Deleuze Challenges Kolmogorov on a Calculus of Problems

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Abstract[1]

In 1932 Kolmogorov created a calculus of problems. This calculus became known to Deleuze through a 1945 paper by Paulette Destouches-Février. In it, he ultimately recognised a deepening of mathematical intuitionism. However, from the beginning, he proceeded to show its limits through a return to the Leibnizian project of *Calculemus* taken in its metaphysical stance. In the carrying out of this project, which is illustrated through a paradigm borrowed from Spinoza, the formal parallelism between problems, Leibnizian themes and Peircean rhemes provides the key idea. By relying on this parallelism and by spreading the dialectic defined by Lautman with its 'logical drama' onto a Platonic perspective, we will attempt to obtain Deleuze's full calculus of problems. In investigating how the same Idea may be in mathematics and in philosophy, we will proceed to show how this calculus of problems qualifies not only as a paradigm of the Idea but also as the apex of transdiciplinarity.[2]

Keywords: calculus, Deleuze, problems, rhemes, transcendentals,

I. Introduction

Transdisciplinarity has two conceptual neighbours or challengers in neighbourhood: pluridisciplinarity and interdisciplinarity. Pluridisciplinarity reaches its apex in anthropology: a man falling from a cliff is a point for mathematics, a body for physics, an organism for biology, perhaps a candidate for suicide for sociology. Interdisciplinarity, if it

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exists, is constantly in danger of falling prey to fashion. In a congress of mathematicians, physicists and epigraphists, the first condition of 'interdisciplinarity' is that mathematicians make mathematics, physicists physics and epigraphists epigraphy; but the second condition is that each also says something in the disciplines about which he or she is supposed to have a lot to learn. In this sense, 'interdisciplinarity' can be said to have as many advantages as squaring the circle does. And so the suspicion is born that mathematicians, physicists and epigraphists go to interdisciplinary jamborees in the rare periods when they really have nothing particular to say in mathematics, physics or epigraphy, respectively.

But what about this *tertium datur* between the Necessary and the Impossible that is transdisciplinarity? It is fairly easy to notice that the whole universe of pluridisciplinarity and interdisciplinarity turns in the circle of Comte's classification of the sciences: mathematics, physics, biology, sociology (and other social sciences). In other words, pluridisciplinarity and interdisciplinarity are by-products of positivism. Since universities, still bearing the vestiges of their medieval mothers, currently include a department of philosophy on a par with the faculties of theology, medicine and law, philosophers are also invited by force or politeness in the interdisciplinary jamborees. But the status of their discourse remains a mystery for all the participants, including themselves.

In the Deleuzian doctrine of Thought, however, we do have a concept for transdisciplinarity. This is the concept of the Idea, taken in the sense that the same Idea may be, for example, an Idea in mathematics, in cinema and in philosophy. In relation, then, to the positivist encyclopaedia, which circumscribes the whole territory of interdisciplinarity as well as pluridisciplinarity, we notice that Deleuze exceeds this Procrustean bed at least in two ways. First, philosophy acquires, in addition to its *de facto* status inherited from the medieval university, a *de jure* status reminiscent of its role in Plato's Academy or Aristotle's Lyceum. Second, the extension of Ideas is not confined to sciences (even *sensu lato*) but includes arts, such as cinema. Starting with this freed space of the Idea, we will focus on the couple philosophy–mathematics. With regard to this couple, the relation of Deleuze to Kolmogorov and the latter's calculus of problems offers more than a paradigm: rather, it opens a royal path, which, incidentally, also encounters the main difficulties surrounding what we will call the problem predicament.

Deleuze's assessment of the calculus of problems in symbolic logic is contained in two sentences that must be quoted in reverse chronological order:

> The 'intuitionist' school (Brouwer, Heyting, Griss, Bouligand, etc.) is of great importance in mathematics, not because it asserted the irreducible rights of intuition, or even because it elaborated a very novel constructivism, but because it developed a conception of *problems*, and of a *calculus of problems* that intrinsically rivals axiomatics and proceeds by other rules (notably with regard to the excluded middle). (Deleuze and Guattari [1980] 1987: 570)

> The calculus of problems as it is defined – notably by Kolmogoroff – still remains traced from a calculus of propositions, in 'isomorphism' with it. (Deleuze [1968] 1994: 322)[3]

The question faced in these two sentences is precisely indicative of the problem predicament. The first sentence contains the Deleuzian synopsis of mathematical intuitionism, which is better, as we shall see, than its counterparts in standard companions on the foundations of mathematics.[4] The second sentence is the Deleuzian challenge to Kolmogorov on the calculus of problems.[5] Yet there is a crucial connection between the two: the 'etc.' following the names of Brouwer and Heyting in the first sentence must be taken to refer primarily to Kolmogorov in the second. For, as it is well known, the calculus of problems, which according to Deleuze is the highpoint of mathematical intuitionism, had been created by Kolmogorov in 1932. This entails that the first sentence, which appears in *A Thousand Plateaus* in 1980, is an important correction of the position taken in 1968 in *Difference and Repetition*. In the latter, the calculus of Kolmogorov was simply seen as illustrating a 'false conception of the category of problem'; accordingly, its value was dismissed (Deleuze [1968] 1994: 322). In *A Thousand Plateaus*, however, the calculus of problems has reached the level of 'great mathematical importance'. As we shall see, this does not mean that the critical assessment is wholly withdrawn. In fact, the main reason behind Deleuze's enduring critical stance is his endorsement of the most celebrated among the Leibnizian 'dreams', that is, 'the great logical dream of a problem calculus'. For the *locus classicus* of this dream is the paragraph where Leibniz precisely anticipates the effect of his 'philosophical grammar' on Philosophy as a whole:

> Then, between two philosophers, there will be no more need of discussions longer than between mathematicians, since it will suffice that they seize their

> pen, that they sit at their computing table (appealing to a friend if they wish) and that they say to each other: 'Let us calculate' [*Calculemus*]. (Leibniz [1687] 1978: 200)[6]

