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THE NEED FOR ABSTRACT ENTITIES IN SEMANTIC ANALYSIS

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We distinguish between a *logistic system* and a *formalized language* on the basis that the former is an abstractly formulated calculus for which no interpretation is fixed, and thus has a syntax but no semantics; but the latter is a logistic system together with an assignment of meanings to its expressions.

As primitive basis of a logistic system it suffices to give, in familiar fashion: (1) The list of primitive symbols or *vocabulary* of the system (together usually with a classification of the primitive symbols into categories, which will be used in stating the formation rules and rules of inference). (2) The *formation rules,* determining which finite sequences of primitive symbols are to be *well-formed* expressions, determining certain categories of well-formed expressions — among which we shall assume that at least the category of *sentence* is included—, and determining (in case *variables* are included among the primitive symbols) which occurrences of variables in a well-formed expression are *free* occurrences and which are *bound* occurrences.[1] (3) The transformation rules or *rules of inference,* by which from the *assertion* of certain sentences (the *premisses,* finite in number) a certain sentence (the *conclusion*) may be *inferred.* (4) Certain asserted sentences, the *axioms.*

In order to obtain a formalized language it is necessary to add, to these *syntactical rules* of the logistic system, *semantical rules* assigning meanings (in some sense) to the well-formed expressions of the system.[2] The character of the semantical rules will depend on the theory of meaning adopted, and this in turn must be justified by the purpose which it is to serve.

Let us take it as our purpose to provide an abstract theory of the actual use of language for human communication — not a factual or historical report of what has been observed to take place, but a norm to which we may regard every-day linguistic behavior as an imprecise approximation, in the same way that e.g. elementary (applied) geometry is a norm to which we may regard as imprecise approximations the practical activity of the land-surveyor in laying out a plot of ground, or of the construc-

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tion foreman in seeing that building plans are followed. We must demand of such a theory that it have a place for all observably informative kinds of communication — including such notoriously troublesome cases as belief statements, modal statements, conditions contrary to fact — or at least that it provide a (theoretically) workable substitute for them. And solutions must be available for puzzles about meaning which may arise, such as the so-called "paradox of analysis."

There exist more than one theory of meaning showing some promise of fulfilling these requirements, at least so far as the formulation and development have presently been carried. But the theory of Frege seems to recommend itself above others for its relative simplicity, naturalness, and explanatory power —or, as I would advocate, Frege's theory as modified by elimination of his somewhat problematical notion of a function (and in particular of a *Begriff*) as *ungesättigt,* and by some other changes which bring it closer to present logistic practice without loss of such essentials as the distinction of sense and denotation.

This modified Fregean theory may be roughly characterized by the tendency to minimize the category of *syncategorematic* notations — i.e., notations to which no meaning at all is ascribed in isolation but which may combine with one or more meaningful expressions to form a meaningful expression[3] — and to reduce the categories of meaningful expressions to two, (proper) *names* and *forms,* for each of which two kinds of meaning are distinguished in a parallel way.

A name, or a *constant* (as we shall also say, imitating mathematical terminology), has first its *denotation,* or that of which it is a name.[4] And each name has also a *sense* — which is perhaps more properly to be called its meaning, since it is held that complete understanding of a language involves the ability to recognize the sense of any name in the language, but does not demand any knowledge beyond this of the denotations of names. (Declarative) *sentences,* in particular, are taken as a kind of names, the denotation being the *truth-value* of the sentence, *truth* or *falsehood,* and the sense being the *proposition* which the sentence expresses.

A name is said to *denote* its denotation and to *express* its sense, and the sense is said to be *a concept of* the denotation. The abstract entities which serve as senses of names let us call *concepts* — although this use of the word 'concept' has no analogue in the writings of Frege, and must be carefully distinguished from Frege's use of 'Begriff.' Thus anything which is

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or is capable of being the sense of some name in some language, actual or possible, is a concept.[5] The terms *individual concept, function concept,* and the like are then to mean a concept which is a concept of an individual, of a function, etc. A *class concept* may be identified with a *property,* and a *truth-value concept* (as already indicated) with a proposition.

