Axiomathes (2009) 19:245–270
DOI 10.1007/s10516-009-9063-6

ORIGINAL PAPER

# Cohomological Emergence of Sense in Discourses (As Living Systems Following Ehresmann and Vanbremeersch)

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Received: 22 February 2009 / Accepted: 14 March 2009 / Published online: 2 April 2009
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**Abstract** As a significant extension of our previous calculus of logical differentials and moving logic, we propose here a mathematical diagram for specifying the emergence of novelty, through the construction of some "differentials" related to cohomological computations. Later we intend to examine how to use these "differentials" in the analysis of anticipation or evolution schemes. This proposal is given as a consequence of our comments on the Ehresmann–Vanbremeersch's work on *memory evolutive systems* (MES), from the two points of view which are characterization of life and use of categorical modeling. It would not be possible to conceive of the "differentials" outside the frame of a MES (and specifically the description of the part played in a MES by the "co-regulators" and their "landscapes" for anticipation and emergence). Furthermore, we hope that this diagram will be useful to exhibit the "emergence of sense" in discourses, and this idea is supported here by a brief examination of how a discourse could be seen as a living system (and then could be studied as a MES).

**Keywords** Novelty · Emergence · Differential · Cohomology · Category · Colimit · Living system · Discourse · Sense

## 1 Introduction

Our epistemological purpose here is a triple examination bringing together living systems, the theory of categories and discourse analysis, in such a way that we can

This article is written in homage to Andrée C. Ehresmann and Jean-Paul Vanbremeersch.

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bring out a precise technical way to analyze discourses as living systems, in order to be able to deal with the "emergence of sense".

In previous works we have shown that the theory of sense could be conceived in terms of the theory of categories, using as a tool either the regimes of assimilations (Guitart 2003) or specular logic (Guitart 1994) and moving logic (Guitart 2005b). In (Guitart 2005a) making-sense and meanings are related to a cohomological style computation, a kind of variation of the logic itself and of the truth of reference, a paralogical cohomology. But the whys and wherefores of the happening of sense remain in parentheses.

In the spirit initiated in the "categorical modeling" (Guitart 2008), it will be possible now to relate discourses and living systems, specifically thanks to the theoretical crossing between category theory and living system theory fully deployed in the works of Andrée Ehresmann and Jean-Paul Vanbremeersch [from their first paper (1987) to their site (1999) and the book (2007)]. So we can consider this paper as an application of their theory of *Memory Evolutive Systems* (we would like to speak here of the Ehresmann-Vanbremeersch's method,[1] shortly the *EV method*).

In paragraph 1 we propose the idea that the theory of categories is a natural tool for the study of living systems, and the idea that a discourse is actually a living system. In paragraph 2 we suggest that nature, both in its biological and its physical aspects, is written in a categorical book. In paragraph 3 we insist on the fact that there are no objects, there are only presentations of objects, and objects exist only as a process. In paragraph 4 we engage in the question of the paradoxality of the idea of emergence, and then in paragrah 5 we can propose a model.

A discourse is a living producer of itself, embedded in the medium named "language". Through this analogy, at the end of this paper we propose a mini-model for the "emergence of sense", in terms of a cohomological measuring dA of the non-identity of a discursive aggregate A with itself. This emergent cohomology appears then as the formal cause of the sense, that is to say the formal cause of the paralogical cohomology with which any achieved discourse is provided. It will exhibit the articulation between "difference" d and "becoming" via the "possible pasts" p.

## 2 Categories, Discourses and Living Objects

Our horizon, in several works, is a theory of meanings and sense keeping a close watch on ambiguity of speeches and on the irreplaceable use of metaphors and narrations. Especially we play down the importance of the semantical content, and in the field of speech we emphasize the role of the gestures of anyone speaking. From a hermeneutic perspective, the sense of a text is what we understand of the

[1] Here we do not include a summary of this EV method, we refer the reader to the book (Ehresmann-Vanbremeersch 2007), or to (Ehresmann-Vanbremeersch 2006) which is enough to follow our comments. A French version of the introduction of Ehresmann-Vanbremeersch (2007) can be found at the url: http://people.math.jussieu.fr/~guitart/docpp/evsem07.pdf. One can also read Ehresmann-Vanbremeersch (1992b).

living process of invention of new interpretations. The sense depends on the imagination of a subject (Fœssel 2007; Ricœur 1986), and the basic operation of such an imagination is collecting and qualifying collections of elements as novelties ["découpages de sous-ensembles et titrage" in Salanskis (2007 p. 15)]. A model of this operation is the idea of "inductive cone" in a category [*i.e.* "collective link" in Ehresmann and Vanbremeersch (2007), p. 53]. From a structuralist perspective, the sense could be understood as the shape of the discourse of a subject considered as an open sub-system of the language, and this shape is given as the geometry of an evolutive diagram.

We think that the theory of categories is itself an open mathematical discourse on the living process of mathematical activities, and so is well relevant for the pertinent modeling of discourses.

In the late forties a book of mathematics for elementary school in France was entitled "The Living Calculus" ("Le Calcul Vivant") (Vassort L and M 1949). The expression "Living Calculus" is related to Freinet and the movement "Ecole Moderne", it is to promote a mathematical learning analogous to the learning of natural language. Here we start from this point of view: calculus is living as language is living, and therefore the living side of calculus could be helpful to indicate the living side of sense. This very real vivacity of sense is of course another thing than the ineffable and transcendent aspects of the human speech.

For discourse analysis, the very methods of categories and cohomology are of interest, as we developed them with moving logic (Guitart 2004). With such a categorical neo-structuralist approach two pitfalls could be avoided. First we are not in the urgency to furnish numerical results, which cancels any access to the vitality of letters and to the theoretical sense of the model. Secondly we are clearly confronted with the ambiguity of the meaning of "model" (Guitart 2008). In some cases a "model" is like an automaton in which computations on inputs produce outputs useful for prediction, with no pretention to intelligibility: it just has to work. In other cases a model is rather a kind of laboratory for symbolic manipulations of ideas to get intellectual explanations. In other cases as well the model is on the side of gestures, it is the proposal of an action at a theoretical level, with symbols, such that its use provides a sensation equivalent to the sensation obtained in acting in the real situation which is modeled. In this case we want to produce an affect, to manage gestures in the place of other gestures, and, as Herbert Holl says, that is characteristic of a living translation process. It is mainly in this last sense (production of affects) that we consider the construction of models. We insist on the fact that not only the work of the person which is modeling is near to a living translation, but also the situation which is the object of the modeling (in our case the discourses) is also to be considered as a living system; in the end it is a question of "living (symbols) for living (objects)".

The "living part" of a discourse depends on a reactive confrontation of the one who is speaking with a grammar (a grammar itself in evolution), on the fact that to invent sense he respects and he transgresses grammatical rules; it also depends on the variable embedding of the discourse in the language and on the structural effect of this embedding in relationship with the pursuit of this given discourse. So, with the terminology of Jacques Lacan, the crucial aspect is more the "significance"

("significance") than the "signification". And moreover this "living part" of a discourse is another thing than the living biological body of the speaker; precisely it is at the point of interaction between the living system of the discourse and the living system of the body that we could get a theory of symptoms in psychoanalysis.

