeit with its more abstract denizens. The main alternative open to a universalist who does not want to consider logical truths as truths about our world is to declare logical truths to be tautologies, as Wittgenstein did.

Thus the predominance of the tradition of language as the universal medium explains why modal logic failed to catch the attention of most philosophers until the late fifties. And in so far as it enjoyed a modest earlier flowering, as in C.I. Lewis's hands, it did so outside the mainstream tradition in logical theory.

Indeed, anyone who takes modalities and modal logic seriously must countenance some version of the view of language as calculus. Here we are obviously approaching Peirce again. Unfortunately, this is not the place to offer an exhaustive study of Peirce's theory of modality. The main facts can nevertheless be registered. They show once again Peirce's firm place in the tradition of language and logic as calculus.

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1 At least in his mature thought, Peirce understood modalities realistically in a straightforward metaphysical sense, without trying to explain them away, for example, epistemically.
2 In his actual work in logic, Peirce developed ways of studying modal logics. In his logical theory in general Peirce used freely ideas that presuppose a multiplicity of possible worlds (or other possibilia).
3 Peirce influenced the subsequent development of explicit systems of modal logic. In particular, he was greatly admired by C.I. Lewis.
4 The philosophically deep point here is that Peirce showed a much keener awareness of the crucial distinction between truth (truth *simpliciter*) and the ill-named concept of 'logical truth' than the members of the universalist tradition.

In the light of these observations, we can see why, for a universalist like Quine, Peirce's anticipations of modal logics were scarcely a recommendation. And by the time the tide had turned and modal logics and algebra-oriented and model-theoretical approaches had again become important, the actual detailed methods of theorizing in logic had surpassed what Peirce could offer.

Thus, to ask Quine to be the main speaker when Peirce's logical work is being discussed is a little bit like asking Cardinal Manning to deliver a eulogy on John Henry Newman.

## 5 Peirce as a Game-Theoretical Semanticist

Furthermore, Peirce's interest in the deeper nature of quantifiers turns out to lend him a remarkable place in the entire model-theoretical tradition. Since I have dealt with this matter at some length in other recent papers, I can be quite brief.

What do quantifiers mean, anyway? What is their logical status? A one-world universalist like Frege is likely to try to get away with saying merely that quantifiers are second-order predicates of a special kind. For instance, the existential quantifier $(\exists x)$ in the sentence $(\exists x)\, S[x]$ is a second-order predicate (predicate of predicates) which says that the (possibly complex) first-order predicate $S[x]$ is not empty. But Frege never gives any explanation of what is special about such second-order predicates. His explanation does not work without a great deal of hand-waving for nested (dependent) quantifiers. And a model theorist will ask in any case for further explanations. What makes a second-order predicate an existential quantifier is not its extension in the actual world, but the way this extension is determined in any old (or new) world. And this 'way of being determined' is never explained by Frege.

Inevitably, when our present-day conception of a quantifier was first con-

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ceived of by the likes of Skolem and Hilbert, it was related to the idea of a choice function (Skolem function). A sentence containing a dependent existential quantifier, for instance

(1) $(\forall x)(\exists y)\, S[x,y]$

in effect asserts the existence of a choice function. For instance, (1) is equivalent with

(2) $(\exists f)(\forall x)\, S[x, f(x)]$

Now the remarkable thing here is that Peirce had this way of looking at quantifiers down pat. He felt called upon to explain the logical behaviour of the quantifier words *some* and *any* in a way which goes beyond the mere formulation of the rules of inference in which they play a part. 'Every *some*, as we have seen, means that under stated conditions, an individual could be specified of which that which is predicated of the *some* is [true], while every *any* means that what is predicated is true of no matter what [specified] individual; and the specifications of individuals must be made in a certain order, or the meaning of the proposition will be changed' (4.483).