Names are to be meaningful expressions without free variables, and expressions which are analogous to names except that they contain free variables, we call forms (a rather wide extension of the ordinary mathematical usage, here adopted for lack of a better term).[6] Each variable has a *range,* which is the class of admissible *values* of the variable.[7] And analogous to the denotation of a name, a form has a *value* for every system of admissible values of its free variables.[8]

The assignment of a value to a variable, though it is not a syntactical operation, corresponds in a certain way to the syntactical operation of substituting a constant for the variable. The denotation of the substituted constant represents the value of the variable.[9] And the sense of the substituted constant may be taken as representing a *sense-value* of the variable. Thus every variable has, besides its range, also a *sense-range,* which is the class of admissible sense-values of the variable. And analogous to the sense of a name, a form has a *sense-value* for every system of admissible sense-values of its free variables.[10]

The following principles are assumed:[11] (i) Every concept is a concept of at most one thing. (ii) Every constant has a unique concept as its sense. (iii) Every variable has a non-empty class of concepts as its sense-range. (iv) For any assignment of sense-values, one to each of the free variables of a given form, if each sense-value is admissible in the sense that it belongs to the sense-range of the corresponding variable, the form has a unique concept as its sense-value. (v) The denotation of a constant is that of which its sense is a concept. (vi) The range of a variable is the class of those things of which the members of the sense-range are concepts. (vii) If S, s_1, s_2, ..., s_m are concepts of A, a_1, a_2, ..., a_m respectively, and if S is the sense-value of a form F for the system of sense-values s_1, s_2, ..., s_m of its free variables x_1, x_2, ..., x_m, then the value of F for the system of values a_1, a_2, ... a_m of x_1, x_2, ..., x_m is A. (viii) If C′ is obtained from a constant C by replacing a particular occurrence of a constant c by a constant c′ that has the same

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sense as c, then C′ is a constant having the same sense as C.[12] (ix) If C′ is obtained from a constant C by replacing a particular occurrence of a constant c by a constant c′ that has the same denotation as c, then C′ is a constant having the same denotation as C.[13] (x) If C′ is obtained from a constant C by replacing a particular occurrence of a form f by a form f′ that has the same free variables as f, and if, for every admissible system of sense-values of their free variables, f and f′ have the same sense-value, then C′ is a constant having the same sense as C.[12] (xi) If C′ is obtained from a constant C by replacing a particular occurrence of a form f by a form f′ that has the same free variables as f, and if, for every system of values of their free variables which are admissible in the sense that each value belongs to the range of the corresponding variable, f and f′ have the same value, then C′ is a constant having the same denotation as C.[13] (xii) If $x_1, x_2, \ldots, x_m$ are all the distinct variables occurring (necessarily as bound variables) in a constant C, if $y_1, y_2, \ldots, y_m$ are distinct variables having the same sense-ranges as $x_1, x_2, \ldots, x_m$ respectively, and if C′ is obtained from C by substituting $y_1, y_2, \ldots, y_m$ throughout for $x_1, x_2, \ldots, x_m$ respectively, then C′ is a constant having the same sense as C. (xiii) If $x_1, x_2, \ldots, x_m$ are the distinct variables occurring in a constant C, if $y_1, y_2, \ldots, y_m$ are distinct variables having the same ranges as $x_1, x_2, \ldots, x_m$ respectively, and if C′ is obtained from C by substituting $y_1, y_2, \ldots, y_m$ throughout for $x_1, x_2, \ldots, x_m$ respectively, then C′ is a constant having the same denotation as C. (xiv) The result of substituting constants for all the free variables of a form is a constant, if the sense of each substituted constant belongs to the sense-range of the corresponding variable.[12] (xv) The sense of a constant C thus obtained by substituting constants $c_1, c_2, \ldots, c_m$ for the free variables $x_1, x_2, \ldots, x_m$ of a form F is the same as the sense-value of F when the senses of $c_1, c_2, \ldots, c_m$ are assigned as the sense-values of $x_1, x_2, \ldots, x_m$.

