This is a pre-published version. The published version appeared first in French as 'Heidegger et Weyl: nombre, mouvement et continuité', in *Noesis* (Jan 2006), 85-102. It was then published in English as Chapter 5 of David Webb, *Heidegger, Ethics and the Ontology of Freedom* (Continuum Press, 2009).

Heidegger and Weyl on the Question of Continuity

David Webb

In his 1924-25 lectures on Plato's *Sophist*, Heidegger refers with admiration to the work of the mathematician Herman Weyl.[1] His remarks come part way through a reflection on arithmetic and geometry as alternative grounds for understanding connection or relation in the context of the dialogue in general, and of Plato's interpretation of the term *heteron*, otherness or being-other in particular. Over and above their contribution to his reading of the *Sophist*, they also provide an indication of Heidegger's attitude towards contemporary issues within mathematics, including those concerning analysis, number theory, and above all continuity.

The idea of handling geometrical problems algebraically in what is now called analytic geometry was first introduced by Descartes and lay behind the development of calculus by Newton and Leibniz. In the latter half of the nineteenth century, Cantor and Dedekind used set theory to give calculus a new expression in what is now called analysis. This involves constructing the continuum from discrete points correlated with real numbers defined by sets of infinitesimals approaching a limit. Although the analysis of problems that had once been the province of geometry proved to be extremely powerful, the appearance of infinity in mathematical operations remained a cause for concern; questions were raised regarding the capacity of analysis to define its terms adequately and there was growing unease over

[1]. M Heidegger, *Plato's Sophist*, trans. R Rojcewicz & A Schuwer (Indiana University Press, Bloomington, 1992), pp.80-81: *Platon: Sophistes*, Gesamtausgabe 19 (Vittorio Klostermann, Frankfurt am Main, 1992), 117-118. Hereafter GA19 followed by the English and German page references respectively.

the rigour its foundation. Another concern, and one shared by Weyl, arose from the apparently twofold nature of the continuum; in our experience of space, time and motion, it emerges as a pure and non-objective intuition to which the analytical reconstruction of the continuum from the discrete elements of number never seems fully adequate.

Heidegger's remarks on arithmetic and geometry in Plato and Aristotle can be read as a thinly veiled comment on these issues. As such, they may appear at first glance to be no more than a rehearsal of this debate; interesting as a testament to his understanding of the issues involved, without offering anything new. In fact, this is not the case; not only does Heidegger display a secure grasp of the main issues in the debate over analysis, he is also prepared to contribute to this debate, albeit in a somewhat covert fashion. Towards the end of this passage, Heidegger insists on the importance of Aristotle's treatment of change, suggesting that mathematicians were beginning to approach the same issues as Aristotle "though from the opposite direction" (GA19 81). My aim here will be to understand what Heidegger means by this and to argue that it is more than just a philosopher's defence of the tradition against the presumed radicality of science. Closer examination of the issues involved will lead us to Heidegger's commentary on Aristotle's account of time in his 1927 lecture course *The Basic Problems of Phenomenology*.[2] It could be argued that Heidegger's appeal to Aristotle here is primarily strategic insofar as he roots his own conception of original temporality back into the philosophical tradition by finding a potential radicality inherent within Aristotle's account that had been covered over and lost. However, set against the background of the mathematical debate over analysis and continuity,

[2]. M Heidegger, *The Basic Problems of Phenomenology*, trans. A Hofstadter (Indiana University Press, Bloomington, 1982), §19: *Die Grundprobleme der Phänomenologie*, Gesamtausgabe 24 (Vittorio Klostermann, Frankfurt am Main, 1975), §19. Hereafter GA24 followed by the English and German page references respectively.

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Heidegger's examination of number, dimension and continuity in the context of the Aristotelian account of change can be read as a covert intervention in the debate over analysis and continuity, and thus as rather more than just an introduction to his own conception of original temporality. Grounding the measurement of time in an existential reckoning is part of an exploration of the ontological dimension of the act of numbering that enriches the mathematical problematic rather than rejecting it.

I will set out the context of Heidegger's comments on Weyl and then look briefly at analysis and Weyl's work on continuity. This will lead on to a consideration of Heidegger's reading of Aristotle's account of time in order to clarify his remarks on the importance of Aristotle's account of motion to the debate over analysis and continuity.