As Risto Hilpinen has shown, Peirce spells out quite explicitly the game-theoretical meaning of quantifiers. If I am right, something like this meaning is the only model-theoretically natural way of dealing with quantifiers. Here Peirce's comments on quantifiers offer further evidence of his model-theoretical stance.

Peirce's comments may be more than an anticipation of the idea of quantifiers as being based on choice functions. The notion of Skolem function (choice function connected with a quantifier) was foreshadowed by Löwenheim's use of double indexing, which he inherited from Schröder. Now Peirce, who in his semiotic theory had the idea of quantifiers as asserting the possibility of choices or 'specifications' of a certain kind, is known to have influenced Schröder's logic significantly. Is it therefore likely that Peirce not only anticipated the idea of quantifiers as choice functions, but in a sense started it? Further historical research is needed before a convincing answer can be given to this question.

It is considerably easier to tell why Peirce, Schröder, and Löwenheim never managed to implement this idea in a way that would have enabled them to put it to use in their logical theory. They had not reached Skolem's idea of codifying the relevant choices in an explicitly mentioned function (Skolem function). Now what is the game-theoretical meaning of such a function? It is a partial codification of a verifier's *strategy*. Before the general game-theoretical con-

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cept of strategy was formulated by von Neumann (or at least before the special case of this concept in the form of a Skolem function was available), there simply was no obvious model for a full-fledged development of Peirce's game-theoretical interpretation of quantifiers.

## 6 Peirce and Metalogic

One particular consequence of the universalist position is that our language and its logic can be neither self-applied as a whole, nor discussed in its entirety in a separate metalanguage (except for its purely formal features, of course). This consequence, which was noted by van Heijenoort in his pioneering paper (1967), offers some of the most useful tests of actual historical membership in the two traditions. If there were any lingering doubts about Peirce's allegiance to the model-theoretical tradition, they would hence be quickly dispelled by his willingness to discuss logic by means of logic. A case in point is offered by Peirce's theory of existential graphs. For instance, in 4.527 he writes:

> I will now pass to another quite indispensable department of the gamma graphs. Namely, it is necessary that we should be able to reason in graphs about graphs. The reason is that reasoning about graphs will necessarily consist in showing that something is true of every possible graph of a certain general description. But we cannot scribe every possible graph of any general description and therefore if we are to reason in graphs we must have a graph which is a general description of the kind of graph to which the reasoning is to relate.

And Peirce goes on to develop in some detail a notation for 'gamma expressions of beta graphs,' which is precisely a logic applied to logic.

One has to be careful here, however. For a universalist, too, has to apply logic to itself, just because it is universal and hence has to apply to everything. What distinguishes the two is (among other things) that a believer in the 'logic as calculus' conception feels free to develop a separate new technique, even a new language (a metalanguage, perhaps) if need be, for the purpose.

That this is what Peirce was in effect doing is clear from his writings, indeed clear from the examples already given.

## 7 The Very Idea of Formalization

One particularly important and particularly subtle set of consequences of the two contrasting approaches concerns the idea of formalism in logic and mathematics. What is subtle here is that the attitude of both the universalists and the model theorists to the idea of formalism is bound to be ambivalent. Take first a

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believer in logic as calculus. As the very term highlights, such a theorist is likely to emphasize the usefulness of calculus-like systems in the study of logic. Indeed, this tradition was at an early stage of its development known as the tradition of the algebra of logic. The idea was to find quasi-algebraic laws by means of which logical formulas could be manipulated in the same (or a similar) way in which algebraic equations were manipulated.

Here we can in fact see one of the many symptoms which show that Peirce was fully and firmly a member of the model-theoretical ('logic as calculus') tradition. As every student of Peirce knows, most of his work in logic belongs historically speaking to the 'algebra of logic' movement. At first sight, the use of the techniques of old-fashioned elementary algebra might seem indicative of a concentration of interest on formal matters rather than interpretational ones. But it is crucial to realize that, but for the believers in the model-theoretical conception, formalism was a servant, not the master. As was found in section 2 above, Peirce in any case distinguished his interests from those of the logical algebraists. They were merely looking for rules to facilitate the drawing of actual logical inferences; Peirce was deeply concerned with the model-theoretical basis of such inferences, especially with analysing them into the shortest and most obvious steps.