To these must still be added principles which are similar to (viii)–(xv), except that substitution is made in forms instead of constants, or that forms and variables as well as constants are substituted for the free variables of a form. Instead of stating these here, it may be sufficient to remark that they follow

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if arbitrary extensions of the language are allowed by adjoining (as primitive symbols) constants which have as their senses any concepts that belong to sense-ranges of variables in the language, if the foregoing principles are assumed to hold also for such extensions of the language, and if there is assumed further: (xvi) Let an expression F contain the variables $x_1, x_2, \ldots, x_m$; and suppose that in every extension of the language of the kind just described and for every substitution of constants $c_1, c_2, \ldots, c_m$ for the variables $x_1, x_2, \ldots, x_m$ respectively, if the sense of each constant belongs to the sense-range of the corresponding variable, F becomes a constant; then F is a form having $x_1, x_2, \ldots, x_m$ as its free variables.

To those who find forbidding the array of abstract entities and principles concerning them which is here proposed, I would say that the problems which give rise to the proposal are difficult and a simpler theory is not known to be possible.[14]

To those who object to the introduction of abstract entities at all I would say that I believe that there are more important criteria by which a theory should be judged. The extreme demand for a simple prohibition of abstract entities under all circumstances perhaps arises from a desire to maintain the connection between theory and observation. But the preference of (say) *seeing* over *understanding* as a method of observation seems to me capricious. For just as an opaque body may be seen, so a concept may be understood or grasped. And the parallel between the two cases is indeed rather close. In both cases the observation is not direct but through intermediaries — light, lens of eye or optical instrument, and retina in the case of the visible body, linguistic expressions in the case of the concept. And in both cases there are or may be tenable theories according to which the entity in question, opaque body or concept, is not assumed, but only those things which would otherwise be called its effects.

The variety of entities (whether abstract or concrete) which a theory assumes is indeed one among other criteria by which it may be judged. If multiplication of entities is found beyond the needs of the workability, simplicity, and generality of the theory, then the razor shall be applied.[15] The theory of meaning here outlined I hold exempt from such treatment no more than any other, but I do advocate its study.

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Let us return now to our initial question, as to the character of the semantical rules which are to be added to the syntactical rules of a logistic system in order to define a particular formalized language.

On the foregoing theory of meaning the semantical rules must include at least the following: (5) *Rules of sense,* by which a sense is determined for each well-formed expression without free variables (all such expressions thus becoming names). (6) *Rules of sense-range,* assigning to each variable a sense-range. (7) *Rules of sense-value,* by which a sense-value is determined for every well-formed expression containing free variables and every admissible system of sense-values of its free variables (all such expressions thus becoming forms).

In the case of both syntactical and semantical rules there is a distinction to be drawn between *primitive* and *derived* rules, the primitive rules being those which are stated in giving the primitive basis of the formalized language, and the derived rules being rules of similar kind which follow as consequences of the primitive rules. Thus besides primitive rules of inference there are also derived rules of inference, besides primitive rules of sense also derived rules of sense, and so on. (But instead of "derived axioms" it is usual to say *theorems.*)

A statement of the denotation of a name, the range of a variable, or the value of a form does not necessarily belong to the semantics of a language. For example, that 'the number of planets' denotes the number nine is a fact as much of astronomy as it is of the semantics of the English language, and can be described only as belonging to a discipline broad enough to include both semantics and astronomy. On the other hand, a statement that 'the number of planets' denotes the number of planets is a purely semantical statement about the English language. And indeed it would seem that a statement of this kind may be considered as purely semantical only if it is a consequence of the rules of sense, sense-range, and sense-value, together with the syntactical rules and the general principles of meaning (i)–(xvi).

