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# Realization and Accumulation in a Marxian Model of the Circuit of Capital

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A mathematical model of Marx's circuit of capital is presented, whose elements, the composition of capital, the rate of surplus value, the rate of capitalization of surplus value, and production, sales and financial time lags, are measurable from the accounts of capitalist firms. The model is solved for exponential paths. A solution with a positive rate of accumulation generally exists. On paths with positive rates of accumulation and positive time lags in the recommital of realized value to production an expansion of credit is required to permit the sale of commodities. A model of demand pull inflation is incorporated in the framework. *Journal of Economic Literature* Classification Numbers: 023, 031, 051, 111, 134.

## 1. INTRODUCTION

In Volume II of *Capital*, Marx discusses the motion of capital value through time in the form of a closed circuit of capital. My aim in this paper is to develop a mathematical model of the circuit of capital and use it to analyze the relations between accumulation and realization on exponential growth paths, which correspond to Marx's analysis of expanded reproduction.

In this paper I abstract from changes in the technical and social conditions of capitalist production, distribution, and circulation. The composition of inputs, wages, and turnover times of production are assumed to be constant so as to permit the model to exhibit steady exponential growth. Since real capitalist production is marked by constant overturning of the techniques of production and conflicts over distribution, this analysis must be viewed as only a step toward a complete understanding of accumulation.

The conclusions that emerge from this exercise are as follows. First, Marx's conception of the circuit of capital can be made into a rigorous and coherent mathematical model, the variables of which are magnitudes

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measurable from ordinary financial accounts of capitalist firms. Second, this model is capable of sustained accumulation on the artificial assumption that the underlying parameters do not change with accumulation itself. Third, there is a precise sense in which accumulation, in the absence of expanding relations of credit, is incompatible with the realization, that is, sale, of produced commodities due to a deficiency in aggregate effective demand. This difficulty can be overcome by positing a constantly increasing borrowing. Fourth, an increase in the scale of this borrowing can increase the rate of accumulation, but there is an upper limit to the achievable rate of accumulation. This limit is defined by the current social relations of production and level of development of productive forces, not by the full utilization of available resources and labor given technology, as in neoclassical growth models. Fifth, it is possible to analyze declines in the value of money as a consequence of the maintenance of actual rates of accumulation very close to this maximal potential rate. This simple model, then, provides a framework based on Marxist economic theory within which we can see the classic questions of macroeconomics in a new light.

This work follows in the steps of Kalecki [3] and Lange [5] in applying the mathematical tools of process analysis to the study of the circuit of capital. Both Kalecki and Lange focus on the fixed capital stock and the dynamics of fixed capital replacement. The present analysis generalizes these models by explicitly considering the whole costs of production, hence circulating as well as fixed capital. The earlier models use the markup to determine a distribution of income which in turn shapes consumption spending. The model of this paper integrates the markup with the labor theory of value, and treats all spending in a unified way through the concept of lags in the flows of value. This method also permits us to see the financial moment of the circuit of capital within the same framework, so that the problem of aggregate demand becomes a problem of the timing of value flows.

Section 2 reviews briefly the basic concepts of the labor theory of value and the circuit of capital analysis in Marx's discussion. Section 3 sets out a simple model of the circuit of capital and discusses its basic properties. Section 4 analyzes the problem of realization and accumulation within this model, and Section 5 shows how the model can be modified to resolve the problems of realization by the introduction of credit relations. Section 6 introduces the possibility of changes in the value of money into the model and studies the resulting solutions. Section 7 contains some concluding remarks.

## 2. Labor, Value, and the Circuit of Capital

The fundamental point of view of the labor theory of value in Marx's interpretation is that the flow of value added for a commodity producing economy is an expression of, or the form taken by the expenditure of abstract social labor.[1] Value, which appears in the mass of variegated commodities produced by the economy as a single homogeneous quantity, is abstracted from the concrete commodities as *money*, whatever form money takes in the particular society under consideration (i.e., commodity money or credit). Individual commodities may have money prices which are larger or smaller than the amount of social labor actually expended in their production, due to unequal exchange, but in the aggregate the value added in the mass of commodities is proportional to the total labor expended on them. I will call the ratio of total labor expended to value added the *value of money* (see [2]).

It is the peculiarity of capitalist production that the power to labor, and hence add value to commodities, itself becomes a commodity, wage-labor, with a money price, the *wage*, and a value, the *value of labor power*, which can be expressed as the product of the wage and the value of money (see [6]). Whether because wages are determined by the cost of subsistence, or because class struggle between workers and capital establishes them at some particular level, the value of labor power is "normally" less than 1. On average this means that the purchase of labor power allows the appropriation of a *surplus value*, the excess of the value added by the expenditure of labor over the value returned to workers in the form of the wage. This surplus value in turn appears as the incomes of managers, salespeople and other unproductive labor, as rent, interest, taxes (not excluding the possibility that some taxes are levied on wages) and a residual profit of enterprise which takes the form of dividends and retained earnings.

Thus the capitalist economy within this framework rests on the constant repetition of a particular motion of value. Value in the form of money is committed to production through the purchase of means of production and labor power; the expenditure of labor power preserves the value of the means of production and adds a value larger than the value paid out in wages in transforming the means of production into a finished commodity; the commodity is then sold, *realizing* its value in money form, returning the capital originally laid out together with a surplus value to be distributed among its various claimants. Some part of this surplus value, either directly

[1] The concept of abstract labor is a complex and qualified idea in Marx's theory. See Rubin [11] for a detailed discussion of these qualifications. For the purposes of this paper I will assume that the appropriate qualifications are present in the reader's mind, and will use the term "labor" as a shorthand for abstract, simple, social and necessary labor.