Space Time and Matter

Before looking at Heidegger's discussion of arithmetic and geometry in Plato and Aristotle in detail, it is helpful to remember that he breaks off from this discussion to acknowledge that mathematicians were once again broaching the question of the continuum, adding that mathematicians were "learning to understand that the continuum is not resolvable analytically but that one has to come to understand it as something pre-given, prior to the question of an analytic penetration" (GA19 80:). Plainly dissatisfied with analysis, he praises Weyl's work on the continuum. This may seem odd at first, since Weyl was an advocate of analysis and Heidegger is quick to reject the idea that the continuum can be constructed in the way analysis proposes. However, Weyl had read and appreciated the work of Bergson and of Husserl, with whom he corresponded; and like Bergson and Husserl, Weyl believed that the continuum underlying our experience of time, space and motion is characterised by a form of original connectivity that analysis, beginning with discrete

elements, could never adequately describe.[3] His declared sympathies with a phenomenological point of view and his openness to philosophical questions would certainly have attracted Heidegger, even if his conception of mathematics was not one that Heidegger shared. However, the work of Weyl's to which Heidegger refers is an interpretative exposition he wrote on Einstein's General Theory of Relativity.[4] What interests Heidegger is the way Einstein's account of the gravitational field introduces a new way of addressing physical being. Rather than treating things as material objects in space and time, they become values of a field which in turn determines the nature of that space itself; its curvature and the geometry appropriate to it. Physical being enters into a dynamical relation with space and time, rather than simply occuring 'in' them. Acknowledging this relation means that geometry alone can no longer determine a priori the nature of space as a fixed and independent form through which matter moves, as was the case in seventeenth century science. Gravitational field theory and General Relativity, supported by the non-Euclidean geometry of n-dimensional manifolds, brings the elements of space and physical being into a dynamical relation. As Weyl himself states in the Introduction to *Space Time and Matter*:

[3]. H Weyl, *The Continuum*, trans. S Pollard & T Bole (Dover Publications, New York, 1987), cf. especially p.93, p.103. Hereafter CO followed by the English page reference. This book was first published in German in 1918. Weyl's views on the foundations of analysis changed considerably over the years. Cf. S Feferman 'The Significance of Herman Weyl's *Das Kontinuum*' *Proof Theory*, V.F. Hendricks et al., eds. (Kluwer Academic Publishers, 2000), 179-194; J Bell, 'Herman Weyl's Later Philosophical Views – His Divergence from Husserl', in R Feist (ed.) *Husserl and the Sciences*, (University of Ottawa Press, 2003).

[4]. H Weyl, *Space Time Matter*, trans. H L Brose (Dover Publications, New York, 1952). Hereafter STM followed by the English page reference.

> Space and time are commonly regarded as the forms of existence of the real world, matter as its substance. A definite portion of matter occupies a definite part of space at a definite moment of time. It is in the composite idea of motion that these three fundamental ideas enter into intimate relationship. (STM 1)

Placing motion at the centre of things in this way would certainly have appealed to Heidegger, though it is less certain that he would have been satisfied by the description of movement as a 'composite idea'. I shall come back to this shortly. First, it is clear that Weyl appreciates the importance of the conjunction of space and time not only for science, but also for philosophy, which had since Kant understood consciousness as exclusively temporal in contrast to the spatiality of the external world. Weyl is optimistic that work on the relation between time and space in mathematical physics may contribute to the resolution of this divide and thereby to the philosophical understanding of the relation between consciousness and the world.[5] However, while it may bring deeper insight, the amalgamation of space and time also brings with it certain risks. To derive the form they share from either one or the other would be reductive and would obscure the specificity of each, thereby undermining Weyl's hope that relativity theory might contribute to the philosophical problem of consciousness and its action in the world (that is, remarkably perhaps, to overcoming the divide between consciousness and the world handed down to us by Kant). It would also be wholly inadequate to the needs of General Relativity, which posits motion as the original form in which space and time are combined and which cannot then derive one from the other, or indeed motion from their combination. Heidegger, too, aims to reach an understanding of motion that does not depend on a prior grasp of space and time as distinct forms, though his guide in this respect is less Einstein than Aristotle. This

