Hans Magnus Enzensberger

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Drawbridge Up



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Mathematik im Jenseits der Kultur
thematics — A Cultural Anathema

Hans Magnus Enzensberger

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Mathematics — A Cultural Anathema

Translated by Tom Artin



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Preface

I am accustomed, as a professional mathematician, to living in a sort of vacuum, surrounded by people who, as in the first paragraph of Enzensberger's essay, declare with an odd sort of pride that they are mathematically illiterate. How astonishing to come to the International Congress and find a distinguished poet and essayist who analyzes this problem with such a deep understanding and sensitivity. This is a beautiful essay and a great delight for a mathematician to read. Here the strange contradictions with which we live are laid bare with the accuracy of a high-resolution microscope. He is onto us and how, on the one hand, we take pride in building an elegant world utterly divorced from the demands of reality and, on the other, claim that our ideas underlie virtually all technological developments of significance. I was particularly pleased to find that he believes progress in teaching mathematics is possible, that it can be made exciting to young minds. I tried (unsuccessfully) to get each high school in which my children were enrolled to go outside during geometry and find out how tall the oak in the yard really is. Instead they buckled under to the educational establishment and even removed that staple of my generation: the odd formulaic Euclidean-style proof in which the pedigree of each step was detailed. This old game, the one piece of high school mathematics often mentioned warmly by people who otherwise hated math, had been found wanting by the mathematical puritans described in the Stewart quote.

where the mathematician is unwilling to "lie a bit, like everyone else does." I hope the right people read this essay, the movers and shakers of school curricula, and that it moves them to let a hundred flowers bloom in the classroom.

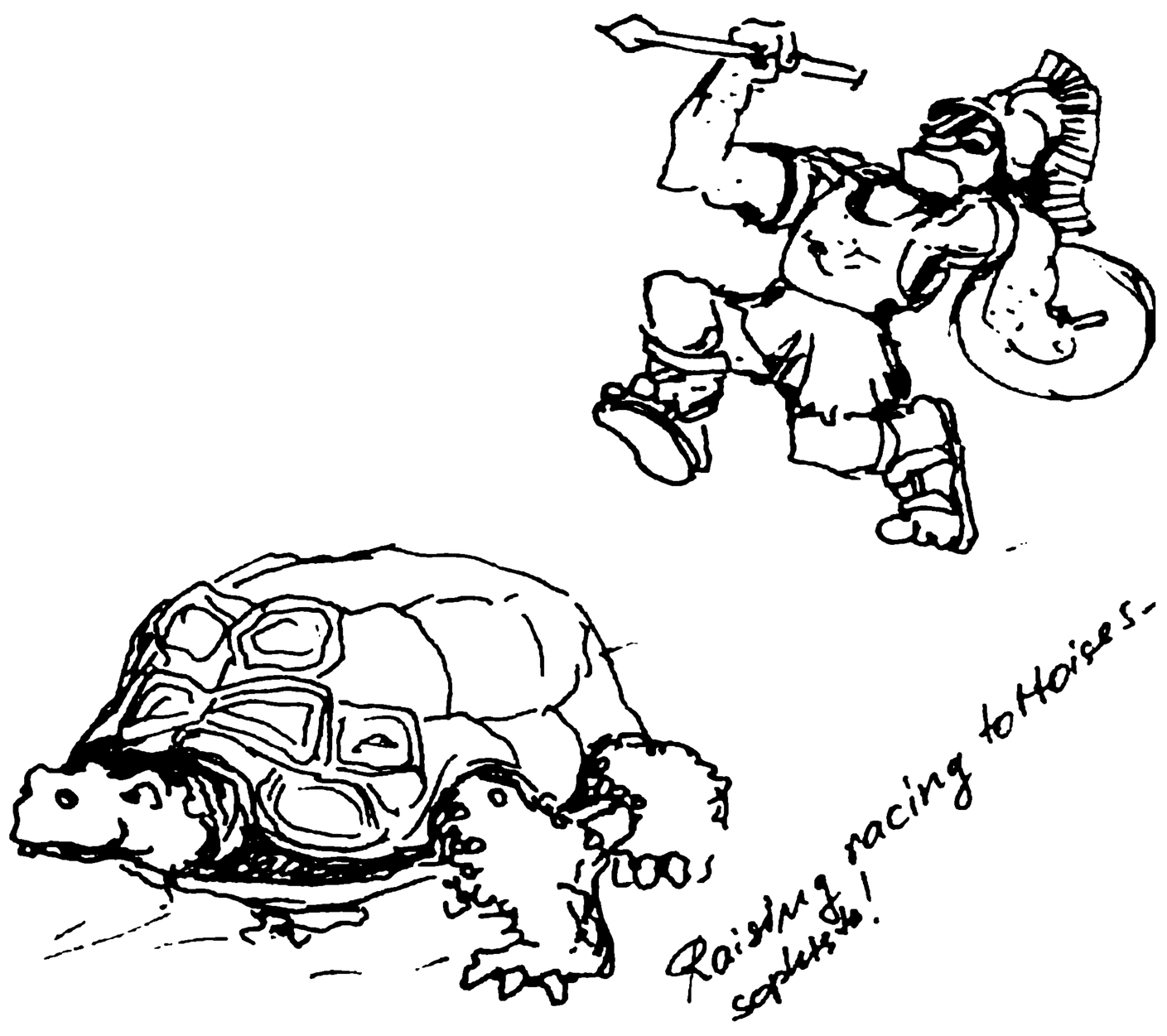
DAVID MUMFORD
President, International Mathematical Union



It's the same old refrain: "Stop, for heaven's sake! I hate math." "Pure torture from the start of school. It's a total mystery how I ever managed to graduate." "A nightmare for me — I have no talent for it, period!" "I can just about figure the tax with a pocket calculator. Beyond that, it's all over my head." "Mathematical formulas are pure poison. They just turn me off."