(in the case of retained earnings) or indirectly through financial intermediaries or financial markets (in the case of incomes of unproductive labor, rents, interest, taxes, and dividends).

From this point of view the capitalist economy appears as a system of *flows* of value through the stages of this circuit. These flows take time to pass through the various phases, so that at any moment a certain amount of value is tied up in each phase of the circuit as a *stock*: some as productive capital, some as finished commodities awaiting sale, and some as money-capital about to be recommitted to capitalist production.

What give coherence and determinacy to this system are two principles that are derived from, and in a sense express the essence of, the labor theory of value: (1) value is created only by the expenditure of labor; (2) value is conserved in exchange.[2] The second principle does not mean that individual agents cannot gain or lose value in an exchange, since they can, as everyday observation tells us. It does mean, however, that the gain of one is the loss of the other, so that the total value involved neither expands nor contracts in exchange.

This way of looking at the capitalist economy gives us a powerful method of analyzing flows of value and their relations to stocks. Flows of value are analytically primary; stocks arise only because it takes time for value to pass through each phase of the circuit.

We can establish a direct correspondence between the income statements and balance sheets of capitalist firms and the theoretical concepts Marx develops out of the labor theory of value to analyze capitalist production. On the income statement costs of production are the flow of capital value into the production process, divided into purchases of raw materials and means of production, which Marx calls *constant capital*, and the wages of production workers, which Marx calls *variable capital*. Sales revenue comprises these two categories of cost and a gross profit or gross margin, which Marx calls *surplus value*. The surplus value in turn appears divided into sales and administrative overhead, interest, taxes, dividends, and retained earnings. The asset side of the firm's balance sheet measures the stocks of value tied up in

[2] Things are more complicated in real capitalist production. The capitalist hires labor-power and gets a certain amount of labor time out of the workers, but only the actual sale of the commodity can establish how much value the labor-time actually created. If the workers used backward techniques, or produced commodities in oversupply, the labor time would produce a smaller than average money value (see [1] for a discussion of the relation between production and realization in the formation of value). In this paper I deal with aggregates, in which individual differences average out. The assumptions of unchanging technical conditions of production and given paths for the value of money also allow me to avoid an explicit discussion of the way in which money value becomes the expression of labor time through realization. That discussion would become necessary once the simplifying assumptions made here are relaxed.

the different phases of the circuit of capital due to the time lags involved in production, sales, and recommital of value to production. *Productive capital* is the stock of value in long-lived plant and equipment, and inventories of raw materials and partly finished goods (valued at costs which include the wages of labor already expended on them), and exists because there is a time lag in the production process. *Commercial capital* is the inventories of finished commodities awaiting sale, and exists because of the time lag involved in selling itself. *Financial capital* is the financial assets of the firm representing value realized in the past in sales but not yet recommitted to production in the form of new costs.

These categories can be computed for any single capitalist firm, or for any aggregate of firms. The categories correspond to the labor theory of value categories only at the level of the whole social capital. At any lower level of aggregation the measured money value added and surplus value may deviate from the actual value produced in the firms because their prices are above or below the true value of the commodities (cases of unequal exchange). These deviations cancel out by definition when we aggregate over the whole social capital; this follows from the definition of the value of money as the ratio of total labor time expended in production to the aggregate value added (see [2]).

It is important to distinguish in this framework between *money* and *value*. Money is one of the forms value takes in the circuit of capital. When the firm pays workers their wages, for instance, money leaves the circuit of capital (only to return through workers' spending, of course) but value does not, since the firm still retains the value equivalent of the wages (in principle) in the labor power it has bought from the workers. When the firm pays dividends or interest or taxes, on the other hand, value leaves the circuit of capital along with the money, since the firm receives no equivalent commodity in exchange. The creation of credit is an alteration of the form of value, not a creation of value itself. From the point of view of the lender, credit transfers money value already produced by the expenditure of labor and realized by sale of commodities in the past to the borrower. From the point of view of the borrower the credit is money available in the present which the borrower intends to repay out of value produced and realized in the future. The creation of the credit claim does not create value from either perspective.

The basic concepts in Marx's analysis of capital, then, are as follows. The idea that value is produced by the expenditure of labor is expressed by the value of money, a coefficient which allows us to translate money value into labor time. The costs of production are divided into *constant* and *variable capital*, which we will measure by the ratio of variable capital (wages) in total capital (total costs), and call the *composition of costs*. The value added is divided into *variable capital* (wages) and *surplus value* (gross profit), a

relation expressed by the *rate of surplus value*, the ratio of surplus value to variable capital. The product of the rate of surplus value and the composition of costs is the ratio of surplus value to total costs, or the *markup*, which expresses how much value can expand on each of its tours of the circuit of capital. The speed with which value travels is measured by a *production lag*, a *sales* or *realization lag*, and a *recommittal lag*, which give rise to stocks of *productive capital*, *commercial capital*, and *financial capital*. The circuit of capital is closed when some fraction of the surplus value is recommitted (along with the costs of previous rounds of production) to production. This fraction we will call the *rate of capitalization* (of surplus value).