[5]. Weyl writes, "The new solution of the problem of amalgamating space and time offered by the theory of relativity brings with it a deeper unsight into the harmony of action in the world" (STM 6).

goes some way towards explaining why Heidegger, having remarked on the significance of gravitational field theory and general relativity, immediately offers up the hope that physicists will "abandon the old prejudices" and return to Aristotle's conception of motion.[6]

Number, Line and Manifold

In the sections leading up to his comments on modern mathematics in the *Sophist* lectures, Heidegger discusses the problem of the separation or abstraction (*chorismos*) of first principles (*archai*) from that which they ground. Following Aristotle, he argues that in order to retain its properly mathematical character, the outcome of the abstraction should be *atopos*, not situated in a place. Such abstraction is fine when one is dealing with mathematical entities such as 'odd,' 'even,' 'curved,' 'line,' and 'number' but, for Aristotle, it is inappropriate when dealing with physical beings. Heidegger underlines that this is because physical beings are essentially determined by their relation to motion ("*phusei onta* are *kinoumena*" (GA19 71:103)), whereas the abstraction of the *archai* to another place divorces them from motion and encourages us to see them as beings (as in Plato's theory of forms); the outcome of this trend is that even *kinesis* is treated as a being, and thus, in Heidegger's terms, ontically. By contrast, writes Heidegger, "it is possible to determine the *archai* of the moving *phusei onta* in such a way that the *archai* are not taken as divorced from motion and, furthermore, that *kinesis* itself is not taken as an idea and hence as *choriston*" (GA19 71:103). The challenge is to understand the process of abstraction in such a way that the formal description of motion can be *atopos* and therefore mathematical, while not stepping

[6]. In this way, the integration of space, motion and physical being stands as a correlate in physical science to Heidegger's deployment of 'world' in overcoming the Cartesian framework for treating physical phenomena in terms of matter in motion through a uniform space. Cf. *Being and Time*, trans. J Macquarrie & E Robinson (Blackwell, Oxford, 1980), §§19-24; *Sein und Zeit* (Max Niemeyer Verlag, Tübingen, 1986), §§19-24.

outside of the framework of motion and placing its *archai* elsewhere. In itself, this means that abstraction cannot be understood in its usual sense as the subtraction of a formal element from what is given to perception.

The question of abstraction is also at the heart of Heidegger's consideration of the difference between geometry and arithmetic. For Aristotle, place (*topos*) belongs to beings as such in the sense that it determines their possibility of being present. However, although both geometry and arithmetic involve an abstraction from place (*topos*), geometry retains a relative sense of orientation, stance or situation (*thesis*) and this is reflected in the determination of the basic elements of the two disciplines: the point (*stigme*) and the unit (*monas*). The unit is what remains 'alone' and 'for itself,' whereas the point carries with it an additional *thesis* that bears on the form of connection that exists between the elements of geometrical figures. As Aristotle notes, a line is not composed out of points (*Physics* VI.1:231a24ff), a surface is not composed out of lines, nor a body from a surface, since between any two points there will always be other points. However many points one begins with, they will never be enough to make a line and there will always be something 'between' the points that hasn't been accounted for. Even though geometrical figures are abstracted from place, they still deal with relations such as above and below, right and left. "Every geometrical object is an *ousia thetos*" (GA19 76:110) and "a determinate kind of connection is required, a determinate kind of unity of the manifold" (GA19 76:111). The form of connection and unity in a geometrical manifold is that of continuity (*suneches*), while that which belongs to an arithmetic manifold is succession (*ephexis*), where elements do not touch and no linkage is required between them. Succession is therefore a simpler relation than geometry with fewer elements and arithmetic is accordingly more original. This priority of arithmetic over geometry is further reinforced by the fact that the Being of every being is determined by its being 'one' (GA19 80:117) and by the way number is disclosed through

logos and *noeisis*, whereas geometrical figures are disclosed in *aisthesis*. Altogether, *arithmos* "acquires for the structure of beings in general a more fundamental significance as an ontological determination" (GA19 80:117). However, this must be set against the fact that what makes number more fundamental than geometry – the absence of *thesis* – also separates it from continuity and thus, apparently, from motion. As long as this is the case, it is unlikely that any approach based on number could, as Heidegger proposes, "determine the *archai* of the moving *phusei onta* in such a way that the *archai* are not taken as divorced from motion" (GA19 71:103).

