Topics in Logic
Proof theory

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1 Office information

411 Malloy Hall
Monday 11am-2pm

2 This course

In the 20th Century, a formal theory of proofs was conceived by David Hilbert and realized by Jacques Herbrand, Kurt Gödel, and Gerhard Gentzen. We hope to appreciate the conception and realization of proof theory as deeply as possible. To that end, we will work through classical results of the proof theory of first order logic and then read the original manuscripts of Herbrand, and Gentzen. We'll also read a bit of secondary historical and philosophical literature on those primary texts. Finally, we'll look at a broader selection of historical papers, some more contemporary results and ideas, according to your interest.

Below is a more detailed outline of the course.

In the initial weeks, I will present the sequent calculus for classical, first order logic and some of the classical results of logic: theorems of completeness, compactness, cut-elimination, Herbrand, Kreisel, Craig, Beth. My exposition will follow *Buss 1998a* and *Buss 1995*. Then I will present a sequent calculus formulation of arithmetic and some basic applications of cut-

elimination in arithmetic: Parikh's theorem (as in *Buss 1998b*), a version of Gentzen's consistency proof.

With that behind us, we will begin reading papers by Herbrand and Gentzen, as well as the surrounding essays.

Then, as your interests lead you, we can look into more topics in logic (deeper results in ordinal analysis or structural proof theory), read later papers by Kreisel, Parikh, etc., or look into some of the ideas of proof-theoretical semantics and inferentialism. Hopefully a number of you will prepare to present some of this material, so that the latter half of the course is maximally collaborative.

3 Principal texts

Note: Entries marked with a "‡" are available on-line. Those marked with a "†" will be circulated at the appropriate time. Readings from *Szabo 1969* should be available as a packet soon. You should endeavor to obtain your own copy of *Goldfarb 1971*.

Bernays, P. 1970†. "On the original Gentzen consistency proof for number theory," in *Kino, et al. 1970*. pgs. 409-17.

Buss, S. R. 1998a‡. "Introduction to proof theory," in *Buss 1998*. pgs. 1-78.

Buss, S. R. 1998b‡. "Proof theory of arithmetic," in *Buss 1998*. pgs. 79-147.

Buss, S. R. 1995‡. "On Herbrand's theorem," in *In Logic and Computational Complexity: Lecture Notes in Computer Science* **960**. Springer-Verlag. pgs. 195-209. .

Gentzen, G. 1935. "Untersuchungen über das logische Schliessen." Gentzen's doctoral thesis at the University of Göttingen, translated as "Investigations into logical deduction," in *Szabo 1969*. pgs. 68-131.

Gentzen, G. 1936. "Die Widerspruchsfreiheit der reinen Zahlentheorie," *Mathematische Annalen* **112**, pgs. 493-565. Translated as "The consistency of elementary number theory," in *Szabo 1969*. pgs. 132-213.

Gentzen, G. 1936a. "Der Unendlichkeitsbegriff in der Mathematik," course-notes for a seminar in the mathematics department at the University of Münster. Translated as "The concept of infinity in mathematics," in *Szabo 1969*. pgs. 223-33.

Gentzen, G. 1938. "Die gegenwärtige Lage in der mathematischen Grundlagenforschung," *Forschungen zur logik und zur Grundlegung der exackten Wissenschaften*, New Series **4**, pgs. 5-18. Translated as "The present state of research into the foundations of mathematics," in *Szabo 1969*. pgs. 234-51.

Gentzen, G. 1938a. "Neue Fassung des Widerspruchefreiheitsbeweises für die reine Zahlentheorie," *Forschungen zur logik und zur Grundlegung der exackten Wissenschaften*, New Series **4**, pgs. 19-44. Translated as "New version of the consistency proof for elementary number theory," in *Szabo 1969*. pgs. 252-86.

Gentzen, G. 1943. "Beweisbarkeit und Unbeweisbarkeit von Anfangsfällen der transfiniten Induktion in der reinen Zahlentheorie." Gentzen's Habilitationschrift at the University of Göttingen, translated as "Provability and nonprovability of restricted transfinite induction in elementary number theory," in *Szabo 1969*. pgs. 287-308.

