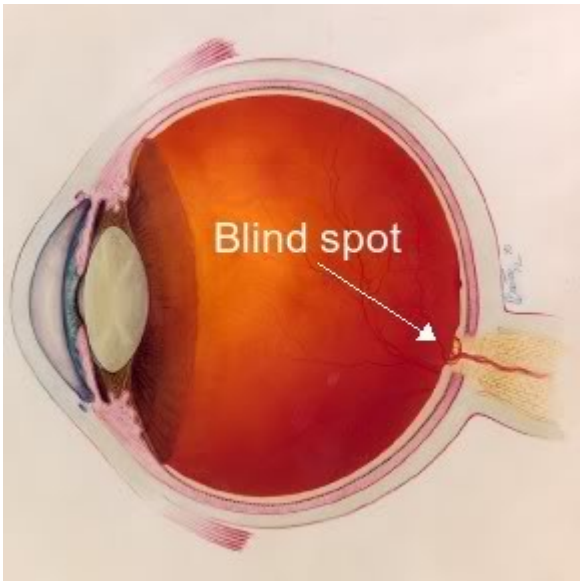
NOTES FOR PRESENTATION IN THE WORKSHOP AT JAN VAN EYCK ACADEMY ON "FORM AND FORMALISM"

**TOPIC: THE CONCEPT OF "BLIND SPOT" (TOWARDS AN OPTOMETRY OF LOGIC)**

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## § 1: THE CONCEPT OF THE BLIND SPOT IN THE BADIOU-MILLER DEBATE

"Science is the Outside without a blind spot."

This is the conclusion Badiou strikes at the end of his critical dispute with Jacques-Alain Miller on the status of science -- a dispute fought out almost exclusively on the terrain of mathematical logic, and its possible articulations onto Lacanian psychoanalysis and Althusserian historical materialism. The conflict played itself out in the *Cahiers pour l'analyse*, a journal directed by a small group of philosophers, linguists and psychoanalysts inspired by the projects of Althusser and Lacan, with respect to which they sought to rearticulate the landscape of epistemology. The group called itself *Le Cercle d'épistémologie;* the Cahiers ran from '66 to '69. It is to logic that I will direct my attention in what follows, leaving largely untouched the full generality of this claim, and its bearing on the sciences in general. Precisely, my question will be: What does it mean, here, to say that logic is *without a blind spot*? What exactly is at stake here?

We'd do well to begin with Althusser, his initial articulation of the concept of "blind spot" and his framing of the problematic of epistemology.

A "blind spot", in the sense that interests us here, is defined by Althusser as **the point at which a field of knowledge, or possible knowledge, has necessary recourse to a structuring principle which it cannot itself articulate within that field**. In *Reading Capital*, this principle is attributed to the *practical form* of the discipline in question, the discipline generating the field of knowledge and its lacuna. Any epistemic endeavour, Althusser notes, has its pattern of stubborn oversights. While these may well be attributable, at times, to mere human weakness, inattentiveness and error, this manner of framing the question is ultimately insufficient. It measures knowledge according to a standard of ideal transparency: the object stands naked before the knower, and delivers to the latter all that it is, at least as a matter of principle. If something remains invisible, recalcitrant, in the object, this fact refers merely to the knower's finitude. Althusser has a slew of epithets for this manner of framing the problem of knowledge, running together with relative indifference expressions such as "realism", "empiricism", "empiricist-realism" (a rather strange sort of hybrid, it always seemed to me), "essentialism", and, most colourfully, "the mirror myth of knowledge". If any headway is to be made in epistemology, he insists, this conception of knowledge must be broken with, decisively. "We must abandon the mirror myths of immediate vision and reading," he instructs us, "and conceive knowledge instead as a production." (RC 24)

To conceive knowledge as production means to conceive its object, in an almost Kantian spirit, as displayed according to structural conditions determined by the form of various production practices. The "visual field" that the knower confronts is not spontaneously given, but labouriously constructed. Expanding on this metaphor, Althusser writes that

1. "Any object or problem situated on the terrain and within the horizon, i.e., in the determinate structured field of the theoretical problematic of a given theoretical discipline, is visible. We must take these words literally. The sighting is thus no longer the act of an individual subject,

> endowed with the faculty of 'vision' which he exercises either attentively or distractedly; the sighting is the act of its structural conditions, it is the relation of immanent reflection between the field of the problematic and *its* objects and *its* problems." (RC 26)

Conversely,

2. "In the development of a theory, the **invisible** of a visible field is not generally *anything whatever* outside and foreign to the visible defined by that field. The invisible is defined by the visible as *its* invisible, *its* forbidden vision: the invisible is not therefore simply what is outside the visible (to return to the spatial metaphor), the outer darkness of exclusion -- but the *inner darkness of exclusion*, inside the visible itself because defined by its structure. [...] And when it happens that, in certain very special critical circumstances, the development of the questions produced by the problematic [...] leads to the *production* of the *fleeting presence of an aspect* of its invisible within the visible field of the existing problematic -- this product can then only be *invisible*, since the light of the field scans it blindly without reflecting on it. This invisible thus disappears as a theoretical lapse, absence, lack of symptom." (RC 28)
   It is Althusser who gives Badiou the directive with which he opens "Marque et manque," the

text from which the declaration that "Science is the Outside without a blind spot" erupts. At Althusser's cue, Badiou writes that

> Epistemology breaks away from the ideological recapture, wherein every science comes to represent its own reflection, by excluding the institutional operator of this recapture – the notion of Truth; proceeding instead according to the concept of a mechanism of production, the structural theory of which is to account for its effect through difference.

How, then, does Badiou arrive at a conclusion so radically opposed to the Althusserian thesis on the structural inevitability of that "inner darkness of exclusion" that is the "blind spot"? To understand this, we must take up the third link in the chain of dispute, and turn to the contributions made by Miller.

Miller's project, in his two key contributions to the *Cahiers*, is to provide a theoretical elaboration of the concept of "blind spot" by means of categories drawn from Lacanian psychoanalysis. In doing so, the concept is generalized beyond the domain of scientific knowledge so as to apply to experience in general, but this generalization need not concern us here directly. In a nutshell, Miller proposes the following theses:

1. Every experience is structured by a structuring principle that cannot be directly represented in the structured field of experience, itself.

2. The structured field is marked by the representationally absent structuring principle in the form of a *lack*, a "blind spot".
3. In order for the structured field to remain *coherent*, the lack must be patched up somehow. (The metaphor of vision is helpful here: think of the way the optical system itself covers over its own lacunae.) This is accomplished by an operation that Miller refers to as a *suture*. It involves the appearance of a "lure" in the place of the lack, a place that cannot rightly be occupied by the inhabitants of the structured field, but which nevertheless serves to *orient* this field.
4. The structuring principle organizes the "visual field" through the mediation of the suturing lure.

These are the theses put forward in "L'Action de la structure". After some initial hesitations voiced in that 1964 text, hesitations with respect to the applicability of this scheme onto the domain of the formal sciences -- logic, for instance -- Miller makes a bold move. In "Suture (elements d'un logique du signifiant)", penned one year later, he elaborates his theory of the blind spot in the context of logic itself -- and specifically with respect to Gottlob Frege's *Foundations of Arithmetic*.