## 3. A Model of the Circuit of Capital

The simplest way to represent this circuit of capital is as a sequence of flows of value, connected by lags representing different phases of the circuit.

Let $C(t)$ represent the total flow of value being committed to production in the form of advances for raw materials, means of production, and wages of productive labor. This flow is an aggregate over all capitalist firms, and is divided into wage advances (a fraction $k$ of the total) and non-wage advances including gross investment in long-lived capital goods (a fraction $(1-k)$ of the total).

This flow results in a flow of value of finished output $Q(t)$. (I will follow the practice of measuring the value of finished but unsold output at cost.) Since different components of the flow of value remain for different lengths of time in the production process, we can express the relation of $Q$ to $C$ as a convolution

$$Q(t) = \int_{-\infty}^{t} C(t')\, a(t - t'; t')\, dt', \tag{3-1}$$

where $a(t^*; t')$ is defined for $t^* \geqslant 0$, is non-negative, and integrates to one. $a(t^*; t')$ is the proportion of the costs incurred at time $t'$ which emerge as finished output at time $t' + t^*$.

The flow of finished product, $Q$, likewise results after some time in a flow of sales, $S(t)$. The sales price of commodities exceeds their costs by an average markup $q(t)$, so that

$$S(t) = \int_{-\infty}^{t} Q(t')(1 + q(t'))\, b(t - t'; t')\, dt', \tag{3-2}$$

where $b(t^*; t')$ has the same properties as $a(\cdot)$ and $b(t^*; t')$ is the proportion of commodities finished at time $t'$ which are sold at time $t' + t^*$.

From the point of view of the labor theory of value, the markup reflects the fact that the value added by the expenditure of labor on the commodities exceeds the wage payments to production workers. The surplus value is obviously $qC$. It is natural from the perspective of the labor theory of value to express the markup $q$ as the product of the ratio of surplus value to the wage share, $e$, and the wage share in total costs, $k$.

The flow of value $S$ is made up of a component corresponding to the advances $S'$, which is value replacing costs (including depreciation of long-lived capital), and a component corresponding to the surplus value, $S''$. The cost component we view as being returned to production, perhaps with a lag. Some fraction, $p$, of the surplus value also returns to production, part directly as retained earnings, part as the savings of households which receive interest or dividends through financial intermediaries or the financial market. We call $p$ the *rate of capitalization*. The remainder of the surplus value is spent by households who receive it as revenue on consumption. We can close the circuit of capital, then, by writing

$$C(t) = \int_{-\infty}^{t} S'(t')\, c(t-t';t')\, dt' + \int_{-\infty}^{t} p(t')\, S''(t')\, c(t-t';t')\, dt'. \tag{3-3}$$

The lags $a(\cdot)$, $b(\cdot)$, and $c(\cdot)$ mean that at any instant there will be a certain amount of value tied up in each phase of the circuit, which will appear as productive capital, $N$, inventories of finished commodities awaiting sale (commercial capital), $M$, and financial assets in the hands of capitalist firms, value realized but not yet committed to production, $F$. These stocks are governed by the laws

$$dF(t)/dt = S'(t) + p(t)\, S''(t) - C(t), \tag{3-4}$$

$$dN(t)/dt = C(t) - Q(t), \tag{3-5}$$

$$dM(t)/dt = Q(t) - S'(t). \tag{3-6}$$

Equations (3-1), (3-2) and (3-3) are a closed, homogeneous system of linear integral equations. In this paper we will examine only exponential solutions to the system. From this point onward, we assume that the lags, $a(\cdot)$, $b(\cdot)$, $c(\cdot)$ are stationary in time, as are the parameters $p$ and $q$.

First consider solutions to the system in which all flows and stocks are growing at the same rate, $g$. Since the system is homogeneous, let $C(0) = 1$, so that $C(t) = \exp(gt)$.

The initial conditions and the growth rate must satisfy the following relations, reasoning from (3-1) to (3-3).

$$Q(0) = a^*(g), \tag{3-7}$$

$$S(0) = (1+q)\, b^*(g)\, Q(0) = (1+q)\, a^*(g)\, b^*(g), \tag{3-8}$$

$$S'(0) = S(0)/(1+q) = a^*(g)\, b^*(g), \tag{3-9}$$

$$S''(0) = qS(0)/(1+q) = qa^*(g)\, b^*(g), \tag{3-10}$$

$$\begin{aligned} C(0) = 1 &= c^*(g)(S'(0) + pS''(0)) \\ &= (1+pq)\, a^*(g)\, b^*(g)\, c^*(g), \end{aligned} \tag{3-11}$$

where $a^*(g') = \int_0^\infty a(t') \exp(-g't')\, dt'$ is the discounted present value of the lag function $a(\cdot)$ at the discount rate $g'$, and similarly for $b^*(g)$ and $c^*(g)$.[3]

If $pq > 0$, the right-hand side of (3-11) is equal to $(1+pq) > 1$ when $g = 0$ (since $a^*(0) = b^*(0) = c^*(0) = 1$, given that $a(\cdot)$, $b(\cdot)$, and $c(\cdot)$ integrate to 1 by definition). $a^*(g)$, $b^*(g)$, and $c^*(g)$ are monotonically declining functions of $g$, since they are present values of non-negative functions. Thus as $g$ increases the right hand side of (3-11) declines monotonically to zero, so that there is a unique $g$ at which

$$1 = (1+pq)\, a^*(g)\, b^*(g)\, c^*(g).$$

We will call this $g$ the accumulation mode of the system.