Thinking of language as a living system (for both the whole system of the language and for the private local language of someone) could be brought closer to the conception of the psychologist Lev Vygotsky (Vygotski 1934). For Vygotsky the language of a child is at first just for social exchange of information, and then it evolves toward a self-centered state, where it regulates actions, descriptions, planning, anticipation. This evolutive language is a mediator for the functioning of thought. The crucial notion in Vygotsky's work is the notion of a "zone of proximal development" which is the difference between what a learner can do without help and what (s)he can do with guidance or help (by adults or other learners). In this theory the development of the child, his language and his thinking process, stays open, follows various paths inspired by different sources in his complex social environment. The learning is organized by "propping up" in the zone of proximal development. So we conceive the discourse as living in the language of the speaker, and this language as living in the social environment.

Some authors (Clot 2001) join together the theory of Vygotsky and the theory of Bakhtin, as they complete each other. The theory of Bakhtin is named "dialogism", and it says that sense and meanings are constructed inside the replying process, by addresses and retorts. The opening of a feasible world of sense results from reactions among interlocutors. In this Bakhtin's manner J. P. Darré [cited in Clot (2001)] indicates that "consciousness, far from existing by itself, is a living orientation of the body toward itself and the others". This is useful for the conception of sense.

We observe here the analogy between the idea of the zone of proximal development (and the associated method for learning), the idea of dialogism as interaction in dialogues and the conception of sense and consciousness as orientation and forecast in interactions, and the categorical modeling in the EV method of anticipation in living systems.

Very often in argumentation, each element is perfectly admissible by each of the opponents, but the difficulty is with the consideration of the elements as a whole A, and the problem is with the concatenation process. Concatenation provides in fact something which is new, a bit of sense of which it would be necessary to debate beyond the isolated meanings of elements. Concatenation is usually modeled by an inductive limit or glueing, and its shape has to be analyzed with respect to logic, grammar and traditions of a given level. Paradoxically the strength of a given argument depends on the obvious acceptance of its novelty by everybone, as if everyone could anticipate this argument. Here we find the operator d: $A \rightarrow dA$ which exhibits the self-variation or proper novelty of A, as will be explained in paragraph 6. So in the case of discourses, dA has to be constructed from logical variations and grammatical exceptions. This was the purpose of logical differentiation in (Guitart 2004, p. 21).

In the book "L'œil mathématique" (Chauviré 2008), Christiane Chauviré explains the conception of Peirce who thinks that a mathematician reasons about

diagrams exhibiting the structure of a given state of mathematical things, that he makes variations on these diagrams, and so he produces new statements which were not explicitly or implicitly contained in the initial data, and which are rigorously proved by this process. In this style, the theory of categories is a calculus with mathematical gestures (Guitart 2005a), and as expressed in (Ehresmann and Vanbremeersch 2007, p. 12), "it reflects some of the main capacities of the brain […] formation, dissolution, comparison and combination of relations between objects […] synthesis of complex objects from more elementary ones (formation of colimits)". So the theory of categories has a natural claim to the analysis of systems, it could explain a moving architecture of things by a moving organization of mathematical objects. The main clue is to understand the possibility in the categorical framework of parsing the variation of diagrams, and the fact that we can do that in a diagrammatic way: the categorical grammar consists in the diagrammatic rules for diagrams constructions and deconstructions. The main rule is the possibility of giving a presentation of an object as a solution of a universal problem, or as a colimit (coherent binding). The clarification of this point using the fact that any mathematical theory could be presented with these tools only is at the root of the theory of sketches (Ehresmann 1968).

From this point of view, today living mathematical discourses are very close to games with diagrams, close to categories and functors, and they are more distant from the standard formulation in terms of first order logic. These discourses, which "are" categories now, are to be analyzed by categorical methods, that is to say mainly by the tools of homological algebra and algebraic topology (the area of mathematics for which in fact the theory of categories was invented).

In the case of a given non mathematical discourse the question clearly is more difficult to grasp, the pleasant thing being nevertheless that we have only to see the categorical side, how we could found a categorical discourse similar to the given discourse, and naturally conceived by (and following) the capacities of our spirit to make mathematical gestures: this mathematical discourse will be a mathematical image of this capacity built over the given non mathematical discourse.

The EV method expands in a very natural and interesting way to questions of cognition, the theory of brain and neural systems, consciousness. Here we do not try to comment on that, we just consider the roots of the method, in some neigbourhood of the question of emergence. For us precisely, the question of cognition is close to this question of emergence. Cognition is situated between discourse analysis and the theory of living systems, especially when we think of cognition in a social science double perspective, in the spirit of Vygotsky and Bakhtin. At first the question is that of the brain as a living system, and of the products of the brain which are discourses of the living system of the things of which we are speaking, and of the living people to whom the discourses are addressed. Cognition is looking for the formal process toward an understanding of sense considered as an event between discourses and worlds, worlds of which the discourses are speaking, worlds to which discourses are speaking. So a parallelism is possible with the situation of living systems in interactions: here we have an interaction between discourses, brains, society, that is to say the affect of sense and the production of sense, the way in which our discourses have influence on others. We meet here the famous problem in

Artificial Intelligence of the relation between the machine seen as a black box and the machine seen from the point of view of its internal logical constitution, as expressed by Kleene's theorem (Barbin 2003), but now we get this problem for the understanding of a living intelligence. So here we get a larger perspective in order to understand discourses as living systems, a perspective which consists in considering a discourse as situated in a living interaction with other discourses, and then the relation between this living situation with other living situations which are the system of the brain and the system of society. So, briefly, we have to claim, in order to be able to study cognition and consciousness, for the elucidation of morphisms between living systems and the determination of a category (in the mathematical sense) of living systems.

The more natural way in order to understand discourses as living systems is precisely to use of the categorical way (as taught by the Yoneda lemma, the ideas of localizations, diagrams and fibrations) for the comparisons of a given discourse to other living systems associated to it. So really we need a category of living systems LS, and then the sense of a discourse D could be thought of as the way it resounds categorically among living systems, something like a shape LS/D (see the description of the shapes hereunder at the end of paragraph 3).

So we have to consider seriously the idea of categorical modeling in biology. Of course we have here a lot of works already accomplished in the style of "System Theory", e.g. (Goguen 1971; Ehrig and Kreowski 1974). But if we restrict to the strict problem of living systems, the first to be cited is Robert Rosen (1958) who thought about categories and biology in connection with the work of N. Rashevsky (1954, 1967) on topological and relational views in biology. In Rosen (1985, 1986) he elaborates upon a significant theory of anticipatory systems. In 1974 I. C. Baianu and M. Marinescu (1974) have provided a categorical model of the metabolism-repair systems of Rosen. Of interest for us also is the work of P. Kainen (2005) who emphasized that the theory of categories by itself is a continuous rewarding process in a kind of hierarchical elaboration of concepts, as in Evolution. Kainen also explained how perception and action in living systems can be expressed with adjunctions, or better (because of the non uniqueness for the biological actions) with weak adjunctions (Kainen 1971). He pointed out also that preservation of colimits is related with anticipation of consequences (Dubois 1998; Ehresmann and Vanbremeersch 1987). Currently I. Baianu (2007) works on "categorical ontology" as formulated by R. Poli (2001). The question of levels is studied with the emerging of complexity, and the analysis of characteristics and level binding is envisaged from the categorical point of view. Baianu observes also the usefulness of colimits for the analysis of a local-to-global approach to the growth of living organisms according to E. Mayr (1970). Recently there have been multidimensional approaches with toposes [see Baianu et al. (2006)].