There, Miller identifies a suturing point in the Fregean definition of **zero**: the extension of the concept "not identical to itself". Miller argues that the supposition that *every object is self-identical* is a necessary condition for the coherence of logic's "visual field", understood as "the field of truth": the reason being that if every object were not self-identical, *then the substitution of an object for itself would fail to preserve truth*. (Substitution *salva veritae* is indeed Frege's *definition* of identity, borrowed from Leibniz.) What this determination of the field of truth necessarily *excludes*, is the **subject**. The subject is that which is *not* self-identical -- a thesis which Miller adopts from Lacan. But the subject is not *simply* excluded from logic's visual field, Miller argues, since it is implicitly *called upon* by the definition of zero, which sights it by means of its concept -- the non-self-identical -- by which the subject is "summoned" only to be "annulled". This action of summoning-and-annullment is then repeated in the Fregean generation of the numerical chain, which defines each number N+1 as *the*

*extension of the concept "equal to N"*, a procedure which would get nowhere if zero itself were not counted as a number. The visual field of logic (or of arithmetic, which for Frege is contained within logic) exhibits a blind spot insofar as its structural coherence depends on its having recourse to a structuring principle -- the subject -- which cannot be made visible within that field, since the explicit appearing of the non-self-identical subject in that field would spell the ruin of truth.

It is this articulation of the notion of "blind spot" onto the terrain of logic that Badiou is challenging in "Marque et manque: à propos du zéro". His argument, in sum, is as follows:

1. "Logic" cannot be understood as a single "visual field". It is, in essence, *stratified*. If we wish to speak of what does or does not appear in the field of logic, we must specify which *stratum* we are referring to. Even in the simplest logical systems, there are at least *three* distinct strata. These include: a mechanism of **concatenation (M1),** which takes as its "input" "a stock of graphic marks, separable and indecomposable, forming a finite (or at most denumerable) set" (MM 152), and which yields as its "output" a set of finite sequences; a mechanism of **formation (M2)**, which receives this set of finite sequences from M1 and sorts them into "well-formed" and "ill-formed" expressions according to certain "grammatical" rules; a mechanism of **derivation**, which receives the output of M2 and sorts it into sets of theorems and non-theorems, according to rules of derivability.
2. When Frege states that the extension of the concept "not-identical-to-itself" is *void*, he has no need to look to "something" *outside of logic's visual field.* The formula ~ (x = x) is concatenated at stratum M1, determined to be well-formed at stratum M2, and sorted into the heap of non-theorems at M3. Nowhere does logic have recourse to anything it does not produce at some stratum or other. With reference to the non-derivable character of ~ (x = x), established at M3, it is possible to introduce a constant, **0**, defined as the extension of this formula on the level of yet another stratum, call it M4.

Badiou then generalizes from these findings, which show Miller's search for logic's recourse to an inarticulable "internal exclusion" to be in vain, and argues that *precisely insofar as logic is stratified* it need call upon nothing it does not itself produce, in the structuring of its "visual fields". It is therefore, definitively, without suture, and without blind spot.

Badiou takes this finding as an occasion to mark a crucial epistemological distinction. The concepts of sutures and blind spots are not to be rejected altogether. We must, however, carefully delimit their legitimate domain of application. This domain, in a word, is *ideology*. From science, qua science, they are decisively excluded, and excluded *without a trace*.

The difficult point, here, is determining what is meant by *science qua science*. Badiou insists, with Althusser, and in general consensus with Le Cercle d'épistémologie, that no final separation between science and ideology is possible. To begin with, science always begins its stratified practices by taking up something as "raw material" -- this something can never be a mere heap of neutral sense-impressions, but is always structured in advance, and structured by *ideology* -- understood as something like a historically determinate, but relatively a priori, "faculty of representation". Ideology, moreover, does not rest inert in the prehistory of science, but accompanies it every step of the way. In this way, the Althusserians of Le Cercle take up Bachelard's category of *epistemological obstacle*. Ideology is the means by which a science represents its own activities to itself, by which it gathers these activities into a practical unity, and orients them with regards to determinate and "alluring" problems. This gathering into unity inevitably effects provisional "suturings" of the scientific signifier. Every genuine step forward in the history of a science, however, consists in a real *disruption* of its ideological envelope -- an event that Badiou, with Althusser and Bachelard, calls an *epistemological break*. Each break effectively reasserts the *stratified* character of scientific practice, undoing the sutures that its autochtonous ideology imposes upon it, only to have new sutures reform on the surface of a newly stratified discourse. These new sutures, however, never succeed in reversing the work of stratification

that incessantly displaces them. And so, while "science is science of ideology, and even science of the ideology of the science of ideology, and so on, [...] ideology never recovers itself in it" (MM 173). Stratification, by Badiou's lights, is sufficient to ensure that whatever blind spots crop up in the course of scientific -- and, in particular logical -- history are essentially ephemeral and destined to be displaced.

In this light, we should note, it is necessary to read Badiou's claim that "Science is the Outside without a blind spot" as a claim which takes "Science" as something like a Kantian Idea, a normative ideal which orients the historical processes of the sciences without ever achieving full actualization. It is, after all, to an "*infinitely stratified*" science that Badiou refers in this claim -- an infinity incommensurable with the finite history of any science, which only ever produces its strata in finite assemblages (by the end of Badiou's argument, we get to *four* strata... infinity of any sort is still a long ways off).

## § 2: The Girardian Concept of Blind Spot (and the problem of soundness)

Logic, then, is at least capable of absolving itself of every ideologically induced blind spot in virtue of the *stratificational* structure of its practice. Insofar as logic is stratified, it everywhere supplies itself with its own materials, and so never needs to call upon an inarticulable, unformalizable background in order to conduct its operations. This is Badiou's thesis.

But is this structural characteristic of logic truly sufficient to absolve it of any lacunae?

These matters are difficult to settle at this level of abstraction and generality. They become far clearer on the terrain of concrete logical problems. One problem where the notions of stratification and lacuna become particularly salient, one which is not too complex to become unmanageable here, is that of determining the **soundness of a logical system**. We'll take this problem up now, with respect to the

classical predicate logic, as taught in any undergraduate logic class.

It has become traditional, after Tarski at least, to frame the question of a logic's soundness in terms of **truth preservation**. A logic is deemed "sound" if, and only if, the proofs permitted by its syntactical rules preserve truth, that is, that *the conclusion of a proof will not be false unless one or more of its premises is false*.

Straightaway, it should be noted that no treatment of **soundness** (framed though it may be as a characteristic of the *syntax* of the logic, a characteristic of the "object language") that proceeds along these lines will be a purely syntactic, immanent treatment. We must first of all install ourselves, as Tarski instructs, in a **metalanguage**, the task of which will be to relate the syntax, the object language, to a class of **models**. So, let there be a metalanguage, separated from the object language by a formal *difference in strata*. In this metalanguage, we state the following criteria for soundness:

We can say that a syntax is *sound* if the *provability* of X **implies** the *truth* of X in all models.

Now, if X is provable, it is because X can be constructed by means of the syntactical rules of the logic on the basis of atomic formulae. The basic idea is to show that if, in these rules, we switch the syntactic relation "proves" for the semantic relation "entails", then we obtain a set of formulae, **in the metalanguage**, which follow from the **truth conditions** by which the metalanguage assigns truth values to the sentences of the object language (relative to a class of models, generally). The "truth conditions," for classical logic, are the following:

A ^ B is true <==> A is true **and** B is true
A v B is true <==> A is true **or** B is true
A → B is true <==> (A is true ==> B is true)
~ A is true <==> A is **not** true
(∀x)A(x) <==> A(*i*) is true **for all** interpretations *i* of x.
(∃x)A(x) <==> A(*i*) is true **for some** interpretation *i* of x.