It is easy to see that $g$ must increase with increases in $p$, the rate of capitalization, or $q$, the markup. It will also rise with any change in $a(\cdot)$, $b(\cdot)$, or $c(\cdot)$ which reduces their present value at every discount rate.

These solutions also allow us to calculate the relative sizes of productive, commercial and financial capital on the exponential path of accumulation from (3-4) to (3-6).

[3] To reach (3-7), substitute $C(t) = \exp(gt)$ and $Q(t) = Q(0)\exp(gt)$ in (3-1) to get

$$Q(t) = Q(0)\exp(gt) = \int_{-\infty}^{t} \exp(gt)\, a(t-t')\, dt'.$$

Let $t^* = t - t'$ be a change of variable in the integral, to get

$$\begin{aligned} Q(0)\exp(gt) &= \int_0^\infty \exp(gt)\exp(-gt^*)\, a(t^*)\, dt^* \\ &= a^*(g)\exp(gt), \end{aligned}$$

which gives (3-7). The other results follow by the same method.

$$N(0) = (1 - a^*(g))/g, \tag{3-12}$$

$$M(0) = (a^*(g)(1 - b^*(g))/g, \tag{3-13}$$

$$F(0) = ((1 + pq)\, a^*(g)\, b^*(g)(1 - c^*(g)))/g. \tag{3-14}$$

In this way we can see how the steady-state income statements and balance sheets of the aggregate of all the capitalist firms will look and how they will change with changes in the underlying parameters of accumulation.

The gross rate of profit for the system of capitals as a whole is the ratio of the flow of surplus value $S''$ to the stock of capital value tied up in production and circulation, $N + M + F$. Using (3-7) to (3-14), we find that

$$N(0) + M(0) + F(0) = pqa^*(g)\, b^*(g)/g \tag{3-15}$$

by (3-11). Hence the gross rate of profit $r$ will satisfy

$$r = S''/(N + M + F) = (gqa^*(g)\, b^*(g))/(pqa^*(g)\, b^*(g)) = g/p \tag{3-16}$$

or

$$pr = g. \tag{3-17}$$

This is the "Cambridge" equation which links the rate of profit, the rate of accumulation, and the rate of capitalization (the latter, in Keynesian terms being the propensity to save out of profit) (see [10]).

The steady accumulation of the system is governed by five fundamental parameters: $q$, the markup, which summarizes the conditions of production and appropriation of surplus value; $p$, the rate of capitalization, the proportion of surplus value reinvested; and the functions $a(\cdot)$, $b(\cdot)$, and $c(\cdot)$ which express the time lags, or rates of turnover of each of the phases of the circuit. Notice that the time lag in production expressed by the function $a(\cdot)$ is *not* the same as the labor time expended in production. The total production time lag includes time during which the commodity is not actively the object of labor (see [9, II, Chaps. XII, XIII]).

This analysis shows how the rate of profit is formed at the level of social capital. The flow of surplus value is actually determined by the fraction of value added production workers receive in the form of wages; because means of production take the form of commodities which have value, they appear as "capital," i.e., value tied up in production. From the point of view of the labor theory of value, attempts to calculate the productivity of capital (or its marginal product) as in production function studies, miss the macroeconomic point, since they confuse the time lags in production and realization with the social conditions of the exploitation of labor and the technical conditions of production expressed in the markup, $q$.

## 4. Realization and Accumulation

The model just outlined implicitly abstracts from the problem of realization, or, in conventional macroeconomic language, of effective demand. It does this by assuming that the time lag between production and sale, $b(\cdot)$, is a given parameter of the system, and that the price at which commodities are sold is not affected by the state of the market. This amounts to assuming that all a firm has to do to sell a commodity is to produce it and wait the appropriate time for it to be taken off the shelf by a buyer.

But realization is not automatic in capitalist production, either for the specific firm, which may have produced the wrong commodity in terms of quality or the existing state of demand, or for social capital as a whole, as Keynes' and Kalecki's analysis of effective aggregate demand reminds us. Where does the effective demand come from to realize the finished product?

From the point of view of the circuit of capital, the money demand which realizes commodities arises from revenues created within the circuit itself. One part of the demand is the expenditure of capitalists on means of production, $(1-k)C$. This spending is a demand for finished commodities to be used as inputs to production or as means of production. A second component of demand comes from wage payments to production workers, which they spend on means of subsistence, perhaps with some dynamic lag, as the life-cycle consumption hypothesis suggests. A third component is the demand of capitalist households and households which depend on revenues of unproductive labor (administrative overhead, sales, finance, and so on) for consumption goods, a demand traceable to incomes which are a part of the surplus value. The government may also demand some finished commodities as well, its revenue being derived from taxes on different parts of surplus value, or on wages.

These are circuits of revenue different from the circuit of capital, in which agents get a money income from selling labor power, or appropriating a part of the surplus value, money they then spend on finished commodities.