In general, like all believers in logic as calculus, Peirce was not only ready to provide an interpretation for their calculi at the drop of a symbol. He could – or thought they could – discuss such changes systematically in an explicit language. The very freedom of choosing a formalism differently on different occasions was sometimes a consequence of their belief that it is the underlying representational realities that really matter, not the formalism. In some typical and important cases, one can see how a formalism and even a temporary adoption of a formalistic attitude in reality served substantive interpretational purposes.

The interplay between formalism and interpretation can be further illustrated by considering Hilbert's often-misunderstood program. Hilbert the axiomatist conceived of his axioms as interpretable systems. They served a purpose as soon as they had one realization, one interpretation which made all the axioms true. But how can we be sure that a given axiom system has such an interpretation? If we have available to us, as Hilbert thought, a complete and completely formal system of logic, then it would suffice to study the axiom system purely formally to answer the question. If it can be shown that one can never derive a contradiction from the axioms in a purely formal way, then the axiom system is consistent also in the interpretability sense, granted of course the completeness of one's formalized logic. What concerns us here is merely the fact that Hilbert's formalism was in this important sense merely a means of reaching interpretational (semantic) results. He was trying to prove the semantic consistency

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of axiom systems by proving their syntactical consistency, one might say.

Peirce, too, found it useful in his logical work to give a 'pure mathematical definition of existential graphs, regardless of their interpretation' (4.414). As with Hilbert, this does not mark any lapse from model-theoretical virtue.

Likewise, a believer in the universality of logic and in the ineffability of semantics is also likely to care for the basic interpretational meaning of his or her interpreted *Begriffsschrift*. But that meaning cannot be discussed in language. Consequently it cannot be theorized about or studied in a systematic fashion. It has to be taken for granted. It is presupposed, not discussed. It can be studied only in so far as it manifests itself in the purely formal behaviour of one's logic and language. Hence a universalist's belief in the ineffability of semantics easily leads him or her, too, to a formalist practice. It is for this reason that Frege the universalist created our current paradigm of a formal system of logic and that Wittgenstein the lifelong believer in the ineffability of semantics became the fountainhead of the idea of a 'logical syntax of language,' in spite of the fact that both had many strong views about the relation of language and the world.

Of course, different philosophers' ideas about formalism are likely to have their mirror image in the same philosophers' ideas about the non-formal aspects of language, that is, about the representative function of language and its logic. One might perhaps expect that an emphasis on calculation in logic might divert a philosopher's attention from the intuitive picture-like representativeness of language. Yet the contrary is usually the case. The free re-interpretability of language according to the calculus conception is apt to focus one's attention on the interpretation and the representative functions of language.

## 8 The Iconicity of Logic

Here I am obviously touching on one of the most characteristic features of Peirce's ideas about language and logic, namely, the role of what he called icons and iconicity.

My readers are all undoubtedly familiar with Peirce's distinction between an *index*, an *icon*, and a *symbol*. I am interested here in the second of these three vehicles of linguistic representation. An icon, Peirce says, represents whatever it represents by resembling it. Moreover, this resemblance need not mean looking alike in a vulgar sense of the expression. The similarity is essentially a *structural* similarity. The parts or elements of an icon are related to one another in a way analogous to the way the corresponding ingredients of what it represents are related to each other.

What is instructive here is that Peirce's model-theoretical attitude is revealed

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by his emphasis on the role of icons in logic, reasoning, and thinking in general. The point is not difficult to appreciate. Another way of expressing the iconic relation of a sign to what it represents is to say that the sign is a model of what it stands for in a sense not completely different from logicians' use of the term. One way of illustrating this iconicity is by reference to the technique (which goes back at least to Henkin) of using suitable sets of formulas (e.g., model sets, or maximal consistent sets) literally as their own models. Hence Peirce's willingness to theorize about icons and to use them in his actual work in logic is but another facet of his model-theoretical approach to language and logic.

