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Modern Science and Refutation of the Paradoxes of Zeno

ADOLF GRUNBAUM

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ABOUT 2500 years ago, a Greek named Zeno of Elea confounded his contemporaries by a series of startling arguments. These were designed to show that the science of geometry is beset by a paradox and that any attempt to provide a mathematical description of motion becomes ensnared in contradictions. So seminal was the scientific challenge bequeathed to posterity by Zeno's polemic that the contemporary philosopher Bertrand Russell paid tribute to him (*1*), saying: "Zeno's arguments, in some form, have afforded grounds for almost all the theories of space and time and infinity which have been constructed from his day to our own."

Authorities disagree on the identity of the adversaries at which our searching Greek aimed his intellectual broadside. Disregarding entirely the question of historical authenticity, we shall consider a version of Zeno's paradoxes directly relevant to modern science. To my knowledge, the arguments that I shall offer in refutation of Zeno have not been given by previous writers. It will be best to examine (i) the geometric paradox, which impugns the consistency of the contemporary mathematician's conception of the relationship between a line and its points, and (ii) the paradoxes of motion by which Zeno attempted to demonstrate the impossibility of motion.

Geometric Paradox

In the geometric paradox, our philosopher asserts that it is self-contradictory to claim that a line segment consists of points, each having zero length (*2*). For if a line segment of, say, 2 centimeters actually does consist of points, then the total length of that segment should be computable by adding the individual lengths of its constituent points. But instead of yielding the required value of 2 centimeters, this computation unavoidably yields the paradoxical result of 0 centimeters, since a summation of zeros can issue in nothing other than zero. By the same token, Zeno argues that it is self-contradictory to maintain that a positive time interval can consist of instants of zero duration. Among those who have taken up their cudgels on this issue, we find such diverse thinkers as Kant (*3*), P. du Bois-Reymond (*4*), William James (*5*), and P. W. Bridgman (*6*). Yet the very conception that Zeno and these writers would proscribe has become commonplace in contemporary science through the work of the German mathematician Georg Cantor (*7*, p. 275). It therefore behooves us to come to grips with this charge of absurdity.

Consider the computation by means of which Zeno deduced his paradoxical result. In order to add the lengths of all the points in an interval of 2 centimeters, we must first form a precise idea of how many points compose such an interval. We note immediately that the number of points here is not finite. But of what avail is the trite observation that it is infinite? To no avail, unless we succeed in giving a mathematically articulate characterization of the particular *kind* of infinite collection with which we are confronted. It was Cantor's achievement to provide precisely this characterization. And since it will give us the means by which to disprove the Zenonian allegation against geometry, we briefly pause to give a statement of its meaning.

Suppose that a very intelligent child who does not, however, know the names of numbers exceeding 10 is confronted with two bags of pennies, each containing more than 10 pennies, and is then asked to determine which of the two bags contains more pennies. By virtue of his limited knowledge of numbers, the child cannot make a separate count of the contents of either bag. But his intelligence will enable him to make the required determination nonetheless: he will pair off each coin in bag *A* with a specific coin in the other bag *B*. If he exhausts the coins in bag *B* and still has coins left over in bag *A*, then he will conclude that there are more pennies in bag *A* than in bag *B*. On the other hand, if the supply in *B* outlasts the supply in *A*,

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he will know that the *B* collection is richer than the one in *A*. And if he comes out even, he will know, *without* having been able to make *separate* counts, that bag *A* contains the same number of pennies as bag *B*. By the same kind of reasoning, we know that, in a monogamous society, there are just as many husbands as wives, although we have not taken a census.

Cantor saw that this method of one-to-one pairing lends itself to the solution of the problem of comparing two *infinite* collections as to cardinality: two such collections are equinumerous, if there is at least one way of pairing off their members so as to secure for each member in either collection exactly one partner in the other. In the event that no such pairing can be achieved, the collection that is always left with some unpaired objects is the *larger* of the two. Now, Cantor was able to show (*7*, pp. 278–280) that the infinity of points in a unit line segment is larger in precisely this sense than the infinity of positive integers. Having called the latter infinity "denumerable," he then called the former "super-denumerable."