But here we reach a paradox. If we posit a regime of smooth, continuous accumulation with time lags at each stage of the circuit of capital, the flow of value at any one point in the circuit corresponds to the flow at an earlier time at the previous stage of the circuit. Thus in Eqs. (3-8) and (3-11):

$$S(0) = (1+q)\,a^*(g)\,b^*(g), \tag{4-1}$$

$$C(0) = (1+pq)\,c^*(g)\,S(0)/(1+q). \tag{4-2}$$

The coefficients $a^*(g)$, $b^*(g)$, and $c^*(g)$ are less than one if $g$ is positive, and become smaller as $g$ increases. But effective demand arises from $C$ and from components of $S$. If there are lags in capitalist respending of the

recovery of costs, or in workers' households spending of wages, or in capitalist households' spending of their share of surplus value, or in the reinvestment of the surplus value, effective demand from these sources must be smaller than sales. Only if $g = 0$, so that the lag coefficients are equal to 1 would there be enough demand to realize the surplus value in the finished commodities. There is a contradiction in this sense between accumulation at a positive rate and the realization of the commodities produced.[4]

This contradiction can be overcome only if some agents in the system, capitalist firms, the State, or households, spend revenue before it is realized. This can happen in two ways: the most obvious and common is for an agent to spend by contracting a debt which it plans to pay out of future revenue. The second is for an agent to draw down stocks of a money-commodity accumulated outside the capitalist production process.

The anticipation of revenue by contracting a debt does not, it is important to keep in mind, involve the *creation* of value. In fact, it amounts to the transfer of value already realized in the form of money from one agent, which is holding the value temporarily in money form, to another agent who will spend it (and thereby realize the value of finished commodities). For the lender the loan represents value created and realized in the past; for the borrower the loan represents value it anticipates receiving in the future. From this difference in point of view comes the possibility of bankruptcy and other disruptions of the credit system.

Normally the lender is in a position to bargain for a share of the surplus value the capitalist borrower plans to appropriate by spending the loan to set in motion a cycle of capitalist production. This share is the interest on loans. Its existence in lending to capitalists means that other borrowers must also pay it to compete for funds. Thus households and the State may borrow without planning to commit the funds to capitalist production and reap a surplus value, but they must pay interest just the same. We would expect normally that the rate of interest could not rise for long above the gross rate of profit, since that rate limits the amount potential capitalist borrowers will offer, and that the trouble and uncertainty of making a loan limits the interest rate lenders will accept. Within these (broad) limits the balance of bargaining strength between lenders and borrowers can shift back and forth.

In periods of credit crisis, of course, other considerations can influence the level of the interest rate: pressure on borrowers to pay off other debts in a period of illiquidity, and uncertainty of lenders as to the ability of borrowers to pay.

Credit transactions appear from the point of view of the circuit of capital to be determined in the first instance by the time delays in spending revenues

[4] This problem has a famous history in Marxist literature. See Marx [9, II, Chap. XXI], Luxemburg [7] and Luxemburg and Bukharin [8].

and anticipations of future revenues of capitalist firms themselves. The State's role, through its fiscal and monetary agents, is to regulate and limit this clash by managing its own borrowing and lending (which do not differ basically from private borrowing and lending in form). (Of course, the State is not typically borrowing as a capitalist to appropriate a surplus value; the interest on State debt is paid out of tax revenues.)

Interest itself, then, is merely one component of the flow of surplus value; not a payment for a valuable commodity or service, except in a metaphorical sense.

In terms of the model we have been developing, when we consider realization explicitly we must drop the assumption that the realization lag $b(\cdot)$ is a given, and allow the sales lag to be determined endogenously by sales, which depend on aggregate demand. We can write sales as:

$$S(t) = (1-k)\,C(t) + \int_{-\infty}^{t} k(t')\,d(t-t';t')\,C(t')\,dt' + \int_{-\infty}^{t} e(t'-t;t')(1-p(t'))\,S''(t')\,dt' + B'(t), \tag{4-3}$$

where $d(\cdot)$ is the time lag in workers' households' respending of wages, $e(\cdot)$ is the time lag in the respending of revenues arising from surplus value which are not accumulated, and $B'(t)$ is State and household spending financed by new borrowing against anticipated revenue. We must also write $C(t)$ as

$$C(t) = \int_{-\infty}^{t} S'(t')\,c(t-t')\,dt' + \int_{-\infty}^{t} p(t')\,S''(t')\,c(t-t')\,dt' + B(t), \tag{4-4}$$

where $B(t)$ is capital spending financed by new borrowing.

Inventories of finished commodities continue to be governed by the law

$$dM(t)/dt = Q(t) - (S(t)/(1+q(t))). \tag{4-6}$$

If we view sales inventories as a first-in, first-out process, this corresponds to the idea that sales are the value of finished commodities after a simple time lag $T$, where

$$dT/dt = S(t)/((1+q)\,Q(t-T)) - 1 \tag{4-7}$$

since the assumption of first-in, first-out amounts to the rule

$$\int_{t-\Delta t}^{t} S(t')/(1+q(t'))\,dt' = \int_{t-\Delta t-T(t-\Delta t)}^{t-T(t)} Q(t')\,dt', \tag{4-8}$$

the differentiation of which yields (4-7).