There is no doubt that this approach *works*. If it is soundness we wish to demonstrate, these "verités de La Palice" will let us do just that. But we need not go far into the details in order to realize

that we are being told virtually *nothing* about the logic itself. The soundness of the rules for **conjunction** (the "and"), for example, is worked out by appealing to **meta-conjunction**, and so on. (More than anything, we're reminded of Molière's Bachelierus, who explains to his audience that opium sedates because it has in it a "sedative power"[1], a "vertus dormitiva".) The dynamic of stratification that seems to be playing itself out in the formal mechanics of the method is underwritten by a monotonous drone at the level of *sense,* at the level of the "intuitive" (that is, the "ideological" or "imaginary") backdrop on which the procedure casts its formal shadows. If we suspend this backdrop, if, taking a page from Husserl's methodology, we *"neutralize"* it we find ourselves plunged into the darkest **nonsense**, the brutest **opacity**. It lends itself well to parody, as Girard is keen to point out (at least a dozen times in print, if I'm not mistaken) with an illustration of what he calls "broccoli logic", which enjoys the following connective:

A *broccoli* B is true <==> A is true **broccoli** B is true

a condition in every way analogous to the classical truth conditions.

This is the point on which Girard brings his notion of the "**blind spot**" to bear. This notion of blind spot can be understood as something of a *permutation* of the concept discussed earlier, the concept denoting those blind spots of which logical discourse absolves itself through stratification. For Miller and Badiou, recall, logic would bear a **blind spot** if had necessary recourse to a non-formalizable structuring principle in order to carry out its own operations. Girard, however, speaks of a **blind spot** occurring at those moments where logic has implicit recourse to its opaque ideological accompaniment in order to give its productions the *appearance of sense or intelligibility.* The function of the notion of truth, in logic, seems to be precisely to reserve a *place* for this ideological supplement, a place that the stratified layering of metalanguage upon metalanguage incessantly patches together,

1 "Mihi a docto doctore Domandatur causam et rationem quare Opium facit dormire. A quoi respondeo, Quia est in eo Vertus dormitiva, Cujus eat natura Sensus assoupire." (Molière, *The Imaginary Invalid*) The same allusion lies behind the title of one of Girard's recent articles, "De la vertu negatrice au surmoi logique".

formalizing and making explicit what could only be gestured towards in prior strata, but at the price of shifting the bump in the rug, **displacing the blind spot**, *the place of ideological supplementation*, further and further "upwards" in the direction of the "meta-". This displacement, however, has the fatal flaw of leaving the blind spot essentially *unchanged*.

As this repetition of "meta-strata" carries on, the blind spot -- the necessary recourse to quasi-intuitive notions of truth and of the sense of the connectives, which that repetition never transforms but only displaces -- this blind spot becomes more and more pronounced. Tentatively, we could venture a rough generalization here, and say that this particular "style" of repetition is a reliable index of the fact that there is a lacuna in our midst. [WORK THIS OVER.]

Can we characterize this "style" more precisely? We should note, first of all, that it is never a matter of *mere* or *pure* repetition. With each new "meta-level", there is, as Tarski understood very clearly, an "essential enrichment" of logical structure. Each stratum in this progression must be somehow more powerful than the last, insofar as it must quantify over "higher types" of objects, for example. (We can't use exactly the same tools to quantify over formulae, which themselves quantify over individuals, as would be sufficient in those formulae themselves.) That the metalanguage must be "logically richer" than its object language is absolutely necessary. If the object language had all the power required to function as its own metalanguage -- if it had the power to formulate its own notion of semantic truth and to quantify over its own formulae -- then it would be *inconsistent,* for it would then be capable of expressing such antinomies as the famous "Liar Paradox" (it could say things like "this sentence is false"). However, despite this progressive enrichment, in the Tarskian stratification of metalanguage upon metalanguage, certain core structures find themselves incessantly repeated without any essential transformation or explication. If there is a genuine production of knowledge in such processes, that knowledge does not concern the *core structure* of the object language -- the structure of the logical connectives, for instance.

Of course, not everything can be explicated at once, and none of this implies that the notion of truth or the Tarskian deployment of stratified metalanguages is *useless* in logic. On the contrary, it can be, and has been, immensely productive, and has served to coordinate a plethora of results in the theory of models over the past century, results which, again and again, have served to explicate, transform and interrelate mathematical concepts which would otherwise have lacked any means of communication. It has served, in short, to "put our concepts to work". However, this fashion of deploying the concept of semantic truth, whatever its utility has been for mathematical production in general, generally leaves the structure of logic transparent and unthought, taken as a means but not as an end in itself, we might say.

The recognition of such a blind spot can elicit two very different kinds of response. One of them, perhaps the most common, is to adopt an attitude that Girard calls "essentialist" (an attitude that strongly resembles what Althusser, with a similar abuse of terminology, calls "empiricist"). The incessant redoubling of the structure of the object language at the level of its metalanguage, by "essentialist" lights, only bears witness to an unsurpassable set of "logical intuitions".

The production of such phenomena, in which an incessant movement of repetition is taken to attest to an unsurpassability intuition, can be seen to issue from what Badiou, in an early study of Althusser's thought, diagnoses as an essential function of *ideology*: to install the repetition of immediate givens (here, the manifest structures of the object language) in a 'system of representation [...] thereby produc[ing] an effect of *recognition* rather than cognition' (RMD 449)." The difference between repetition and recognition is bridged by ideological representation by articulating the former against a backdrop of sense that serves to validate it even as that backdrop, itself, refuses all explication. It suffices to neutralize this backdrop to dispel the effect of *recognition* and reveal the incessant form of *repetition* that it underwrites, revealing it in all the barren uncanniness that we find in Girard's parodical "broccoli logic".

The other attitude is to take the blind spot not as the index of an irreducible intuition -- which, as Cavaillès once said, is nothing but an abdication of thought (TC 271/469) -- but as an *imperative for explication*. Here, we ask: What is the *mechanism* that allows these brute and transparent processes to work? How can these mechanisms be decomposed and recombined? What, beyond their obvious and known effects, is their potential?

This second attitude does not at all call for a full-out prohibition of semantic investigation! It does, however, orient the *use* of semantics. In the introduction to his groundbreaking work on Linear Logic, Girard distinguishes two uses of semantic inquiry, roughly correlated with the two attitudes indicated here:

1. Some people [he writes] think that the purpose of semantics is to interpret sets of equations in a complete way; in some sense, the semantic comes like an *official blessing* on our language, rules, etc.: they are perfect and cannot be improved. The success of this viewpoint lies in Gödel's completeness theorem which guarantees, for any consistent system, a complete semantics.
2. A more controversial view consists in looking for *disturbing* semantics, which shed unexpected lights on the systems we know. Of course, this viewpoint is highly criticizable since there is no absolute certainty that anything will result from such attempts. But the stakes are clearly higher than in the hagiographic viewpoint (1) and a change of syntax (as the one coming with linear logic) may occur from *disturbing semantics*. (Girard, LL 14-15)

Insofar as it bears upon the blind spot to which the Tarskian treatment of soundness is concerned, this second attitude (of which the search for disturbing semantics is just one possible prescription) can carry us quite a ways. It demands a radical shift in perspective. It summons us to grasp the concept of soundness without letting ourselves be captivated by the metalinguistic play of mirrors into which the lure of an official blessing might seduce us. The clearest route forward, in the current context, is to frame our problem by means of the formalism deployed by Gentzen's **Sequent Calculus**.