Goldfarb, W. 1971a. "Introduction to Jacques Herbrand's Logical Writings," in *Goldfarb 1971*. pgs. 1-20.

Herbrand, J. 1930. *Recherches sur la théorie de la démonstration*. Herbrand's doctoral thesis at the University of Paris. Translated by W. Goldfarb, except pgs. 133-88 trans. by B. Dreben and J. van Heijenoort, as "Investigations in proof theory" in *Goldfarb 1971*. pgs. 44-202.

Herbrand, J. 1930a. "Les bases de la logique hilbertienne" *Revue de métaphysique et de morale* **37**, pgs. 243-255. Translated by W. Goldfarb as "The principles of Hilbert's logic" in *Goldfarb 1971*. pgs. 203-214.

Herbrand, J. 1931. "Sur le problème fondamental de la logique mathématique" *Sprawozdania z posiedzeń Towarzystwa Naukowego Warszawskiego, Wydział III* **24**, pgs. 12-56. Translated by W. Goldfarb as "On the fundamental problem of mathematical logic" in *Goldfarb 1971*. pgs. 215-271.

Herbrand, J. 1931c. "Sur la non-contradiction de l'arithmetique" *Journal für die reine und angewandte Mathematik* **166**, pgs. 1-8. Translated by J. van Heijenoort as "On the consistency of arithmetic" in *Goldfarb 1971*. pgs. 282-98.

Hilbert, D. 1921†. "Neubergründung der Mathematik. Erste Mitteilung" translated by William Ewald as "The new grounding of mathematics: First report" in *Mancosu 1998*. pgs. 198-214.

Mostowski, A. 1965†. "Theorems of Herbrand and of Gentzen," in Mostowski *Thirty Years of Foundational Studies*, Acta Philosophica Fennica. Helsinki. 1965. pgs. 43-50.

Szabo, M. E. 1969a. "Introduction to the collected papers of Gerhard Genzten," in *Szabo 1969*. pgs. 1-23.

van Heijenoort, J. 1982†. "Jacques Herbrand's work in logic in its historical context," in *Selected Essays*, Bibliopolis. Napoli. 1985. pgs. 99-121.

4 Supplimentary texts

Barwise, J. 1977 (ed). *Handbook of Mathematical Logic*. North Holland. New York.

Buss, S. R. 1998 (ed). *Handbook of Proof Theory*. North Holland. New York.

Buss, S. R. 1999‡. "Bounded arithmetic, proof complexity, and two papers of Parikh," *Annals of Pure and Applied Logic* **96**. pgs. 43-55.

Craig, W. 1956‡. "Review of 'On Padoa's Method in the Theory of Definition' by E. W. Beth." *The Journal of Symbolic Logic* **21**(2), pgs. 194-195.

Craig, W. 1957‡. "Linear reasoning: a new form of the Herbrand-Gentzen theorem." *JSL* **22**(3). pgs. 250-68.

Craig, W. 1957a‡. "Three uses of the Herbrand-Gentzen theorem in relating model theory and proof theory." *JSL* **22**(3). pgs. 269-85.

Girard, J. L. 1987. *Proof Theory and Logical Complexity*. Vol. 1. Bibliopolis. Napoli.

Gödel, K. 1938†. "Lecture at Zilsel's," in *Kurt Gödel: Collected Works - Volume III*, S. Feferman et al, (eds.), Oxford University Press. New York. pgs. 86-113.

Goldfarb, W. 1971 (ed). *Jacques Herbrand: Logical Writings*. Harvard Univeristy Press. Cambridge.

Herbrand, J. 1931a."Unsigned note on *Herbrand 1930*" *Annales de l'Université de Paris* **6**, pgs. 186-9. Translated by W. Goldfarb in *Goldfarb 1971*. pgs. 272-6.

Herbrand, J. 1931b. "Notice pour Jacques Hadamard." Translated by W. Goldfarb as "Note for Jacques Hadamard" in *Goldfarb 71*. pgs. 277-81.