## 5. Realization and Accumulation

We can summarize the model which takes account of realization explicitly as follows:

$$Q(t) = \int_{-\infty}^{t} C(t')\, a(t-t';t')\, dt', \tag{5-1}$$

$$S(t) = (1-k(t))\, C(t) + \int_{-\infty}^{t} k(t')\, C(t')\, d(t-t';t')\, dt' + \int_{-\infty}^{t} (1-p(t'))\, S''(t')\, e(t-t';t')\, dt' + B'(t), \tag{5-2}$$

$$C(t) = \int_{-\infty}^{t} S'(t')\, c(t-t';t')\, dt' + \int_{-\infty}^{t} p(t')\, S''(t')\, c(t-t';t')\, dt' + B(t), \tag{5-3}$$

$$dF/dt = S'(t) + S''(t) - C(t) + B(t), \tag{5-4}$$

$$dN/dt = C(t) - Q(t), \tag{5-5}$$

$$dM/dt = Q(t) - S'(t), \tag{5-6}$$

$$dT/dt = S(t)/((1+q(t-T))\, Q(t-T)) - 1, \tag{5-7}$$

$$S'(t) = S(t)/(1+q(t-T)), \tag{5-8}$$

$$S''(t) = q(t-T)\, S(t)/(1+q(t-T)). \tag{5-9}$$

We have nine variables, $Q$, $C$, $S$, $S'$, $S''$, $F$, $N$, $M$, and $T$ with nine equations, so that the system is well-determined given the lag functions, $p$, $q$, $B$, and $B'$.

Equations (5-7) to (5-9) simply keep track of the first-in first-out dynamics of inventories of finished commodities awaiting sale. This establishes the "vintage" of the output which is being sold at any moment, and hence the division of sales into recovery of costs and surplus value. (The markup $q$ in this model is determined at the time of production, since $q = ek$. The composition of costs, $k$, is a result of production decisions. We measure the rate of surplus value $e$ as $(1-wm)/wm$, where $w$ is the money wage per hour, and $m$ the value of money at production time, so that $e$ is also determined in production. In this section we assume that the value of money, $m$, is constant over time.) Given this division (5-2) and (5-3) form an autonomous subsystem (of circulation), linked to (5-1) (production) by inventories of finished commodities awaiting sale, measured by $M$, or $T$, the sales lag.

Thus at this stage we have a model whose actual rate of accumulation depends on conditions of realization, essentially the lags $c(\cdot)$, $d(\cdot)$, $e(\cdot)$ and new borrowing $B$ and $B'$. Assuming the system is on an exponential path (so that $B$ and $B'$ are also growing exponentially at the same rate as the other variables) we can for given stationary lags $a(\cdot)$, $b(\cdot)$, $c(\cdot)$, $d(\cdot)$ and $e(\cdot)$ and constant $p$ and $q$ calculate the relative sizes of the flows of value. From (5-2) we derive

$$S'(0) = (1 - k(1 - d^*(g)) + B'(0))/(1 + q(1 - (1 - p)\, e^*(g))), \quad (5\text{-}10)$$

and from (5-3)

$$1 = c^*(g)(1 + pq)\, S'(0) + B(0). \quad (5\text{-}11)$$

Equations (5-10) and (5-11) establish the relation between the actual rate of accumulation $g$, and the scale of new borrowing, $B(0)$ and $B'(0)$, for given lags in spending and given rate of capitalization and markup. If there are no lags in workers' spending and in the spending of surplus value, we would have $d^*(g) = e^*(g) = 1$ for all $g$. If we further assume that only capitalist firms borrow, so that $B'(0) = 0$, we find that $S'(0) = 1/(1 + pq)$. In this case (the classic Marxian case) (5-11) becomes

$$B(0) = 1 - c^*(g), \quad (5\text{-}12)$$

and it is clear by inspection that the realized growth rate $g$ corresponds to the proportion of capitalist costs financed by new borrowing, $B(0)$; the higher is $g$ the larger must be $B(0)$.

This result is striking, since it establishes a link between the realized rate of accumulation and the expansion of credit. In this way the circuit of capital model clarifies the link between the analysis of Keynes and that of Marx, and incidentally shows one possible resolution of the problems raised by Luxemburg [8] in her critique of Marx's models of expanded reproduction. Although the present analysis is limited to an examination of steady state paths, this relation could be the basis for understanding the role credit policy can play in stabilizing the rate of accumulation, and the relation between credit crises and crises in production.

At growth rate $g$ we must have

$$Q(0) = a^*(g), \quad (5\text{-}13)$$

$$N(0) = (1 - a^*(g))/g, \quad (5\text{-}14)$$

$$M(0) = (a^*(g) - S'(0))/g, \quad (5\text{-}15)$$

$$F(0) = ((1 + pq)\, S'(0) - (1 - B(0)))/g, \tag{5-16}$$

$$D(0) = B(0)/g, \tag{5-17}$$

where $D(t)$ is the stock of debt of capitalist firms.

On any realized growth path the capitalist firms borrow back the money value accumulated as financial capital, $F$, plus any net money accumulation of other sectors. In the classic Marxist case, for example, we have

$$F(0) = B(0)/g = D(0). \tag{5-18}$$

To simplify the discussion from this point on, we will examine only the case where the spending lags and non-capitalist borrowing do not exist, so that $d^*(g) = e^*(g) = 1$, and $B'(0) = 0$.