Complaints such as these are heard all the time. Thoroughly sensible, educated people express them routinely, with a remarkable blend of defiance and pride. They assume their listeners' sympathy, of which there is no dearth. A consensus has evolved that tacitly but forcefully determines the public's attitude toward mathematics. That its exclusion from the cultural sphere amounts to a kind of intellectual castration seems trou-

bling to no one. Anyone who bemoans this situation, who utters so much as a word about the charm, the significance, the far-reaching influence, or the beauty of mathematics, is marveled at as an expert. If he admits he's just an amateur, he will in the best case be written off as an eccentric, who occupies his time with an exotic hobby — rather as though he were to breed turtles, or collect Victorian paper-weights.



How rarely, on the other hand, do we encounter a person who asserts vehemently that the mere thought of reading a novel, or looking at a picture, or seeing a movie causes him insufferable torment; that since graduating from high school he has scrupulously avoided all contact with the arts in whatever form; that he would rather not recall past experiences of literature or painting. It's virtually unheard of for music to be broadly anathematized. To be sure, there are some who — not unrealistically, perhaps — proclaim themselves unmusical. One person may tend to sing harshly and off pitch, another may be incapable of playing an instrument, and certainly only the exceptional music lover rushes to a concert with the score tucked under his arm. But who would seriously contend that he didn't know a single song? Be it the Spice Girls or the National Anthem, Techno or Gregorian chant, no one is entirely immune to music. And for good reason. The ability to produce and hear music is genetically based — it is one of the anthropological universals. This does not mean, of course, that we are all equally musically talented. Like all other gifts and characteristics, this aspect of our genetic equipment conforms to the normal Gaussian distribution. It is just as rare in any given population to find extraordinarily musically talented people as to find people who are musically totally deaf — the statistical maximum appears mid-field.

Exactly the same relation, of course, is true of mathematical abilities. They too are established genetically in the human brain, and they too distribute themselves in

all populations exactly along the model of the bell curve. It is thus an illusion born of superstition that mathematical thinking is a rare, exceptional occurence, an infrequent whim of nature.

So we confront a mystery. How does it happen that mathematics has remained as it were a blind spot in our culture — alien territory, in which only the elite, the initiate few have managed to entrench themselves?

A Certain Isolation

It would be easy to say that mathematicians themselves are to blame. This explanation does have the advantage of simplicity. It confirms as well the stereotype the world at large has long held of the professional practitioners of the discipline. The mathematician is imagined as a secular high priest, jealously guarding his esoteric grail. On the mundane things of this world, he turns his back. Occupied exclusively with his mystifying problems, communication with the outside world is difficult for him. He lives a life withdrawn, deprecates the joys and sorrows of human society as burdensome interruptions, and positively indulges in a solitude verging on misanthropy. With his logical pedantry, he for his part grates on the nerves of his contemporaries. Above all, though, he inclines toward a form of arrogance that is hard to take. Intelligent though he may be — no one disputes his sheer brainpower — he views the helpless struggles of others to grasp this or that concept with demeaning condescension. For this reason, it would

never occur to him to try to woo the public over to his cause.