Thus as derived semantical rules rather than primitive, there will be also: (8) *Rules of denotation,* by which a denotation is determined for each name. (9) *Rules of range,* assigning to each variable a range. (10) *Rules of value,* by which a value is determined for every form and for every admissible system of values of its free variables.

By stating (8), (9), and (10) as primitive rules, without (5),

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(6), and (7) there results what may be called the *extensional part* of the semantics of a language. The remaining *intensional part* of the semantics does not follow from the extensional part. For the sense of a name is not uniquely determined by its denotation, and thus a particular rule of denotation does not of itself have as a consequence the corresponding rule of sense.

On the other hand, because the meta-linguistic phrase which is used in the rule of denotation must itself have a sense, there is a certain sense (though not that of logical consequence) in which the rule of denotation, by being given as a primitive rule of denotation, uniquely indicates the corresponding rule of sense. Since the like is true of the rules of range and rules of value, it is permissible to say that we have fixed an *interpretation* of a given logistic system, and thus a formalized language, if we have stated only the extensional part of the semantics.[16]

Although all the foregoing account has been concerned with the case of a formalized language, I would go on to say that in my opinion there is no difference in principle between this case and that of one of the natural languages. In particular, it must not be thought that a formalized language depends for its meaning or its justification (in any sense in which a natural language does not) upon some prior natural language, say English, through some system of translation of its sentences into English — or, more plausibly, through the statement of its syntactical and semantical rules in English. For speaking in principle, and leaving all questions of practicality aside, the logician must declare it a mere historical accident that you and I learned from birth to speak English rather than a language with less irregular, and logically simpler, syntactical rules, similar to those of one of the familiar logistic systems in use today — or that we learned in school the content of conventional English grammars and dictionaries rather than a more precise statement of a system of syntactical and semantical rules of the kind which has been described in this present sketch. The difference of a formalized language from a natural language lies not in any matter of principle, but in the degree of completeness that has been attained in the laying down of explicit syntactical and semantical rules and the extent to which vaguenesses and uncertainties have been removed from them.

For this reason the English language itself may be used as a convenient though makeshift illustration of a language for which

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syntactical and semantical rules are to be given. Of course only a few illustrative examples of such rules can be given in brief space. And even for this it is necessary to avoid carefully the use of examples involving English constructions that raise special difficulties or show too great logical irregularities, and to evade the manifold equivocacy of English words by selecting and giving attention to just one meaning of each word mentioned. It must also not be asked whether the rules given as examples are among the "true" rules of the English language or are "really" a part of what is implied in an understanding of English; for the laying down of rules for a natural language, because of the need to fill gaps and to decide doubtful points, is as much a process of legislation as of reporting.

With these understandings, and with no attempt made to distinguish between primitive and derived rules, following are some examples of syntactical and semantical rules of English according to the program which has been outlined.[17]

(1) Vocabulary: 'equals' 'five' 'four' 'if' 'is' 'nine' 'number' 'of' 'planet' 'planets' 'plus' 'round' 'the' 'then' 'the world' — besides the bare list of primitive symbols (words) there must be statements regarding their classification into categories and systematic relations among them, e.g., that 'planet' is a common noun,[18] that 'planets' is the plural of 'planet,'[19] that 'the world' is a proper noun, that 'round' is an adjective.

(2) Formation Rules: If A is the plural of a common noun, then 'the'⌢'number'⌢'of'⌢A is a singular term. A proper noun standing alone is a singular term. If A and B are singular terms, then A⌢'equals'⌢B is a sentence. If A is a singular term and B is an adjective, then A⌢'is'⌢B is a sentence.[20] If A and B are sentences, then 'if'⌢A⌢'then'⌢B is a sentence. — Here singular terms and sentences are to be understood as categories of well-formed expressions; a more complete list of formation rules would no doubt introduce many more such.

(3) Rules of Inference: Where A and B are sentences, from 'if'⌢A⌢'then'⌢B and A to infer B. Where A and B are singular terms and C is an adjective, from A⌢'equals'⌢B and B⌢'is'⌢C to infer A⌢'is'⌢C.