Regarding our question, then, in *Difference and Repetition*, Deleuze essentially draws on two tenets:[7] on the one hand, he endorses 'the great logical dream of a combinatory or calculus of problems' he ascribes to Leibniz (Deleuze [1968] 1994: 157);[8] on the other, he says about Carnot's *Reflections on the Metaphysics of the Infinitesimal Calculus* that '[by] invoking the notions of "problem" and "problem conditions" ... Carnot opened up for metaphysics a path which went beyond the frame of his own theory' (177). So that with regard to what we have called the problem predicament, the whole Deleuzian doctrine may be described as outlining a metaphysical calculus of problems. As we shall see, this view is maintained until *What Is Philosophy?* and it is precisely what subtends the definition of a properly Deleuzian dialectics. For according to Deleuze, 'Dialectic is the art of problems and questions, the combinatory or calculus of problems as such' (157). And in this dialectics, the metaphysical calculus of problems is the spearhead, and the theory of problems is more specifically called 'problematics'.[9]

Based on the above, our whole exposition will have two main, though unequal, parts. In the first part, which is relatively brief, we shall explain the Kolmogorov calculus and its discussion by Deleuze. In the second part, relying on scattered data in Deleuze, we shall disclose, through their systematisation, his full metaphysical calculus of problems.

II. The Kolmogorov Calculus of Problems

We have a double reason to expound upon and discuss the calculus of Kolmogorov. We must understand why Deleuze arrived at a positive assessment of it; and we must also find in it the target of his lasting critic on the problem predicament, explaining why the very same calculus falls short of the 'great logical dream'.

In order to appraise the Deleuzian judgement in its two pronouncements, we may condense the Kolmogorov calculus of problems in the four following definitions:

1. $A \wedge B$ is the problem of solving problem A *and* problem B.
2. $A \vee B$ is the problem of solving *at least* one of problems A *or* B.
3. $A \rightarrow B$ is the problem of reducing a solution of problem B to a solution of problem A (conditional problem).

4. ¬ *A* is the problem: assuming a solution of problem *A* exists, deduce a contradiction.

Based on the logic so defined, we shall try to answer two questions: first, what is the 'great importance in mathematics' of this calculus 'notably with regard to the excluded middle'? Second, why does it nonetheless miss, according to Deleuze, the true 'conception of the category of problem'?

According to the standard story, as concerns the calculus of problems Kolmogorov is credited with having provided the 'semantics' of intuitionist logic. This is not false, but when Deleuze asserts that the calculus of problems 'intrinsically rivals axiomatics and proceeds by other rules (notably with regard to the excluded middle)' (Deleuze and Guattari [1980] 1987: 570), he is pointing to a much more important performance. As we shall see, the main contribution of Kolmogorov to intuitionist logic is a real foundation of this deviant logic in his calculus of problems. Taken in this perspective, 'intuitionist' logic is better called 'constructive' logic.

In order to see this point more clearly, it should suffice to deepen the comparison between two of the best-known laws in formal logic, here presented in a Kolmogorov dress. We mean the *tertium non datur* (or principle of 'the excluded middle')

$$A \vee \neg A$$

and the *modus ponens*

$$(A \wedge (A \rightarrow B)) \rightarrow B.$$

Between these two laws, the universal validity of the *tertium non datur* was notoriously questioned by Brouwer; and the first contribution of Kolmogorov to intuitionist logic is in his 1925 paper 'On the Principle of Excluded Middle'. Indeed, in the seemingly homogeneous web of the propositional calculus where these laws are selected, the Kolmogorov calculus of problems will reveal an unsuspected break.

Suppose the problem $A \wedge (A \rightarrow B)$ has a solution. This means that we have a solution to problem A and a solution to problem $A \rightarrow B$. But a solution of problem $A \rightarrow B$ means that the solution of problem B may be reduced to a solution of problem A. And, by hypothesis, we have a solution of problem A. So we have also a solution of problem B. All this means that the solution of problem B may be reduced to a solution of problem $A \wedge (A \rightarrow B)$ – that is, the exact case of $(A \wedge (A \rightarrow B)) \rightarrow B$. In other words, the *modus ponens* works in the calculus of problems

exactly like it does in the standard propositional calculus. But with the *tertium non datur*, it is a different ball game! Because here we hit at the difference between

$$A \vee \neg A$$

and the crude

$$p \vee \neg p.$$

The classical $p \vee \neg p$, for its part, means only that in any couple $\langle p, \neg p \rangle$ of contradictory propositions one at least is true. But with $A \vee \neg A$, it is quite another story because in this case we have either a solution of problem A or a solution of problem $\neg A$. And having a solution to any problem is an epistemic endorsement. Thus, the *tertium non datur* which, in a calculus of truth functions is a valid law, reveals itself to be subject to an 'impeachment' when the calculus of problems enters the landscape.[10]

Let us illustrate the above point with a celebrated example revisited by Deleuze: 'Will there be a naval battle tomorrow?' (Deleuze [1985] 1989: 130). This is a problem. As Aristotle says, 'it is necessary that there will be a sea-battle tomorrow or not' (Aristotle, qtd in Anscombe 1956).[11] This gives us an illustration of $p \vee \neg p$. But $A \vee \neg A$ is the problem of solving *at least* one of problems A *or* $\neg A$, where $\neg A$ is the problem: assuming a solution of problem A exists, deduce a contradiction. So that in this case $A \vee \neg A$ is the problem of having either a demonstrably true prognostic of a sea-battle tomorrow or a *reductio ad absurdum* of such a prognostic. We sublunar mortals are far off this epistemic performance. In other words, we may say that in the practice of problems, the transmission of obtained solutions along the channel of implication can conduce to the detachment of a solution according to the step of the *modus ponens*, but that on the fork of a disjunction there is no adjudication of solutions isomorphic to a *tertium non datur* (this alternative adjudication being confined to the truth values).