Chiefly here we are looking at the relation between biology and its categorical modeling according to the works of Andrée Ehresmann and Jean-Paul Vanbremeersch, for more than 20 years. Their elaboration does not consist of an elementary use of some basic notions of categories (objects, morphisms, functors, adjoints) but they make deep use of the categorical craft with diagrams in such a way that their ideas result directly from manipulations and computations with

drawings. From the very beginning in 1987 they used "sophisticated" tools: diagrammatic completions by colimits, and fibrations. The colimits (inductive limits) are a glueing process, the fibrations are lax-colimits or 2-glueings; and so the essential starting point in Ehresmann-Vanbremeersch's method (EV) is this use of glueings at two levels, with the analysis of the formation of the 1-glueing inside the 2-glueing, an analysis in terms of coherence and variation. Metaphorically at least it is like in the constitution of Newton's law as $(mx')' = F$: there also we are looking to a variation $(\text{-})'$ of a variation $(.)'$. In fact from 1963 to 1965, Andrée Ehresmann (Bastiani) had already studied the so called preferential guidable systems in connection with the Bellman principle, with the help of categories and species of structures (Ehresmann 1963, 1964, 1965) [See a synthesis and the relations with studies on MES in Ehresmann (2008)]. And now, with living systems, we are again really in this type of question of a categorical version of a qualitative self-guidance.

## 3 Life and Machine, the Metaphor

Before going on with categorical things and the EV method, we would like at first to briefly clarify the meaning of "living organism" or "living systems". In the course of History, several answers have been proposed for the question: what is life? (Morange 2003; Tirard 1997, 2000, 2008a, b).

On the one hand, in the early eighteenth century, Georg-Ernst Stahl promoted animism, a preliminary variant of vitalism. A living being is an organism, i.e. some mixed physico–chemical compound equipped with an extra datum, which is its finality; and with respect to its own finality it exists as a very precise and delicate complex. Living beings are characterized by their growth and their perishability, coming from their soul which is the product of the vital source in them; and by this process they escape the laws of physics and chemistry [see Pichot (1993, pp. 453–502)]. In the continuation of Stahl, the French vitalists (Paul-Joseph Barthez, Théophile de Bordeu, Xavier Bichat) abandon the idea of the soul, but retain the fact that a living being contain a principle in conflict with the physical and chemical laws [and these laws are contrary to life (Pichot 1993, p. 325)]. So Bichat wrote about living bodies: "Without a permanent principle of reaction, they would succumb. This principle is the principle of life" (Bichat 1962, p. 43) and we know his famous: "Life is the system of functions which resists death".

On the other hand, we find the mechanism, as expressed by Ernest Kahane in the twentieth century in these words: "Life does not exist" (Kahane 1962). Already in 1922 Jean Nageotte argued radically against vitalism and the "finalist illusion" (Nageotte 1922): "Nothing in what we know allows us to understand life as an other thing than a natural physico–chemical process in the living body". The problem of life, for Nageotte, lies in the "utmost complexity" of situations, and he argues against any emergentism: "We must not assign properties to sets of forces which do not have their source in the constituents".

In fact, paradoxically, each attempt to define life takes its source in the tension between science of matter and the science of life itself. From a lecture of S. Tirard (2008a, b) some important steps are given by texts written by Buffon, Lamarck

(Recherches sur l'organisation des corps vivans, 1802), Thomas Huxley (1868), Oparin (1924) and J. B. Haldane (1929), Schrödinger (1944) (where arguments are elaborated on the questions of information from one generation to the other and of macromolecules), J. Monod (1970), E. Schoffeniels (1973), M. Morange(2003).

In fact as an answer to the question: "what is life?" very often we get rather an answer to the question: what are the origin and the history of life, how did life start and construct itself? As a prolongation of chemical synthesis in the nineteenth century (starting with the synthesis of urea by Wöhler in 1828) in the first half of twentieth century, in particular by Oparin and Haldane, Lipman, Harvey, is introduced the scenario of an evolution from a Prebiotic Soup toward the elaboration of proteins and complex organic matter, as in the complex coacervats of Bungenberg de Jong in 1932, the Stanley Miller experiment in 1953 (Miller 1953; Maurel 2003, p. 43; Bada and Laczano 2003) which succeeded in producing amino acids, supporting of the idea of an analyzable and reconstructible life. Today G. Joyce (1992) sustains the point of view of Directed Molecular Evolution: "Life is a self-supported chemical system, capable of a Darwinian evolution". This conception comes with the NASA program in exobiology, founded on variation and a selection process. In this line of thought in 2002 there was a chemical synthesis of poliovirus (Cello et al. 2002).

The approach through the question of "origin" is in some sense as a "stammer", useful for the question of emergence. So let us admit with Marie-Christine Maurel that "the living ought to be capable of reproduction, but with some whims (or "errors" as someone could say inaccurately) which are the mutations needed for evolution. There is life only where there is an evolution process. Life transforms energy and matter in complex structures, and life is limited in space and time". (Maurel 2003, p. 38). In this view life is a complexification process by trial and necessary errors, and by selection of novelties; the system has to be self-informed at least at two levels. Then the question is: what is the cause of the complexification process from which emerges the new objects, and why do we have such an emergence process related to a retroactive system? The approach through the question of "origin" reduces to"how do such systems emerge which are capable of an emergence process?"

Opposite to the question of the very definition of life or the question of its origin, we can ask about the exhibition of criteria for living situations. Morange says that we can define only theoretical entities, and we cannot define real objects of the world as living organisms; these organisms are rather defined in a functional way, by the fact that they are a (living) system. We have to emphasize the modalities of connections among the characteristics of living systems. A majority of biologists (Morange 2002) agree on three characteristics: reproduction with variation, molecular complexity, activity of metabolic exchange.

It is precisely in the line of these characteristics that we would like to construct a theory of the sense of any given discourse, and not on the analysis of codes and transmissions. For us the sense comes from an excess over encoding procedures, as life is an excess over physico–chemical reactions; in both cases the point is to detect such an excess in the "significance" ("signifiance") and through the emergence of

new structures. So a metaphor in a discourse comes as the emergence of a new possibility of amalgamation.

For the continuation of our observation in terms of categories, we have to noticed the definition by Robert Rosen [cited in Baianu (2006); Rosen (1991), p. 244]: a material system is a living organism if it is capable of metabolism and repair, and if, at the same time, it is "closed to efficient causation", that is to say that the function of any component of the system is explainable inside the system itself: the system is the whole of its efficient causation, and it contains the reasons for its functional organization. Furthermore, a distinction is possible among three ideas (Chemero and Turvey 2006): closure to efficient causation, operational closure (Varela 1979) (when every effect of the system is also a part of the system), catalytical closure (Kauffman 1993) (every product of the system is also a catalyst in the system). Chemero and Turvey explain these distinctions and Rosen's ideas with the help of hypersets (as defined in Aczel 1988).

So we keep the idea that in living objects there is certainly a "matter" organized in interaction with the environment and with itself, there are catabolism and anabolism, internal reproduction and autonomy. Mainly we retain the thema of complexity and hierarchy, variation, proliferation and emergence, regularity and singularity, catastrophe. Life and death, emergence and fracture, in the terms of (Ehresmann and Vanbremeersch 2007). In fact the emergence is a true emergence under the Rosen's condition that the merging object comes from the inside of the system, and this implies that the death of the system could be conceived as an internal consequence of the system's composition, because of missed emergence, fractures generated by the system, as in the EV method. About the question of the determination of what is a living system, the EV method is compatible with the ideas of Rosen on anticipation, as exposed in (Ehresmann and Vanbremeersch 2006).