By and large, Peirce's model-theoretical attitude, in contrast to Frege's universalist stance, shows up in his actual work on (and with) formal rules of inference in the following way: Both logicians obviously rely on tacit interpretational ideas to set up the basic rules of inference. But Frege is suspicious of such semantic ideas, undoubtedly because they apparently cannot be expressed in a correct *Begriffsschrift* and apparently hence cannot be subject to the same standards of argumentative rigour and clarity as that which can be so expressed, including (Frege thought) mathematical concepts.

Moreover, Peirce's idea of the iconicity of logic highlights still another difference between Frege and Peirce. For Frege, finding the right principles of logic is a matter of thought, not of intuition. By the same token, when we proceed from logical axioms to logical theorems, we cannot expect any bottom-line help from intuition, and we should not trust other interpretational ideas very much, either. Hence, all told, for Frege there is only a limited role for interpretational considerations in pure logic, and properly speaking no real role for intuitions at all. They are at best dispensable auxiliary aids to logical thought.

Thus, Frege writes in *Foundations of Arithmetic* (section 60): 'Time and again we are led by our thought beyond the scope of our imagination, without thereby forfeiting the support we need for our inferences.'

In contrast, Peirce's characteristic logical techniques are iconic and graphic, and in that sense involve intuition (in the general sense in which sense-perception is a species of intuition). Sometimes he even denies that 'demonstrative reasoning is something altogether unlike observation' (3.641). And the foundation of this claim is precisely the iconic element in reasoning: 'The intricate forms of inference of relative logic call for such studied scrutiny of the representations of facts, which representations are of an *iconic* kind, that they represent relations in the fact by analogous relations in the representation, that we cannot fail to remark that it is by *observation* of diagrams that the reasoning proceeds in such cases' (3.641).

Peirce's iconic methods did not, as far as I can see, anticipate the specific ideas which subsequently guided the model-theoretically oriented axiomatiza-

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tions of logic, such as the Beth-Hintikka idea of construing a (first-order) logical proof as a frustrated attempt to construct a counter-example. Such later ideas are closer to the spirit of Peirce's philosophy than to its letter. This spiritual kinship is nevertheless unmistakable. One of its many symptoms is Peirce's comment on the usefulness of icons: 'The Icon does not stand unequivocally for this or that existing thing, as the Index does ... But there is one assurance that the Icon does afford in the highest degree. Namely, that which is displayed before the mind's gaze – the Form of the Icon, which is also its object – must be *logically possible*' (4.531). This is essentially just the other side of the interpretational coin minted by Beth and Hintikka. If proofs are frustrated attempts to construct a counter-example, then a completed construction will *ipso facto* show that a counter-example is indeed logically possible. (Such uses of the techniques of logical argumentation are still being neglected by philosophers.)

The Beth-Hintikka idea is precisely to interpret a logical proof as an attempt to construct an iconic counter-example to the putative conclusion. What the logician who carries out this construction hopes is that this attempted construction is seen to fail inevitably, whereby the result to be proved is shown to be logically necessary.

## 9 Theorematic versus Corollarial Inferences

In some specific respects, however, Peirce did anticipate later semantics-friendly conceptualizations in logic.

Indeed, an excellent case study of the role of iconic thinking in Peirce is offered by a distinction to which he himself attached considerable importance. It is a distinction between what Peirce called corollarial and theorematic reasoning in logic. (See Hintikka 1983.) The paradigm case is geometrical reasoning, which of course is often overtly iconic, relying on geometrical figures as an aid to reasoning. In elementary geometry, familiar to many readers from their school-days, some theorems can be proved by considering simply the configuration of geometrical objects mentioned in the statement of the theorem. Such proofs Peirce calls *corollarial*. But in other cases, a proof is possible only by reference to other geometrical objects not mentioned in the original statement. In the traditional jargon of elementary geometry, we need 'auxiliary constructions' in order to be able to carry out the proof. Such proofs Peirce calls *theorematic*. Peirce's choice of terms is in fact very much in keeping with traditional geometrical terminology.