(4) Axioms-Theorems: 'if the world is round, then the world is round'; 'four plus five equals nine.'

(5) Rules of Sense: 'round' expresses the property of roundness. 'the world' expresses the (individual) concept of the world. 'the world is round' expresses the proposition that the world is round.

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(8) Rules of Denotation: 'round' denotes the class of round things. 'the world' denotes the world. 'the world is round' denotes the truth-value thereof that the world is round.[21]

On a Fregean theory of meaning, rules of truth in Tarski's form — e.g., "'the world is round' is true if and only if the world is round" — follow from the rules of denotation for sentences. For that a sentence is true is taken to be the same as that it denotes truth.

NOTES

1 For convenience of the present brief exposition we make the simplifying assumption that sentences are without free variables, and that only sentences are asserted.

2 The possibility that the meaningful expressions may be a proper subclass of the well-formed expressions must not ultimately be excluded. But again for the present sketch it will be convenient to treat the two classes as identical—the simplest and most usual case. Compare, however, footnote 13.

3 Such notations can be reduced to at most two, namely the notation (consisting, say, of juxtaposition between parentheses) which is used in application of a singulary function to its argument, and the abstraction operator λ. By the methods of the Schönfinkel-Curry combinatory logic it may even be possible further to eliminate the abstraction operator, and along with it the use of variables altogether. But this final reduction is not contemplated here—nor even necessarily the simpler reduction to two syncategorematic notations.

4 The complicating possibility is here ignored of *denotationless names*, or names which have a sense but no denotation. For though it may be held that these do occur in the natural languages, it is possible, as Frege showed, to construct a formalized language in such a way as to avoid them.

5 This is meant only as a preliminary rough description. In logical order, the notion of a concept must be postulated and that of a possible language defined by means of it.

6 Frege's term in German is *Marke*.—The form or *Marke* must of course not be confused with its associated abstract entity, the *function*. The function differs from the form in that it is not a linguistic entity, and belongs to no particular language. Indeed the same function may be associated with different forms; and if there is more than one free variable the same form may have several associated functions. But in some languages it is possible from the form to construct a name (or names) of the associated function (or functions) by means of an abstraction operator.

7 The idea of allowing variables of different ranges is not Fregean, except in the case of functions in Frege's sense (i.e., as *ungesättigt*), the different categories of which appear as ranges for different variables. The intro-

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duction of *Gegenstandsbuchstaben* with restricted ranges is one of the modifications here advocated in Frege's theory.

8 Exceptions to this are familiar in common mathematical notation. E.g. the form x/y has no value for the system of values 0,0 of x, y. However, the semantics of a language is much simplified if a value is assigned to a form for every system of values of the free variables which are admissible in the sense that each value belongs to the range of the corresponding variable. And for purposes of the present exposition we assume that this has been done. (Compare footnote 4.)

9 Even if the language contains no constant denoting the value in question, it is possible to consider an extension of the language obtained by adjoining such a constant.

10 The notion of a sense-value of a form is not introduced by Frege, at least not explicitly, but it can be argued that it is necessarily implicit in his theory. For Frege's question, "How can a=b if true ever differ in meaning from a=a?" can be asked as well for forms a and b as for constants, and leads to the distinction of value and sense-value of a form just as it does to the distinction of denotation and sense of a constant. Even in a language like that of *Principia Mathematica*, having no forms other than propositional forms, a parallel argument can be used to show that from the equivalence of two propositional forms A and B the identity in meaning of A and B in all respects is not to be inferred. For otherwise how could A $\equiv$ B if true (i.e., true for all values of the variables) ever differ in meaning from A $\equiv$ A?

11 For purposes of the preliminary sketch, the meta-language is left unformalized, and such questions are ignored as whether the meta-language shall conform to the theory of types or to some alternative such as transfinite type theory or axiomatic set theory. Because of the extreme generality which is attempted in laying down these principles, it is clear that there may be some difficulty in rendering them precise (in their full attempted generality) by restatement in a formalized meta-language. But it should be possible to state the semantical rules of a particular object language so as to conform, so that the principles are clarified to this extent by illustration.