In view of this, and the importance of continuity for phenomenology in general and Heidegger in particular, it is perhaps surprising to find him so readily endorsing the priority of arithmetic. That he does so, albeit strictly following Aristotle's lead, should make us think twice before casting him too quickly and unambiguously as an opponent of analysis. It is all the more striking, therefore, that the passages in which Heidegger acknowledges the priority of arithmetic are followed directly by the remarks addressed to Weyl and contemporary work on the continuum, where Heidegger states explicitly that the continuum cannot be resolved by analysis, thus identifying a fatal weakness in the analytical approach. An intriguing situation has arisen in which arithmetic is ontologically more fundamental, yet unable to deal with the continuum, which is for Heidegger an ineliminable feature of time, space and motion. Against this background one can see how Heidegger might have regarded analysis with qualified approval; bringing the ontologically more original use of number to problems previously treated by geometry could result in the best of both worlds, but only if the tendency for abstraction to lose sight of motion as such can be overcome.

It is in this context that Heidegger remarks that Aristotle "arrived at facts natural scientific geometry is striving for today, though from the opposite direction" (GA19 81). The most straightforward way to interpret Heidegger's comments here is as a further endorsement of

the originality of the continuum, and thus as a rebuttal of analytical attempts to construct the continuum on an arithmetical base. However, the passages before and after these brief comments on relativity theory and Weyl's work on the continuum make it clear that Heidegger does not simply reject number and the analysis of matter, motion, time and space in favour of a phenomenological approach. On the contrary, phenomenology is enlisted overcome an impasse reached by the mathematical treatment of space and time in dealing with what is, for Heidegger, a genuine problem. What an analytical treatment of the relation of space and time in motion requires is an account of the application of number to motion as original. Moreover, Heidegger insists that this should be done in such a way that motion does not become an abstraction, a concept divorced from the physical setting that gives it meaning and whose ontological structure it articulates. Since Aristotle provides just such an account in his determination of time in *Physics* IV.11, Heidegger can argue that the General Theory of Relativity has led modern mathematics back to an Aristotelian problematic, at the heart of which lies the relation between continuity and number.

Weyl and the Continuum

In this section, I shall briefly set out the mathematical framework of Weyl's treatment of space, time and motion in order to shed light on the limitations of analysis from Heidegger's point of view, and indeed from Weyl's own as well.

Riemann's development of the geometry of n-dimensional manifolds provided a mathematical tool that was perfectly suited to the description of space-time in Einstein's General Theory of Relativity. As a purely formal description, it should allow for the determination of space-time without recourse to our existing intuitions regarding space and time. However, its independence may also be regarded as a weakness: certainly Weyl was concerned at the extent to which the analytical construction of the continuum was removed from the continua of space and time as we meet them in our experience and feared that

mathematics had become remote from what it purported to describe.[7] Moreover, the absence of a clear foundation in intuition left the elements of analysis in need of definition in order to be well founded, but Weyl argued that many of its key concepts were caught up in vicious circles that left the whole edifice of analysis looking like a house built on sand.[8] For example, each real number *n* is determined by a set of rational numbers that are all less than *n*, yet with no single greatest number. If real numbers are then placed in a direct correspondence with points, one can generate an analytical description of the continuity of a line. However, the adequacy of the definition of the continuum depends on the adequacy of the definition of real numbers using set theory and the idea of the Dedekind cut, over which Weyl had serious reservations. Poincaré had concluded that the definition of an object which assumes the existence of a totality that includes that object as a member was fine so long as the totality *precedes* the definition. But if the totality does not precede the definition, then the definition is called impredicative and contains a vicious circle. Unfortunately, this is precisely what Weyl saw happening in the definition of number that underpins analysis.[9]

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[7]. As Weyl writes, "the intuitive and the mathematical continuum do not coincide: a deep chasm is fixed between them" (CO 93). For others, this separation from experience was to be welcomed. Bachelard, for example, celebrated the capacity of modern mathematics to break with the habits embedded in experience and to invent new formal objects and operations.