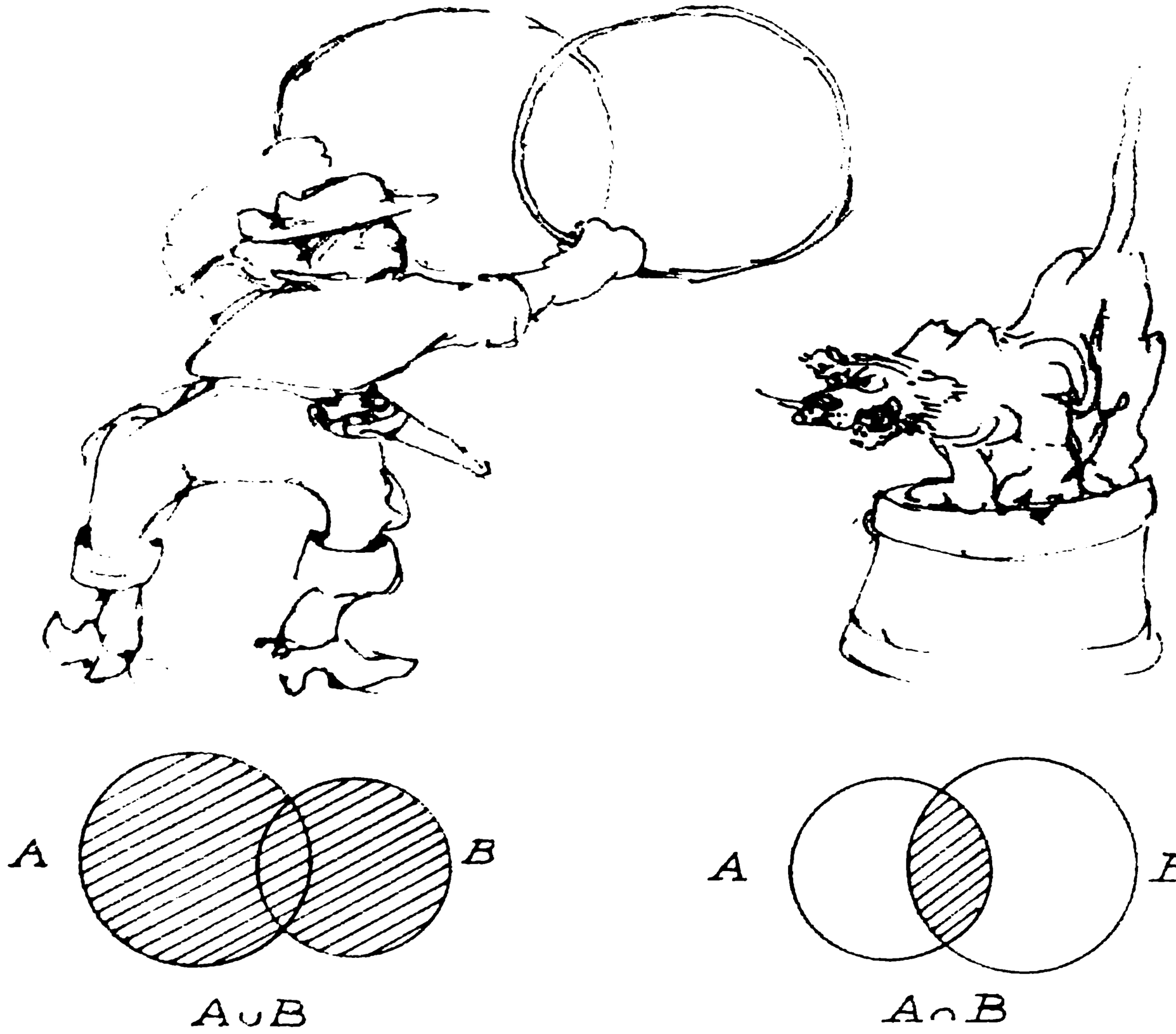
So much for the caricature, which is certainly often enough taken for gospel. It's nonsense, of course. Apart from their profession, mathematicians differ little from other people; I know men and women in the field who are joyful, urbane, witty, and occasionally even foolish. Nonetheless, just a kernel of truth does, as usual, reside within the cliché. Every profession has its own hazards, its own particular pathologies, its *déformation professionelle*. Miners suffer from black-lung, writers, from narcissistic neuroses, directors, from delusions of grandeur. All these defects can be traced back to the working conditions under which the patients function.

In the case of mathematicians, their occupation demands above all intense and sustained concentration. The lumber they are given to drill through, so to speak, is peculiarly hard and extremely thick. No wonder, under such circumstances, that every external annoyance is experienced as an unreasonable demand. Second, the fact is that the era of the universal mathematician with the stamp of an Euler or a Gauss is now long past. No one today commands a grasp of all the areas of his discipline. This means in turn that the circle of fellow research professionals whom one is able to address has shrunk materially. Work that is truly original is initially comprehensible only to a handful of colleagues — it gets circulated via E-mail among a dozen readers between Princeton, Bonn, and Tokyo. This exclusivity does occasion a certain isolation. Researchers such as these have

long since given up the effort to make themselves understood by outsiders, and it may well be that this aloof posture is adopted not so legitimately by other, less highly advanced laborers in the vineyard of mathematics.

Typifying this attitude is a phrase heard routinely starting with the first semester of any course in beginning calculus or linear algebra. This derivation or that correspondence is said to be "trivial," and is left at that. Further elucidation, being superfluous, would be, as it were, beneath the dignity of the mathematician. Of course, it is in truth both tedious and boring to spin out afresh each step in the chain of a proof over and over again. Thus, mathematicians are trained to leap-frog over recurring intermediate steps — that is, simply to assume a validity that has already been proven thousands of times. There is doubtless an economy to such methodology. It does, however, affect communicative behaviour in a very particular way. Only those few for whom the trivial really is trivial — that is, self-evident — gain admission to the discussion with the experts. The rest of us — more or less 99 percent of humanity — are in this regard hopeless cases, with whom it is simply pointless to engage in conversation.

To make matters worse, mathematicians don't just — like other scientists — employ a specialized professional jargon; they also use a notation, quite different from ordinary writing, that is indispensible to their intercommunication. (Here too, one may evoke an analogy with music, which likewise has evolved its own special-



ized notation.) Now, most people go into a panic at the mere sight of a formula. It's difficult to determine the source of this flight-reflex, which is in its turn quite incomprehensible to the mathematician. He is of the opinion that his notation is beautifully clear and far superior to any merely verbal language. Thus, he cannot fathom why on earth he should bother to translate his ideas into German or English. Such an effort seems to him a terrible distortion.

So are the mathematicians themselves to blame for the insularity of their science? Have they themselves turned their backs on society, and willfully raised the drawbridge to the island of their discipline? To settle on so simplistic an answer is to underestimate the scope and importance of the problem. It is simply implausible to lay the blame on a handful of experts, while the over-

whelming majority of mankind happily renounces the acquisition of a cultural capital of immense significance and enormous charm.