The paradoxical problem here is to get a coherent holding for two opposite aspects: on the one hand the "internal" causation of life and death, the system itself produces the fact that it is living and then that it will die; on the other hand there are "external" agents in the environment of the system which act on the states of the system, but nevertheless these external agents are not the efficient radical causes of the life and the death of the system. Of course the naive cause of the death could be an external agent (e.g. a virus) but we have to understand why this agent is harmful for the structure of the system, how this "possible death" is inscribed in the growth of the system, how the effect of this agent could be anticipated by the system.

In fact the idea of the living serves as a guiding metaphor to think about complex systems, that is to say systems which are not only elementary mechanisms, in which (Zwirn 2006) we find holism and these situations where "the whole is more than the sum of its parts", self-organization and emergence (the richness of interactions allows the system to get a spontaneous organization), adaptability, evolution and selection. In this way we think of biology of course, but also economics and linguistics, and also Artificial Intelligence. We think of our brain as a living system not only because it is really a biological living system, but also because of its complexity, and the complexity of what it does. For Marvin Minsky the functioning of our brain as the functioning of our society is a matter of diversity and complexity.

He said that "very few of our actions and decisions come to depend on any single mechanism. Instead, they emerge from conflicts and negotiations among societies of processes that constantly challenge one another" (Minsky 1988). For him, "minds are simply what brains do", and the complexity of brains explains the complexity of minds.

In this perspective enlarged from the strict living (as far as we know what it means) to complex systems, the point that remains active is the idea of emergence. Of peculiar interest in this area are ideas on cellular automata, neural networks, swarm intelligence [see Bonabeau et al. (1999) and Eberhart and Kennedy (2001)] in which new objects are produced as an effect of the collective action only, and are not thinkable from each component. But the newness in such a swarm system could be very complex whatever the complexity of the swarm system itself, and so it is like a structural proliferation, a cancer, an exuberant behavior. The result there is announced as complex (and new) by comparison with the initial simplicity of the system.

In a different configuration, in the spirit of the Stahl's conceptions of a living organism as a too complex and unstably stable system (from which the necessity of the death is understandable), we could relate the fact of being a living system to the native complexity of the system (and not to the complexity of its productions). Here we meet the theory of complexity of Chatin, and the proofs of Gödel's Theorem in the direction of an analysis of complexity, with a type of argument absolutely different from Gödel's by a self-reference or diagonal process. In fact the sentences which are too complex with respect to a given system of proof would not be proved [see Chatin (1982); Calude and Jürgensen (2005); Delahaye (2009)]. Always out of a given mechanism there will be some too complex sentences inaccessible by means of reason; the emergence of such an unprovable true sentence could be thought of as being something new, and in some sense something living and/or autonomous (out of a specified external guiding system). In fact such a "living" thing exists in our classical physics, because of the uncomputability of the stability of some solutions of equations (Barrow 2006; Da Costa and Doria 1991).

In the study of classical mathematical physics in the "old fashion", these aspects of living systems are not taken into account a priori; on the contrary we are looking there for simple laws, simple objects, stability, and a strict reproduction of an object exactly as it was, so we can understand the position of Stahl in his time. But now in physical research the situation is a little different, with the consideration of probabilities (and laws with a few exceptions), and with the analysis of the distribution of exception for any proposed law, the description of how a law is more or less satisfied. After the over-all picture changes further when we enter in the idea that the law for regular situations comes from the knowledge of exceptions, singularities, catastrophes. Numerical analysis loses a part of its interest when we look for qualitative models, when we try to avoid cumputations and to analyze ("déconstruire") calculus, thus arriving at a second order computations level.

Nowadays the worry of complexity and irregular objects is present in physics. However when these worries are present in physical theory it is precisely intuitively supported by a metaphor with living systems; the theoretician plays the game in which everything is as if physical nature was living. And at the same time, some

biologists have the reciprocal dream of a living nature as an incredible super-machine. Leibniz decided to put the calculus of infinity inside algebra, and in some way this dream is almost achieved by the nineteenth century; now we can dream of constructing the laws of living systems in the dependence of possible laws for machines, in such a manner that living systems could be naturally called "machines"! But at the end the question which remains is exactly the question of how, in these "machines" causation is managed. In Rosen's perspective, the living systems are under a non-classical logic, and they look like physical systems only locally in space and in time. These living systems would be like manifolds constructible by glueing of classical physical systems, and so the very principle of living systems exactly resides on the point of the glueing process among a game of classical physico–chemical models, and of its variation in time and in space. So the core of a living system would be a glueing process, and the main functional aspect of the system would be a glueing process in it, from which the emergence emerges. This is precisely accomplished in the EV method, with the consideration of fibrations for the first glueing, and of colimits for the second one. We observed that at the end of paragraph 2.

In (Sinding 1993), Christine Sinding examines the use of metaphors in biology, especially in endocrinology the metaphor of Ernest Starling about the "wisdom of the body", and some other cognitive metaphors as those which allow speaking of receptors and messengers. The theory of life grows as a picture of our conception of knowledge, and as an effect of this conception. E. F. Keller studies the part played by metaphors in the invention of theoretical tools for biology (Keller 1999). At first she underlines that "the classical distinction between literal and metaphorical sense is no more valid in scientific language than in the ordinary". Behind the invention of the gene we foresee the metaphor of the homunculus, and with the conception of an encoded text (Schrödinger) we have in fact "a localization of the Maxwell's demon inside the molecular structure of the gene". So some tools from discourse analysis work as incentives and stimuli for theoretical thinking about life. To think in terms of genes and encoding leads to the view of a living system as a kind of super-text, written in a mathematical language. But this time, in contrast with the famous Galileo sentence, the mathematical language in question is not the one of euclidean geometry, it is rather the language of the mathematical theory of words and languages. So we get the temptation to identify the living with the sense of this super-text.

So the physical explanation of mechanisms and the biological explanation of the living systems fall at the feet of the metaphor of a Mathematical Book, in a geometrical or in a linguistical style. We accept the idea that something here is to be unified, if we use a mathematical system which unifies geometry and the theory of languages and theories. Such a system in mathematics exists, it is the theory of categories, with which we could study movements as well as encodings. It is our job then to distinguish in the big book of nature between the mechanical and the living, between what is dead and what is alive. Taking the consideration in the opposite way, our proposal is to study the sense of a discourse as its living part, while its truth is conceivable as its basical inertial constitution. The modeling of the living is thus at the tension point in the Categorical Book of nature, between algebra and

geometry; and this meeting point is precisely the place of cohomological and homological algebra, the place of algebraic analysis of shapes and figures, and very especially this involves the categorical analysis of shapes and co-shapes.

The categorical theory of shapes is remarkably simple and efficient. We consider a functor J: $\underline{M} \rightarrow \underline{X}$, and with each object X in $\underline{X}$ the category of objects over X via J, denoted by $\underline{J/X}$, with a fibered functor [J/X]: $\underline{J/X} \rightarrow \underline{M}$. Then we get a functor [J/]: $\underline{X} \rightarrow \underline{Fib(M)}$ from $\underline{X}$ to fibrations over $\underline{M}$, a kind of parser for objects of $\underline{X}$ seen as agregates of objects in $\underline{M}$. We assume that objects in $\underline{M}$ are simple or elementary bricks with which we can build objects of $\underline{X}$, and we see [J/X] as being the information about X observable from $\underline{M}$, the so called "shape" of X. For us this «shape calculus» constitutes the first ingredient of any categorical modeling. With the properties of [J/-] we just have a relative and dynamical version of the Yoneda's lemma. The homology and the cohomology of an object X are in fact associated to a choice of a point of view "J", and depend only on the shape (or co-shape) of this object X through this point of view.