Hilbert, D. 1923. "Die logischen Grundlagen der Mathematik" *Mathematische Annalen* **88** pgs. 151-65. Translated by W. Ewald as "The logical foundations of mathematics" in *From Kant to Hilbert: Readings in the Foundations of Mathematics* edited by W. Ewald. Oxford University Press. 1996. pgs. 1134-48.

Hilbert, D. 1926. "Über das Unendliche" *Mathematische Annalen* **95**, pgs. 161-190. Translated by E. Putnam and G. J. Massey in *Philosophy of Mathematics: Selected readings* Second Edition edited by P. Benacerraf and H. Putnam. Cambridge University Press. 1983. pgs. 183-201.

Hilbert, D. 1928. "Die Grundlagen der Mathematik" translated by S. Bauer-Mengelberg and D. Føllesdal as "The foundations of mathematics" in van Heijenoort *From Frege to Gödel: A sourcebook in mathematical logic 1879-1931*. Harvard University Press. Cambridge. pgs. 464-79.

Hilbert, D. 1931. "Die Grundlegung der elementaren Zahlentheorie," *Mathematische Annalen* **104**, pgs. 485-94. Translated by W. Ewald as "The Grounding of Elementary Number Theory" in *Mancosu 1998*, pgs. 266-73.

Kino, A., J. Myhill, R. E. Vesley. 1970 (eds). *Intuitionism and Proof Theory: Proceedings of the summer conference at Buffalo, N. Y. 1968*. North Holland. Amsterdam.

Kreisel, G. 1951‡. "On the interpretation of non-finitist proofs I," *The Journal of Symbolic Logic* **16**, pgs. 241-267.

Kreisel, G. 1958‡. "Mathematical significance of consistency proofs," *The Journal of Symbolic Logic* **23**, pgs. 159-182.

Kreisel, G†. "Principles of proof and ordinals implicit in given concepts," in *Kino, et al. 1970*. pgs. 489-506.

Mancosu, P. 1998 (ed). *From Brouwer to Hilbert: The debate on the foundations of mathematics in the 1920's*. Oxford University Press. New York.

Parikh, R. 1971‡. "Existenecy and feasibility in arithmetic," *JSL* **36**. pgs. 494-508.

Scanlon, T. M. 1973‡. "The Consistency of number theory via Herbrand's theorem." *The Journal of Symbolic Logic*, **38**(1), pp. 29-58.

Scwhichenberg, H†. "Some applications of cut-elimination," in *Barwise 1977*. pgs. 867-95.

Sieg, W. 1985†. "Fragments of arithmetic," *Annals of Pure and Applied Logic* **28**, pgs. 33-71.

Statman, R†. "Herbrand's theorem and Gentzen's notion of direct proof," in *Barwise 1977*. pgs. 897-912.

Szabo, M. E. 1969. *The Collected Papers of Gerhard Gentzen*. North Holland. London.

Tait, W. 2005‡. "Gödel's reformulation of Gentzen's first consistency proof for arithmetic: the no-counterexambple interpretation," *BSL* **11**(2). pgs. 225-38.

5 Requirements

To earn credit for this seminar, attend and participate regularly and do at least one of the following: write a paper, present some material, or think of something else that I can approve for credit. I encourage you to follow your own interests and chart your own course. You can also audit the course, if you're mostly interested in hearing what others have to say.

6 Note

Please be aware of the University's policies regarding academic honesty, anti-discrimination, and access to education for students with disabilities.

Here is the web-page of the office for students with disabilities:

`http://www.nd.edu/~osd/NEWHOMEPAGE.htm`

Here is the Philosophy Department's web-page devoted to academic honesty, with links to information about plagiarism and the University's honor code:

`http://philosophy.nd.edu/undergraduate-program/honesty/`

In addition I am someone you can approach if you have concerns about discrimination or proper scholarly behavior, whether or not the concern is related to this course.

7 Important dates

March 12	no meeting	Break
April 9	no meeting	Passover
April 16	no meeting	Passover