Between Utility and Elegance

We know that ignorance is a cosmic force of truly insuperable power. Most people seem to be convinced that one can get along quite nicely without any mathematical knowledge at all, and that this science is inconsequential enough to be left to the scientists. Many even harbor the suspicion that we are dealing here with an unprofitable occupation, whose usefulness is far from obvious. They feel corroborated in this error by the views of many mathematicians, who defend the purity of their endeavours in no uncertain terms. Thus, the eminent English number theorist Godfrey Harold Hardy made the following notorious declaration: "I have never done anything 'useful.' No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world. I have helped to train other mathematicians, but mathematicians of the same kind as myself, and their work has been, so far at any rate as I have helped them to it, as useless as my own. Judged by all practical standards, the value of my mathematical life is nil; and outside mathematics it is trivial anyhow." There it is again, that ominous word *trivial*, with which the author brands everything he disdains. "I have just one chance," Hardy went on, "of escaping a verdict of complete triviality, that I may be

judged to have created something worth creating. And that I have created something is undeniable: the question is about its value." (*A Mathematician's Apology*, Cambridge, 1967).

Wonderfully put! — with a sort of modesty scarcely distinguishable from aristocratic arrogance. Nothing is more foreign to a mathematician like Hardy than courting the approbation of his contemporaries by appealing to the practical utility of his work. In this, he is at once justified and not. His attitude approaches the artist's. Judged strictly along the criteria of industrial management, not just Ovid and Bach, but also Pythagoras and Cantor would have been found wanting. Their work would hardly have yielded that automatic 15 percent that these days goes as the standard under the rubric *shareholder value*. To be sure, most human endeavours fall short of that benchmark. (Be it noted, incidentally, that compared economically with other cultural accomplishments, mathematics surely represents one of the best values. While the new particle accelerator of the Genevan CERN is estimated at 4 to 5 billion DM, the Max Planck Institute for Pure Mathematics in Bonn, a world reknowned research center, expends a mere 0.3 percent of the budget of the Max Planck Society. Great mathematicians like Galois and



Abel were dirt poor their whole lives. Genius at a cheaper price would be hard to find.)

The autonomy Hardy requires for his basic research finds its counterpart in the arts, and it is no coincidence that most mathematicians are thoroughly comfortable with aesthetic criteria — a proof needs not just to be conclusive, the mathematician aspires to "elegance." The word expresses a quite particular aesthetic sensibility that has characterized the mathematical enterprise since its earliest beginnings. (Of course, this only raises once more the conundrum why the general public should value gothic cathedrals, Mozart's operas, and Kafka's stories so highly, but not the Method of Infinite Descent or Fourier analyses.

With respect to social utility, however, Hardy's assertions are easily refuted. The engineer set to designing a quite ordinary electric motor employs complex numbers as a simple matter of course. Wessel and Argand, Euler and Gauss could not have guessed anything of the kind when, around the turn of the 18th century, they created this elaboration of the number system. Without the binary number code developed by Leibnitz, today's computers would be unthinkable. Einstein could not have formulated his Relativity Theory had Riemann's work not tilled the ground for him, and without group theory, quantum mechanics, crystallography, and communications technology would all be pretty hollow endeavours. The exploration of prime numbers, a branch of number theory of truly infinite beauty, was long held to be an esoteric specialty. Over a couple of



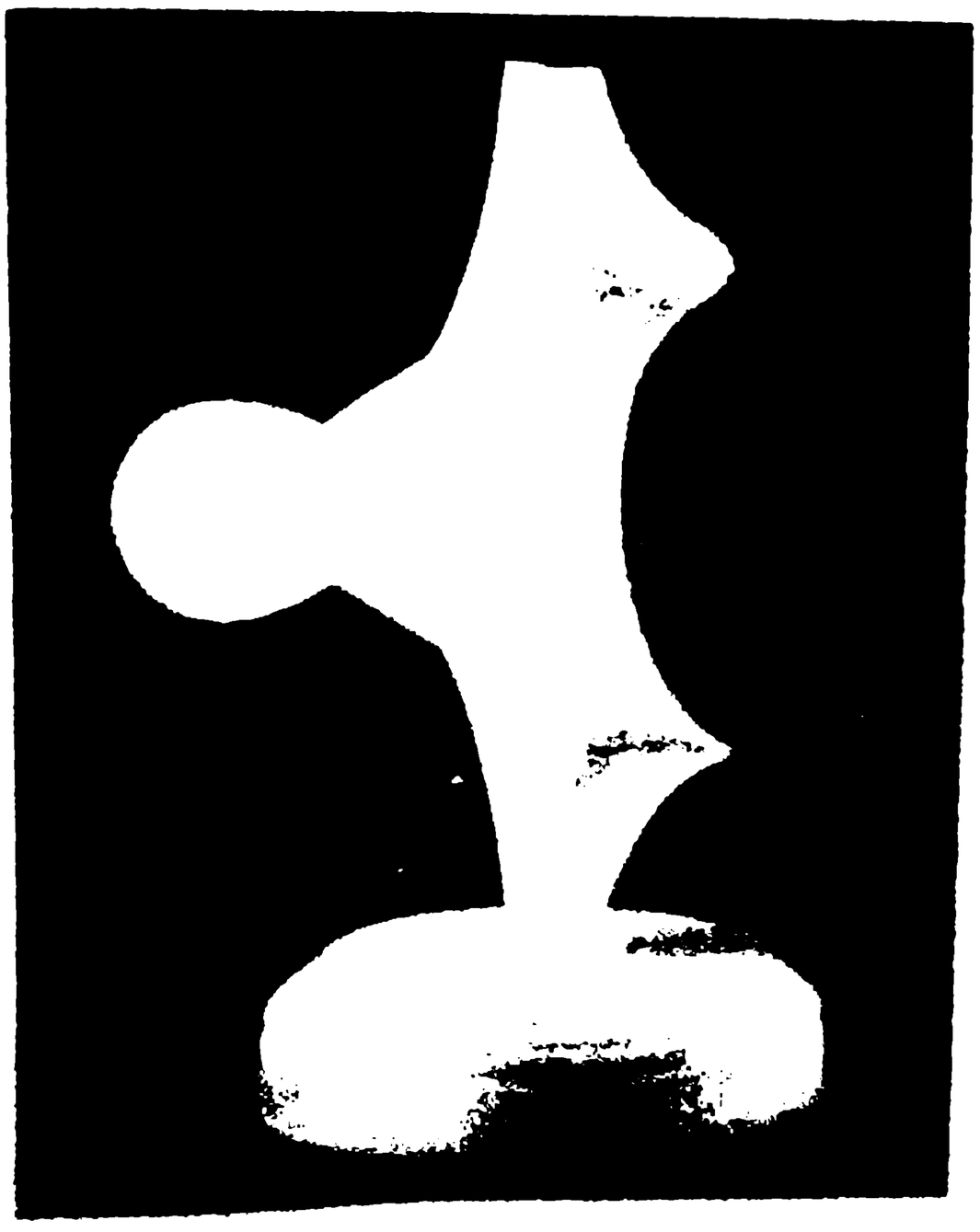
milennia, from before the time of Eratosthenes and Euclid, the best brains had been occupied with these highly capricious numbers, without ever being able to say what they might be good for — until suddenly in our century secret service people, computer programmers, military men, and bankers all recognized that with factorization and trapdoor codes they can carry on business and wage wars.