With shape theory we have a deep and very flexible tool for categorical modeling. A reason for that, in our opinion, is that, in the first place, with the theory of exact squares (Guitart 1980) we have a natural unification of various categorical specifications (adjonctions, limits, extensions, faithfulness, cofinality, and so on), and that, in the second place, these exact squares are the natural morphisms between theories of shapes. So with exact squares as morphisms we have a natural category of theories of shapes, and there the process can be repeated, we can introduce the "shape of a given shape theory", and so on. So that the shape method could be applied to itself.

## 4 Time, Computations, Objects, Individuations

Let us distinguish the "clear letter" used in geometry, of which the signification is visible, from the "dark letter" in algebra, that we can only touch. For Hamilton, algebra is time, and the creative emergence of thought arrived in the process of blind computation. The time in a modeling is the time of the computation process, the space is the spacing that we understand at distance from a figure. This tension is dialectically resolved in the categorical setting for these two poles; and actually the two opposite intuitions of time and space are simultaneously expressible in categories, in the diagrammatic way. In fact, in categorical constructions, time and space proceed by quotient from a more primitive datum which is the rhythm (in the spirit of Pierre Lusson).

We do not forget also the strong paradoxical flexibility for modeling in the idea of an arrow. At some moment an arrow means a differentiation, a distinction, and at some other moments it could mean an identification or assimilation. So, in the same categorical modeling we are able to put in interaction these two heterogeneous aspects, the differentiation and the assimilation, both as relations among agglomerates or clusters of objects.

But the more decisive argument is the following: the theory of categories, as a general theory of mathematical practice and gestures, started in parallel with a

historical lack of interest in the search for «concrete» mathematical objects in the mathematical world; in a categorical setting, an object doesn't exist really in a full autonomous way, it is in fact just a name at a crossing point within a category. We do not need substantial things, just we need functional operators.

The idea of a foundation for mathematics, as emerged at the end of the nineteenth century, was already distanced from the claim to a concrete existence for mathematical objects. The belief that it is necessary to maintain a connection between mathematical constructions and objects in the world, has lost much of its meaning in the erea of axiomatics. Often in mathematical society today, the belief is held that mathematical objects as avatars of platonic ideas exist, but this is mainly from an aesthetic perspective. In fact the "objects" have no referents in the real world or in an ideal world. In fact if we turn to the functional approach of mathematics, objects such as numbers, functions, sets, spaces, disappear with their specific constructions as soon as we are thinking at their theoretical function. The object is just an abstract intersection of morphisms, it does not have an internal substance, it is an empty place, fully turned toward its function in its category, and its existence is nothing else than the stable existence of this function. And above all, we have to emphasize the fact that we could never really "give" a mathematical object, the only possible data being *presentations of objects* and not objects stricto sensus. The presentations of an object X in a category C are objects P of another category D, and the first object X could be announced as an equivalence class among presentations P. And furthermore the category C itself, where X is living is given in fact by a presentation, and so on.

In fact there are no objects, only presentations of objects, and a practice of computations and discussion with presentations; these practices at first are homological, analysis of equivalences, comparison of presentations: in a wide sense this is homological algebra. And it is much worse: the computation exist only partially, and we have to use presentations of computations; homological algebra itself is a kind of deconstruction of the computations. A case of interest is the description of a limit as a quotient of a lax-limit in the category of categories, by inverting some arrows [see details in Guitart (1978)]; by this observation, a finer analysis of emergence in the EV method could be possible.

These remarks are related also to conceptions of Gilbert Simondon about individuation (Simondon 1964). From the point of view of Simondon, it is not enough to have the concept of stable individual objects, we also have to think of these objects as unfinished, in progress towards a future. The essence of an individual rests on the metastable process of its individuation, by composition of a lot of changing interactive factors, by resonances, distinctions of contents in shapes. In this line of thought, Henri Van Lier considers that "A real view of the Universe presupposes an axiomatic of the becoming, and so a general theory of exchanges and modifications of states, an "allagmatic" (allateîn = to change, to exchange). Therewith, any form (Gestalt) or formation (Gestaltung) exists in accordance with its emergence on an older background in which it was in germ, and in accordance with its immersion toward a new background. It will be more or less complex, more entropic or more allotropic, according to good or bad chances in Evolution" (Van Lier 2006). In our modeling of living systems we have to take into account this fact

that there are no true individuals but only individuals in a precarious becoming. This is a transposition of our previous observation that there are in fact no objects, and precisely a presentation of an object is, in some sense, the object seen as a becoming from looking backward. In the EV method the formation of objects as colimits is in fact presentations of these objects, and more generally it is what brings the analysis by "shapes". In (Ehresmann-Vanbremeersch 1992a) is already exposed the description of a co-regulator CR in a system which does not classify objects by strict isomorphisms, but according to their shapes; conversely we can say that CR is objectively known only as a sub-category of the shape of CR, and this sub-category is the *effective shape* of CR. So CR do not exist as objects, they exist only as diagrams and according to their effective shapes.

To go further in this direction in future studies, we propose that in the modeling of a living system (or of a discourse) every object B is associated to a non-empty family of glueings, i.e. equipped with p(B), a diagram of diagrams such that B is the "aggregate" of each diagram which is an object in p(B). So p(B) is the system of heaps preceding B, of which B is the possible becoming in the category surrounding B. So "p" is the "possible past", a reciprocal of the becoming "b". We write $A \in p(B)$ if and only if $b(A) = B$. We find there a function for outlines ("ébauches"), machines and hypermodules (Guitart 1978), and for the functor of shapes [J/-]: $\underline{X} \to \underline{Fib(M)}$ (met in paragraph 3).

Modeling supposes that some objects or semblances of objects are incorporated into a computation with chronological variations, in order to produce values and affects for the user. In the case of biology as in the case of discourse analysis we meet here a serious difficulty: what should be the true objects and the correct chronology guiding their development? With the categorical approach we can suspend the answers to the question of the substantial nature of objects, emphasizing functional potentialities in the game of morphisms; and we can put the accent on the natural timing related to compositions, aggregations and disaggregations of objects, under the supervision of the rules for progression and factorization of morphisms; and as a natural external timing we have the chronology of observations and actions by functors coming in and out of the system. The system itself "computes", it does some internal modifications, agglutinations and dispersions (metabolism and catabolism), emergences and fractures; and out of the system we also compute, in order to understand the properties of the system, to note success and failures among emergences: these computations are for us as observations and measures of changes and identifications, their natural setting should be homological algebra.

## 5 The Emergence, the New, the Anticipation

A clear paper on "emergence" with an extensive bibliography is (Stanford Encycl. Phil. 2006). See also for the history (Blitz 1992). An emergent property is novel, it has a new qualitative aspect which is not the resultant of the properties previously existing, and it is unpredictable, the exhaustive knowledge of the properties of previous beings is not enough to derive the new property. In the nineteenth century J. S. Mill (1843) introduced the idea of heteropathic effects (with these two

characteristics: novelty and unpredictability) of life: "it is certain that no mere summing up of the separate actions of those elements will ever amount to the action of the living body itself" (Mill 1843, Bk. III, Ch. 6, §1, p. 268). For essentially the same idea G. H. Lewes (1875, p. 412) introduced the term of "emergence". On this subject it is important also to read A. Bain and H. Bergson. After 1920 emergence is worked out by S. Alexander (1920), L. Morgan (1923, 1926, 1933), and R. W. Sellars (1922). We also notice the conceptions of Charlie D. Broad (1925) with the emergentism, a middle way between vitalism and mechanism, the emerging comes from the lower levels, but it is new. Broad is also interested in the similar paradox of free will. Here we have to be very careful with this paradoxical tension emergentism/reductionism, it is a key for mathematical examination of emergence.