"The Sequent Calculus" refers to a certain style of formalism in which various logical systems, including the system of classical predicate logic, can be presented. Its basic unit, the sequent, is

composed of two collections of formulae (or of sets of formulae), themselves written in the standard form, on either side of a symbol "|–", read "proves". The left hand side of the sequent expresses a conjunction of the premises of the "proves" relation, while the right hand side expresses a disjunction of consequences following from those premises. So,

[DRAW THIS:]

A, B, C |– D, E, F

means that (A and B and C) implies (D or E or F). The reason for treating the two sides of the sequent as conjunctive and disjunctive, respectively, will become clear in what follows, as we get a sense of the remarkable symmetries that it offers us.

Certain operations we are about to perform on this logic will be much easier if we adjust the presentation slightly, by exploiting just those symmetries. These adjustments will not affect the machinery of the logic itself, but will make it far more manageable. They both have to do with negation.

- We will hold that, strictly speaking, negation may be applied only to atomic formulae. We will still write down negated complex formulae, but take these as "abbreviations" for complexes of positive and negative atoms. This can be formalized by way of the De Morgan transformations:

  ~ (A v B) := ~A ^ ~B
  ~ (A ^ B) := ~A v ~B
  ~ (~ A) := A
  ~ (A → B) := ~ (~ A v B) = ~ ~ A ^ ~ B = A ^ ~ B
  ~ (∀x) A := (∃x) ~ A
  ~ (∃x) A := (∀x) ~ A

- It would make things simpler if we had to deal only with one-sided sequents. Here, negation comes to our aid. We know that the right side of a sequent expresses a conjunction of terms, while the left expresses their disjunction. We also know, in virtue of the De Morgan transformations, that conjunction and disjunction are **dual**, that it suffices to **negate a conjunction to transform it into a disjunction**, and vice versa. Therefore, anything appearing

on the left hand side of a sequent can be shifted onto the right hand side by replacing it with its negation. And there we have it. Everything can be placed on the side of consequence.

These two operations reveal to us something about the essence of negation, something far more interesting and profound than the "essentialist" dictum by which "*not*-A is true" means "A is **not** true": classical negation *is* the exchanging of premise and conclusion, of input and output, of left and right in the sequent calculus. There is, here, "*a clear paradigm shift: rather than concentrate on the intended sense of negation, we work instead on its geometry, which happily mocks our intentions*" (PA, 52).

Thus adjusted, the rules for the classical sequent calculus can be given as follows:

$$\frac{}{\vdash \neg A, A}\ (identité) \qquad \frac{\vdash \Delta, A \quad \vdash \neg A, \Pi}{\vdash \Delta, \Pi}\ (cut)$$

$$\frac{\vdash \Delta}{\vdash \tau(\Delta)}\ (X) \qquad \frac{\vdash \Delta}{\vdash A, \Delta}\ (\vdash W) \qquad \frac{\vdash A, A, \Delta}{\vdash A, \Delta}\ (\vdash C)$$

$$\frac{\vdash A, \Delta}{\vdash A \vee B, \Delta}\ (l \vdash \vee) \qquad \frac{\vdash A, \Delta \quad \vdash B, \Delta}{\vdash A \wedge B, \Delta}\ (\vdash \wedge)$$

$$\frac{\vdash B, \Delta}{\vdash A \vee B, \Delta}\ (r \vdash \vee)$$

$$\frac{\vdash A, \Delta}{\vdash \forall x A, \Delta}\ (\vdash \forall) \qquad \frac{\vdash A[t/x], \Delta}{\vdash \exists x A, \Delta}\ (\vdash \exists)$$

[EXPLAIN RULES BRIEFLY, ad lib]

In presenting these rules, we have the freedom to choose from several, logically equivalent versions. We could, for instance, have presented the rule for conjunction in the format:

$$\frac{\vdash A, B, \Delta}{\vdash A \wedge B, \Delta}$$

In the classical sequent calculus, the distinction between this format and the other is a distinction without difference. The two formats are, as I said, logically equivalent. Of course, to say that they are

logically equivalent means, precisely, that their equivalence is guaranteed by certain, specifiable *rules* of transformation. In particular, it is assured by the **structural rules** of *contraction* and *weakening* ((|– C) and (|– W), respectively). This fact will become significant in a moment.

So, now that we have our logic spread out in front of us, consider what it might mean to say that, in purely syntactical terms, that the logic is **sound**. This notion of soundness should, indeed, *agree* with its traditional ("Tarskian") counterpart, but should be formulated in such a way that the inquiry into the soundness of a logic tells us something about the logic itself, explicating its structure, rather than merely holding it up to its own reflection (however "essentially richer" that reflection might be in certain respects). One promising approach would be to establish that any proof in the sequent calculus can be *rendered analytic*, or "normalized". An analytic proof is one which enjoys the "subformula property", which means, in a nutshell, that all of the information contained in the endsequent, the conclusion, can be found on the "exterior surface of the proof" (Joinet cite), that is to say, on the uppermost sequent, the "premises" of the proof. Analyticity, beautifully, *implies soundness*. The task, then, is to demonstrate that any proof in the classical sequent calculus can be rendered analytic, *and to demonstrate this without running afoul of the "blind spots" that plague the Tarskian method.*

The uppermost line of a sequent proof is always given *ex nihilo* by the AXIOM rule, and so is of the form: A |– A (or, monolaterally, |– ~A, A). Consider, now, the sequent presented on any line below the uppermost line of a proof. Call it |– Γ, Δ. Since it is not the top line, it must have been produced by means of another rule of the calculus. Knowing only this, can we say anything about its premises, anything about the line above it? "It is clear," Girard notes, "that we can say nothing if the last rule is a CUT, because this rule has the effect of making a statement *disappear* (which is the origin of the expression 'cut')" (PA 53). If we know that CUT was not used in producing this line, however, we know at least that the line above |– Γ, Δ must be composed of formulae "simpler" than those in the

present line; moreover, we know it must be composed of subformulae of Γ and Δ. We know this just by inspecting the rules of the calculus. So, analyticity can be quite easily established for any proof, *provided that we can demonstrate that the CUT rule is everywhere dispensible*. We must demonstrate that we can, in principle, eliminate CUT from any proof.

This is the burden of Gentzen's *Hauptsatz*, his *fundamental theorem*. It's worth remarking on the strangeness of this theorem. A proof without cuts is essentially a proof that does without what, in the traditional (Hilbertian) axiomatic treatment of logic, is the sole rule of inference, the *modus ponens*. This rule was traditionally treated as essential to deduction, and for good reason. Modus ponens, or CUT, essentially encodes the use of *lemmata* in a proof. Virtually every *actual* proof in mathematics or logic, any proof of real interest, is a tissue woven from lemmata. A fully analytic, cut-free proof, is a proof without lemmata, a proof, Girard likes to say, without "ideas". It's remarkable, then, that CUT is eliminable at all.