A distinction of this kind can be drawn only on the basis of some sort of iconic representation of reasoning, which in the case of geometry is provided by the familiar geometrical figures. When geometrical reasoning was 'formalized'

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in the sense of being considered as strictly logical reasoning, many philosophers, for instance Bertrand Russell, thought that any conceptualization that is made by reference to figures is dispensable and off the mark in rigorous geometry. They were wrong because they overlooked the potential iconicity of all logical reasoning.

Indeed, Peirce's brilliant insight was that all logical reasoning is at bottom iconic. This is the basic reason why the contract between corollarial and theorematic reasoning can be applied to *all* logical reasoning. For since all such reasoning is iconic, it involves representatives of the entities involved in the reasoning, either representatives of some particular individuals or else of suitable sample individuals the reasoner is considering. Hence we can always distinguish such (theorematic) reasoning as requires more complex configurations of entities than are mentioned in the result to be proved (or in the premises one has available) from such (corollarial) reasoning as merely involves reshuffling an iconic configuration that has already been given. Obviously, a general distinction of this kind is possible only if one can consider all logical reasoning as an iconic or model-theoretical process.

A long time ago (cf Hintikka 1973), I came independently upon the same distinction in my work in the philosophy of logic, only to discover that the same insight had been reached by Peirce.

Peirce's place in the history of logical theory is vividly illustrated by the incomprehension with which his distinction between corollarial and theorematic reasoning was met. This incomprehension was undoubtedly due to the prevalence of the universalist tradition at the time. As was indicated, this rejection of ideas like Peirce's distinction was based on the formalization of all mathematical reasoning as a sequence of purely logical conclusions. Since, this tradition maintained, all reasoning in theories like elementary axiomatic geometry can (and must) be capable of being represented in the form of strictly logical inferences codifiable in terms of a completely formalized logical system, conceptualizations like Peirce's which rely on geometrical constructions (or on other iconic representations) are irrelevant.

Russell is a representative example of thinkers adhering to this line of thought. Unfortunately for them, they miss what in reality is Peirce's deep insight, to wit, that even the kind of apparently purely symbolic reasoning we carry out in formal logic is in the last analysis iconic. This is the same insight as is expressed in a different way by the Beth-Hintikka idea of considering all first-order logical arguments as frustrated counter-model constructions. As soon as we realize this iconicity of logical reasoning, we can make the Peircean distinction, which therefore is a telling example of his way of thinking in general.

Ironically, my unwitting reconstruction of Peirce's distinction has met with

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precisely the same misunderstanding as that which led the vast majority of logicians and philosophers to overlook the significance of Peirce's discovery. For instance, Hookway (1985: 199–200) considers this reconstruction of Peirce's distinction merely as a way of 'developing Peircean themes' in specialized contemporary logical theory and rejects it as not being able to 'help us to get to the heart' of Peirce's thought. Yet the evidence Hookway himself marshals tells eloquently against his judgment rather than for it. He mentions several wider issues in Peirce to which my reconstruction is alleged to be irrelevant. They include 'the use of abstract reference as crucial to the most important and widespread forms of theorematic reasoning' (ibid. 200). In reality it is the introduction of new objects by existential instantiation that creates the most interesting problem of abstract reference, namely, the problem of the status of the 'arbitrary objects' or 'witness individuals' that are apparently referred to by the 'dummy names' introduced in existential instantiation. And this problem, far from being neglected in my reconstruction, is thrust into prominence by it. Moreover, I have pointed out that the problem of understanding existential instantiation is a time-honoured one, going back to the interpretation of the logical and mathematical notion of *ekthesis* in Aristotle and Euclid (Hintikka 1974).