The emergent supervenes [on supervenience see Davidson (1993)], as the spirit supervenes upon the matter of the body, it is something added to the ordinary aggregate of elements during the process. At a level n°1, coming out of the process the emergent product (structure, schema, pattern, property) is something more than the ordinary predictible result of inputs at level n°0. As it does not come from the initial external inputs, the emergent seems to come from the inside of the process itself, from its abilities of self-organization [see for example Ali and Zimmer (1997) and Goldstein (1999)].

Frédéric Fabre (2005) looks at the emergence of new significations, and for him the inner workings are some modification of representations of concepts and theories. For example Mandelbrot discovered the concept of fractal as a consequence of a change of the classical concept of dimension (which became the concept of fractal dimension). The emergence results from a rearrangement and is the casting of a presentation (of an object, of a conception). In mathematics with a change of presentation of an object we can see it in another way, we can modify its theoretical or practical significance, and so new possibilities of computations and results arrive.

So it is possible to envisage a modeling of emergence starting from the idea of self-distortion of a process and the autonomous guidance of this distortion through a learning process. In the theory AMAS (Adaptative multi-agent systems) (Capera et al. 2003) the emergent object is the fact that the system modifies its function in order to fit its environment (seen as a given constraint). Each part of a system generates a partial function which is an element of the global function, and the global function is a glueing of these partial functions. As the glueing is guided by the geometry of parts of the system, a modification of the function could be reached by a modification of this geometry. The emergent object is not explainable by the ingredients, but by the addition of an action, namely the action of modification of the geometry. As is the case in chemistry or in cookery, where the order and the proximities of ingredients act upon the final result. In these cases a hidden modification of order or proximities (with respect to a canonical obvious disposition) could make the result seem to be an emergent object.

This principle at the root of AMAS, or also the idea of Fabre on the modification of representations, could be brought closer to the EV method. In the EV method the emergent object is more global than the local parts of which it is composed, and comes from something which is not specified as a local ingredient, that is to say the

order of disposition of the ingredients. In this way a meta-reductionism is possible, and a mathematical version of the situation is given by the analysis of the shape of glueings, with the help of homological algebra. So via homological arguments we could quantify emergence.

However, in the EV method things are more advanced. By anticipating the completions that we could need in future, we could consider collections and assemblages which will be interesting to glue at some stage as being data in the system ab initio; but the properties among these collections would stay strongly intermingled because, as is explained by the EV method, the true constructions are not only made of free successions of free completions, we have to add some specific limits, to force some given cone to become a limit, to cancel some objects [see "A characteristic of Complexity", (Ehresmann and Vanbremeersch 2007, theorem 7, p. 141)]; and these intermingled limits are again causes of emergences. The hierarchical complexification by levels of constructions of limits (as used by Ehresmann and Vanbremeersch) is indispensable, and it could be visualized [see Obtulowicz (2007)] in terms of membrane systems of Paum (2000).

After the fact, we could consider the whole of the system with its history, and there every ingredient of the history would be an element, notably the subsets of the system which have been used during the story. Once the history is finished, reductionism will be almost possible. But even at this moment we cannot get a real anticipation from the beginning a priori (absolute reductionism) as long as the system has not selected in real time among various possibly incompatible complexifications. On this point the theory of MES is good, sophisticated as it is necessary.

There are two modalities of emergence well noted for a long time. The first one is "vertical" and presents a spatial flavour. Its cause is for instance a changing of scale or of structural hierarchical level; for perception an example is when you move back from a stage and you discover something which is too big to be seen nearby; or when we can see on a map the name of the country only in broad perspective. The other one is "horizontal" and has a temporal signification, it needs time and it is related to adaptation and selection and modification over time. In modeling by the EV method the two directions are clarified, the first one supports the hierarchical complexification, and the other one is made explicit by the action over time of co-regulators (CR) and by evolution guided by them in reaction to the environment. This distinction is interesting as soon as we understand that the horizontal temporal regulation consists of action tending to modify the vertical hierarchical complexity. The emergence results from the co-regulation by the CR, and it is visible from the outside of the system as a vertical variation of the complexity with a horizontal cause. For references in Hierarchy Theory see Allen (2006).

The point of view of Alex Ryan (2007) is closer to the one of Y. Bar-Yam (1997 and 2004), and he suspects that the explanations by emergence and levels of complexification could create a vicious circle or a tautology. Furthermore our hierarchies are rather epistemic, and we should carefully differentiate between epistemological or ontological emergence. For Ryan the explanation by hierarchy creates a confusion, but this could be repaired if we replace the idea of levels by a system of resolutions (and scales), scopes (defined by a spatial boundary in the

broadest sense of the word, with both true spatial and temporal dimensions), and states. A state of the system is "the information that distinguishes between alternative system configurations up to some resolution at one moment in time. Macrostate and microstate denote sets of states with two different resolutions and scopes". The emergence then is a consequence of relations between the scopes of description of one given macrostate and one given microstate: "a property is emergent iff it is present in a macrostate and it is not present in the microstate".

This clarification by Ryan is compatible with the EV method, as we can see in the example that is given on the mathematical side: a Möbius strip can be triangulated to show it has novel emergent properties. The triangular components of the Möbius strip are orientable (two sided) but the strip is not orientable, it has the new property of being "one sided". So, as a glueing of its components the strip is new. Furthermore we can relate the idea of Ryan to the undecidability of mechanics (paragraph 3) as in Da Costa and Doria (1991): perhaps this undecidability could be explained by a drastic changing of scale, "which cannot be overcome" by the computations.

For Rosen (1985) an anticipatory system contains a predictive model of itself and its environment, and so it can change state at a given moment $t$ in agreement with the model expected for the next moment $t+1$. So in an anticipatory system actions are conditioned by anticipated phenomena and expectations about the future. In fact perhaps this property is not really a characteristic of living systems (Dubois 1998, 2000). From Poli's study of the relations of anticipation with levels of reality (Poli 2001) it is clear that anticipation works not only in the case of biological living systems, but also for systems in sociology or in psychology, where anticipation plays a role in the onset of conflicts (Poli 2009). Let us remark finally that there are two complementary orientations for anticipation: one way is by prediction of consequences of possible actions, the other way is by estimation of actions which could lead to some objective.

In the EV method, the co-regulators are not objects but diagrams of objects, sub-systems of the system. They all work together, and each of them chooses the modifications it would like to obtain in the state of the system that it is able to observe (the landscape $\mathrm{L}(t)$ at the moment $t$ for a given CR). The co-regulator CR is a sub-system, its landscape $\mathrm{L}(t)$ is a sub-category of the category of objects over CR in the completion of the system in which CR becomes an object; so the landscape describes also how the shape of the CR is partially analysable by the system at a given moment. Let us notice this reversal of data, where we think of the knowledge of the system by the CR as a functional determination of the CR in the system. The landscape $\mathrm{L}(t)$ is seen as a referent built by the CR, an epistemic map of the system [see Vallée (1986)]. At a given moment $t$, there is a comparison functor $\mathrm{L}(t) \rightarrow \mathrm{S}(t)$ from the landscape to the system. In a second step, a CR has to choose an objective on the landscape, and an admissible procedure to realize this objective.