But eliminable it is. Gentzen proved his theorem (employing a several lemmata, of course, like any logician). The trouble here, and I hope you'll excuse the fair bit of handwaving I'll have to have recourse to for the next five minutes or so, the trouble is that the various proofs of the CUT ELIMINATION THEOREM, the Hauptsatz, tend to participate in the same tendencies for which Girard indicted Tarskian semantics. [HERE SOME DETAILS WOULD BE NICE] They all seem to find themselves forced into the same lacunic terrain. [DETAILS NEEDED, AT LEAST *ONE* DETAIL!!!] An analysis of these proofs, however, reveals something very interesting: the culprits, the rules which seems to throw us back again and again into various blind spots, come into view. They are two of the Structural Rules of the calculus: WEAKENING and CONTRACTION, the rules which permit us to **repeat** formulae at will, once they have entered into a sequent, and to **discard these repetitions** as we see fit. An intriguing question therefore arises for us: what would happen if we *suspended* these rules, put them out of play? This question forms the first prong of the tactic Girard will adopt.

Conversely, suppose we take the form of the process of CUT ELIMINATION as our focus. Since the discovery of the Curry-Howard isomorphism, we have known that we can transform any proof of the sequent calculus into a functional *programme*, a sort of abstract computer programme, written in a pure theory of functions called the LAMBDA CALCULUS. In the lambda calculus, the process of cut elimination maps onto the process of lambda term normalization, which, basically, is the process by which any lambda term -- any nested complex of functions -- can be reduced to its unique simplest form, if it possesses such a form. Not all lambda terms do, however, but there are subsystems of the lambda calculus, the typed lambda calculi, in which normalization is always possible. One of Girard's key discoveries has been a model for a version of the typed lambda calculus which assigns to each term a topological structure that is *invariant* with respect to the transformations induced by the normalization process. This model is given by the *theory of coherent spaces*. Upon discovery of this theory, Girard sought to isolate the most primitive algebraic operations suggested [W/C?] by this topology. These, remarkably, turned out to form a system of *logical operators*, from which we can abstract a full syntactic system.

More remarkable still, is that both prongs of attack arrive at the same logic! "A logic behind logic", Girard calls it, the "secret blackness" of the classical "milk".[2] We pass from classical logic to its strange and hidden substrate, obtained by suspending the rules which blinded us to its deeper structure, or else by uncovering the most basic operators that could be found in the topology induced by the cut elimination process. As we do so, the very shape of the logic seems to transform before our eyes.

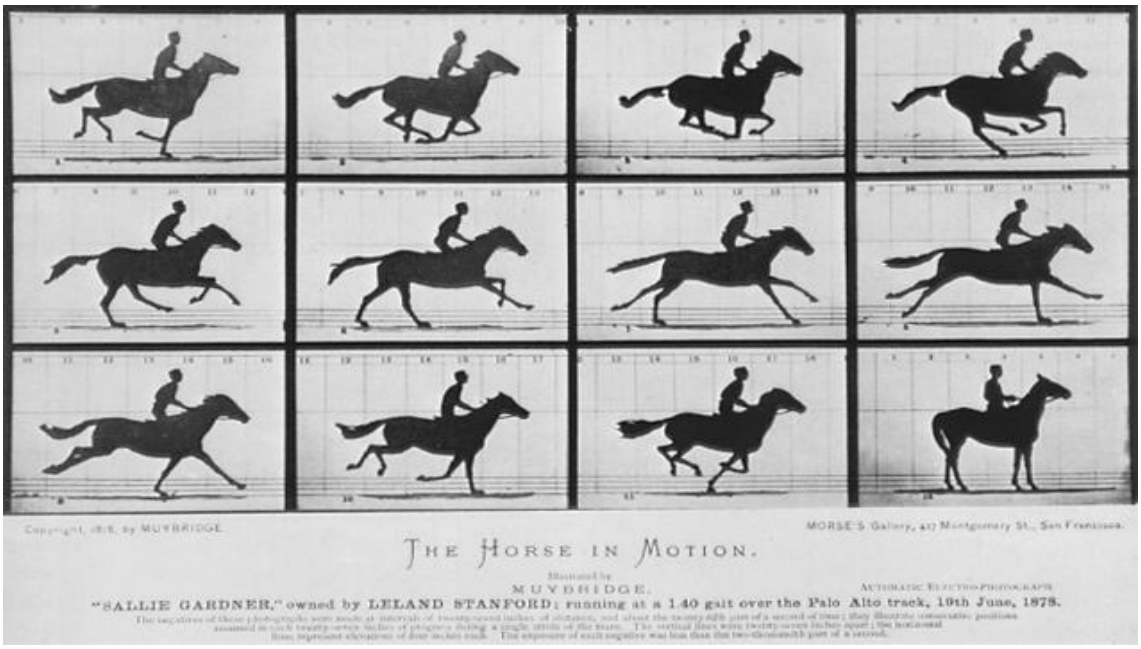
- In the absence of CONTRACTION and WEAKENING, formulae lose their "eternal" character -- their capacity for indefinite repetition -- but take on the appearance of something like "resources" that are consumed at the moment of their utilization.[3] Suppose I am given premises

2 "La secrète noirceur du lait..." This phrase of surrealist poet Jacques Audiberti forms the epigraph of Girard's seminal text, *Linear Logic* [CITATION INFO].

3 Linear Logic, the logic at issue here, is, unfortunately, often introduced as a sort of "special logic" of resource management. That it *can* be applied to such problems as managing resources indeed makes it a valuable tool for various

A –o B and A. I may well infer B from these premises, but in doing so I destroy both A and (A –o B). Extracting an analogy from linguistics, Girard will say that what has been unearthed here is something like a "**perfective**" stratum of logic.

- Formerly equivalent presentations of connectives split apart, stripped of the equivalence in which the two structural rules united them. Conjunction and disjunction each split into two distinct varieties, which Girard terms "multiplicative" and "additive".




Like the legs of Muybridge's horses, logical movements "invisible 'to the naked eye" (Joinet, "Introduction" to *Ouvrir*, p.48) become apparent to us for the first time.

These "optical effects", which invite us to coordinate, in an informal poetic image or representation, what might otherwise appear as discrete formal systems with no organic relation to one another, these are without a doubt "ideological" in the strict sense. But not all ideologies function alike,

pursuits, many of which are central concerns to computer scientists and programmers, for example, but this one possible application of the logic should not let us see Linear Logic as one more among the many logics formulated for such special concerns (such as epistemic logic, deontic logic, modal logics, etc.). What is essential to it, rather, is that it emerges from classical logic by means of an attempt to get at classical logic's obscure subterrain. It is not a special logic, not a logic designed for this or that special purpose, but an attempt to descend to a logic *that is more universal than the most universal of logics at our disposal*.