The Brain and the Cosmos

The unforeseen utility of mathematical models is somewhat puzzling. It is by no means clear why highly precise mental productions, devised entirely in isolation from empirical reality — somewhat akin to *l'art pour l'art* — should be so capable of explaining and manipulating the real world around us. Many have marveled at "the unreasonable effectiveness of mathematics." In eras more grounded in faith, the idea of such pre-established

harmony did not present a problem; Leibnitz declared confidently that, through mathematics, we "achieve a happy insight into the divine ideation," simply because the Almighty himself was the first mathematician. Philosophers nowadays have considerably greater difficulty with this question. The ancient debate among the Platonists, the Formalists and the Constructivists appears to be petering out into an exhausting stalemate. Mathematicians hardly concern themselves in their praxis with such questions. One explanation that presents itself — though not especially popular among the guardians of tradition — might be that one and the same evolutionary process has produced the universe at large and our brain, so that a weak anthropic principle determines that we observe the same operating rules in physical reality and in our own thought processes.

Konrad Knopp stated triumphantly in his inaugural address at Tübingen in 1927 that mathematics was "the basis of all knowledge and the bearer of all higher culture." Grandiosely and pompously put, but not false. It's just that the tangible utility, the technological application of mathematical research normally makes its appearance only much later, largely behind the backs of the pioneering mathematicians, who, like Hardy, recklessly go their



own ways that lead to destinations no one could have predicted. It is frequently difficult to perceive the connections between pure and applied mathematics; this may be another reason why the status of mathematical research is hugely undervalued in today's society. In addition, there is surely no other field in which the cultural time lag is so enormous. Popular consciousness trails research by centuries. Indeed, one can state dispassionately that great segments of the population have never progressed beyond the mathematical level of the ancient Greeks. An equivalent backwardness in other fields — medicine, say, or physics — would arguably be perilous. Less directly, this could be said of mathematics also, for never has a civilization been so infused with mathematical methodology — right down to its everyday life — and so dependent on it as ours.

The cultural paradox of which we are speaking may be becoming increasingly critical. For one might with good reason argue that we are living in a golden age of mathematics. Contemporary achievements in this field are at all events spectacular. The visual arts, literature, and the theater, too, I'm afraid, come off rather poorly by comparison.

It is beyond my competence to elaborate more precisely on such an assertion. As one of those hopeless lay persons, I can follow the arguments of the mathematicians only in their broadest outlines. Often I have to content myself with grasping merely the jist of the question. For me too, the drawbridge to their island is raised. That doesn't keep me, though, from glancing now and then at

the opposite shore. And with what I can make out from here, I'll butress my argument with several examples.

Most people have probably never heard of the Class Number Problem, one of the greatest puzzles in number theory. Formulated by Gauss in 1801, it was, after protracted groundwork, definitively solved by Zagier and Gross in 1983. It took just as long to prove the so-called Classification of the Simple Groups. This entailed ordering the infinite diversity of the simple groups, whose name is a complete misnomer, since they are in reality decidely complicated entities. It was 180 years after the foundations of group theory were laid before Aschbacher and Solomon were able to unearth the keystone. I'll spare the reader further examples. Both Incompleteness Theorems of Gödel, arguably the century's most brilliant logician, are well enough known. I suppose word has also gotten around that Fermat's Last Theorem, over which three centuries have gnashed their teeth, was proved in 1995 by Andrew Wiles. I'd like to see the World Cup football match that could compete with triumphs such as these — not to speak of the Documenta exhibitions and our international theater festivals of recent years.