The procedures are intended to complexify the system, mainly (Ehresmann and Vanbremeersch 2007, p. 118) in three ways: a graph to be absorbed, an object or a diagram to be eliminated, a pattern to be bound (a colimit to be constructed). In a further step, when the procedure is choosen, the CR can "compute" an anticipated landscape $\mathrm{AL}(t+d)$ for the moment $t+d$, and we get a "comparison" partial functor $\mathrm{AL}(t+d) \rightarrow \mathrm{L}(t+d)$. We insist on the fact that this "computation" by the CR is a

physical computation, in real time, it needs the delay $d$, and its result is available only at the moment $t+d$. If the comparison functor is an isomorphism we consider that the objectives are satisfied, otherwise we suppose that there is an "error". An "error" could be the result of a problem in delay, or a conflict with requests by another CR [see Ehresmann and Vanbremeersch (1990)]. So a colimit could not be constructed because one of its components has been destroyed by another CR, or because the construction needs more time than the delay d allotted. If the comparison functor itself cannot be achieved and has to stay in a partial state (for a lack of convenient procedure or because of a perturbation by another CR in a higher level, or because of external perturbations), we will say that the CR has a *fracture*. Starting from this notion of a fracture, and examining the delay necessary for repairing, it is possible to study the degeneration of the system.

A simple question remains: how is the decision about the anticipation produced, how is the objective selected? In the system MES the effective manner for the choice is not specified, it could be by a stochastic process for instance (Ehresmann and Vanbremeersch 2007, p. 194). Whatever would be in concrete terms the process of choice, in the EV method the critical point is that reasons for the choice come from the memory of the system; this memory provides information to the CR about former successes and failures, and the procedure is then by trial and error. An implementation of the procedure in the context of the theory of "cat-neurons" of Ehresmann-Vanbremeersch would be necessary, and a first attempt in this direction is (Monteiro et al. 2009), where they consider neuronal dynamical models given by the Hodgkin–Huxley equations and the recent model of Izhikevitch (2008).

A variant would be to imagine that, with the help of memory, the CR knows another landscape $J(t)$ considered as an ideal state to be reached. This could be modeled by the datum of a landscape morphism $j(t) : J(t) \to L(t)$, and the choice of the procedure by the CR would be guided by the objective to force $j(t)$ to become an isomorphism once translated at the time $t+d$. Also a possibility of a computation with multiple outputs would be via the locally free diagram construction and the associated presentation of algorithms (Guitart 1986). This would allow simultaneously modeling the construction of $AL(t+d)$ and the construction (partial or not) of the comparison $AL(t+d) \to L(t+d)$.

## 6 Emergence and Cohomological Observation of Sense and Novelty

Aristotle wrote "The whole is something over and above its parts, and not just the sum of them all, there is a cause for the unity of the whole" (Aristote 1991, H, 1045: 8–10). The unity of the whole is emergent from the system of its parts, and "emergence" is in fact a very old problem. As we have explained in paragraph 5, the modern idea of emergence of "qualitative novelty" started with Mill and Lewes. It is linked today to the questions of downward causation and complexity. A review on the state of art in emergence is Boschetti (2005).

But the main difficulty with the notion of « emergence » rests in the possibility or the impossibility of a scientific reduction of the explanations furnished by "emergence".

We claim that everything can, in principle, be understood in a reductionist way, from the knowledge of its parts, *plus* the knowledge of the behaviors of each of these parts and of how these parts interact. The question is handling the system of interactions and relative dispositions of parts from an efficient perspective. The emergence of such a perspective is an epistemological emergence of a structure over the initial situation, and in this way we need only epistemological emergence (or emergence of structures) in order to explain the whole of emergence. This seems to agree with the epistemological position in the EV method of a "reductionist emergence" [as explained in Ehresmann and Vanbremeersch (1993)].

An example given by Francis Crick [cited in Corning (2002)] is the case of the benzene molecule which is made of six carbon atoms arranged in an hexagonal ring with a hydrogen atom attached to each. It has many distinctive and new properties ("new" according to classical criteria), but these can be explained in terms of quantum mechanics. So we get this question: how is quantum theory introduced in the system of our explanations about benzene?

Clearly at the end we get a scientific reduction of the properties of benzene, using an external coherent extension of the usual classical physics of atoms, namely quantum theory. The efficient structured perspective is the one in which the ultimate explanation could be formulated in terms of dispositions and interactions of parts. In the case of benzene it is not the initial phenomenological level P of the ordinary formula $C_6H_6$, nor the second level E′ of the more expanded spatial hexagonal formula, but it is the much more structured level E of a quantum figuration. In E the "parts" are not only the classic atoms, and the dispositions and interactions are more sophisticated. We have to consider the final knowledge of the benzene molecule as an object in E, which determines a more contracted object in E′ and then in P, which is the initial version $C_6H_6$ of the benzene molecule.

So we have a sequence $E \to E' \to P$ of forgetful functors, elaborated from P in order to provide a context E in which things could be explained, and in particular the glueings of objects could really be constructed as colimits. In modeling, our reductionist method consists, in the construction from the datum of a phenomenological level P, of a true epistemological level E and a functor $E \to P$ over P, such that "constructions" in P could be presented as colimits in E.

In the EV method there is a game on the hierarchical levels (quantum, atomic, molecular) and a localization at the "elementary level" of the quantum level (which is included in the next ones) from where the emergence started and propagated by iterated complexifications and multiplicities (Ehresmann and Vanbremeersch 2002). This conception is like a mirror image in the internal evolution of the system of the idea on our epistemological system that we have explained above (where the quantum is the last step in the knowledge process, as a covering of the previous steps).

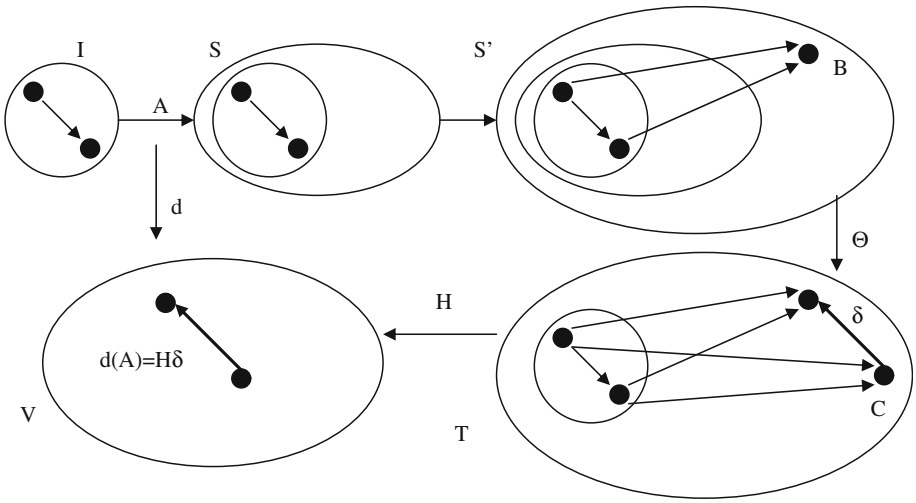
Then in the EV method, the system S is considered as being at the epistemological level $E = S$, and the phenomenological level too $P = S$, and so glueings are actually colimits (in fact it is a little more subtle, because of the hierarchical localization of the origin of emergence, as explained above); we have to assume this hypothesis in the definition below for differentials dA. Otherwise it would be necessary to add an epistemological level $E \to E'$ over the

phenomenological level S → S′, and to compute colimits in E′. Furthermore, in the manner of our discussion in paragraph 4, we keep in mind the possibility of using the shape [J/X] of an object X in S instead of the object X itself. With these precautions it is possible, among other constructions, to include in our setting cohomological glueings, homotopical colimits, and so on.