Again, Hookway places considerable emphasis on the connection between Peirce's distinction and questions of decidability. This connection is nevertheless a mere corollary to my interpretation and hence tells in its favour. Not only is it the case that corollarial reasoning is mechanizable and decidable. On my reconstruction, the decidability of a theory becomes simply the problem of telling which 'auxiliary constructions' to carry out – or even merely how many of them will be needed for a proof. Predicting this number is equivalent to the decision problem of the theory in question.

Even Peirce's at first somewhat surprising speculation that 'the need for theorematic reasoning reflects the current state of mathematical ignorance' turns out to have an interesting point. For if the mathematical theory we are dealing with happens to be (deductively) complete, decidability follows from axiomatizability. Hence Peirce's conjecture might simply reflect a (mistaken but natural) surmise that decidability (completeness) entails the possibility of corollarial reasoning.

Hence, far from forcing us to ignore such Peircean claims and conjectures, my reconstruction offers an excellent framework for discussing and even evaluating them. In reality, it is Hookway who fails to 'get to the heart of' Peirce's thought, which in this case is the iconicity of even purely formal inferences of symbolic logic.

In emphasizing the importance of diagrams, graphs, and icons in reasoning, Peirce is not primarily talking about some new fancy forms of reasoning differ-

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ent from the traditional modes of reasoning in geometry or other kinds of mathematics or from the modes of logical reasoning developed since Peirce. To allege this is to miss completely his deeper point. His point is that all these different modes of reasoning are at bottom iconic and diagrammatic. This result may have striking philosophical consequences – as, for example, Kettner maintains – but they are not tied to any particular method of doing logic. To claim that they are is comparable to accepting a logical argument only as long as it is expressed in French, but not in English translation. The philosophical consequences in question must be capable of being argued for in terms of any method of logical systematization. To try to connect them to Peirce's idiosyncratic and quaint techniques in logic is to misunderstand completely Peirce's point and to block the path of inquiry in logical theory.

Peirce's idea of the iconic character of logic can be illustrated by a different comparison. It is an index of the subtly ambiguous relations between formalism and representation that a universalist thinker like the early Wittgenstein could also maintain the iconicity of language under the guise of his ill-named 'picture theory of language.' Thus an emphasis on iconicity does not *per se* put Peirce in the model-theoretical camp. The true differences between the two traditions are revealed by more elusive clues. But even though elusive, these clues are real. Not only does the pictorial character of language *apud* the Wittgenstein of the *Tractatus* belong to what can only be shown but cannot be said, which for Wittgenstein implies that we cannot speak about the iconicity of our language in the language itself. Wittgenstein's sometime picture conception of language is completely static, completely timeless. It serves only to account for the way sentences (both atomic and complex sentences) represent the world. In contrast, in Peirce, iconicity is the key to logic and logical inference. 'Icons are specially requisite for reasoning,' he wrote (4.531).

## 10 The Role of Human Action in the Constitution of Meaning

The role of game-theoretical concepts in the model-theoretical tradition is but a special case of a wider phenomenon. On the universalist conception, the representative relations have to be there before any one particular use of language. One can use them, but one cannot change or generate them – or at least one cannot systematically discuss in one's own language such changes and generations nor theorize about them. Hence, particular concrete human actions cannot be a constitutive element in such semantic relationships. Consequently, for a believer in the universality of language there is likely to exist a sharp contrast between the study of the semantics of language (in so far as it is possible) and

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the study of the use of language. This distinction is of course the one generally known as the contrast between the semantics and the pragmatics of a language. As a matter of historical fact, a belief in the inexpressibility of the semantics of language (as maintained by the Viennese positivists) was the background of Morris's creation of the semantics versus pragmatics contrast (Morris 1938, 1946). What is remarkable about the distinction is not just the postulation of some sort of boundary between two theoretical enterprises, but the idea that pragmatics inevitably involves the peculiarities of the language users in question and hence inevitably belongs to the psychology and sociology of language rather than its logical and philosophical study.