The difference between the respective destinies of the *modus ponens* and the *tertium non datur*, when the calculus of problems makes the difference, is of a huge significance. It means that formal logic, on the level of the propositional calculus, is crossed by an essential heterogeneity. The *modus ponens* is paradigmatic of the logical principles as laws of thought. With the *tertium non datur* we are on another territory. As the sea-battle predicament shows, the *tertium non datur* is a law of being and becoming. The calculus of problems reveals the objective difference between the two kinds of laws by epistemological criteria. And when the epistemological considerations are predominant,

as in mathematical intuitionism, the verdict of the epistemological criteria is promoted to the status of a logical touchstone: so much so that the principle of excluded middle is rejected as a logical rule.

In all this, as it will be obvious by now, the isomorphism duly registered by Deleuze has an essential function: it is because the symbolic formulas of the calculus of problems, literally taken, are indiscernible from the corresponding formulas in the propositional calculus that the calculus of problems has a discriminating power on the propositional calculus. In other words, the calculus of problems detects differences in the propositional calculus on condition that it is (morphologically) identical to it. 'Difference' is detected by (literal) 'repetition'. But on the level of the whole predicament, this means that the Kolmogorov calculus is a kind of medal, with its reverse and its obverse. For in the Deleuzian balance, the same isomorphism that motivates the critical attitude on the calculus is also a key condition of the discriminating power which constitutes its value. Nevertheless, there is no inconsistency between the two faces of the medal. On the contrary, the more the Deleuzian assessment recognises the unquestionable value of the Kolmogorov breakthrough, the more its reservations are well anchored, with its own doctrine well erected.

III. The Metaphysical Calculus of Problems

It must be noted that Kolmogorov does not define the notion of 'problem' but gives only some examples. These are the paradigmatic problems according to Kolmogorov. One of these examples is the following:

Find four positive integers x, y, z, n, such that $x^n + y^n = z^n$ and $n > 2$.

Here we can already recognise the target of the Fermat–Wiles theorem. A truly tiny target for a truly titanic proof! But this paradigmatic problem of mathematics leads us to an exegetic problem in philosophy: where in Deleuze do we find an example on the level of Kolmogorov's?

To our knowledge, taking into account that the Deleuzian concept of a calculus derives in part from the 'metaphysics of the calculus' in Carnot and that the paradigmatic problem of Kolmogorov is a case of exponentiation, this is an *apax*:[12]

> Hegel shows that variability in the function is not confined to values that can be changed (2/3 and 4/6), or are left indetermined ($a = 2b$), but requires one of the variables to be at a higher power ($y^2/x = P$). For it is then that a relation can be directly determined as differential relation dy/dx, in which

> the only determination of the value of the variables is that of disappearing or being born, even though it is wrested from infinite speeds. A state of affairs or 'derivative' function depends on such a relation: an operation of depotentialisation has been carried out that makes possible the comparison of distinct powers starting from which a thing or a body may well develop (integration). (Deleuze and Guattari [1991] 1994: 122)

These lines from *What Is Philosophy?* are directly referred to a page by Hegel in the *Science of Logic* [1816] (2010).[13] In fact, they are an abstract of two or three pages of Hegel's on the differential and integral calculus – a strange abstract, no less, because it admits of a double reading.[14] Upon a first reading, it can be said to follow some lines on the representation of velocity and acceleration in the famous diagram of Nicole Oresme, so that we have in two pages a philosophical epitome of the infinitesimal calculus considered in its genesis and evolution, a subject which Deleuze had studied at length in *The History of the Calculus and its Conceptual Development* by Carl B. Boyer.[15] A second reading is induced by a tiny anomaly of the text. In his abstract of Hegel on the calculus, Deleuze retains the examples given in the *Science of Logic* but with one exception. Hegel begins his series of examples by the equality $2/7 = 4/14$. Without any warning, Deleuze replaces it by the equality $2/3 = 4/6$. Why? We are in a book where Spinoza is proclaimed 'the prince of philosophers' (Deleuze and Guattari [1991] 1994: 48). So we can recognise here 'the proportion 2, 4, 3, 6', taken as example from the *Treatise on the Emendation of the Intellect*.[16] And this proportion illustrates a general request: 'Three numbers are given – it is required to find a fourth, which shall be to the third as the second is to the first.' That is, we are here before a paradigm of the problem. In the *Ethics* (II, XL, n. 2), this paradigm is even made both more simple and more explicit: 'one, two, three, being given, everyone can see that the fourth proportional is six'. In this version, the paradigm is more simple since it begins with the three first numbers; and it is more explicit as a case of problem since only the given numbers 1, 2 and 3 are named, leaving the solution of the problem in the dark or, better, as a blank. So, if we put this paradigm in the same form as $2/3 = 4/6$, we obtain

$$1/2 = 3/\ .$$

This is the Deleuzian paradigm of problem. And now that we have this paradigm, we may proceed to its dogmatic exposition. It has the nice form of an ABC.

Problems, Rhemes and Themes in the Procession of a Same Theme

We are also in a chapter of *What Is Philosophy?* where the first sentence declares: 'The objects of science are not concepts, but functions . . . '. The sequel of this thesis is to be found in the first page of the following chapter, where Deleuze, speaking about logic, says that 'following the route marked out by Frege and Russell, it wants to turn the concept into a function' (Deleuze and Guattari [1991] 1994: 135). Deleuze adds the condition of this transgression: 'Thus a new, specifically logical type of function must be invented', the 'propositional function'. And he gives an example of propositional function:

x is human.

But this is only the parvis of the temple. Because Deleuze knows as well the paradigm of the giving in C. S. Peirce (Deleuze [1983] 1986: 197). He writes it as:

A 'gives' B to C.

