

The Dangers and Promises of Comparative History of Science

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"The Dangers and Promises of Comparative History of Science"

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I. Introduction: Needham's opposition to Spengler's vision of history of science

Anyone doing research outside East Asia on the history of science in China owes a debt to Joseph Needham (1900-1995). In the history of science, he is well-known for many essay-articles as well as for being the author, or rather the editor, of *Science and Civilisation in China*, a multi-volume encyclopaedia whose publication began in 1954 and which deals with virtually every aspect of the history of science and technology in China. However, this is not the sum of Needham's career and broader activities. To evoke a few other dimensions of this multi-faceted man, Christian-Socialist Needham began his scientific life as a biochemist, specializing in embryology. He experienced and denounced the rise of fascism and National Socialism in Europe, later contributing to the war effort as the Director of the Sino-British Cooperation Office in Chongqing between 1942 and 1946. This position was his first significant engagement for a worldwide scientific cooperation, a task he carried on while being, from 1946 to 1948, the first head of UNESCO's Natural Sciences Section.[2]

Given this background, one tends to pay close attention, when in various Parts of Volume 5 of *Science and Civilisation in China*, published in 1974, 1976 and 1980, he repeats warnings such as the following:[3]

[1] I have pleasure in extending my thanks to Professor Rubens, president of the Sarton Committee, and to the entire Committee for having invited me to hold the Sarton chair in the 2013-2014 academic year. I am grateful to Liesbeth De Mol, for her heartfelt plea and her reading of this text. Professor Paul Van Cauwenberghe, Professor Marc Boone, as well as my colleagues from Ghent, Maarten van Dyck, Albrecht Heeffer, Rita Malfliet, and all the others, have been extremely helpful and contributed in making the stay in Ghent the most pleasant experience. I would also like to thank Richard Kennedy for kindly helping me prepare the final version of this article. The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC Grant agreement n. 269804.

[2] For biographical details and a historically informed approach to Needham's ideas in the history of science, see [1], in particular the chapters by Gregory Blue and Patrick Petitjean. See also Petitjean's articles mentioned below.

[3] The publication of *Science and Civilisation in China* began in 1954 and was still unfinished when Needham died in 1995. Needham divided it into seven "Volumes." From Volume 4 onwards, the Volumes were in their turn divided into "Parts," each of which had the size of a thick book. In this article, the use of "Volume" and "Part" in relation to *Science and Civilisation in China* follows Needham's terminology. A Volume usually began with an "Author's note." Starting from Volume 4, except for Parts entirely written by a single other author (like Part I of Volume 5, authored by Tsien Tsuen-Hsuin), Needham's "Author's note" was regularly repeated at the beginning of each Part, with variations that are quite meaningful. This is the case for Parts II, III, IV and V of Volume 5. The quotation that follows occurs in [2: xxii, 3: xxiv-xxv, 4: xxxvii], that is, Parts II, III, and

> "There is a danger to be guarded against, the danger of (…) denying the fundamental continuity and universality of all science. This could be to resurrect the Spenglerian conception of the natural sciences of the various dead (or even worse, the living) non-European civilisations as totally separate, immiscible thought-patterns, more like distinct works of art than anything else, a series of different views of the natural world irreconcilable and unconnected. Such a view might be used as the cloak of some historical racialist doctrine, the sciences of pre-modern times and the non-European cultures being thought of as wholly conditioned ethnically, and rigidly confined to their own spheres, not part of humanity's broad onward march. *However*, it would leave little room for those actions and reactions that we are constantly encountering, *those subtle communicated influences which every civilisation accepted from time to time.*"[4]

To my knowledge, throughout *Science and Civilisation in China*, these are the first times Needham mentions Oswald Spengler (1880-1936) —twenty years or more after the beginning of the publication in 1954. Why does Needham refer here to Spengler, this anti-democratic thinker who was so influential, in Germany and beyond between the two world wars, in particular for his book *The Decline of the West*, whose two volumes appeared in 1919 and 1922, respectively, and whose English first translation was published in 1926?[5] What is the context that incites Needham to return to this author? Addressing this question will lead us to reflect about comparative history and its various practices in a way that might prove useful.