At any actual rate of growth $g$ we have

$$M(0) = (a^*(g) - 1/(1 + pq))/g \tag{5-19}$$

and if $dT/dt = 0$ (as it must be on an exponential path)

$$(1 + q)\, Q(0) \exp(-gt) = S(0) \tag{5-20}$$

or

$$T = (1/g) \ln((1 + pq)\, a^*(g)). \tag{5-21}$$

These equations establish an upper bound to the realized growth rate $g$, since must have $M(0)$ and $T \geqslant 0$. This upper limit $g^*$ is defined by the relation

$$(1 + pq)\, a^*(g^*) = 1. \tag{5-22}$$

This result that the maximal rate of accumulation $g^*$ depends on the markup $q$, the rate of capitalization, $p$, and the production time lag $a(\cdot)$, is of central importance from the Marxist point of view. The markup is the product of the composition of costs, which, along with $a(\cdot)$, reflects the level of development of forces of production, and the rate of surplus value, which reflects the social relations between capitalists and workers. The maximal rate of accumulation need not depend on the rate of growth of the labor force, or on the full employment of available resources, as in neoclassical growth theory. (Of course, the state of the labor market might indirectly influence the maximal rate of accumulation through its effects on the rate of exploitation. A tight labor market, if it drove up the wage share, would lower $e$ and $q$, and as a consequence lower the maximal rate of accumulation.) Value can expand only by the markup on each of its tours of the circuit; the maximal rate of growth corresponds to the most rapid movement of value around the circuit, when the realization lag is zero. This case corresponds to

the "von Neumann" growth path, since accumulation is limited entirely by the time of production and the given capacity of the system of capitals to expand.

For the classic Marxist case we can easily see the relations between the parameters of the circuit of capital model and the realized and maximal rates of accumulation. The lower the markup, the lower the rate of capitalization, and the longer the production lag, the lower the maximal rate of accumulation. The higher is the realized rate of accumulation, the larger must be the proportion of costs financed by new borrowing, the shorter the lag between production and sale, hence the smaller inventories of finished commodities awaiting sale.

This model combines and shows the relation between the Keynesian analysis of effective demand and the Marxist analysis of production. As Keynes pointed out, an expansion of effective demand will increase the realized rate of accumulation, but, in the absence of changes in the underlying profitability of capitalist production, does so only by accelerating the motion of value through the circuit of capital by reducing inventories (or possibly by increasing the utilization of fixed capital).

## 6. Changes in the Value of Money

Up to this point we have assumed that the value of money, the amount of abstract social labor time represented by a dollar, is constant. Let us now consider the situation where that parameter of the system is changing over time in a smooth exponential fashion.

At any moment the value of money $m(t)$ translates money quantities into labor time quantities. The value of money is not the inverse of the price level as usually measured, since the price level is the money cost of a certain bundle of goods and services. The inverse of the value of money is the product of the price level in this sense and the productivity of labor, which measures the amounts of goods that can be produced by an hour's labor. Understanding the motions of the value of money is a first step toward understanding the price level in the usual sense, but must be supplemented by a theory of changes in the productivity of labor to yield a theory of the price level.

We adopt the convention that the value of money is measured at the time of realization of commodities; it is the ratio of the necessary labor expended on the average commodity sold to the money value added realized from the sale. If labor productivity were changing, the value of the commodity when sold would be the labor currently necessary to reproduce it, not the historical labor embodied in it (see [11, Chap. 16], and [9, I, Chap. 1]).

Changes in the value of money leave the value relations in the circuit of capital unaltered. This result follows assuming that the rate of surplus value and the value of labor power do not change as the value of money changes. This amounts to assuming that money wages adjust fully and instantaneously to the change in the value of money (so that the product of the money wage and value of money, the value of labor power, remains constant). This assumption may or may not hold in particular historical episodes.

The value of the flow of finished commodities at cost is still

$$Q(t) = \int_{-\infty}^{t} a(t - t'; t')\, C(t')\, dt'. \tag{6-1}$$

If we want to calculate the money value of $Q$ at any time, we must divide it by the value of money. Putting an asterisk before a variable to denote the *money value* of the variable, we have

$$^*Q(t) = Q(t)/m(t). \tag{6-2}$$

We will study paths on which the value of money is falling exponentially at rate $u$:

$$m(t) = m(0) \exp(-ut). \tag{6-3}$$

The realization submodel (the equivalent of (5-2) and (5-3)) is now in money terms. Clearly the *money* rate of accumulation is higher than $g$ by the rate of depreciation of the value of money, $u$. In the classic Marxian case, where there are no spending lags, and no borrowing except by capitalist firms, we have

$$^*S(t) = {}^*C(t), \tag{6-4}$$

$$^*C(t) = \int_{-\infty}^{t} {}^*S(t')\, c(t - t'; t')\, dt' + {}^*B(t). \tag{6-5}$$

If the system is on an exponential path, the money flows will all be growing at the rate $g + u$. The relation between the money rate of accumulation and the scale of new borrowing (cf. (5-13)) becomes

$$^*B(0) = 1 - c^*(g + u). \tag{6-6}$$

The higher is the rate of depreciation of money, the lower is the rate of real accumulation for any given $^*B(0)$, the fraction of money costs financed by new borrowing.