In the context of modern mathematics, Zeno is thus defying us to obtain a result differing from zero upon adding all the lengths of the super-denumerable infinity of points that compose a unit segment. This means that we are being asked to add as many zeros. To Zeno's mind, it was axiomatic that the result of any addition of zeros would be zero, regardless of the cardinality of the set of zeros to be added. But he could not anticipate that the addition of a *super*-denumerable infinity of numbers, be they zero or positive, presents a problem altogether different from adding *either* a finite sequence of numbers such as 3, 4, 7 *or* a *denumerable* infinity of numbers such as 1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, ... Although arithmetic has evolved a definition of the "sum" of a *denumerable* infinity of numbers by a consistent generalization of the concept of finitary sum on the basis of the limit concept, this definition is utterly useless and irrelevant when the addition of a *super*-denumerable infinity of numbers is called for. Thus, the addition of the lengths of all the points composing a unit segment is a meaningless (undefined) operation in analytic geometry and hence cannot be used to compute a value for the total length of the segment. In particular, the deduction of Zeno's paradoxical result zero for the length of a unit segment is precluded by this arithmetical fact. And what holds for the unit segment also holds for a segment of 2 centimeters and for any other, for, *mirabile dictu*, Cantor demonstrated that no matter what their length, all line segments contain the same super-denumerable infinity of points. Therefore, if the concept of super-denumerable infinity is itself free from contradictions, then Zeno's charge of inconsistency is false, both in regard to the decomposition of a line into mathematical points and with respect to the resolution of a time interval into instants of no duration (*8*).

Proponents of Zeno's view might still argue that this *arithmetical* rebuttal is unconvincing on purely geometric grounds, maintaining that if extension (space) is to be composed of elements, these must themselves be extended. Specifically, geometers like Veronese objected (*9*) to Cantor that in the array of points on the line, their extensions are all, as it were, "summed geometrically" before us. And from this geometric perspective, it is not cogent, in their view, to suppose that even a super-denumerable infinity of unextended points would be able to sustain a distance, especially since the Cantorean theory can claim arithmetical consistency here only because of the obscurities that obligingly surround the meaning of the arithmetic "sum" of a super-denumerable infinity of numbers.

Is this objection to Cantor conclusive? I think not. Whence does it derive its plausibility? It would seem that it achieves persuasiveness via a tacit appeal to a *pictorial* representation of the points of mathematical physics in which they are arrayed in the consecutive manner of beads on a string to form a line. But the properties that any such representation imaginatively attributes to points are not even allowed, let alone prescribed, by the formal postulates of geometric theory. The spuriousness of the difficulties adduced against the Cantorean conception of the line becomes apparent upon noting that not only the cardinality of its constituent points altogether eludes pictorialization but also their dense ordering: between any two points, there is an infinitude of others. Thus, in complete contrast to the discrete order of the beads on a string, *no* point is immediately adjacent to any other.

These considerations show that from a genuinely geometric point of view, a physical interpretation of the formal postulates of geometry cannot be obtained by the inevitably misleading pictorialization of *individual* points of the theory. Instead, we can provide a physical interpretation quite unencumbered by the intrusion of the irrelevancies of *visual* space, if we associate *not* the term *point* but the term *linear continuum of points* of our theory with an appropriate body in nature. By a point of this body we then mean nothing more or less than an element of it possessing the formal properties prescribed for points by the postulates

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of geometry. And, on this interpretation, the ground is then cut from under the geometric *parti pris* against Cantor by the modern legatees of Zeno.

Apart from the *metrical* consistency of the modern conception of an interval as an aggregate of unextended points, mathematicians have succeeded during the current century in proving, by means of the *topological* theory of dimension, that there is also no contradiction in regarding the one-dimensional line as consisting of zero-dimensional points (*10*). This proof is not redundant with the foregoing analysis, since it turns out that the concepts of length and one-dimensionality are not at all the same.