It is not meant that the list of principles is necessarily complete or in final form, but rather a tentative list is here proposed for study and possible amendment. Moreover it is not meant that it may not be possible to formulate a language not conforming to the principles, but only that a satisfactory general theory may result by making conformity to these principles a part of the definition of a formalized language (compare footnote 12).

12 In the case of some logistic systems which have been proposed (e.g., by Hilbert and Bernays), if semantical rules are to be added, in conformity with the theory here described and with the informally intended interpretation of the system, it is found to be impossible to satisfy (viii), (x), and (xiv), because of restriction imposed on the bound variables which may appear in a constant or form used in a particular context. But it would

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seem that modifications in the logistic system necessary to remove the restriction may reasonably be considered non-essential, and that in this sense (viii), (x), (xiv) may still be maintained.

In regard to all of the principles it should be understood that non-essential modifications in existing logistic systems may be required to make them conform. In particular the principles have been formulated in a way which does not contemplate the distinction in typographical style between free and bound variables that appears in systems of Frege and of Hilbert-Bernays.

In (x) and (xi), the condition that f′ have the same free variables as f can in many cases be weakened to the condition that every free variable of f′ occur also as a free variable of f.

[13] Possibly (ix) and (xi) should be weakened to require only that if C′ is well-formed then it is a constant having the same denotation as C. Since there is in general no syntactical criterion by which to ascertain whether two constants c and c′ have the same denotation, or whether two forms have always the same values, there is the possibility that the stronger forms of (ix) and (xi) might lead to difficulty in some cases. However, (ix) as here stated has the effect of preserving fully the rule of substitutivity of equality—where the equality sign is so interpreted that $[c_1 = c_2]$ is a sentence denoting truth if and only if c_1 and c_2 are constants having the same denotation—and if in some formalized languages, (ix) and (xi) should prove to be inconsistent with the requirement that every well-formed expression be meaningful (footnote 2), it may be preferable to abandon the latter. Indeed the preservation of the rule of substitutivity of equality may be regarded as an important advantage of a Fregean theory of meaning over some of the alternatives that suggest themselves.

[14] At the present stage it cannot be said with assurance that a modification of Frege's theory will ultimately prove to be the best or the simplest. Alternative theories demanding study are: the theory of Russell, which relies on the elimination of names by contextual definition to an extent sufficient to render the distinction of sense and denotation unnecessary; the modification of Russell's theory, briefly suggested by Smullyan (in *The Journal of Symbolic Logic*, vol. 13 (1948), pp. 31-37), according to which descriptive phrases are to be considered as actually contained in the logistic system rather than being (in the phrase of Whitehead and Russell) "mere typographical conveniences," but are to differ from names in that they retain their need for scope indicators; and finally, the theory of Carnap's *Meaning and Necessity*.

Though the Russell theory has an element of simplicity in avoiding the distinction of two kinds of meaning, it leads to complications of its own of a different sort, in connection with the matter of scope of descriptions. The same should be said of Smullyan's proposed modification of the theory. And the distinctions of scope become especially important in modal statements, where they cannot be eliminated by the convention of always taking the minimum scope, as Smullyan has shown (loc. cit.).