We need one further relation to determine $u$, the rate of depreciation in the value of money. Many different theories of inflation could be incorporated in

the circuit of capital framework. To demonstrate this possibility I will work out here a version of a "demand pull" model of inflation in which the pressure of demand appears as a reduction of inventories of commodities awaiting sale. As a first approximation, assume that $u$ depends on $T$, the lag between production and sale. The notion here is that when $T$ becomes short capitalists perceive demand as high and markets as amenable to price increases, but when $T$ is large, firms experience considerable difficulty in selling commodities and are deterred from increasing prices, or forced to cut them. Notice that $T$ is a macroeconomic variable; it is a measure of the *average* difficulty *all* capitalist firms are having in selling commodities, not a measure of the competitive advantage of any particular firm. Of course in reality the firms will find themselves distributed around this average because of competitive differences; some firms may be facing tight markets and raising prices when on average there are conditions of overproduction and very sluggish sales. $T$ itself is still determined by the markup, the rate of capitalization, and the actual rate of growth:

$$T = (1/g)\ln((1+pq)\,a^*(g)), \tag{6-7}$$

$$u = u(T) \qquad \text{with} \quad u' < 0. \tag{6-8}$$

Equations (6-6) to (6-8) allow us to find $g$, $u$, and $T$ given $p$, $q$, $^*B(0)$ and the production and recommittal lags, $a(\cdot)$ and $c(\cdot)$. It is clear that a rise in $^*B(0)$ corresponds to a rise in $g+u$; this increase is divided between the real rate of accumulation and the depreciation of money in proportions determined by (6-7) and (6-8). In fact, we can substitute (6-7) in (6-8) and write:

$$u = u((1/g)\ln((1+pq)\,a^*(g))) \tag{6-9}$$

Differentiation of (6-9) shows that $u$ rises with $g$ given the other parameters, and *falls* with an increase in the markup or the rate of capitalization for a given $g$.

Here we have a complete linkage between production and circulation through the realization lag and the depreciation of money. A larger effective demand (represented in this model by a larger $^*B(0)$) increases the rate of accumulation measured in money. One part of this increase raises the realized rate of accumulation of value; the remainder results in higher rates of depreciation of money. As the realized rate of accumulation approaches the maximal rate determined by social relations and the level of development of forces of production, more and more of the increase in money rates of accumulation appears as depreciation of money.

An interesting result of this model is that a fall in the markup, which lowers the maximal rate of accumulation, will be inflationary because it tends to lower inventories of finished commodities awaiting sale and to

shorten the realization time lag. Since the rate of capitalization, $p$, always appears multiplied by the markup, the same results fall from a fall in the rate of capitalization.

This result shows that the management of aggregate effective demand by manipulating the level of new borrowing has effects on the realized rate of accumulation, but that those effects are limited by the underlying conditions of profitability and accumulation.

It is a straightforward exercise to extend the model to the full case where there are lags in workers' consumption and consumption out of surplus value, and where other sectors besides capital incur debt.

## 7. Conclusions

The concept of the circuit of capital provides us with an alternative method of macroeconomic analysis based on the labor theory of value in Marx's interpretation. It is a general framework of analysis which can incorporate hypotheses from other theories in a testable form. For example, the "real balance" effect could be incorporated by making $p$, the rate of capitalization, depend on household wealth. Keynesian theories of the dependence of investment on the interest rate could be expressed in the framework of the circuit of capital by making new borrowing, $B$, or the recommittal lag, $c(\cdot)$ depend on the interest rate. Since the variables in the circuit of capital model are observable in aggregate balance sheet and income account data, we could in principle see how important these hypothesized macroeconomic effects are in particular periods.

In the simple linear form in which I have presented it in this paper, the circuit of capital model reflects basic Marxian insights into the aggregate process of capital accumulation. The maximal rate of accumulation is limited by the social relations of production, expressed by the rate of surplus value, and the level of development of forces of production, expressed in the composition of costs and the production lag. Limited supplies of labor or other resources, which fundamentally determine the potential growth rate in neoclassical growth models, will have an influence on the maximal rate of accumulation only through these social variables. The actual rate of accumulation corresponds to a certain level of system-wide new borrowing. A fall in the underlying profitability of capitalist production lowers the maximal rate of accumulation and the realization lag. If the rate of depreciation of the value of money rises as the sales lag becomes shorter, a lower markup will correspond to a more rapid depreciation of the value of money. If the value of money cannot depreciate more rapidly, the system must adjust to a lower actual rate of accumulation, a process which may precipitate a crisis in accumulation and credit, but which could be studied

only by going beyond the exponential paths analyzed here. These conclusions express a basic idea of Marx's that macroeconomic phenomena like changes in the value of money and the rate of accumulation reflect changes in the underlying social relations which govern production.

The circuit of capital model can be viewed as synthesizing a number of theoretical approaches to economic growth. Keynes' idea that economic activity depends on aggregate effective demand is expressed in the relation of the actual rate of accumulation to the expansion of new borrowing. The circuit of capital model explains the dynamics behind this relation; higher effective demand accelerates the flow of value through the circuit by reducing the realization lag. (One could imagine systems where the production lag also shortened as the realized rate of accumulation increased. This shortening would appear as an increase in the rate of utilization of fixed capital.) The circuit of capital model shows how the role of effective demand interacts with underlying limitations on the maximal rate of accumulation. This maximal rate of accumulation is a particular case of the von Neumann growth rate, since when the realization lag is zero, the system is limited by the ability of value to expand in production. From another point of view, one can see the circuit of capital model as a generalization of the Harrod–Domar type models, with a more explicit treatment of the financial market.

Though the model studied here is purely linear, it should be possible to study non-linearities in the lag structures and parameters within the same framework, and to move from the analysis of exponential paths of the system to a study of its full dynamic properties.

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