In addition to refuting Zeno's geometric paradox, our analysis yields the following result: unless substantial modifications are made simultaneously throughout the body of analytic geometry, the proposal of some writers that we replace the Cantorean conception of the line as continuous by the postulate that it consists of only a *denumerable,* discontinuous infinity of points must be rejected on logical grounds of inconsistency alone (*11*). For the length or measure of a denumerable point set is zero, as can be seen upon denumerating any such set and then applying the familiar arithmetic definition of the sum of a denumerable sequence to the sequence of zeros representing the lengths of its members.

Paradoxes of Motion

Among the four paradoxes with which Zeno sought to discredit the possibility of physical motion, only two are of relevance to contemporary mathematical physics. Their pertinence derives from that discipline's affirmation that the time variable ranges over the real numbers just as the space variable does. More particularly, it is the *denseness* of the ordering of these numbers, when they are arranged according to magnitude, that provides the point of application for Zeno's polemic. The claim that for any point on the path of a moving object, there is *no* next point, anymore than there is an immediately following or preceding instant for any instant during the motion, enables Zeno to ask incisively: In what sense can the events composing the motion be significantly said to succeed one another temporally, if they succeed one another densely rather than in the consecutive manner of a discrete sequence? This question takes the form of asking (i) how can a temporal process even begin, if, in order to survive the lapse of a positive time interval T, a body must first have endured through the passage of an infinite series of subsidiary time intervals $T/2^n$ ($n = \ldots, 3, 2, 1$), which has *no first term* because the denseness postulate entails infinite divisibility, and (ii) how can a temporal process be completed in a finite time, if its completion requires the elapsing of an endless progression of temporal subintervals $T/2^n$ ($n = 1, 2, 3, \ldots$) in which there is *no last term* as a consequence of the denseness postulate?

The more familiar versions of these two queries are the "Dichotomy" paradox and the paradox of "Achilles and the Tortoise." In the former, whose argument was endorsed by A. N. Whitehead and P. Weiss (*12*), Zeno contends that the mathematical theory of motion entails the impossibility of the very process that it purports to describe, for if there is indeed no point *next* to the starting point on the path of a runner that he can occupy immediately after leaving the starting point, then an infinity of points and, hence, intervals inevitably always keep interposing themselves between the starting point and *any* other point to which the runner would move. And thus the motion is nipped in the bud, as it were. On the other hand, even if, *per impossibile,* the motion had begun, the paradox of Achilles shows that its completion in a finite time is quite unachievable. Having granted a head-start d to the slow tortoise, which moves with velocity v, Achilles can never catch up with the turtle, despite his greater speed V: the fleet-footed warrior will *not* overtake the reptile after a time $d/(V-v)$, because there is no last term in the series of time intervals

$$\frac{d}{V} + v\,\frac{d}{V^2} + v^2\,\frac{d}{V^3} + \ldots,$$

a series whose terms represent the successive times required by Achilles to traverse the successive distances separating him from the tortoise, beginning with the head-start d of the tortoise.

According to a view that is as widespread as it is erroneous, Zeno's argument is no more than a mathematical anachronism. We are told that if he had only known, as we do today, that the arithmetic sum of an infinite convergent series is finite rather than infinite, then he would have recognized that he had merely posed a pseudo-problem. More particularly, the sum formula for a geometric series would have convinced him at once that the sum of the series considered in the Dichotomy is actually T and that the series of times needed by Achilles to come abreast of the tortoise does add up to the finite value $d/(V-v)$. This retort is based on the fact that in modern mathematics, the arithmetic "sum" of an infinite con-

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vergent series is *defined* as the limit of the sequence of its partial sums. An illustration will serve to introduce my contention that this rebuttal fallaciously attempts to settle a question of physical fact by invoking a *definition.*

Suppose that there were no limitations on the rapidity of human speech or thought, so that a man could recite the numbers 1, 2, 3, 4, . . . one by one, taking 1 minute to utter the first, ½ minute for the second, ¼ minute for the third, and continuing to require for each successive vocal act only half the time of its predecessor. The times taken to recite the infinite series of positive integers would therefore form the corresponding infinite series 1, 1/2, 1/4, 1/8, 1/16, ... Remembering that the series 1, 2, 3, 4, . . . has *no last term,* we now ask: Will it take our hypothetical man a finite or infinite amount of time to recite the totality of positive integers? Clearly, this question is one of physical fact. And no such question can ever be settled by having recourse to an *arbitrary* definition of what we shall mean by the "sum" of an infinite convergent series. Mathematically, it is entirely a matter of stipulation, as H. Hankel has explained (*13*), whether we define that sum to be finite in the customary manner or infinite. In any physical situation, it is therefore an open, empirical question which one of these two logically possible definitions is relevant.