In Peirce we find the following writing:

–gives ¬ to – (Peirce [1898] 1992: 154)

And Deleuze has adopted this writing by hyphens on the case of the Other as a paradigm of concept with two components: 'its concept is that of an other – a subject that presents itself as an object – which is special in relation to the self' (Deleuze and Guattari [1991] 1994: 16). Peirce explains this kind of writing in the definition of his concept of rheme, also written 'rhema':

> If parts of a proposition be erased so as to leave blanks in their place, and if these blanks are of such a nature that if each of them be filled by a proper name the result will be a proposition, then the blank form of proposition which was first produced by the erasures is termed a rheme. According as the number of blanks in a rheme is 0, 1, 2, 3, etc., it may be termed a *medad* (from *μηδ*, nothing), *monad*, *dyad*, *triad*, etc., rheme. (Peirce [1903] 1998: 299)

> In a complete proposition there are no blanks. It may be called a *medad*, or *medadic relative*, from *μηδαμός*, none, and *–άδα*, the accusative ending of such words *μονάς*, *δυάς*, *τριάς*, *τετράς* etc. A non-relative name with a substantive verb, as '– is a man', or 'man that is –', or '–'s manhood' has one blank; it is a *monad*, or *monadic relative*. An ordinary relative with an active verb as '– is a lover of –' or 'the loving by – of –' has two blanks; it is a *dyad*, or *dyadic* relative. A higher relative similarly treated has a plurality of blanks. It may be called a *polyad*. The rank of a relative among these may be called its

> *adinity*, that is the peculiar quality of the number it embodies. (Peirce 1933: *CP* 3.465, 294)

But all this means that the rheme of Peirce is exactly the same thing as the propositional function of Russell as a case of the function of Frege. 'A gives B to C' is only a notational variant of '*x* gives *y* to *z*'. And since the Deleuzian writing of the concept is borrowed from the Peircean writing of the rheme, the conclusion seems unavoidable: Deleuze himself has remelted the concept of Concept in the mould of the function. What Peirce has labelled 'adinity', now called 'arity' by the logician, is in the vocabulary of Deleuze a case of *multiplicity* (Deleuze and Guattari [1991] 1994: 15).

We must, however, remark that this reasoning does not go without saying for the Peircean step:

– gives ¬ to –

As we may notice, in the Peircean paradigm of the giving, we have to distinguish three heterogeneous 'roles': the donor *A*, the donee *C*, and the gift *B*. And the juridical difference between the persons and the things is appropriately marked by a difference between the allotted hyphens '–' and '¬ '. This implies that here the limits of formal logic are exceeded, since the difference donor/donee, for example, pertains to the matter of the meaning. And such a difference of roles defines, as we shall see, a stepping stone for the Deleuzian logic of problems.

The key concept, here, is exhumed by Deleuze when, in opposition to the Kolmogorov calculus, he takes up[17] a notion that Leibniz himself had borrowed from 'some logicians of the reformation century': the concept of theme which immediately divides itself in incomplex theme and complex theme according to the distinction between 'ideas or terms' and 'propositions or truths' (Leibniz 1992: IV, i, § 2).

In these Leibnizian terms, an incomplex theme is 'a thing or idea', such as 'in love', and a complex theme is 'a thesis, proposition or truth' such as 'Candide is in love' or 'Candide is in love with Cunegonde'. As it is well known, from a propositional function such as '*x* has a daemon', propositions are obtained by what Frege has described as the saturation of the function, that is, either by the substitution of constants for the variables – as in 'Socrates has a daemon' – or by quantifying on the variables – as in '$\exists x$. *x* has a daemon' ('There is an *x* such that *x* has a daemon'). This means that the conceptual couple incomplex theme/complex theme is the ancestor of the conceptual couple

propositional function/proposition (x is in love/everybody is in love, etc.).

But Deleuze quotes the following lines from Leibniz:

> There are indeed 'themes' which can be said to be midway between an idea and a proposition, namely *questions*. Some of these ask for the Yes or the No, and these are the closest to propositions; but there are others which ask the how and the circumstances, and so on, and more must be added to these if they are to become propositions. (Leibniz 1981: IV, i, § 2)

For example, we have the following procession on a same theme:

[a] — is in love

[b] Is Marcel in love?

[c] Marcel is in love with Albertine.

In this paradigm, [a] is a term, [b] is a question (formulation of a possible problem) and [c] is a proposition. But the procession [a]–[b]–[c] as a whole is on a same theme. From the Deleuzian point of view, this is the fundamental fact of logic. And in the conceptual conflict with Kolmogorov, this is exactly what challenges the isomorphism so well exploited by Kolmogorov, for example between '$p \wedge q$' and '$A \wedge B$'. As its name recalls, iso*morph*ism is identity of form. The thematic identity, on the contrary, is an identity of matter, content or topic. It is an 'iso*hyl*ism' or, better, iso*top*ism.

Let us compare the formulas of the following thematic table:

(1) $1/2 = 3/x$	[1] Pleasure/$x = y/z$
(2) $1/2 = 3/6$	[2] Pleasure/Act = Beauty/Youth in its flower
(3) $x/2 = 3/6$	[3] x/ Act = Beauty/Youth in its flower
(4) $x/y = 3/6$	[4] x/ y = Beauty/Youth in its flower
(5) $x/y = z/6$	[5] $x/y = z$/Youth in its flower
(6) $x/y = z/w$	[6] $x/y = z/w$
(7) 2/y = z/w	[7] Queen/y = z/w
(8) $2/4 = z/w$	[8] Queen/Monarchy = z/w
(9) $2/4 = 5/w$	[9] Queen/Monarchy = President/w
(10) $2/4 = 5/10$	[10] Queen/Monarchy = President/Republic

In this table, we must begin with line 6, where (6) = [6], being a propositional function with four free variables. From this starting point, we can then proceed either downwards or upwards. If we proceed

downwards, along the series from (5) to (10) or [5] to [10], following the method recalled by Deleuze when he says that we arrive at '$f(a)$ for a variable x', we shall obtain finally, by the progressive elimination of all the free variables, a proposition that is true or false such as (10) or [10]: '2/4 equals 5/10' or 'The queen is to a monarchy what the president is to a republic'. This is the logical genesis of propositions by quantification binding the free variables of the propositional function or by substitution of constants for these variables. Here, it is illustrated by the second method, the more fundamental one.