Moreover, in its present form it would seem that the Russell theory

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requires some supplementation. For example, 'I am thinking of Pegasus,' 'Ponce de Leon searched for the fountain of youth,' 'Barbara Villiers was less chaste than Diana' cannot be analyzed as '$(Ec)[x$ is a Pegasus $\underset{x}{\equiv}$ $x{=}c]$ [I am thinking of c],' '$(Ec)[x$ is a fountain of youth $\underset{x}{\equiv}$ $x{=}c]$ [Ponce de Leon searched for c],' '$(Ec)[x$ is a Diana $\underset{x}{\equiv}$ $x{=}c]$ [Barbara Villiers was less chaste than c]' respectively—if only because of the (probable or possible) difference of truth-value between the given statements and their proposed analyses. On a Fregean theory of meaning the given statements might be analyzed as being about the individual concepts of Pegasus, of the fountain of youth, and of Diana rather than about some certain winged horse, some certain fountain, and some certain goddess. For the Russell theory it might be suggested to analyze them as being about the property of being a Pegasus, the property of being a fountain of youth, and the property of being a Diana. This analysis in terms of properties would also be possible on a Fregean theory, though perhaps slightly less natural. On a theory of the Russell type the difficulty arises that names of properties seem to be required, and on pain of readmitting Frege's puzzle about equality (which leads to the distinction of sense and denotation in connection with names of any kind), such names of properties either must be analyzed away by contextual definition—it is not clear how—or must be so severely restricted that two names of the same property cannot occur unless trivially synonymous.

[15] Here a warning is necessary against spurious economies, since not every subtraction from the entities which a theory assumes is a reduction in the variety of entities.

For example, in the simple theory of types it is well known that the individuals may be dispensed with if classes and relations of all types are retained; or one may abandon also classes and relations of the lowest type, retaining only those of higher type. In fact any finite number of levels at the bottom of the hierarchy of types may be deleted. But this is no reduction in the variety of entities, because the truncated hierarchy of types, by appropriate deletions of entities in each type, can be made isomorphic to the original hierarchy—and indeed the continued adequacy of the truncated hierarchy to the original purposes depends on this isomorphism.

Similarly the idea may suggest itself to admit the distinction of sense and denotation at the nth level and above in the hierarchy of types, but below the nth level to deny this distinction and to adopt instead Russell's device of contextual elimination of names. The entities assumed would thus include only the usual extensional entities below the nth level, but at the nth level and above they would include also concepts, concepts of concepts, and so on. However, this is no reduction in the variety of entities assumed, as compared to the theory which assumes at all levels in the hierarchy of types not only the extensional entities but also concepts of them, concepts of concepts of them, and so on. For the entities assumed by the former theory are reduced again to isomorphism with those assumed by the latter, if all entities

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below the nth level are deleted and appropriate deletions are made in every type at the nth level and above.

Some one may object that the notion of isomorphism is irrelevant which is here introduced, and insist that any subtraction from the entities assumed by a theory must be considered a simplification. But to such objector I would reply that his proposal leads (in the cases just named, and others) to perpetual oscillation between two theories T_1 and T_2, T_1 being reduced to T_2 and T_2 to T_1 by successive "simplifications" *ad infinitum*.

16 As is done in the forthcoming revised edition of my *Introduction to Mathematical Logic, Part I.*

17 For convenience, English is used also as the meta-language, although this gives a false appearance of triviality or obviousness to some of the semantical rules. Since the purpose is only illustrative, the danger of semantical antinomies is ignored.

18 For present illustrative purposes the question may be avoided whether common nouns in English, in the singular, shall be considered to be variables (e.g., 'planet' or 'a planet' as a variable having planets as its range), or to be class names (e.g., 'planet' as a proper name of the class of planets), or to have "no status at all in a logical grammar" (see Quine's *Methods of Logic*, p. 207), or perhaps to vary from one of these uses to another according to context.

19 Or possibly 'planet' and 's' could be regarded as two primitive symbols, by making a minor change in existing English so that all common nouns form the plural by adding 's.'

20 If any of you finds unacceptable the conclusion that therefore 'the number of planets is round' is a sentence, he may try to alter the rules to suit, perhaps by distinguishing different types of terms. This is an example of a doubtful point, on the decision of which there may well be differences of opinion. The advocate of a set-theoretic language may decide one way and the advocate of type theory another, but it is hard to say that either decision is the "true" decision for the English language as it is.

21 But of course it would be wrong to include as a rule of denotation: 'the world is round' denotes truth. For this depends on a fact of geography extraneous to semantics (namely that the world is round).

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