and it is possible to classify the ideological functions that (without exception) accompany scientific production into various types. (Badiou attempts to do just this in distinguishing between positivist and materialist ideological recoveries of the concept of model.) Without, for the time being, carrying this classification of ideological functions any further, I suggest that they can fruitfully be classified according to the *transformations* that they inflict on their blind spots, and according to the resulting reorientations they impose on the course of scientific research. A *reactionary* ideology revels in reflection, and at phase of the research it accompanies, it preserves the basic structure of that research's blind spot. Insofar as the blind spot of the science is precisely the point at which it must have recourse to an ideological stratum of sense, this strategy conserves the ideology as it is and secures its tenure as the science's official accompaniment -- but at the price of retarding the movement of the science itself, by a sort of counter-heuristic restriction of its horizons. What could we say characterizes a *progressive ideological function*? This is, in all honesty, a point I am still grappling with. For now I can offer nothing less cryptic than saying that the distinction as stake here is something like the distinction between the *vertus dormitiva* and *la secrète noirceur du lait.* I wish I had something more satisfactory. [ANY IDEAS NOW? AD LIB?] [see p.21 of notebook] [INPUT FROM PSYCHOANALYSTS WELCOME]

So, to return to this metaphor, what does the secret blackness of classical logic look like, once we suspend the movements of the structural rules that hid it from us? The rules for this "hidden" logic, to which Girard names "**Linear Logic**," are as follows:

**Identity/Negation**

$$\frac{}{\vdash \sim A, A}\ (identity) \qquad \frac{\vdash \Gamma, A \quad \vdash \sim A, \Delta}{\vdash \Gamma, \Delta}\ (cut)$$

**Structure**

$$\frac{\vdash \Gamma}{\vdash \Gamma'}\ (exchange)$$

**Logic**

$$\frac{}{\vdash 1}\ (one) \qquad \frac{\vdash \Gamma}{\vdash \Gamma, \underset{\sim}{\perp}}\ (false)$$

$$\frac{\vdash \Gamma, A \quad \vdash B, \Delta}{\vdash \Gamma, A \otimes B, \Delta}\ (times) \qquad \frac{\vdash \Gamma, A, B}{\vdash \Gamma, A \parr B}\ (par)$$

(*no rule for zero*) $\qquad \frac{}{\vdash \Gamma, \top}\ (true)$

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, A \oplus B}\ (left\ plus) \qquad \frac{\vdash \Gamma, A \quad \vdash \Gamma, B}{\vdash \Gamma, A \,\&\, B}\ (with)$$

$$\frac{\vdash \Gamma, B}{\vdash \Gamma, A \oplus B}\ (right\ plus)$$

$$\frac{\vdash \Gamma, A[t/x]}{\vdash \Gamma, \exists x\, A}\ (there\ exists) \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, \forall x\, A}\ (for\ all)$$

We might wonder what the point is in this bizarre mutilation of classical logic. Is it anything but an exercise in mathematical surrealism? How can it shed any light on the problem with which we began, that of the soundness of *classical* logic? What is the interest in chasing the problem of classical soundness into this rabbit hole if, in doing so, we completely deform the problem we were pursuing?

This is certainly a deformtion, *but nothing is lost.* It is possible to supplement this "perfective" logic with two new modal operators, which Girard names the "exponentials". The addition of these new modal operators, which permit us to replicate or discard a formulae when needed, along with four new constants (which arise from the classical constants "true" and "false" when they are split by the suspension of the structural rules) completes the description of this new logic, to which Girard has given the name, **Linear Logic.** The rules for the exponentials are as follows:

$$\frac{\vdash ?\Gamma, A}{\vdash ?\Gamma, !A} \quad (of\ course) \qquad \frac{\vdash \Gamma}{\vdash \Gamma, ?A} \quad (weakening)$$

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, ?A} \quad (dereliction) \qquad \frac{\vdash \Gamma, ?A, ?A}{\vdash \Gamma, ?A} \quad (contraction)$$

A proof in which all formulae are exponentialized (so that premises are marked with a ! and conclusions with a ?) **is** a classical proof. (Likewise, the multiplicative and additive variants of the classical connectives collapse back into their classical counterparts once the formulae in which they occur are exponentialized.) Classical logic is therefore a *subsystem* of **Linear Logic**, and something like a "limiting case" of the latter.

And so we have the **old lacunae** before us still, but embedded in a richer and subtler environment, which permits us to grasp them "from the outside", so to speak. In the discrete and manipulable literality of the exponentials, the "transparency" of the imperfective is reduced "to *an opaque modal kernel*" (PA 12). What's remarkable, here, is that Girard counts this reduction -- of lacunic transparency to literal opacity -- as a moment of progress. Something is accomplished, at the very least, in rendering a blind spot "opaque". But what is perhaps more significant is that this "reduction to opacity" -- the concentration of the structural rules of contraction and weakening in the modal operators !A and ?A -- opens the possibility of, first of all, attending to the interactions that take place between the perfective and the imperfective elements of the logic, specifying far more finely than

was previously possible the exact "behavioural" characteristics of the imperfective as concentrated in these operators, and finally, "modulating the principles of perennity"(Girard, "De la vertue negatrice au surmoi logique", p.3) -- subjecting these finely specified characteristics to *variation*, conducting various experiments in the "range" these variations open up between the perfective and the imperfective, the empheral and the eternal.

But let's return to our initial problem, that of determining the **soundness** of a logic.

For at least a *fragment* of Linear Logic -- the fragment composed of the *multiplicative* formulae -- an elegant and informative treatment of the question of **soundness** is possible by means of the theory of **proof nets** -- a theory devised by Girard in response to this very question. The remainder of the logic -- in particular, those limiting conditions under which it reproduces the structure of classical logic -- can be treated along similar lines, but some rather serious difficulties are encountered along that path which require rather complex modifications of the concept of proof net as a result. There, again, logic encounters the **points of opacity**, those resistances to formalization, which the struggle to dissolve the **blind spots** of classical logic repeatedly encounters. I'll venture a few remarks in that direction in a moment, but first we'd do well to get a sense of how the proof-net treatment of the multiplicative fragment of the logic reformulates the problem of **soundness** in its domain.

To generate a proof net, we first take the final conclusion of a sequent calculus proof (in the multiplicative fragment of linear logic), expressed entirely "right handedly" (which, recall, we can do since (1) linear negation is "involutive", i.e. it regulates the duality between premise and conclusion, such that a premise is merely a negated conclusion, and (2) the sequent calculus treats the right hand side as a *disjunction* of formulae).

Suppose that we wish to produce a proof net for the sequent representing the linear version of *Modus Tollens*:

$$A \multimap B \vdash \sim B \multimap \sim A$$

We begin by shuffling everything to the right hand side, putting everything into the conclusion by means of negation:

$$\vdash \sim (A \multimap B), \sim B \multimap \sim A$$

In virtue of the linear De Morgan transformations, and the definition of linear implication (fig. 8 on the handout), this sequent is equivalent to:

$$\vdash (A \otimes \sim B), (B \parr \sim A)$$

To draw the proof net for this sequent, we proceed as follows:

1. We draw two lines stemming from the multiplicative conjunction "A times not-B", connecting it to its two subformulae. This gives us the "times" link.

2. We draw another two lines stemming from the multiplicative disjunction "B par not-A", connecting it to its subformulae. This gives us the "par" link.

2. We draw lines connecting each atomic subformula to its negation. This gives us the two "axiom" links.

**<< DRAW THE PROOF NET FOR MODUS TOLLENS >>**

Our (rather simplistic) net is now complete. It now becomes possible to verify the **soundness** of the net by inspecting *global geometric properties* of the graph itself. This can be done as follows.