Nevertheless, no storm of public adulation seems poised to break forth, which brings us back to the initial question of my ruminations. And on this point, only one scapegoat remains to be singled out, namely the institution of our intellectual socialization, the school system. It is not a question merely of the excessive workload with which this institution is currently overtaxed. The failures



Intellectual socialisation

we are dealing with lie deeper and have more ancient roots. One might well ask whether the first five years of the curriculum even include such a thing as mathematical instruction. The subject taught in those years is what used justly to be designated not math, but "arithmetic." To this day, children are tormented almost exclusively, from one end of the year to the other, with mind-numbing routines, a pedagogy that goes back to the era of industrialization and is now utterly obsolete. Up until the mid-twentieth century, the labor market required of most workers just three basic skills: reading, writing, and arithmetic. Elementary school served the function of providing a minimally literate work force. This may explain why a purely functional relation to mathematics established and entrenched itself in the schools. Now, I won't dispute the usefulness of mastering the times table, or of

knowing how to carry out simple fractional arithmetic or long division. But these have nothing to do with mathematical thinking. It's as though one were to acquaint people with music by having them practice only scales year in and year out. The result would undoubtedly be a lifelong hatred of this art.

Childish Fascinations

In the upper grades, for the most part, the situation isn't much better. Analytic geometry is treated largely as a cookbook of formulas — integral calculus, likewise. As a result, students are able to get good marks without truly understanding what they are doing. Nonetheless, the high school graduate should be awarded his diploma, particularly since he hasn't had the least say in the curriculum or in the teaching method. However, it should come as no surprise that this kind of instruction ultimately fosters mathematical illiteracy. The curriculum's original utilitarian rationale has long since vanished, since technology and the exigencies of the workplace have been decisively transformed over the last several decades. Today's sixteen-year-old cannot fathom why he should waste his time figuring tedious sums that any pocket calculator can accomplish faster and more accurately.

Normal mathematical instruction is not merely boring, however — above all it insufficiently challenges the intelligence of students. It seems a pedagogical *idée fixe* that children are incapable of abstract thinking. That is of

course pure nonsense. More likely the opposite is true. The concept of the infinitely large and of the infinitely small, for example, is intuitively accessible at once to any 9- or 10-year-old. Many children are distinctly fascinated by their discovery of zero. Surely one can explain the concept of a limit, and the difference between convergent and divergent sequences is immediately apparent to them. Many children show a lively interest in topological problems. They can even be amused with elementary problems of group theory or the theory of combinations if only one taps into their inborn sense of symmetry, and the like. Most likely, a child's capacity for grasping mathematical ideas naturally exceeds that of the average adult, who, having already been run through the conventional academic mill, has suffered damage in the process that will in all likelihood prove permanent.

It would admittedly be unfair to hold the math teachers alone accountable for this educational debacle. These people, deserving of our pity, are not only defeated by the handicaps pedagogy imposes on them; they are forced to operate at the end of a bureaucratic tether that prescribes quite brutal curricula and scholastic goals. Perhaps their civil service status is to blame for the inclination of teaching staffs toward slavish obedience, so evident in the example of the so-called "Orthographic Reform." A definite timidity hinders many from exploiting the freedom their *de facto* tenure offers them. Nevertheless, it must be said that there are teachers who resist the obsolete routines they are saddled with, and who manage to introduce their students to the beauties,



the riches, and the challenges of mathematics. Their successes speak for themselves.

Outside the educational system, too, we find scattered encouraging indications that the absolute nadir of mathematical ignorance is perhaps behind us. First, some changes seem to be appearing in the attitudes of the scientists themselves. Today's generation of mathematicians corresponds less than ever to the stereotype of the introverted odd-ball, whose back is turned on the world. This is true particularly in the Anglo-Saxon world. And it is not only the obvious pragmatic motivations such as competition for research funding that bespeak this shift in consciousness. It is grounded first of all within mathematics itself. The so-called Crisis in the Foundations of Mathematics of the first half of the century may have helped bring about the less rigid disposition now in the ascendency. Also, the traditional distance between pure and applied research has shrunk now that both entrepreneurs and investors have realized how profits can be made from basic research faster than ever. Experimental, computer-supported mathematics

has opened up entirely new possibilities, even though its methodology has long suffered under suspicion of insufficient rigour. And with respect to the traditional arrogance of the discipline, I have the impression that these days it is undermined by a healthy dose of irony. More than ever, mathematicians are aware of their own fallibility. They know well enough that their cathedral will never be finished and that, for such an edifice, no perfectly faultless blue-print can even exist. Many are even ready to talk to non-mathematicians.

Semantic Convergences

It is no wonder that this should lead to difficulties in communication. Fortunately, in recent decades there have been greater numbers of interpreters, who specialize in translating the technical jargon of the field into common language. This is an exceptionally delicate, but also an extremely rewarding undertaking. In this area, too, English speaking writers have set the pace. Well-known builders of these metaphorical bridges such as Martin Gardner, Keith Devlin, John Conway, and Philip Davis have done pioneering work; in Germany, journals like *Spectrum der Wissenschaft* and publicists like Thomas von Randow have rendered important service as conduits to the general public. Occasionally, even the mass media has gotten hold of a mathematical story, such as in 1976, when Appel and Haken solved the four color problem, though it was perhaps less important than notorious. Indeed, the risk one runs in such instances of

faddish distortions, as in the case of chaos and catastrophe theories, has to be taken into account. Here, it is not only semantic misunderstanding that plays a role. The Sokal affair has demonstrated how ridiculous things can get when dilettantes mix scientific concepts into their gibberish, without knowing what they are talking about. On the other hand, it is a promising development that *Fermat's Last Theorem*, a thoroughly serious scientific thriller by Simon Singh, has become an international best-seller.