But if we proceed upwards, we penetrate into another territory. First, we must notice that a formula such as (1)

$$1/2 = 3/x$$

is virtually ambiguous. If it is taken in a book of formal logic, it is probably, as (9), a propositional function with one free variable, on a par with the Deleuzian example 'x is human'. If it is taken in a book of mathematics, it is probably a proposition. This proposition may be formulated, as in Kolmogorov, under the form of an imperative:

$$\text{Find one integer } x \text{ such that } 1/2 = 3/x.$$

But it may also take the form of an interrogative proposition:

$$\text{What is the number } x \text{ such that } 1/2 = 3/x?$$

In its two forms, the proposition enunciates a problem by an equation. In a propositional function the variables are dummies disposed of in order to be clothed by constants. In an equation they are the unknown variables, here to be suppressed by their solutions. (By 'virtually ambiguous', we mean that the ambiguity cannot cause any misunderstanding since propositional functions with free variables have no use – but only mentions. Consequently, the amphibology is recommended as an economy!)

The case of '$1/2 = 3/x$' may be generalised. When in our table we proceed from (6) to (3) and (1) or from [6] to [3] and [1], we obtain in each column some examples of (soluble or insoluble) problems. The propositions (2) and [2] contain respectively the solutions of problems (1) and [1]. Therefore, we arrive at the following conclusion: the arity or adinity of propositional functions extends to the problems, taking the form of the number of their unknown variables. In Deleuzian terms, this means that the multiplicity characteristic of the rhemes extends to the problems or, better, that the same multiplicity extends indifferently to propositional functions and to problems. Since problems are naturally

expressed in questions and since the logic of questions is called erotetic logic (from the Greek *erotesis*, questioning) we shall call this thesis of a same arity shared by all themes (in rhemes and problems) the principle of rhema-erotetic arity.

This thesis must be appreciated in its full significance. Formal logic, in its totality, is constituted by the propositional calculus and quantification theory. But since propositions are nothing but saturated propositional functions and since quantification is the binding of variables in propositional functions, the concept of propositional function is the arch-concept of logic in its totality. The fact that the arity of propositional functions exceeds formal logic and qualifies also the problems, therefore, may be said the fundamental fact of logic.

In taking from Spinoza the $1/2 = 3/x$ as paradigm of problems, endorsing the hyphen symbolism of Peircean rhemes and recycling the Leibnizian themes, Deleuze has discovered on the path problem–rheme–theme the principle of rhema-erotetic arity as the thesis of isotopy.

In Kolmogorov's calculus, the only problems at stake are mathematical problems. But, as the thematic table shows, the principle of isotopy is not confined to mathematics. As the example from Aristotle attests, philosophical problems illustrate the same law. The problem [1] may be rephrased more explicitly:

[1bis] Found a good analogy exhibiting the place of pleasure in human life.

In order to reach the full generality of a calculus of problems, how can we neutralise the difference between philosophy and mathematics? Following Coquand in his commentary on Kolmogorov,[18] we shall introduce the conceptual character imagined by Hilbert: **Aristide** = $\tau_x A$. The so-called Aristide is defined by Hilbert in the following way: if A is the predicate 'to be corruptible', then $\tau_x A$ would be a man 'of such absolute integrity that, if he happened to be corruptible, all men in general would be corruptible' (Hilbert [1923] 1996: 1140–1; translation modified). With such a definition, Hilbert's Aristide is obviously an amphibian between mathematics and philosophy.

But in order to reach such a result, and even if, as Kolmogorov, one confines the calculus to mathematics, the fundamental fact of logic itself is not sufficient. Deleuze declares:

> Sense is located in the problem itself. Sense is constituted in the complex theme, but the complex theme is that set of problems and questions in relation to which the propositions serve as elements of response and cases of solution. (Deleuze [1968] 1994: 157)

This thesis precedes the reference to Leibniz[19] which leads us to his division of the concept of theme into complex themes and incomplex themes. As we have seen, in Leibniz, an incomplex theme is for example 'in love' or 'in love with Cunegonde', and a complex theme is, for example, 'Candide is in love' or 'Candide is in love with Cunegonde'. That is, in Leibniz, incomplex themes are terms to be conceived as rhemes or propositional functions, and complex themes are propositions. But when we penetrate into the Deleuzian dialectics, the opposition incomplex/complex acquires an entirely new meaning.

In order to understand the Deleuzian dialectics in its full significance, we have to enlarge our conceptual apparatus and consider at least the following conceptual triplet that we shall call the entire dialectic triad:

$$\begin{array}{c}\text{Problems}\\ \text{Terms} = \text{Rhemes} = \text{Propositional functions}\\ \text{Propositions}\end{array}$$

In the dialectic triad, the layer of rhemes represents the level of incomplex themes. But the main point of Deleuzian dialectics is that, from this layer of incomplexity, two kinds of logical 'complexity' can be reached; two kinds that are not only different but entirely heterogeneous.

There is the well-known complexity of propositions, which qualifies them as complex themes in the Leibnizian sense. This is the 'complexity' obtained by the saturation of rhemes as propositional functions. For example, from the incomplex

$$x/y = z/w$$

there is the increasing complexity of

$$\begin{array}{l}x/y = z/\text{Aristide}\\ \exists x.x/y = z/\text{Aristide}\\ \text{Etc.}\end{array}$$

But there is a quite different complexity in the bargain: the complexity qualifying the problems.

Lautman in Bergsonian territory: incipit Albert Lautman

From here onwards, the Deleuzian doctrine on problems is mainly borrowed from Lautman.[20] In mathematical philosophy, the Lautman posture is an *apax*. On the category of problem, according to Deleuze, Lautman is grasped in a kind of analogy with the doctrine of Bergson on intuition, intelligence and instinct. According to Bergson, between

intelligence and instinct we find the following chiasmatic paradox:

> *Intelligence sets problems of which it has no solution.*
> *Instinct has solutions of problems it has never set.*

The function of intuition follows:

Intuition has the function of giving to the problems of intelligence the ready-made solutions worked out by instinct.

Mathematics is, among others, a case of this Law of Bergson. In mathematics, as in other disciplines, the solutions are the affair of an 'instinct'. And the royal art is the weaving of unforeseeable problems by intelligence. This is the true story of mathematical 'intuition'.