Among the key theses that *The Decline of the West* promoted, and to which Needham alludes here, was the idea that "cultures and civilizations" were the key meaningful entities with which to analyse history. According to Spengler, like biological organisms, these entities experienced an ascending period (their "birth", to which Spengler referred as "Kultur"), and a phase of decay, which he called "civilization," before they eventually died. In particular, Spengler was prophesying the "decline and fall of modern civilization."[6] Moreover, in Spengler's view, these entities each developed their own analogous arts, sciences and other cultural artefacts, all completely singular, specific to the culture in which they took shape, and all incommensurable with those of the others. Finally, for him, these entities were all bound to die out and disappear, including "modern science," perceived as the science of the West.

Needham's declaration against Spengler's theses that I find particularly striking restates some of Needham's main tenets, which inspired the writing of *Science and Civilisation in China*. And in it, each word, I claim, was carefully chosen. Let me comment on some of them.

Most significantly, Needham insists on the "continuity and universality of all science", and here it is important not to misunderstand what he meant. The theme of the "continuity of science" is regularly reasserted in *Science and Civilisation in China*, which helps us interpret what is at stake for Needham. It has often been understood that in such statements Needham primarily emphasized the "universality of science."[7] In my view, here his stress is more on

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IV of Volume 5, and not in Part V, that is, [5]. I quote the 1980 version, using italics to indicate where this version differs from other versions.

[4] Italics are mine and indicate changes between versions of this statement. In Parts II and III, instead of "However," we find "Moreover." In Parts II and III, the last sentence reads: "Moreover it would leave little room for those actions and reactions that we are constantly encountering, deep-seated influences which one civilisation had upon another. "

[5] For an assessment and analysis of Spengler's ideas, and the reception of his book, written by a witness who will be important for us below, see [6], reprinted in [7: 119-43].

[6] The thesis is explicitly mentioned in [4: xxxvi].

[7] Typically, in the extremely interesting section entitled "The threads of universality", Blue [8: 51-6] examines the issue of

what he calls the "human unity", that is, on the fact that humanity holds together and cannot be meaningfully divided. One of the virtues of the history of science, for Needham, is that it highlights the solidarity between all parts of mankind.

In relation to this emphasis on continuity comes the idea that, in the end, the purpose of the history of science is —for example, in Volume 5— to write, the "history of man's enquiry into chemical phenomena as one single development throughout the old world cultures." Clearly, such a vision stands in opposition to the idea that "the sciences of pre-modern times and the non-European cultures" could be "thought of as … rigidly confined to their own spheres".[8] On one side, for Spengler, mankind is divided into distinct groups, on the other, knowledge is in the plural. These are in fact two sides of the same coin. Indeed, Spengler makes sense of the history of science by considering mankind divided into various groups, separate from each other, and seeing, in Needham's words, "the natural sciences of the various … civilisations" —let me emphasize the plural in "sciences," here like above— "as *totally separate*, immiscible thought-patterns, …irreconcilable and *unconnected*." (my emphasis).

In the same few pages of his "Author's note," and also repeated in different Parts of Volume 5, Needham feels the need to distance himself more than once from Spengler, in essentially the same terms as those analysed previously. In the context of a vision such as Spengler's, he emphasizes, the project of a single history, —we would say a "global history," which Needham conceives as a history showing how various bodies of knowledge could be subsumed into a single science—, is meaningless, relativism and incommensurability reigning supreme.[9] Interestingly, in the same pages and for similar reasons, Needham distances himself from Thomas Kuhn's view on the history of science.[10]

Needham is ready to grant that works of art or other cultural artefacts could possibly be incommensurable. However, in his view, "for mathematics, science and technology, the

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universality in detail, while the related issue of continuity remains in the shadow.

[8] In fact, in several places in which Needham reasserts the "absolute continuity" of the human effort in the development of knowledge, he opposes this view to Spengler's. An example thereof is found in the same pages as those in which our key statement occurs ([2: xxi, 3: xxiii, 4: xxxv, 5: xxvii-xxviii], where we read: "Throughout this series of volumes it has been assumed all along that there is *only one unitary science of Nature,* approached more or less closely, built up more or less successfully and *continuously*, by various groups of mankind from time to time. This means that one can expect to trace an *absolute continuity* between the first beginnings of astronomy and medicine in Ancient Babylonia, through the advancing natural knowledge of medieval China, India, Islam and the classical Western world, to the break-through of late Renaissance Europe when, as has been said, the most effective method of discovery was itself discovered. Many people probably share this point of view, but *there is another one* which I may *associate with the name of Oswald Spengler*, the German world-historian of the thirties whose works, especially *The Decline of the West*, achieved much popularity for a time." (my emphasis)

[9] In these pages, Needham repeats: "According to him [Spengler], the sciences produced by different civilisations were like *separate* and irreconcilable works of art, valid only within their own frames of reference, and *not subsumable into a single history* and a single ever-growing structure." [2: xxi, 3: xxiii, 4: xxxv, 5: xxviii] (My emphasis) Note that again, in the position ascribed to Spengler, "sciences" is in the plural, which contrasts with the singular of Needham's "single ever-growing structure". Needham's position has to be distinguished from that of a naïve assumption that science is a priori universal.