It follows that the adoption of the first of these two definitions by the arithmetician cannot contain the answer to the telling question that Zeno raised when he asked, in effect: How can the description of motion resulting from the adoption of the customary limit definition of the "sum" of an infinite series in arithmetic be shown to be the physically true one in the face of the physical difficulties exhibited by the Dichotomy and Achilles paradoxes? To be sure, *if* it can be shown, independently of the meaning that arithmetical theory decrees for the term *sum,* that there are no *physical* reasons after all that would preclude the inception and completion of the motion in a *finite* time, then, of course, the familiar summation procedure used in arithmetic will tell us the length of the required time interval. It will not do to object at this point by saying that a mere glance at a unit line segment establishes that the customary limit definition of "sum" is the physically relevant one, as is seen upon subdividing the unit segment into an infinite sequence of diminishing subintervals ½, ¼, ⅛, . . . , all of which must be contained in the original unit segment. This objection offers a false analogy between a static whole already present in its entirety and subjected to division merely in retrospect, on the one hand, and, on the other, an as yet unbegun or uncompleted temporal process, whose very inception or consummation is first at issue, as we saw in the case of the recital of the set of positive integers.

What is required in order to refute Zeno's objections to the mathematical theory of motion is a proof that neither the denseness of the ordering of the constituent events of the motion nor such features of this process as are entailed by this denseness property constitute obstacles to its inception and consummation.

To lay the groundwork for this proof, we ask: What is the basis for the view that the very meaning of temporal succession involves that events follow upon one another *seriatim,* like the consecutive beats of the heart, and not densely? The answer can be none other than that this feeling derives from a tacit appeal to the properties of the time flow experienced intuitively in human consciousness. Since each act of thought takes a positive amount of time rather than a mere instant of zero duration, it is inevitable that upon analyzing the stream of consciousness into a succession of constituent moments or "nows," these elements are experienced as occurring in a discrete sequence. No wonder therefore that on such an intuitively grounded meaning of temporal succession, there is an ever-present feeling that if *physical* events are to succeed one another in time, their order of occurrence must also be discrete, if it is to be a temporal order at all. It follows that refutation of Zeno will be at hand, if the *psychological* criterion of temporal sequence can be supplanted by a *strictly physical* criterion whose definition of "event *B* is later than event *A*" does *not* entail a discrete temporal order but allows a dense order instead!

Fortunately, the definition of time order and direction made possible by a careful interpretation of the second law of thermodynamics provides a criterion with precisely these required properties. Subject to a qualification to be mentioned presently, this definition can be stated as follows: Of two entropy states of a closed nonequilibrium system, the state of higher entropy will be *called* the temporally *later* state. And we see now that the entropies of a closed system, whose values are given by *real numbers,* can entirely intelligibly characterize a set of physical states that form a linear Cantorean continuum and are thereby densely ordered with respect to the relation "later than."

The qualification to be mentioned is made necessary by the fact that the so-called "periodicity" and "reversibility" objections of statistical mechanics have shown that the entropy of a single closed sys-

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tem does not increase monotonically with time. A viable definition of time order on the basis of the second law of thermodynamics must therefore utilize a more complicated definition that makes reference to the statistical behavior of a *class* of closed systems rather than merely to a single such system. For the numerous relevant details, I must refer the reader to other publications (*14*), where he will also find my reply to the operational critique of the entropic definition of time order offered by P. W. Bridgman (*15*).