This paradoxical situation is paralleled by the main relation between philosophy and mathematics as it is conceived by Lautman. According to Lautman, philosophy sets latent problems to which it has no solution, and mathematics offers patent solutions of some problems it has never set. The task of 'mathematical philosophy' follows: mathematical philosophy must give to the latent problems of philosophy their patent solutions in mathematics:

> Nowhere better than in the admirable work of Albert Lautman has it been shown how problems are first Platonic Ideas or ideal liaisons between *dialectical* notions, relative to 'eventual situations of the existent'; but also how they are realised within the real relations constitutive of the desired solution within a *mathematical*, *physical* or other field. It is in this sense, according to Lautman, that science always participates in a dialectic which points beyond it – in other words, in a meta-mathematical and extra-propositional power – even though the liaisons of this dialectic are incarnated only in effective scientific propositions and theories. (Deleuze [1968] 1994: 163–4)

The mathematical philosophy of Lautman, itself, is exfoliated on two Platonic plateaus: on a first plateau, Lautman holds that even 'at the mathematical level' there is a real 'drama' which is 'played' in the heights, and that it qualifies as a 'logical drama' (Lautman [2006] 2011: 167, 189). On a second plateau, this logical drama, played at the level of mathematics, is in its turn explained by an even more secret history or action, developing itself 'at the background' of the drama, or, more exactly, above it. This double doctrine is contained in the following lines:

> Mathematics, and above all modern mathematics, algebra, group theory and topology, have thus appeared to us to tell, in addition to the constructions in which the mathematician is interested, another more hidden story made for

> the philosopher. A dialectical action is always at play in the background … (Lautman [2006] 2011: 91)
>
> One may say that distinct mathematical theories participate to a common dialectic which dominates them … participation of mathematics to a higher and more hidden reality which, in my opinion, constitute a real world of ideas. (Lautman, qtd in Cavaillès [1939] 1946: 596, 605; also Cavaillès 1994: 596, 605; my translation)

This action, which is played above the Lautmanian drama, will be called the Lautmanian *gesta*. Corresponding to these two plateaux of Lautman's theory, Deleuzian dialectics will itself develop in a drama crowned by a *gesta*.

The proper complexity of problems, taken in its main lines, is described by Deleuze as follows: 'The complete determination of a problem is inseparable from the existence, the number and the distribution of the determinant points *which precisely provide its conditions* (one singular point gives rise to two condition equations)' (Deleuze [1968] 1994: 163; [1969] 1990: 54).[21] Here we have an ascending dialectic of four floors:

> 4th The singular determinant points giving rise to the equations.
> 3rd The equations of conditions.
> 2nd The conditions of the problem.
> 1st The problem.

In this ascending dialectics, the conditions of a problem and the equation of conditions are borrowed by Deleuze from Carnot (Deleuze [1968] 1994: 175):

> after having introduced these quantities in the calculus in order to facilitate the expression of the conditions of the problem, and having treated them, in the equations which expressed these conditions, as zero by comparison with the proposed quantities, in order to simplify these equations, it suffices to eliminate these same quantities … (Carnot [1881] 1970: 11)

The 'conditions of a problem' are a case of dialectical amphibian. Suppose that the problem is

$$x = 100 - a.$$

Here, a 'condition of the problem' is that $a \leq 100$, *if* we are confined to natural numbers. But if we have at our disposal the algebraic numbers, positive or negative, the 'condition' evaporates. And you know that the same tale may be told for the invention of $\sqrt{2}$, 0, $\sqrt{-1}$, etc. In other words, the whole history of mathematics may be rewritten as

the history of superstitious 'conditions' and their successive dissolutions in the ocean of heroic generalisation. But this positively means that, since the quadrature of the circle, the problems are the erotetic fountain from which the whole stream of mathematics flows with 'the rule and the compass' as paradigm of obsolete 'conditions' for a construction problem.

At the other extreme of the dialectical spectrum, the same 'conditions of a problem' are a matter of algebraic routine in the standard cooking of equations. Here, the paradigm is an interdict in mosaic form: 'Never divide by zero' ('Par zéro jamais ne diviseras!'). If this condition is not fulfilled, this is the problem ('set in equation') which evaporates. But in a descending dialectic, Deleuze relies first on a paradigm borrowed from Lautman, with two illustrations of the determinant points:

> [a] The cases that we are going to envisage[22] are those of the functions admitting 3 singular points: the points 0, 1 and ∞. It can still in effect be obtained by a linear transformation that the singularities are produced at these points here. (Lautman [2006] 2011: 179)

> [b] The points of indetermination[23] where $P = Q = 0$, constitute the singularities of the vector field. (Lautman [2006] 2011: 259)

The main fact here is that the two illustrations lead to the same conclusion on the singular points:

> [a*] the nature of singular points on a domain determines, at each point of the domain of the variable z, the existence of solutions of the proposed equation. (Lautman [2006] 2011: 181)

> [b*] The existence and distribution of singularities are notions relative to the vector field defined by the differential equations. The form of the integral curves is relative to the solutions of this equation. (Lautman [2006] 2011: 259)

So Deleuze can conclude:

> Albert Lautman has clearly indicated this difference in kind between the existence and distribution of singular points which refer to the problem-element, and the specification of these same points which refer to the solution-element. (Deleuze [1968] 1994: 324)

> For example, in the theory of differential equations, the existence and distribution of singularities are relative to a problematic field defined by the equation as such. As for the solution, it appears only with the integral curves and the form they take in the vicinity of singularities inside the field of vectors. (Deleuze [1969] 1990: 54)

As for the 'equations of conditions',[24] when the problem is in the differential equation

$$w'' + p_1(z)\,w' + p_2(z)\,w = 0$$

the two alluded 'equations of conditions'[25] are, at the vicinity of the point $z = a$ on the complex plane **C**:

$$w_1 = (z-a)^l_1 P(z-a)$$

and

$$w_2 = (z-a)^l_2 P^*(z-a) + \text{eventually a logarithmic term.}$$

But where is there a 'drama'? This term 'drama', used by Lautman, finds its best ratio in Proclus: 'Proclus defines the problem by means of the events which come to affect a logical subject matter (sections, ablations, adjunctions, etc.)' (Deleuze [1969] 1990: 54). In proper Proclean terms:[26] 'it is in imagination that the constructions, sectionings, superpositions, comparisons, additions and subtractions take place . . .' (Proclus 1970: 64 [78]). But here, the problems whose complexity is exhibited by Lautman are typically equations. This means that we remain in the domain of mathematics. So, we do not see so far how this description could be valid about philosophical problems. In order, then, to reach the level on which the Deleuzian dialectics embraces as well philosophical and mathematical problems, we must move, in its Lautmanian top, from drama to *gesta*.