First, we define the concept of a *trip* through the proof structure. Imagine a particle travelling through the structure along the lines connecting each node (each instance of a formula). The structure is imagined to be something like a circuit board, with each line working like a pair of wires. A particle begins at an arbitrary node, call it X, and pursues a circuitous path through the structure, passing through each node it encounters on its journey, and looping back when it reaches a terminal node (the "conclusions" of the proof structure, which are found at the "bottom" of the graph). The "upwards" movement can be understood, if you like, as a *questioning* of a given *answer* -- the regression from a

conclusion to its premises, as prescribed by the relevant sequent-calculus inference rule; the "downwards" movement, by the same token, can be seen as an *answering* of a given question -- the progression from premises to conclusion, again in accordance with the relevant rule.

Nodes marked by a ⊗ or a par ⅋ are endowed with a "switch", which can be set to either *left* or *right*. Whatever the setting, it remains fixed for the duration of the "trip". The two switches behave somewhat differently. If a switch at (A ⊗ B) is set to *left*, then the particle enters (A ⊗ B) through its "left" premise (A), but then exits (A ⊗ B) to enter the "right" premise (B), and conversely. However, if a switch at (A ⅋ B) is set to *left*, then the particle enters (A ⅋ B) through the "left" premise (A) and then exits (A ⅋ B) to *head back* to the left premise once again, and conversely for a *right* switch.

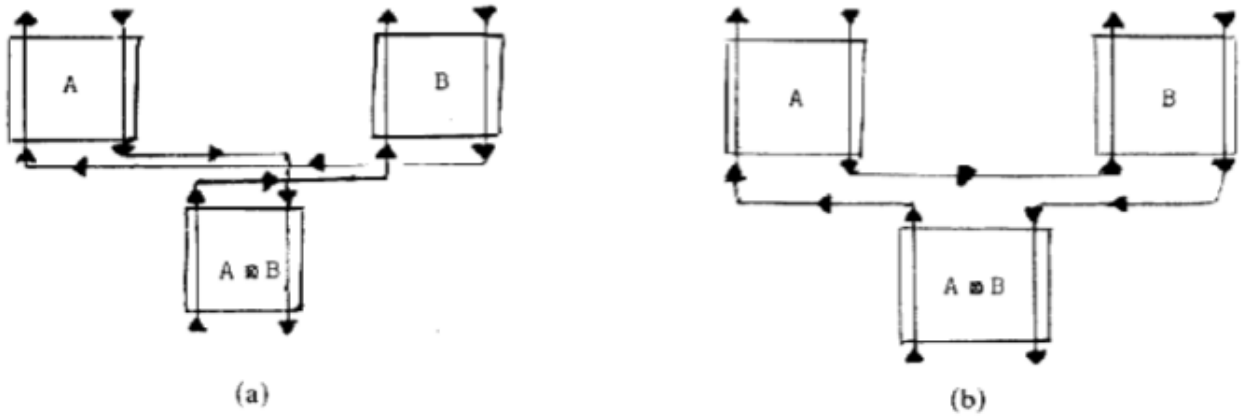



*Illustration 1: ⊗ Link, with switch set to left (a) and right (b)*

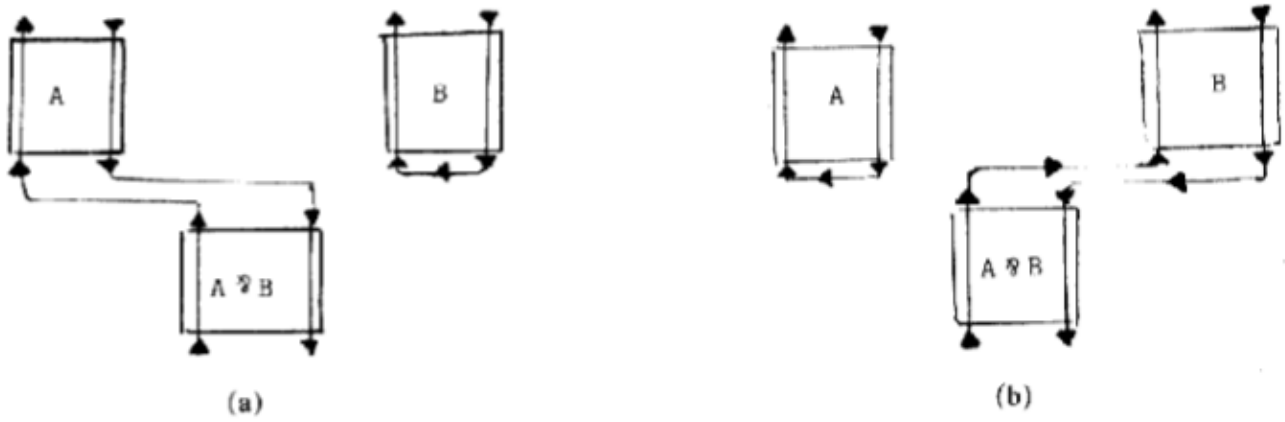



*Illustration 2: ⅋ Link, with switch set to right (a) and left (a)*

Given this framework, Girard writes, "the soundness requirement is simple: for any positioning of the switches, the trip is long, i.e., it goes twice through all the formulas, once up, once down; in other words, the whole structure has been visited, and every question has been answered, every answer has been questioned" (LL 9).

It can be seen that *modus tollens* satisfies the soundness requirement in this sense. To do this, we must work out $2^2$ different trips -- one for each possible combination of switches in the two connective links. To give a sense of how this would work, here's how one such trip could be drawn.

**<<DRAW A TRIP DIAGRAM FOR MODUS TOLLENS, WITH A TIMES ~B SET TO LEFT, AND B PAR ~A SET TO RIGHT. >>**

I should mention, too, that the net-theoretic modelling of proofs gives us a very simple way of treating *cut elimination*. All that need be done to transform a proof-net using the CUT rule (i.e. a net using the CUT link) into a CUT-free proof net is to *delete* the CUT link and its nodes, and then recompose the relevant links between the remaining formulae of the net (these links, in general, may be either additional CUTs obtaining between simpler formulae, or else AXIOM links, when appropriate). What makes CUT ELIMINATION so difficult to manage in the linear sequent calculus is that any given sequent proof can be rewritten in a number of different ways differing only with respect to the order in which the rules are used. The "same" cutting operation can be instantiated, therefore, under a multiplicity of trivially different forms. The reason for the simplicity of the CUT- ELIMINATION procedure in the theory of proof nets, as compared to the sequent calculus, is that nets, unlike sequent proofs, are *invariant under permutations of the order* in which sequent calculus rules are used. [PERHAPS DRAW THE EXAMPLE FROM PA, p.240]

This elegant framework for determining soundness, however, works far more smoothly for the multiplicative and propositional fragment of Linear Logic than it does for the remainder of the system. For the rest, we encounter various resistances, and points of opacity both relative and absolute. These

resistances, I would like to say, should not be seen as mere weaknesses of the method, but points where it displays its epistemic and heuristic *strength*. This point requires some elaboration.