[10] "Just recently a relevant polemical discussion has been going on among geologists. Harrington (I, 2), who had traced interesting geological insights in Herodotus and Isaiah, was taken to task by Gould (I), maintaining that 'science is no march to truth, but a series of conceptual schemes each adapted to a prevailing culture', and that *progress* consists in the *mutation of these schemes*, new concepts of creative thinkers resolving anomalies of old theories into new systems of belief. This was evidently a Kuhnian approach, but no such formulation will adequately account for the *gradual percolation of true knowledge through the successive civilisations, and its general accumulation*. Harrington himself, in his reply (3), maintained that 'there is a singular state of Nature towards which all estimates of reality converge', and therefore that we can and should judge the insights of the ancients on the basis of our own knowledge of Nature, while at the same time making every effort to understand their intellectual framework." (Needham 1974: xxii; Needham 1980: xxxvi), footnote a (my emphasis). In statements such as the one underlined above, exegetes have tended to read Needham's emphasis on the cumulative nature of science. In the light of my analysis here, I suggest that the emphasis lies equally on the continuity between all civilizations. Note how at the end of this quotation, Needham suggests the historian's focus should remain twofold. The quotation faithfully illustrates his understanding of the reasons why circulation and accumulation are possible.

case is altered." The "essentially constant" "properties" of the "environment" in which "man has always lived" should warrant that, in these specific cases, knowledge of this environment has validity beyond the frames of reference in which this knowledge was gained.[11] Perhaps here, Needham considers the case of mathematics in a simplistic and slightly misleading way. We return to the question later. Note, however, that on this issue too, Needham is more nuanced than the theses that have been frequently ascribed to him. In his view, "the ancient and medieval sciences [bore] an obvious ethnic stamp."[12] Moreover, for him such a "stamp" had been eliminated through their incorporation into modern science —a debatable point, which I shall leave uncommented. Further, most interestingly, Needham demanded that "ethnic characteristics" be considered symmetrically, that is, that from this perspective ancient Greece, as well as modern Europe, should be considered in the same way as other areas of the planet.[13] The key issue of his opposition to Spengler thus appears to be Needham's refusal to consider parts of mankind as disconnected and science as possibly the object of parallel histories.

II. Guarding us against a form of comparative history, or Spengler's ghost

On the basis of this preliminary analysis, we can now return to our question: why does Needham formulate his opposition to views like Spengler's explicitly in Volume 5, while he had kept silent on this question up to that point in *Science and Civilisation in China*? Two main reasons can be identified, and to me they seem interesting to ponder.

The first reason relates to the fact that in Part V of Volume 5, Needham deals with the history of physiological alchemy. This topic brings about historiographical difficulties that, for Needham, call forth Spengler's ghost. The point appears clearly in the fifth Part of Volume 5, in the passage following the two paragraphs just evoked. In these paragraphs, we saw Needham restating his rejection of Spengler's views, and claiming that with respect to the issue of incommensurability, "for mathematics, science and technology the case is altered". These topics are precisely those treated in the previous Volumes of *Science and Civilisation in China*. The two paragraphs in question shed light on the type of challenge that, by contrast, physiological alchemy presents to him and which he addresses immediately after

[11] "(…) while one can easily see that artistic styles and expressions, religious ceremonies and doctrines, or different kinds of music, have tended to be incommensurable, for mathematics, science and technology the case is altered—man has always lived in an *environment* essentially *constant* in its *properties*, and his knowledge of it, if true, must therefore tend towards a *constant structure.*" (my emphasis) (Needham 1974: xxi; Needham 1976: xxiii; Needham 1980: xxxvi; Needham 1983: xxviii.)

[12] The following quote clearly shows how in 1973, Needham [9: 418] holds both views at the same time: "the ancient and medieval sciences (though bearing an obvious ethnic stamp) were concerned with the same natural world and could therefore be subsumed into the same oecumenical natural philosophy". In the same vein, Needham asks the following revealing question : "When in history did a particular science in its Western form fuse with its Chinese form so that all ethnic characteristics melted into the universality of modern science?" [10], quoted through its republication as [11: 397].