We would like to emphasize here one possible application of the calculus with respect to emergence and anticipation. Our proposal is compatible with the EV method—of which it is in fact a derived product—and compatible also with various approaches to emergence, and mainly the ideas of Rosen, Ryan, Fabre, and Capera et al., as evoked above. See also here in paragraph 2 our remarks on the proper novelty or internal difference dA of an argument A, and logical differential in the sense of (Guitart 2004), in relation to cohomological computations.

We start with the EV method: a diagram A indexed by I in a system S bind together its links in a new state S′ of the system, under the form of an inductive limit B. Let us suppose now that in S′ we dispose (as a "feature") of a functor Θ from S′ toward a category of ideal types T, and of a functor H from T toward a category V of values. The idea is that, for any object Y of T, HY retains from Y a computable part of the information about the internal organization of the sense of Y viewed in T.

Then there is a comparison map $\delta : C \rightarrow \Theta B$ between the inductive limit C in T (which is assumed to exist) of the image under Θ of the diagram A and the image Θ B under Θ of the inductive limit B in S′. A computable part of this "difference" $\delta$ is $H\delta = d(A)$. So finally we construct as a "measure of emergence" a functor d from the category $S^I$ of I-diagrams in S to the category $V^2$ of arrows in V: $d: S^I \rightarrow V^2$ which will be named a *differential*.




The object B is said to be weakly emergent from A according to HΘ, or according to d, if d(A) *is not* an isomorphism; moreover we say that B is truly

emergent according to d if for any strict sub-diagram A′ of A it is the case that d(A′) is an isomorphism. Short we say that B is *a novelty according to* d *via* A.

And if the information d(A) can be retro-injected as an object or an arrow in the system S, then we get a retroaction loop useful for anticipatory control. But it has to be associated with a system of possible pasts as in paragraph 4. We leave this question for another work.

The datum (H, Θ) could be considered as a precise version of an "observational mechanism" in the sense of (Baas 1996). Later it will be important to decide if this mechanism is or is not a part of the system: it could be possible to think of it as just an external explanation, provided that we integrate in the system the operator of differentiation denoted here by d.

Our first idea here is the case where Θ is a functor to a category T of homotopy types, and H a homological functor to an abelian category. For example we can consider for the object B *a borromean link*, constructed as a diagram of three unknotted circles, with the additional property that any two of these three circles are not knotted also. So in the sense of our definition we can say that here we have the emergence of a novelty, that is to say of a "holding" which was not present in any sub-diagram. We recover also the example given by Ryan (in the previous paragraph) of the Moebius band, where the emerging property is the non-orientability, detectable as a difference d(A).

Another example comes from the analysis of Jacques Ricard (2003a, b, 2008) who is looking for characteristics of information in complex systems in relation to complexity itself and emergence. For a system B = XY where two sub-systems X and Y are associated, let us admit that we know the numerical functions H(B) = H(X,Y), H(X) and H(Y) measuring the number of degrees of freedom or the potential richness of the systems. There is an emergence if we have a strict inequality $H(X) + H(Y) < H(X,Y)$. So the difference between H(X) + H(Y) and H(X, Y) plays the part of our $\delta$: C→ ΘB. Then Ricard constructs H in terms of probabilities as an entropy, and studies the enzymatic reaction of two substrates.

Of course in some sense the undecidable has to be thought of as a drastic novelty. So in line with our scheme we also think of the ideas about the undecidable via the complexity evoked in paragraph 3, with, instead of our d(A), the complexity measure $\delta_g(x)$ of (Calude and Jürgensen 2005, th.4.6.).

In Guitart (2004) we introduced a calculus of a "logical differential" dA for a speculation A over the meaning of a sentence B, in such a way that dA is in some way the paradoxical sense of B. This was a special case of the general scheme given here.

Andrée Ehresmann has attracted our attention to another application of this calculus, in a way sketched in EV as a method for the computation of the strength of a fracture. If we take for S the landscape L($t$) and for S′ the next anticipated landscape AL($t+d$), with the comparison functor from AL($t+d$) to the real next landscape L($t+d$), then d(A) measures the error in the realization of A (with for H a functor associating to any arrow its weight).

In this approach we get a better understanding of an important strong point in the EV method. The EV method allows understanding and computation of the fact that *an object* B *is not necessarily identical with itself*, because the identity map of B is not reducible to elementary identities of elementary components, as soon as the

constitution of B is not fixed once for all. In Ehresmann and Vanbremeersch (1996), this point is made clear (multiplicity principle) by their distinction between elementary links from A to B (those constructed by binding links between diagrams of which A and B are inductive limits) and complex links (e.g. compositions of elementary links, which are not elementary). As it happens an object B may be obtained as an inductive limit in several non comparable ways, and then the identity of B is a complex link.

In the above schematic we have the possibility for the same B to be the inductive limit in S′ of several diagrams A, A′; and then to have several C, C′, $\delta$, $\delta'$, d(A), d(A′). So the novelty of B depends on the specification of A. A better formulation could be that B is a novelty if there is an A for which B is new for d via A. After all, the novelty of B is to be constructed as a category $\underline{\text{Nov}}$(B) with objects the (A, (H, $\Theta$)) such that d(A) is not an isomorphism while this is not the case for any sub-diagram of A. But in fact an axiomatic approach to the differentials here constructed is desirable, and will produce a more direct description of $\underline{\text{Nov}}$(B).

## 7 Conclusion

Our discursive intention here was to join three sources: the theory of categories, and of cohomology, discourse analysis, the theory of life. On the one hand we have started with our previous proposals about sense considered in relation with some aggregates of truths, and then we examined the history of the development of these three sources; on the other hand, and this is the main point, we have exposed and slightly extended the ideas of Ehresmann and Vanbremeersch on the critical part played by the notions of emergence and fracture, and the fact that these notions could be handled with categorical tools. The theory of MES explains how the living can be handled with categorical computations. So, we can hope that a cohomological approach to sense is possible, which, at the end, would present sense as a living calculation. Such a calculation is not at all compositional, and not yet a reduction, and it is not only a computational tool for the science of sense and meanings, but it is properly homologous to sense itself.

The technical development of a discourse analysis with the help of this philosophy has to be pursued later. We have only seen here the theoretical intertwining between discourses, categories and mathematics, and living systems. For example we understand that if we consider a discourse as a category, and more precisely as a MES, then starting from an element B in the discourse, it is possible to conceive a mathematical construction of the novelty of this element (namely $\underline{\text{Nov}}$(B)), and to look at this novelty as an added value of sense for the developing discourse. Of course, as $\underline{\text{Nov}}$(B) is a complete *and* open presentation of the novelty of B, this category is much too large, and in fact for any particular work it will be essential to restrict our interest to a small area in $\underline{\text{Nov}}$(B) denoted by $\underline{\text{Nov}}_{\omega}$(B), and such a restriction has to be understood as a limitation of the understanding that any given observer $\omega$ can have. After that it will be necessary to construct the dialectical tension between the "difference" d (*cf.* paragraph 6) and the "becoming" via the "possible past" p (*cf.* paragraph 4).

In the past, our idea of logical differentials had allowed a trans-logical explanation of the sense of a discourse, linked to the paradoxality needed for its construction. But here now, passing through the characteristics of living systems (complexity, unstable stability, change of scale and evolution, emergence of new properties, self-regulation, unpredictable and undecidable) with the eye of a categorician and with the help of the theory of Andrée Ehresmann and Jean-Paul Vanbremeersch, we have obtained a much more general use of "differentials", and so we get a natural tool for the idea that we would like to support which is the following one: the main source for the sense of a discourse (what we are saying when we speak) is its "*difference from itself* ".

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