Our analysis has shown that we are absolved from the necessity of answering "how" a succession of events can occur by exhibiting a discrete sequence of occurrence. Upon freeing ourselves from the limitations of the psychological criterion of time order by means of the constructive elaboration of an alternative, autonomous physical criterion, it becomes clear that the dense temporal ordering of the constituent point-events of a motion is no obstacle whatever to either its inception or its completion in a finite time. And thus it is seen to be entirely unwarranted to ask "how" the motion can occur despite the dense temporal order, or what the runner does immediately after leaving his point of departure.

Given that it is established now that the entire dense set of events constituting the motion can occur, we can, of course, subdivide it mathematically into various kinds of spatial and temporal subintervals. Thus, the distance traversed can be subdivided into an arbitrarily large finite number of *equal* subintervals. By the same token, the dense ordering allows us to subdivide the motion mathematically into a regression of decreasing time or space subintervals, having no *first* subinterval, as in the case considered in the Dichotomy. Or, we can mathematically effect a subdivision into a progression of decreasing subintervals, which has no *last* subinterval, as is illustrated by the mathematical decomposition of the motion of Achilles. But it would be a grievous error to infer that the individual terms of these infinite series denote *physically distinct* steps into which the motion has been chopped up, and which the runner executes *staccato* rather than *legato,* as musicians would say. For no infinite set of *distinct,* consecutively ordered operations would constitute a physical process that is temporally continuous in the mathematical sense, as the runner's motion is assumed to be. And any such *discontinuous* physical process would indeed defy completion in a finite time: the execution of the geometric progression of physically distinct acts of reciting the positive integers in the manner described earlier would take forever (*16*).

Zeno can therefore not embarrass us by his purely mathematical decompositions of the motion into a discrete denumerable infinity of subintervals. Neither can he create difficulties by pointing out that if we consider the series of individual point-events that divide one subinterval from the next in the Achilles paradox, instead of considering these subintervals themselves, then indeed we *are* confronted with an *unending* progression of physically distinct events, all of which must have elapsed by the time the motion is completed. For by Zeno's own standards, as enunciated in connection with the geometric paradox of extension, no less than by the results of modern measure theory, the total time required for the occurrence of that discrete denumerable sequence of *point*-events is *zero*. Accordingly, this sequence, although unending, can hardly preclude the occurrence of the terminal event of the motion after a finite time.

Quantum Theory

What is the essential bearing of the quantum theory on the argument offered here? The hypothesis of the atomicity of change has taken the form of postulating that there are minimal processes in nature in the sense that no changes occur and remain below certain spatial and temporal minima. The proposed minimal displacement or length, the "hodon," is of the order of 10^{-13} centimeter, the diameter of an electron. And the minimal duration, or "chronon," is of the order of 10^{-23} second, the time required by electromagnetic disturbances, the fastest in nature, to traverse a hodon. Thus, space, time, and change are quantized in the same contingent physical sense in which masses are quantized by the existence of an elementary particle of minimum mass. Being extensive magnitudes, whose values are given by real numbers, "atoms" of space or time presuppose logically all the constituent parts of which they can be regarded to be the sum. In *each* of the finite number of hodons that the runner traverses, he goes through a continuum and hence a dense infinity of points and instants! The "atomicity" of physical processes affirmed by quantum theory is therefore anchored in a mathematical continuum. And the results of that theory do not render our refutation of Zeno either superfluous or invalid.

In regard to theories of space or time quantization that are not grounded in a mathematical continuum but are thoroughgoingly atomistic, it will suffice to cite the following perceptive comment by H. Weyl (*17*): "So far, the atomistic theory of space has always remained mere speculation and has never achieved sufficient contact with reality. How should one understand the metric relations

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in space on the basis of this idea? If a square is built up of miniature tiles, then there are as many tiles along the diagonal as there are along the side; thus the diagonal should be equal in length to the side."