Surely it is an audacious undertaking to attempt to interpret mathematics to a culture distinguished by such profound mathematical ignorance. I can't resist quoting from the dialogue with which Ian Stewart, a professional mathematician and a superb writer, has prefaced his book *The Problems of Mathematics*. A mathematician is chatting with the fictional layman "Seamus Android."

"Mathematician: It's one of the most important discoveries of the last decade!

Android: Can you *explain* it in words ordinary mortals can understand?

Mathematician: Look, buster, if ordinary mortals could understand it, you wouldn't need mathematicians to do the job for you, right? You can't get a feeling for what's going on without understanding the technical details. How can I talk about manifolds without mentioning that the theorems only work if the manifolds are finite-dimensional para-compact Hausdorff with empty boundary?

Android: Lie a bit.

Mathematician: Oh, but I couldn't do that!

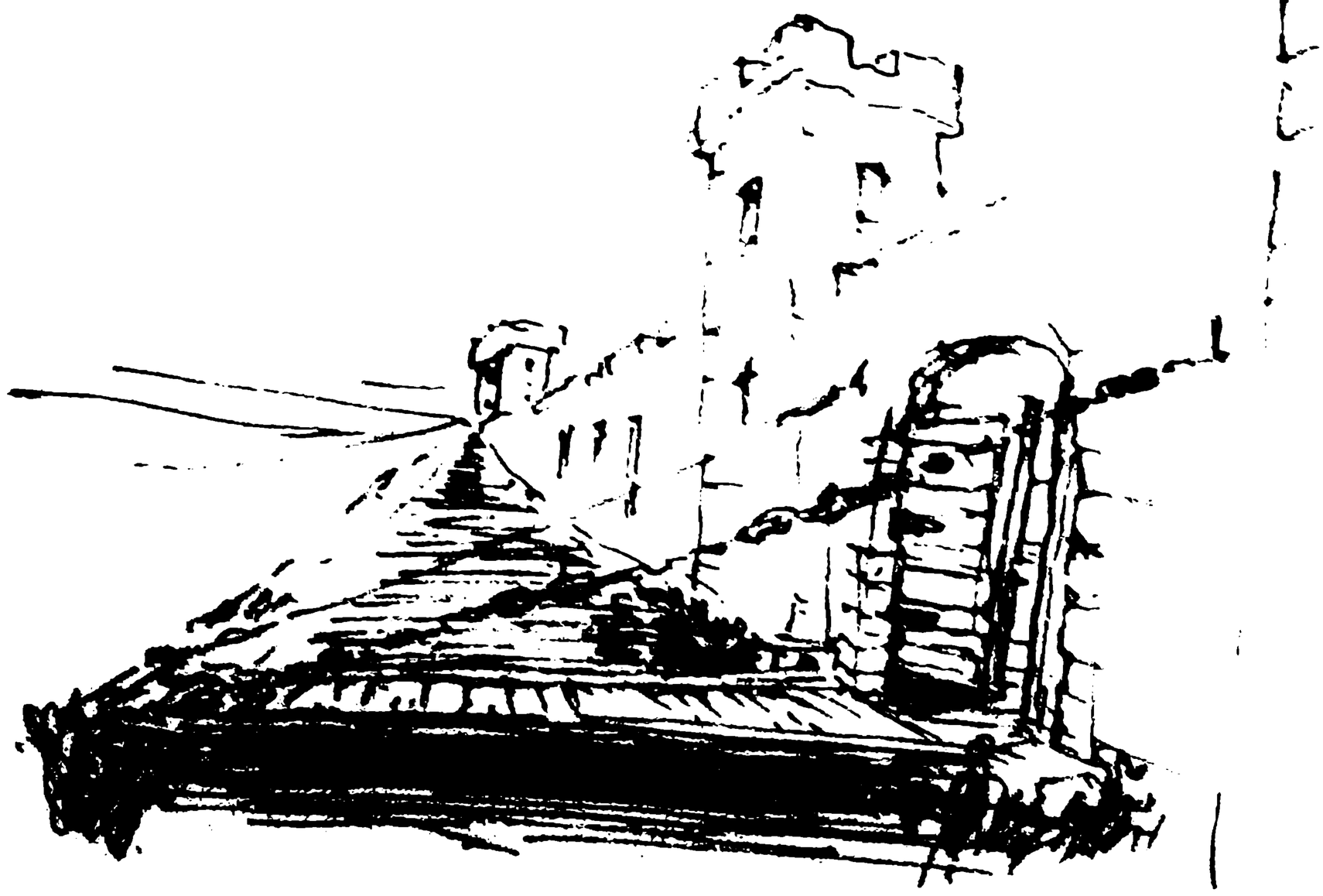
Android: Why not? Everybody *else* does.

Mathematician (tempted, but struggling against a lifetime's conditioning): But I *must* tell the truth!