[8]. Weyl expresses his intention to show that the house of analysis is built on sand in the Preface to *The Continuum* (CO 1). His demonstration of the circularity it involves is at pp.28ff.

[9]. Giuseppe Longo encapsulates the problem in the following passage. "Firstly, there is always an infinity of (positive) rationals smaller than whichever (positive) real: hence we need to use, when defining it, the collection *N* of all the integers, in its totality. And the classical definition of this totality has the following structure: *N* is the smallest set that contains zero and which if it contains *n* it contains *n + 1*. Said in a different way, *N* is the intersection of all sets that contain zero and that are closed under the successor operation. But *N* has also this property: to define it using the phrase 'all

Weyl then also rejected three possible alternatives to this approach. For although he was determined to avoid the prevailing tendency to treat mathematical axioms as stipulations with no origin in experience, and did not believe that mathematics could appeal to a fundamental intuition of its basic concepts, neither did he want to replace the dangerously circular reasoning of impredicative definitions with a rigidly stratified approach, such as one finds in logicism.

The question of circularity in the definition of real numbers is also echoed in the idea of the continuum as a totality of points, insofar as each point is understood in terms of a totality that cannot itself be defined independently of the of the points that constitute it; thus, the point on a line is understood in terms of the line as a whole and the now in terms of time. However, this is not necessarily a bad thing. Impredicative definitions, for all their shortcomings, lend themselves easily to the idea that the continuum is essentially a set not of *things* but of *relations*, and that the individual relations cannot be treated in isolation from one another; to do so would be to reduce the point to thing and thereby also to risk the reification of the continuum as the totality of such things.[10] Since we intuitively accept the relational character of the continuum, we have the curious situation in which the circularity of impredicative definitions actually brings analysis closer to our experience of space, time and motion. Being at once critical of the flaws in the foundations of analysis and

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the sets that...' we quantify over a collection that contains *N* itself. The *definiens* uses the *definiendum*." Giuseppe Longo, 'The Mathematical Continuum: from Intuition to Logic' in J Petitot et al (eds.) *Naturalizing Phenomenology* (Stanford University Press, 1999).

[10].The question of the ontological status of mathematical objects is too big to deal with here, but on the reification of the continuum, see: M Panza, *De le Continuité comme Concept au Continu comme Objet*' in J-M Salanskis and H Sinaceur (eds.) *Le Labyrinthe du Continu* (Springer-Verlag France, Paris, 1992), pp.16-30.

yet also determined to set its path on as close a parallel to experience as possible, Weyl looked for an alternative approach that could retain some of the intuitive sense of the impredicative approach while placing it on a more secure footing. To do this, he took the more straightforward system of natural numbers as a basic given, replaced the idea of a 'set' with that of a 'sequence' and then developed an essentially genetic approach to the construction of analysis through the iteration of simple operations.

Weyl's account of analytical construction initially follows the model we have already outlined: a line is broken down into points each of which corresponds to a real number, or in the case of a line marked on a plane, a pair of numbers standing for variable coordinates x and y on perpendicular axes. The shape of the line is determined by a function describing how x varies with regard to y. Weyl argues that in order to decide whether such a function is continuous or not one has to inspect the totality of sets arising from a complex application of the principles of definition; e.g. the basic axioms and the rules of combination applicable to the recursive generation of more complex functions. But if the principles of definition form an open system then the question of the continuity of the function must remain open in its turn.[11] To illustrate this, Weyl takes as an example the function 'position of a point of mass,' which can be represented as a continuous function of time; that is, for all real values of time, the function stating position "takes on values belonging to a certain region" (CO 87). In everyday terms, for all points of time within a given period, the position will remain within a local region (things do not disappear and then reappear suddenly somewhere else). However, things are not that straightforward, as Weyl's consideration of a pen lying on the

[11]. See CO 17-19, where Weyl argues against the prevailing tendency to interpret mathematical axioms as stipulations. This, he claims, means that a derived statement that 'there is' a given object does not refer back to any "cognitive reality" and is "devoid of any meaning that has cognitive significance" (CO 18).