Hence, the received distinction between semantics and pragmatics is not only inspired by a universalist vision of language, but loses its *raison d'être* if the universalist paradigm is given up. Of course, such a violation of the semantics versus pragmatics distinction takes place in game-theoretical semantics, once again showing its inseparable ties with the model-theoretical tradition.

Peirce is quite explicit about the role of certain interpersonal activities in the semantic interpretation of uttered propositions in general, not just in the case of quantifier sentences. He is also aware of the game-like nature of those interpersonal activities. 'The utterer is essentially a defender of his own proposition, and wishes to interpret it so that it will be defensible. The interpreter, not being so interested, and being unable to interpret it fully without considering to what extreme it may reach, is *relatively* in a hostile attitude, and looks for the interpretation least defensible' (MS9: 3–4).

This passage has been quoted earlier by Risto Hilpinen (1983:267), who adds: 'Peirce occasionally calls the interpreter of a proposition its "opponent" (e.g. in MS515). Thus the language-game played by the utterer and the interpreter with respect to an indeterminate proposition is, according to Peirce, a zero-sum game.' It is of a particular interest to us here to see how according to Peirce an interpretation of an utterance, far from being fixed ahead of time, comes about during the 'language-game' between the utterer and the interpreter.

This shows vividly once again the closeness of Peirce's ideas to those codified in game-theoretical semantics. At the same time it can be seen how Peirce's game-theoretical interpretation of quantifiers that was discussed in section 5 above is merely a special application of more general ideas connected with his pragmatism.

Moreover, Peirce practises in his actual work in logic what he preaches in his semiotic theory. His entire theory of existential graphs is, as the very name suggests, an exercise in iconic representation. Peirce's semiotic ideas, such as his distinction between icons, indices, and symbols, sometimes occur in the midst of his purely logical work and are made use of there (cf, e.g., 4.447).

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Here we are obviously touching on some of the most characteristic general features of Peirce's theories of language and meaning.

## 11 The Model-Theoretical Attitude as a Presupposition of Pragmatism

To put the general point even more bluntly, Peirce's pragmatism (or pragmaticism, if you prefer) is predicated on a denial of any absolute separation of semantics and pragmatics and hence predicated on some version of what I have called the calculus conception. For without it, the use of a word or a sentence either is irrelevant to its meaning in any theoretically interesting sense or else cannot be discussed in language.

A comparison with Wittgenstein might be instructive here. Like Peirce, Wittgenstein believed that the use of language is constitutive of its meaning. And like Peirce, Wittgenstein does not mean by 'use' a mere verbal use, that is, a 'game' whose moves are language-acts. Both are emphasizing the use pragmatically in the sense of its language utilization. (Witness, for instance, how Wittgenstein compares language to a box of tools.) But there the similarities end. As a universalist, Wittgenstein could not officially say anything about language-games in general or even develop a real theory of any particular kind of language-games, in complete contradistinction to Peirce. What is even more striking, as far as their semantic role is concerned, is that Wittgensteinian language-games were essentially static. Their role was to constitute the network of meaning relations that connects language with the world. Paradoxically, they do not help very much to understand how we humans actually use those relations. For all his fame as a theorist of logic, Wittgenstein never had anything interesting to say about logical *inference*, which was precisely one of Peirce's long suits. For Wittgenstein, logic has to be considered as a mere calculus, without any references to a semantic foundation that could be discussed in language.