[13] In the "Author's note," repeated in all Parts of Volume 4, we read: "As for the discoveries and inventions which have left permanent mark on human affairs, it would be impossible even to summarise here the Chinese contributions. (…) In these circumstances it seems hardly believable that writers on technology have run up and down to find reasons why China contributed nothing to the sciences, pure or applied. (…) Another method is to admit that China did something but to find a satisfying reason for saying nothing about it. Thus a recent compendious history of science published in Paris maintains that the sciences of ancient and medieval China and India were *so closely bound to their peculiar cultures that they cannot be understood without them*. The sciences of the ancient *Greek world*, however, were truly *sciences as such, free of all subordination to their cultural matrix* and fit subjects with which to begin a story of human endeavour in all its abstract purity. It would be much more honest to say that while the social background of Hellenistic science and technology can be taken for granted because it is quite familiar to us from our schooldays onwards, we do not yet know much about the social background of Chinese and Indian science, and that we ought to make efforts to get acquainted with it. In fact, of course, *no ancient or medieval science and technology can be separated from its ethnic stamp,*[d] and though that of the *post-Renaissance period* is truly universal, it is *no better understandable* historically *without a knowledge of the milieu* in which it came to birth." Quoted from [12: xlvii-xlviii]. The statement is repeated verbatim in Part I, (published in 1962, pp. xxv-xxvi) and Part II (published in 1965, pp. xlv-xlvi). Footnote d refers to [13: 448], where the issue of culture-rootedness and universality of science had already been discussed.

in the "Author's note" inserted in Part V. The way in which he formulates the challenge is telling:

> "*Nevertheless* (…) we are conscious that [this section on physiological alchemy] is rather different from those which have gone before it (KC: that is, mathematics, physics, technology…) and from those which will follow it. In order to understand the physiological alchemy of China, one has to *enter a world of natural philosophy entirely unlike that of Western tradition* (…). The sheer *un-European-ness* of Chinese physiological alchemy deeply impresses. (…) It was very clearly itself and nothing else (…)" [5: xxviii] (my emphasis)

These expressions betray Needham's discomfort: for him, physiological alchemy definitely belongs to the scope of the history of science, and yet he feels uneasy that this case could justify the claim that a body of knowledge might be potentially "rigidly confined to its own sphere" and "culturally bound." In other words, as Needham appears to concede, he feels that dealing with this new topic threatens to compel him to side with Spengler. What allows him, in this case, to ward off Spengler's ghost is proof that this knowledge nevertheless circulated westwards, that it has echoes in several modern disciplines, and that it thereby demonstrated it was "not wholly antithetical to modern science."[14]

The fact that for Needham, the evidence that knowledge circulates is a key point in refuting Spengler's vision also appears unambiguously at the end of the quotation with which I began this article, when he writes, to counter Spengler —I quote again: "*However*, it would leave little room for those actions and reactions that we are constantly encountering, *those subtle communicated influences which every civilisation accepted from time to time*." The phenomenon of the circulation of knowledge manifestly plays a key role in his conception of the unity of mankind. We return shortly to this issue.

The second reason why, in Volume 5 of *Science and civilisation in China*, Needham's "Author's note" repeatedly returns to his antagonism towards Spengler brings us to our main topic in this article. Needham manifestly disagrees with a contributor to this section of the enterprise. *Science and civilisation in China* is an encyclopaedic endeavour, bringing together contributions by various collaborators. Needham mentions their names in the title page of each Part. In Part IV of Volume 5, Needham publishes a contribution by Nathan Sivin devoted to "the theoretical background of elixir alchemy." However, Needham feels the need to distance himself from the form of comparative history advocated by Sivin. The correlation between his developments about Spengler and this issue is easily demonstrated.