table shows. Simple observation tells him that it remains on the table *throughout* a given period. If time can be taken as a series of "rigidly punctual nows," each of which corresponds to a real number, the continuous presence of the pen on the table requires that the spatial position be localised for every value of time as a real number between given limits. But for this to mean that the pen really is on the table *throughout* the period in question the set of real numbers must account for every time point in this period; that is, there must be a direct correspondence between time points and real numbers, with no time points unmatched to a real number, and there must be a set that is the analytical equivalent to the continuum spanning the period in question. For this to be assured of working, we would have to be able to justify the analytical construction on the basis of an intuition, which Weyl regards as impossible bacause our intuition of time does not include "rigidly punctual nows" comparable to points and the classical definition of real numbers. Indeed, insofar as our intuition of space, time and motion yields any reliable results at all, it suggests that breaking them down into discrete elements is wholly mistaken. Spatial points blur from one to the next; time flows; motion is seamless. Although we cannot easily put this continuity into words, perhaps because we cannot put it into words, we strive for its exact expression in mathematics. For Weyl, this effort can never fully realize its aim, since the mathematical expression cannot be traced directly back to an original intuition of continuity. Nonetheless, he argues that it should be regarded as a theory "which must establish its own reasonableness (beyond its mere logical constistency) in the same way as a physical theory" (CO 93).

The integration of the form of space-time with the dynamical relations between material objects means that the form of space and time is no longer fixed and uniform. Above all, it cannot be determined independently from motion, and so the geometry appropriate to it must be able to map the effects of dynamical relations on the form of space-time itself.

Indeed, the idea of differential geometry developed by Riemann involves determining of the metric of space by the iteration of infinitesimal displacements.[12] But to generate a line by a sequence of infinitesimal displacements is quite different to treating it as a ideal object in a uniform and above all independent space. Continuing his review of the analytical construction of the continuum, Weyl therefore distinguishes between a line in the orthodox sense and the path traced by a moving point.[13] The path is "a one-dimensional continuum of '*path-points*'" and while the path-points coincide with the points of the line, they are not equivalent to them (CO 102). This difference comes down to the necessity of involving movement and is therefore significant when we are thinking about the analytical construction of the continuum as a means of dealing mathematically with time and space. Weyl adds:

> The path-points, as 'stages' of the movement, stand in relations of 'earlier' and 'later' to one another, just as time-points do; in the movement, the continuum of path-points *spreads over* the continuum of time-points in a continuous monotone manner. This conception allows the 'path' to be separated, in a way, from the movement which produces it. (CO 102)

What Weyl points out here is the need to isolate movement as the key term on the basis of which it becomes possible for differential geometry to address the continuum as extended in both space *and* time. Now, we can see that the analytical representation of points by real numbers needs to capture the connectivity of the path, rather than just the set of points in a line. Discussing the temporal continuum, Weyl shows how analysis deals with each now as a

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[12]. In fact, Weyl drew attention to a residue of Newtonian 'action at a distance' in Riemann's acceptance that one can compare two line elements at different points of space

[13]. Weyl compares a line to a road and a path or curve to the route taken along it by a pedestrian (CO 101-102).

transition (rather than as rigidly punctual) by determining each real number in terms of an infinite sequence of ever more narrowly defined neighbourhoods. Although this extends the promise of something indistinguishable from smooth continuity, it is by his own admission still a "reduction of continuous connectedness" (CO 106) to a capture the connectivity essential to the continuum.[14]

Leaving aside the difficulty facing analysis in the construction of continuity, it is worth noting that the introduction of the concept of 'path' does not mean that the continuum is being treated spatially first of all and then only then transposed to the problem of time. If this were the case, serious doubts would arise over whether the account could ever avoid imposing spatial figure onto time and thereby failing to grasp it in its specificity. This is of course the concern that motivated Bergson to develop the idea of the *durée*, which Weyl repeatedly commends for its fidelity to our intuitive experience (CO 90). However, for Bergson, the price of securing the specificity of time is the impossibility of treating it mathematically, since the application of number apparently replaces pure continuity with its reconstruction from discrete elements. Similarly, Bergson is often credited with breaking away from the Aristotelian tendency to make time dependent on motion. Yet the theory of General Relativity regards motion as the original element in relation to which one undestands space, time and matter. For Weyl and Heidegger, then there is every reason to *not* to separate time from motion and indeed to preserve the Aristotelian order of dependency. Weyl's difficulty – which is the direct counterpart to Bergson's – is that the

[14]. Moreover, it creates new difficulties of its own, since the 'neighbourhood' introduced must have a definite shape. The fact that the shape chosen turns out to be quite arbitrary highlights once again what Weyl calls the "inescapable discrepancy between the genuine continuum and a set of isolated elements" – no matter how creatively treated by mathematics (CO 106-107).

price of treating the space-time continuum mathematically is to cut it off from anything we can find in our experience.