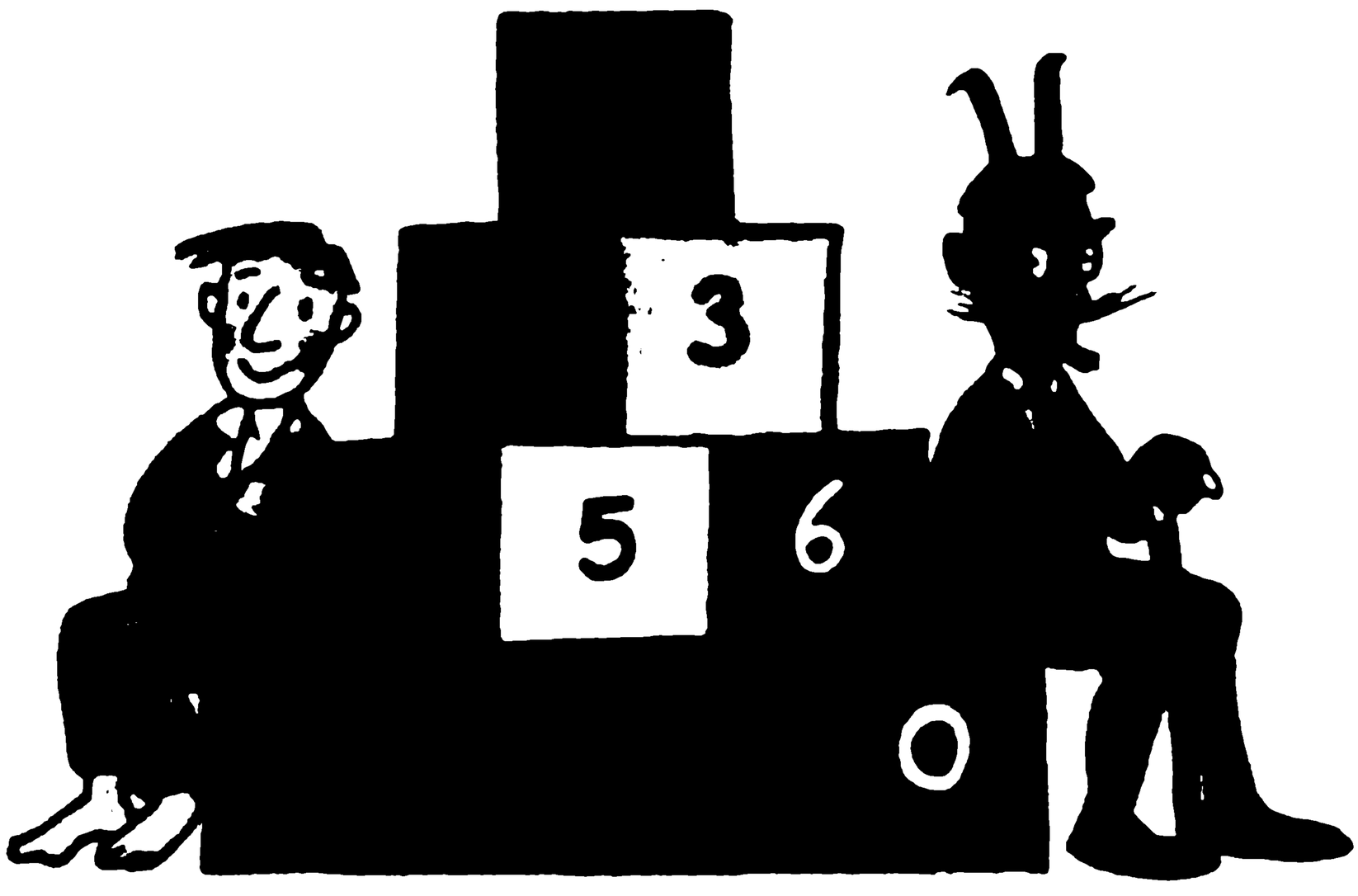
Android: Sure. But you might be prepared to bend it a little, if it helps people understand what you're doing.

Mathematician (sceptical, but excited at his own daring): Well, I suppose I could give it a *try*."

The effort in question is nothing short of the achievement of mathematical literacy — a protracted but highly auspicious project, which, if it is begun at an early age, will furnish our all too sluggish brains with a kind of athletic work-out and yield to us a variety of pleasure to which we are entirely unaccustomed.



Hans Magnus Enzensberger geboren 1929 in K
heute zählt zu den renommiertesten Schriftstelle
deutschen Literatur seit 1945 Neben seine
und Essays die oft aktuelle Fragen in einem
greifenden Zusammenhang behandeln ist besonders
von ihm erstmals 1961 herausgegebene Kinder
Sammlung Allerleirauh bekannt geworden Sie
bis heute große Beachtung Mit dem Zahlenteufel
Zahlenreich Ein Kopfkissenbuch für alle die Angst
Mathematik haben Carl Hanser Verlag München
wendet er sich wieder an ein junges Publikum ohne
Welt der erwachsenen Leser zu vernachlässigen



HANS MAGNUS ENZENSBERGER is a unique intellectual wh
speaks eloquently and writes with wit and insight on th
broadest range of subjects from politics to poetry
Known and respected world-wide, many of M
Enzensberger's books have been translated into English
including *The Number Devil: A Mathematical Adventur*
(Metropolitan Books, Henry Holt and Company, Inc.
New York, 1998), an international bestseller, and his first
book for children and other thinking beings.

The great mathematicians do not allow themselves to make their science comprehensible to us beginners. Is it because they imagine that others have the same degree of understanding that they do and hence they consider it superfluous to expound those statements that form the bases of their arguments? Or is it that they consider that those who do not have as much sagacity as they do are incapable of understanding the mathematical sciences and therefore they think it not worth their while to instruct them? Or is it that they feel it is unpleasant and repulsive to explain matters which, in their opinion, are just too simple?

Georg Ludewig Spohr