The Lautmanian Gesta and its Themes

Over and above the Lautmanian dialectical drama, what is the Lautmanian dialectical *gesta* of problems? The kind of themes providing the plots of this *gesta* are described by Lautman in the following terms:

> the materials of which the universe is formed are not so much the atoms and molecules of the physical theory as these great pairs of ideal opposites such as the Same and the Other, the Symmetrical and Dissymmetrical, related to one another according to the laws of a harmonious mixture. (Lautman [2006] 2011: xxviii)

This thesis must be labelled the Lautman Law. The 'great couples of ideal contraries' alluded in the Lautman Law have a celebrated register in a Pythagorean table:

1. Limit & Unlimited
2. Odd & Even

3. One & Many
4. Right & Left
5. Male & Female
6. Rest & Move
7. Straight & Curve
8. Light & Darkness
9. Good & Bad
10. Square & Oblong

Lautman simply adds to this table two homogeneous lines:

11. Same & Other
12. Symmetric & Dissymmetric

Here we understand why the theory of problems which dominates mathematics according to Plato and Lautman is called a dialectic by them. It is a dialectic because it is a doctrine of dualities. But in order to obtain in this dialectic a calculus of problems at the level envisioned by Leibniz, that is a calculus covering not only of mathematical but also of philosophical problems, the 'great couples of ideal contraries' alluded to by Lautman must extend to the whole register that is due to Plato. This list is drawn up in a dialogue between Socrates and Theaetetus:

> **Soc.** To which class, then, do you assign being; for this, more than anything else, belongs to all things? **Theaet.** I assign them to the class of notions which the soul grasps by itself directly. **Soc.** And also same and other [likeness and unlikeness] and identity and difference? **Theaet.** Yes. **Soc.** And how about beautiful and ugly, and good and bad? **Theaet.** I think that these also are among the things the essence of which the soul most certainly views in their relations to one another, reflecting within itself upon the past and present in relation to the future. (Plato 1921: 186ab)

With this ultimate adjunction:

> **Soc.** Is it possible for one to attain 'truth' who cannot even get as far as being? **Theaet.** No. (Plato 1921: 186c, pp. 163–5; translation modified)

This is in advance an inventory of the transcendental terms, enumerating the objects of philosophy. So, with these 'great couples of ideal contraries' enlisted in their complete enumeration, Lautmanian dialectics reaches its top level and the calculus of problems can satisfy the requisites of the Leibnizian programme. Because, from Plato's listing of transcendental terms to a calculus, there is only one step. This can be shown if we realise that Hilbert's Aristide is a kind of Platonic or Lautmanian 'Idea'. Aristide, as we may recall, is 'of such absolute

integrity that, if he happened to be corruptible, all men in general would be corruptible' (Hilbert [1923] 1996: 1140–1; translation modified). Similarly, according to Plato, the Idea of the Good is of such absolute goodness that, if it happened to be bad, all things in general would be bad. Hilbert's Aristide, therefore, is only a personalisation of the Platonic Idea. So, along with Aristotelian or Stoic logic, with its Boolean and Fregean offspring, Hilbert has forged a Platonic logic. And Lautman, with his dialectics, has maximised the efficiency of Hilbert's Aristide.

Consequently, reconsider our paradigm of an amphibian problem:

$$x/y = z/\text{Aristide}.$$

According to Plato's analogy of the line, this paradigm must be vertically restated.[27] In this new manner it takes the following form:

$$\begin{array}{c} \text{Aristide} \\ \text{- - - - - - - - - - -} \\ z \\ = \\ y \\ \text{- - - -} \\ x \end{array}$$

And on this ladder, at its top rung, since Aristide is only a personification of the Platonic Idea, he can be replaced by any analogous term, with its two main species, mathematical and philosophical. In the mathematical realm, a paradigm is already offered by Plato:

$$\begin{array}{c} \text{(4th Elevation at the Square)} \\ \text{- -} \\ \text{3rd Intelligible Square} \\ = \\ \text{2nd Visible Square} \\ \text{- -} \\ \text{1st Shadow of Square Body} \end{array}$$

Here, in mathematics, we follow an ascending dialectics: the stages x, y and z are given, and the problem is to find the 'fourth proportional' $= Power2 = (\,)^2$. In the philosophical realm, this is the other way round: in the whole vertical procession, Aristide is given at the top level, and the xyz series follows by measured participation to the Idea, in a triad of

hypostases. For example, if Aristide = the Other:

$$\frac{\text{Other}}{x = y}$$
$$z$$

So that, in its Lautmanian empyrean, the calculus of problems permits the deduction of the Platonic dialectics with its two inverse moves of ascending and descending dialectics. Accordingly, the metaphysical calculus of problems in its full development may be exposed in the following Lautmanian table:

	Lautman	Spinoza	Hilbert in Plato	Deleuze
Invisible	*Dialectics*	*x*	Aristide	**Same and Other**
Invisible	Mathematics	3	*a*	*Mathematics*
Visible	Natural philosophy	2	*b*	*Natural philosophy*
Visible	Physics	1	*c*	*Physics*

In this table, the essential fact to be read and emphasised is the difference between the three first columns and the fourth. In the Lautman column, physics, natural philosophy, and mathematics are *given*; the problem, situated at the level of dialectics, is the determination, as 'fourth proportional', of the 'X' which qualifies as the object of dialectics. In the Spinoza column, the *x* is the algebraic unknown variable to be found. In the Hilbert and Plato column, Aristide himself, after all, is only a dummy for Göttingen geometers. But in the Deleuzian column, all this indetermination is suppressed. With the Same and the Other (or similar transcendental terms), the Lautmanian dialectics receives its adequate Object, qualifying it as a philosophical calculus (philosophical, since Aristide becomes a transcendental term; and calculus since the entire column retains the algebraic form of its Spinoza counterpart). And all this means that the entire table functions as a mathematical proportion, where the fourth column is the solution, obtained as 'fourth proportional': QED.