These observations throw some ironic light on Peirce's place in history – or perhaps on the way his place in history is conceived of by later philosophers. Much of the recent resurgence of interest in Peirce and in particular in his theory of language has been in the context of traditions which emphasize pragmatics and, more generally, the roots of language and meaning in the language community, in its customs, rules, and practices. Such an emphasis is apt to lead to many valid insights into Peirce's thought. However, in so far as this tradition presupposes a contrast between semantic approaches to language, including explicit formal semantics, and use-oriented approaches, it is in one important respect foreign to Peirce's thought and likely to distort it. For Peirce, there cannot be any impenetrable boundary between pragmatics and a formal study of language, or between semiotics and logic. As his own words show (e.g., in the application to

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the Carnegie Foundation in 1902), logic was for Peirce formal semiotics. It is not just that according to Peirce we have to study the pragmatics of language over and above its syntax and formal semantics. A rightly understood semantics, even the most formal one, *is* part of pragmatics, and even the most purely syntactical (formal) rules of logical inference are anchored in the semantics and even pragmatics of the symbols involved in it. Conversely, if there ever was an idea that Peirce would have found foreign, it is the idea that we could study semiotics without sooner or later needing the help of symbolic logic. Peirce's ideas about the need of an interpretant or about the game character of quantifiers are not calculated to replace a logical and formal study of language, but to show what such a study is supposed to capture. In a later jargon, Peirce is not maintaining a contrast between pragmatics and semantics (including logical semantics), but is pointing to their ultimate unity. In this respect, too, Peirce's place in history can be defined more sharply than has been done in the earlier literature.

This point is not disproved by the often striking similarities between Peirce and the later Wittgenstein. Wittgenstein made certain rule-governed activities, his 'language-games,' the basis of all meaning. The result was a denial of the presence of any sharp logic in our actual language. But if so, how could Peirce find a foundation for his exact logic in essentially the same human activities? The answer is that those *prima facie* similarities are in some cases only skin deep. Admittedly, there are genuine kinships between the two, such as the role of human activity in linguistic meaning and anti-scepticism, manifesting itself as a reliance on truths which are indubitable because they are not doubted. Similarities between Peirce and the later Wittgenstein can be used as evidence against the unity of logic and semiotics only if Wittgensteinian language-games are conceived as social games of using language in the sense of speech-acts and other language-acts. Only then can these language-games be contrasted to logic and logical theory. This is, however, a rank misinterpretation of Wittgenstein, as I have shown elsewhere (cf Hintikka and Hintikka 1986).

But another question arises here. How can Wittgenstein, unlike Peirce, maintain the ineffability of semantics if that semantics is based on rule-governed human activities? Surely we can talk about such activities, including their rules, all the time in our language! The answer lies in Wittgenstein's peculiar holism. In his mature view, entire language-games are conceptually primary in relation to their rules. We do not understand a language-game by guessing its rules, according to Wittgenstein. We can only grasp the rules of the game by learning to master the entire game. And that game as a whole is ineffable. In Peirce, in contrast, there is no assumption of irreducible holism.

Once again Peirce and Wittgenstein turn out to be at one and the same time very close to each other and yet worlds apart.

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## 12 Placing Peirce in History

So what *is* Peirce's place in the history of logical theory? He was a working member of a tradition which was largely suppressed in his own time and in the next couple of decades. Because of this suppression, few if any of his most interesting ideas were developed by others. By the time the model-theoretical tradition was revitalized again among logicians and philosophers, some of Peirce's problems and ideas had been superseded and even forgotten. Others were found to be brilliant anticipations of ideas that were later rediscovered independently. Undoubtedly there may be further ideas in Peirce that still deserve to be taken up and developed further. (For instance, it is turning out that Peirce's graphical notation is well suited for the purpose of avoiding a certain important trap into which Frege fell in developing his notation.) A genuinely interesting and important interpretation of Peirce has to be able to sort out his ideas into these categories. And that can be done only on the basis of a solid knowledge of the subsequent history of the ideas Peirce had and of the substantive issues involved in his work. At the same time, the forays into the details of Peirce's logical and philosophical views that I have reported in this paper (see especially sections 5 and 9) provide examples of how interesting and important Peirce's ideas are even from a contemporary perspective.

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