References and Notes

1. B. Russell, *Our Knowledge of the External World* (Allen and Unwin, London, 1914), p. 183.
2. See H. Hasse and H. Scholz, *Die Grundlagenkrisis der griechischen Mathematik* (Pan, Charlottenburg, 1928), p. 11.
3. I. Kant, *Critique of Pure Reason,* N. K. Smith, Trans. (Macmillan, London, 1929), pp. 203–204.
4. P. du Bois-Reymond, *Die allgemeine Funktionentheorie* (Laupp, Tübingen, 1882), Pt. I, p. 66.
5. W. James, *Some Problems of Philosophy* (Longmans, Green, London, 1911), pp. 185–186.
6. P. W. Bridgman, "Some implications of recent points of view in physics," *Rev. Intern. Phil.* **3**, 490 (1949).
7. G. Cantor, *Gesammelte Abhandlungen,* E. Zermelo, Ed. (Springer, Berlin, 1932).
8. For details on the relevant mathematical and philosophic issues, see A. Grünbaum, "A consistent conception of the extended linear continuum as an aggregate of unextended elements.," *Philosophy of Science* **19**, 288 (1952).
9. See E. W. Hobson, *The Theory of Functions of a Real Variable,* (Cambridge Univ. Press, Cambridge, ed. 2, 1921), vol. I, pp. 56–57.
10. See W. Hurewicz and H. Wallman, *Dimension Theory* (Princeton Univ. Press, Princeton, N.J., 1941), p. 18, and, in reference 8, pp. 290–295. See also P. S. Aleksandrov, "The present status of the theory of dimension," *Am. Math. Soc. Translations 2,* **1**, 1 (1955).
11. For details on such proposals, see sec. 7, pp. 302–304 in reference 8. I am indebted to the National Science Foundation and to the Lehigh Institute of Research for support of research in the philosophy of science.
12. A. N. Whitehead, *Process and Reality* (Macmillan, New York, 1929), pp. 53, 96, 105–107, and *Science and the Modern World* (Macmillan, New York, 1925), p. 186; P. Weiss, *Reality* (Princeton Univ. Press, Princeton, N.J., 1939), chap. 6 and 7. See also Galileo Galilei, *Dialogues Concerning Two New Sciences,* H. Crew and A. de Salvio, Trans. (Macmillan, New York, 1914), pp. 162–164.
13. H. Hankel, *Theorie der komplexen Zahlensysteme* (Voss, Leipzig, 1867).
14. E. Schrödinger, "Irreversibility," *Proc. Royal Irish Acad.* **53** (sec. A), 189 (1950); H. Reichenbach, "Les fondements logiques de la mécanique des quanta," *Ann. inst. Poincaré* **13**, 140–158 (1953); A. Grünbaum, "Carnap's views on the foundations of geometry," in P. A. Schilpp, Ed., *The Philosophy of Rudolf Carnap,* Library of Living Philosophers, vol. X (Tudor, New York), in press; A. Grünbaum, "Time and entropy," *Am. Scientist* **43**, 550 (1955).
15. P. W. Bridgman, *Reflections of a Physicist* (Philosophical Library, New York, 1950), pp. 162–167.
16. See A. Grünbaum, "Relativity and the atomicity of becoming," *Rev. Metaphysics* **4**, 143–186, esp. pp. 176–184 (1950); "Messrs. Black and Taylor on temporal paradoxes," *Analysis* **12**, 146–148 (1952).
17. H. Weyl, *Philosophy of Mathematics and Natural Science* (Princeton Univ. Press, Princeton, N.J., 1949), p. 43. For a discussion of the Democritean conception of *mathematical* atomism and its reception in antiquity, see S. Luria, "Die Infinitesimaltheorie der antiken Atomisten," in *Quellen u. Studien zur Gesch. d. Math. Astr. u. Phys.,* Abt. B: Studien, II (Springer, Berlin, 1933), pp. 106–185.

A great deal of the joy of life consists in doing perfectly, or at least to the best of one's ability, everything which he attempts to do. There is a sense of satisfaction, a pride in surveying such a work—a work which is rounded, full, exact, complete in all its parts—which the superficial man, who leaves his work in a slovenly, slipshod, half-finished condition, can never know. It is this conscientious completeness which turns work into art. The smallest thing, well done, becomes artistic.—WILLIAM MATHEWS.

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