Heidegger's intervention can be read as an attempt to go beyond the respective positions of Bergson and Weyl, and is characterised by two basic features. In spite of the undeniable importance of time for his understanding of ontology and the originality of is own analyses, Heidegger is in fact happy to concede a *certain* priority to motion – or rather, to a certain conception of motion that he finds in Aristotle. This makes it possible in principle to develop an account of space and time as integrated in a single continuum, as relativity requires, but only if it is possible to demonstrate two basic and related points: first, that it is possible to apply number to continuity without thereby spatialising the account to the detriment of time; second that the application of number does not necessarily involve the sacrifice of connectivity essential to the continuity of either space of time. We can now turn to Heidegger's reading of Aristotle's account of time to see how these conditions are met.

Heidegger on Number and Motion

Aristotle prefaces his account of time by dismissing the misleading idea that it can be understood as literally composed from 'bits' of time such as now points.[15] Time, he concludes, does not exist in the way that things exist but neither does it belong in the soul alone, and the drama of his account is played out in the tension between these two positions. Examining the conditions under which we are aware of time then leads him to consider its relation to motion and to the magnitude or dimension (*megethos*) that underlies all motion in the broadest sense – including qualitative change. The continuity of time (which Aristotle accepts without question) is said to be grounded in the continuity of change, which

[15]. Aristotle, *The Physics* Vol I, trans. P H Wicksteed & F M Cornford (Harvard University Press, Cambridge Mass., 1980). The account of time is in Book IV.10-14.

is in turn dependent on the continuity of the underlying dimension. Time itself is famously declared to be:

> the calculable measure or dimension (*arithmos*) of motion with respect to before-and-afterness. (219b1-3)

Time arises when motion is numbered; that is, when change is marked by the observation that things 'now' are otherwise than they were a moment ago. which time is. Everything depends on the relation between number and motion, and in particular, on how we interpret the relation between number, the now by which we mark change, and the act of numbering by which we do so.

In a passage apparently directed against a crude conception of analysis, Heidegger writes that the juxtaposition of nows to form a manifold is wholly inadequate to the problem of time and misunderstands the essential characteristics of the now. Like time itself, the now refers to the no-longer and the not-yet and is characterised by an open dimension. Just as Weyl recognized that continuity had to be embodied in the now itself, so Heidegger goes on to describe the now as intrinsically *transition* (GA24 249:). He goes on to note that if the now is itself time (rather than that from which time is composed), then Aristotle's definition of time entails that the now is number. This brings Heidegger to what he regards as the most crucial and difficult aspect of Aristotle's account.

> Time is number and not limit, but as number it is at the same time able to measure that with reference to which it is number. Not only is time counted, but as counted it can itself be something that counts in the sense of a measure. Only because time is number in the sense of the counted now can it become a *mensural* number, so that it itself can count in the sense of measuring. This distinction between the now as number in general or what is counted and as the counting counted, is the essential

> content of the difficulty place in Aristotle's essay on time, into which we shall enter only briefly. (GA24 250)

Heidegger reflects here on Aristotle's observation that time is number in the sense of what is counted, rather than number in the sense of the scale of measurement we apply (219b1-3). This is unproblematic when we are dealing with the application of number to discrete entities; we can simply say that number is how many apples there are, rather than how many we count in the bowl. But the situation is hardly so straightfoward when dealing with number and continuous magnitudes, such as motion. By endorsing the idea of number as what is counted, Heidegger of course excludes any interpretation of time as a subjective phenomenon or as a feature of consciousness distinct from the external world, but equally we cannot say that the number of motion is already demarcated prior to its being numbered as time. Time is number as the condition for the application of number in measurement; but first one has to understand how time reveals motion as numerable. What is the relation between time as the disclosure of motion and its demarcation into a dimension to which a scale of measurement can be applied? For Heidegger, this problem is concentrated on the act of counting, in which temporal markers first disclose motion prior to the application of a determinate scale of measurement. It is this dimension of the problem that Heidegger tackles when he describes the way we reckon with time, taking account of time as we guide ourselves towards the accomplishment of activity (GA24 256-261:362-370). As we orient ourselves within the series of overalapping involvements that make up the fabric of our world, judge that we are near or far, that we have too much time or too little. But these judgements are not originally directed towards established magnitudes of space and time. Their condition lies in motion, which is neither spatial nor temporal in itself. This is borne out by Heidegger's insistence that the dimension underlying motion has to be understood in "a completely formal sense" (GA24 242:).