IV. Conclusion

On the problem predicament, the Kolmogorov calculus of problems was received by Deleuze as the summit of mathematical intuitionism and as a challenge to the Leibnizian *Calculemus* in philosophy. The challenge is taken up at least in two results:

1. The thesis of isotopy disclosing the procession of a same theme in the Peircean rhemes and the Leibnizian problems.
2. The deduction of a Lautmanian dialectics where the Hilbertian Aristide is the variable whose values are the transcendental terms defining the objects of philosophy.

On the level of transdisciplinarity *qua* transdisciplinarity, this double result has a huge significance. Since the objects of philosophy are the transcendental terms (*esse*, *unum*, *verum*, *bonum* and *pulchrum*), philosophy transcends all categories, which are the top of all *generality*. And since the territory of all theoretical sciences is generality, philosophy transcends the whole territory of theoretical science. So, philosophy is 'transdisciplinary' by itself. But in the territory of science, mathematics is the top of apodicticity. Hence the Leibnizian programme of the *Calculemus*: to transport into philosophy the apodicticity of mathematics. In solving the proportional problem exposed in the Lautmanian table, Deleuze has accomplished the decisive step in this transfer.

Notes

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2. Chryssa Sdrolia saw immediately that the oral version of my paper was better than the original written version. The following text is revised according to this trusty verdict.
3. In this paper, we will follow the alternate spelling 'Kolmogorov'.
4. With the exception of Carl Posy (2005) 'Intuitionism and Philosophy', in Stewart Shapiro (ed.), *The Oxford Handbook of Philosophy of Mathematics and Logic*, Oxford: Oxford University Press, but for very different reasons.
5. When Deleuze wrote that sentence, the unique source he referred to on the calculus of Kolmogorov was a Note by Paulette Destouches-Février to the French Academy of Sciences, 'Rapports entre le calcul des problèmes et le calcul des propositions', published in 1945.
6. 'Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos

in manus sumere sedereque ad abacos, et sibi mutuo (accito si placet amico) dicere: Calculemus' (Leibniz [1687] 1978 '*Scientia Generalis, Characteristica*'; my translation).

7. From a genetic point of view, we must recall that *Difference and Repetition* was in Deleuze's PhD the 'thèse principale' and that its initial subject was the 'idea of problem' (Dosse [2007] 2010: 116–17).
8. *Différence & Répétition*, p. 204; (translation slightly modified).
9. He stipulates that 'the use of the word *problematics* as a substantive' is 'an indispensable neologism' (Deleuze [1968] 1994: 323).
10. As Coquand says: 'The solution of the problem B must be reduced to the solution of the problem $A \wedge (A \to B)$. So, suppose a solution of this problem is given. This means that one has both a solution of problem A, and a solution of problem $A \to B$, i.e. one may reduce the solution of problem B to that of problem A. Hence, as desired, we can solve problem B.

    On the other hand, the principle of excluded middle in the form

    $$A \vee \neg A$$

    can not be justified. Indeed, a solution would be a general method which, given a problem A, would either give a solution or show that a contradiction follows from the assumption that problem A is solvable. As Kolmogorov states, unless one is bold enough to consider oneself omniscient, it must be recognized that there is no solution to this problem' Coquand [2004] 2007: 31–2.
11. Aristotle, *On Interpretation*, ch. 9, 19a[30], trans. G. E. M. Anscombe, in Anscombe 1956.
12. As in *hápax legómenon* ($\pi\alpha\xi\ \lambda\varepsilon\gamma\acute{o}\mu\varepsilon\nu o\nu$).
13. With an erroneous pagination. Deleuze writes: '*Science de la logique*, Ed. Aubier, II, p. 277'. But the page is in volume I of this translation by Sandor Jankélévitch.
14. See Hegel [1816] 2010: 213–15.
15. Cf. *The Logic of Sense*, Deleuze [1969] 1990: 339.
16. See Spinoza 1992: § 23.
17. See Deleuze [1968] 1994: 322.
18. See Coquand [2004] 2007: § 1.1.
19. See Deleuze [1968] 1994: 322.
20. See Deleuze [1968] 1994: 322. Albert Lautman (2006) *Les Mathématiques, les idées et le réel physique*, Paris: Vrin. For a general introduction to Lautman, see my review of this re-edition (Dumoncel 2008).
21. 'A problem is determined only by the singular points which express its conditions' (Deleuze [1969] 1990: 54).
22. On the differential equation $w'' + p_1(z)w' + p_2(z)w = 0$.
23. On the differential equation $dy/dx = \mathrm{Q}(x, y)/\mathrm{P}(x, y)$.
24. See Lautman [2006] 2011: 179–80.
25. More precisely, these two equations are themselves the two *solutions* of an equation.
26. As Deleuze alludes, it is a conception from which Proclus, at least takes his distance.
27. Cf. J. L. Austin (1979) 'The Line and the Cave in Plato's Republic', in *Philosophical Papers*, ed. J. O. Urmson and G. J. Warnock, Oxford: Oxford University Press; J. C. Dumoncel (1992) 'La théorie platonicienne des Idées-Nombres', *Revue de Philosophie Ancienne*, X :1, pp. 1–34, reprinted in J. C. Dumoncel (2002) *La tradition de la* Mathesis Universalis, Paris: Unebévue-éditeur.

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