Exactly the same paragraphs that in Part V of Volume 5 were followed by the embarrassed explanations we have just evoked about physiological alchemy are followed, in Parts II, III, IV, by the following unambiguous statement:

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[14] To illustrate these points, let me quote fully the paragraphs added by Needham in Part V of Volume 5, immediately after the two statements quoted above in footnotes 9 and 11 respectively: "*Nevertheless*, in presenting to the world this part of Volume 5 [KC: i.e., on physiological alchemy**]**, we are conscious that it is rather different from those which have gone before it and from those which will follow it. In order to understand the physiological alchemy of China, one has to enter a world of natural philosophy entirely unlike that of Western tradition, and to attune oneself to a theology and a realm of religious feeling quite foreign to the common presuppositions of the 'Peoples of the Book'.(…) The sheer un-European-ness of Chinese physiological alchemy deeply impresses. True, it had some connections with Indian thought and belief, yet it was very clearly itself and nothing else, essentially materialist in character (…). In view of the *deep contrasts between Western and Eastern spirituality, a leap of sympathetic understanding* is required in approaching Chinese physiological alchemy, a readiness for new experience of the 'other', as was so well seen by C.G. Jung (…). *Yet* physiological alchemy was *not wholly antithetical to modern science*, as has sometimes been thought. (…) For all these reasons, we believe that most of physiological alchemy merits the name of proto-science rather than pseudo-science." (Needham1983: xxviii, my emphasis)

> "This point [KC: that is, the point about Spengler and incommensurability] would not perhaps need emphasis if certain scholars, in their anxiety to do justice to the *differences* between the ancient Egyptian or the medieval Chinese, Arabic or Indian world-views and our own, were not sometimes tempted to follow lines of thought which might lead to Spenglerian pessimism.[15] (…) For example, our own collaborator, Nathan Sivin (…) " [16]

The point is not for us to discuss whether Needham is right or not, nor to determine whether Sivin was mistaken or not. It is rather to understand, through Needham's perception of it, the problems attached to a form of comparative history of science. Let me thus examine, in this perspective, Needham's argumentation a bit further.

Sivin, Needham says, urges us not to speak of "Chinese biology." Contrary to what one might assume, the point here is *not* to avoid speaking about "something Chinese", but rather to avoid using the term "biology," namely, a category, Sivin insists, that is foreign to the actors. The point cannot be denied. And yet, separation is already at play. The problem for Needham is the program he understands Sivin suggests developing as an alternative for a "fruitful comparative history." It requires, Needham points out, to take China separately and to restore "integral complexes of ideas with their interrelations and articulations intact,"[17] thanks to the historian's recovery of the specific goals the practitioners were after. The problem such an agenda raises in Needham's view is, I believe, clear when he emphasizes that, according to this approach, "Chinese science must, in other words, be seen as *developing out of one state of theoretical understanding into another* (…)." [18]

If we follow Needham, this practice of history of science has several implications. First, there would be a "Chinese science," I can testify to the fact that I often still hear colleagues speaking in these terms. Secondly, it would be legitimate to consider China a priori as separate, and even, methodologically speaking, China *should* be first considered separately, "in its own frame of reference."[19] Thirdly, it would make sense to consider the development of Chinese science "out of one state of theoretical understanding into another", as if it were legitimate, again, to consider this process as occurring separately. We may assume that for Needham such a program, again, left "little room for those actions and reactions that we are constantly encountering, *those subtle communicated influences which every civilisation accepted from time to time.*" Against the background outlined above, we understand how he perceived, in such a program, the threat of seeing a vision such as Spengler's resuscitated.

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[15] Interestingly, this is the point where Needham inserts the note about Kuhnian approaches mentioned above (see footnote 10).

[16] (Needham 1974: xxi-xxii; Needham 1976: xxiii-xxiv; Needham 1980: xxxvi.) We see how in the quotation Needham shows mankind divided in relation to the interest in "differences" he attributes to some scholars.

[17] Needham puts this statement between quotations marks. (Needham 1974: xxii; Needham 1976: xxiv; Needham 1980: xxxvi-xxxvii.)

[18] My emphasis. (Needham 1974: xxii; Needham 1976: xxiv; Needham 1980: xxxvi-xxxvii.)

[19] The following quotation shows that this is one of Needham's worries and how he relates this point to Spengler. He writes: "In a different place Nathan Sivin has written: 'The question of why China never *spontaneously* experienced the equivalent of *our scientific revolution* lies of course very close to the *core of a comparative history of science*. My point is that it is an utter waste of time, and distracting as well, to expect any answer until *the Chinese tradition has been adequately comprehended from the inside*.' The matter could not be better put; we must of course learn to see instinctively through the eyes of those who thought in terms of the Yin and Yang, the Five Elements, the symbolic correlations, and the trigrams and hexagrams of the *Book of Changes*. But here again this formulation might suggest a purely internalist or ideological explanation for the failure of modern natural science to arise in Chinese culture. *I don't think that in the last resort we shall be able to appeal primarily to inhibiting factors inherent in the Chinese thought-world considered as an isolated Spenglerian cell*." (my emphasis) (Needham 1974: xxii-xxiii; Needham 1976: xxv; Needham 1980: xxxvii).