Opening up the problem in this way overcomes the crude opposition between an external or objective time and an inner or subjective time, though it leaves the balance tilted towards subjective – or in Heidegger's terms towards the perception and activity of Dasein. This is to say that the discussion of time-reckoning brings us closer to the original disclosure of motion through number, without quite describing this event itself. However, if we are still approaching the event from the side of Dasein, we can at least say that the temporal constitution of Dasein as care (*Sorge*) provides the structures through which Being is disclosed.[16] Therefore motion, as what acquires temporal form through numbering, is more than just the transition of a thing from one state to another; it is also the ontological condition for the disclosure of beings in and through Dasein's Being-in-the-world. This brings Heidegger's account closer to Aristotle's fundamental intuition that motion is the original framework in terms of which beings are disclosed; or in simple terms, that motion is essential to the Being of the physical world. As Heidegger writes, time, and in particular the now as transition, "is capable of making motion accessible *as motion*" (GA24 251:354).

To number is to give temporal form to motion before we apply pre-determined units of measurement. It is to disclose motion by marking difference, where this difference itself depends on the mark. – which is to say that there would be no motion, and therefore no dimension underlying motion, without its being numbered as time. Since the continuity of time is underpinned by the continuity of motion and of the dimension underlying motion, we can therefore say that continuity itself would not exist without the application of number. We need only recall that the now, like time itself, is characterised by Aristotle in terms of both *synthesis* and *diaresis*, taking together and setting apart (220a5). There should be no question, therefore, of seeing the application of number to motion as the division of a

[16]. For an account of care (*Sorge*) as the ontological structure of Dasein, see *Being and Time* §§39-44 and §65.

pre-existent continuity. It is only through the numbering of motion which occurs when time is marked by the now that the very continuity of motion itself is established.

Providing that one accepts the basic orientation of his reading, Heidegger arrives at a more original understanding of number and continuity than does analysis and highlights what is, at least from his point of view, its single biggest mistake. When it declines the support of an original intuition of the continuum, analysis leaves itself committed to the definition of the concept of number on which it is based. In spite of the sophistication of Weyl's definition of number, analysis therefore applies a pre-determined conception of number *to* motion, rather than disclosing their relation to one another as an original moment.

In the light of this, it is tempting to suppose that we can read Heidegger as providing a resolution to problems within analysis, but this would in the end be too one-sided an account. Phenomenology delves into the structures of conscious experience, whereas analysis proceeds by developing a numerical treatment of problems and then translating this treatment to geometry via a system of co-ordinates. To appreciate the incompatibility of the two methods we need only recall Weyl's remark that the co-ordinate system is what is left when consciousness has been excluded: each method demands that the essential tools and milieu of the other be excluded. In spite of Weyl's enthusiasm for a reconciliation, the discrepancy is probably too great for it to be overcome. Moreover, mathematicians would by no means all welcome the re-interpretation of mathematical formalism that fitting it to a phenomenological basis would entail. Weyl himself, though happy to applaud the progress that phenomenology had made towards a genuine philosophy of the continuum, came eventually to believe that it could not provide a foundation for analysis, above all because

intuition remained too restricted a source from which to develop the extraordinarily rich diversity of mathematical formalism.[17]

Notes

[17]. T A Ryckman, 'Weyl's Debt to Husserl: the Transcendental Phenomenological Roots of the Gauge Principle.' This paper was delivered to the Oxford Symmetry Workshop in 2001 and can be found on-line at: www.philo**s**ophy.ox.ac.uk/conference**s**/ graduate/_2000-2001_phil_physics/ryckman.rtf