If we wish to produce proof nets of linear sequent calculus proofs that traverse not only the propositional and multiplicative fragment of Linear Logic, but which also make use of the quantifiers, the additive connectives, or the exponential modalities, the simplest way of doing this is to first isolate those bits of the proof that don't stick entirely to the propositional multiplicative fragment. These would be those sections of the proof in which we find [DRAW ON BOARD] ⊕ (*plus*), & (*with*), ∀ (*for all*), ∃ (*there exists*), ! (*of course*), or ? (*why not*) -- in short, sections in which we find any rule in Fig. 7 found beneath the second row of the "**Logic**" section. We then put these sections into "boxes" -- we "black box" them -- in which the relevant sections of the sequent proof are enclosed. The endsequents of these sections, the "bottom lines" of the bits of sequent proof in question -- when these bottom lines are purely propositional and multiplicative in form -- are written at the bottom of each box. [DRAW AN EXAMPLE ON THE BOARD: Fig. 11 from LL, p.43] These are the "doors" to the box, and work like any other set of plugs in our circuit board. These boxes "are treated in a perfectly modular way: we can use the box **B** [...] without knowing its contents, i.e., another box **B'** with exactly same *n doors*, $A_1$, ... , $A_n$ would do as well. This," Girard remarks, "is the principle of the *black box*; in order to check the correctness of a proof-net involving a box **B**, we have to check the proof net without knowing anything about **B** *and* check **B** itself" (LL 43).

As it happens, by tampering somewhat with the theory of proof nets, in some cases only slightly, in some cases rather severely, it is possible to break open *some* of these boxes and transform their contents into net-like structures in which their internal order is reduced to permutation invariant graphs, just as we do with the multiplicative propositional fragment of the logic. While there is a sense in which all non-multiplicative boxes exhibit a sense of "opacity" relative to the optics of proof net theory, this opacity varies in degrees, relative to the changes that would need to be made to the theory

of proof nets in order to accomodate the explication of various kinds of boxes, the explication that would allow their contents *to commute* with the other nodes in the net. I'll leave the details aside. What matters here is that a box employing the exponential modality ! (of course) is "absolute". It *cannot* be explicated into a net. "It is," Girard remarks, "the sole connective which absolutely merits a box (for the others, one could take the boxes as a sort of logical stammering). The connective ' ! ' is violently non-linear, and this non-linearity expresses itself by way of an aggressive sequentialization" (PA 258). The *of course*, the !, the operator which allows us to repeat its argument indefinitely, this mark of the "repetition automatism" internal to Linear Logic in the form of an exception to its general structure, forms an ineliminable *obstacle* to the net-theoretic treatment of linear proofs, a stone that can be circumnavigated but not dissolved. This offers us one sense, not the only one, but one which we can make at least somewhat clear today, one sense in which to understand Girard's claim that with the "of course", the !, Linear Logic reduces a blind spot of classical logic to "an opaque modal kernel" (PA 12).

Why should we be inclined to view this failure, this opaque impasse to a certain effort at formalization (the theory of proof nets), as a strength rather than a weakness of the Girardian approach to the problem of soundness? The classical, Tarskian treatment of the problem of soundness, by contrast, seems to glide over the surface of classical logic without resistance, and encounters none of these difficulties. Why not take this as a sign of the clear superiority of the Tarskian approach? If we are to claim superiority for the stammering of the Girardian method over the Molierian eloquence of Tarskiism, the reason should be this: these kernels of resistance throw the logic into relief, they give a sense of the play of conceptual tensions from which the logic is composed, and they mark out strategic points at which we can imagine, and perform, a *transformation* of the logic in question. The smoothness with which the Tarskian is able to glide over the surface of the logic unopposed is bought at the price of a certain degree of *conceptual inactivity,* inertia. I don't mean to devalue the genuine accomplishments of Tarski, of course, or the positive logical *work* of which his approach to semantics

has shown itself to be capable. But in this work, certain structural aspects of the working logic remain purely transparent, purely instrumental, mere means to the labour of explication that the semantics perform on the mathematical structures whose theories and models they coordinate.

In the opaque stutterings they give voice to, the points of resistance encountered by the Girardian approach to logic are points at which conditions for the invention of new logical forms become evident. We could ask, for example, how the principle of perennity expressed by the "of course" could be altered, adjusted in order to allow itself to be explicated in the geometry of nets. Such questions have been answered, elsewhere in Girard's work, by the invention of "Light Linear Logic", and gesture towards the ongoing development of a geometrical reforging of logic as such, a development which leads onwards to the theory of Ludics and its horizons. Here, however, I reach the limits of both my current understanding of logic and the time I have to discuss it with you, so I'll wrap things up now without going any further in this direction.

I want to conclude, here, with a few final remarks.

1. What can we say, *in general*, of the epistemological function of the concept of "blind spot" that we find at work in Girard's research? It would be completely wrongheaded to imagine that, by following the lines of inquiry adumbrated here, we could someday arrive at a pure and transparent logic, a logic *without blind spots*. No less than in Badiou's early epistemology, we must view the Idea of a *logic without blind spots* as a regulatory Idea in the Kantian sense. It is, I'd wager, an ideal goal for the logician, but one whose actualization would likely be impossible.
2. This is easy enough to see with respect to the Badiousian notion of blind spot: a science without blind spot would be one which had definitively absolved itself of ideology, a science which is *actually* stratified to infinity. But this would be, in all strictness, an *impracticable science*! Not only because no human intellect could navigate its way through, or produce, an infinitely

complex hierarchy of levels, but because such a science would lack any point of contact with its own representation in ideological space, would lack any capacity to formulate norms and problematics for itself, and would lack any dimension of historicity, understood as the *dialectic* between formalization and ideological representation. When every possibility of an epistemological break -- a destruction of a blind spot through the production of a new formalizing gesture -- is behind a science, the life of that science has run its course. It would be left with all the rigour of an inanimate crystal, but that rigour would be impracticable, and it is hard to see how it would even be deserving of the name. Rigour is essentially impure.

3. Can the same be said of the notion of blind spot we find articulated in Girard's work? I think that it can. That the notion of a logic without blind spots *de facto* operates in Girard's work as a regulatory, Kantian Idea -- as a norm which cannot be actualized -- is evident from the very equivalence he supposes between the *appearance of transparency* and *the existence of an unknown blind spot*. Speaking of Ludics, for example, he will say that, despite all the ground that it has gained over the classical approach to the problems of completeness, etc. with respect to breaking apart and explicating the old blind spots, "ludics stumbles on a point: its space of questions/answers (the actions) is pre-constituted, which permits the description of the onion through all ways of peeling it. This set of processes is an ultimate possibility of transparency, of semantics, *of which we must find the blind spot*" ("Le fantôme de la transparence", p.13). The procedure of uncovering a blind spot -- a point at which a semblance of sense is given to what is, in fact, mere repetition of form --- of rendering it into an opaque kernel that can then be tranformed so as to yield to explication, this procedure would seem to have the appearance of a truly transparent logic as the sign of its success. To take that very appearance as the sign that the entire procedure must be taken up again is precisely to at least treat the notion of genuine transparency (a logic without blind spots) as rigourously unrealizable norm. If, moreover, logic

*could* arrive at such a state, it would cease to be of interest. "Something explicit has no value in itself," Girard remarks, "what do you think of 'yes'? Of course this word is more interesting when not said by a yes man, but corresponds to a question. The question corresponds to the implicit, the answer to the explicit, and they are linked by explication. [... "Explicit Mathematics" ...] is a perfect oxymoron: reaoning is precisely about the implicit. Mathematics is neither explicit nor formal, but it should be explicable and formalisable" (*Locus Solum,* p.123).