Here, Needham exposes the assumptions of a certain practice of comparative history, in ways that are worth reflecting on. By its very project, this practice assumes entities as separate and forming wholes that have to be understood as such. Further, such a form of comparative history aims at "confronting" —this is the word ascribed to Sivin— the wholes thereby constituted, looking primarily for differences and contrasts.[20] One understands the concern that such a use of history might contribute to the constitution of entities like peoples or cultures as separate, and, I add, as uniform.

In Needham's view, the concern was not merely theoretical. It was also inspired by developments at the time, which he perceived as "similar." In these developments, history of science encouraged the view that there were different sciences and that different peoples should live by different sciences, embodying different communitarian values. This is what Needham points out, when, in the same pages, he mentions the example of Said Husain Nasr, for whom there are "Islamic sciences," which should be viewed as separate and should be considered "from an Islamic viewpoint." Not surprisingly, Needham deplores in such an approach "two fatal drawbacks: it denies the equality of the forms of human experience, and it divorces Islamic natural science from the grand onward-going movement of the natural science of all humanity."[21] We recognize the themes emphasized above. Since the time when Needham was writing these lines, similar trends have become quite widespread in all societies around the world, and they thrive today.[22] How, in our work as historians of science, we provide material fuelling these developments is a question Needham forces us to consider.

To summarize, Needham's concern, as I understand it, was the possibility of contributing a division of mankind through the way in which we write history of science. This is what he sees at the horizon of a practice of comparative history, whose focus is placed on differences and whose method assumes a priori that separate wholes exist, understandable only within their own frames of reference.

III. Back to the earlier years of UNESCO

Needham's emphasis on the history of humanity as a whole, and more precisely on the unity of mankind in history, has a history, which is interesting to explore for my purpose. It strongly evokes the context of UNESCO in the years immediately after its creation, that is, at the end of the 1940s. The stress on the solidarity of all parts of humanity evokes in particular the discussions around UNESCO's project of a *Scientific and cultural history of mankind*, which Patrick Petitjean has recently studied and in the eventual definition of which Needham played an important role.[23]

The project had been officially initiated in 1947, following upon an earlier suggestion by historian Lucien Febvre. Febvre, a historian specialized in early modern history and one of the founding fathers, with Marc Bloch, of the *Annales* School, was involved in many international institutions after the Second World War, including UNESCO. From its inception, this project of a *Scientific and cultural history of mankind* had also been supported and promoted by biologist Julian Huxley, who had been elected to be the first Director General of UNESCO in 1946, as well as by Needham, who was the head of that organization's "Natural Sciences Section" at the time.

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[20] Ito [14] discusses the emphasis on contrasts and differences in certain forms of historiography dealing with East Asia. In the same pages Needham goes on discussing along the same lines Sivin's conception of comparative history (Needham 1974: xxvi-xxvii; Needham 1976: xxix; Needham 1980: xli.)

[21] (Needham 1974: xxiv; Needham 1976: xxvi; Needham 1980: xxxviii-xxxix).

[22] See, for instance, [15], on the movement of "Vedic mathematics" in the Indian subcontinent.

[23] In this section, I rely on Patrick Petitjean's recent research on the topic. See in particular the documents published, and the arguments made, in [16, 17, 18]. However, I can only evoke a few aspects of this history, referring the reader to these publications for a more complete treatment.

The choice of focussing on a history of science and culture was intimately connected with the purpose of writing a history that would promote the unity of mankind and, as a result, peace. As Febvre emphasized in a note published in the first months of 1949, in order to write not on what had divided mankind, but rather on what had united it, one had to do away with a history focused on political events and wars and concentrate rather on science and culture as domains in which peaceful exchanges had never ceased.[24] Interestingly enough, these domains were precisely those to which historians working in the framework of the *Annales* School had called attention.

On May 5, 1949, Febvre presented a first outline for the work. He formulated the aims he assigned to the *Scientific and cultural history of mankind* in quite striking terms, as follows: "Now, to speak precisely, the book of which I come to give an outline is not a book of ordinary science. Its objective is to act upon the minds of people in order to extirpate the fatal virus of war. To act upon the minds of men and of women, to be sure, but, above all, upon the minds of children."[25]

This is precisely the concern driving the project of inquiring into the unity of mankind in a historical fashion, echoed as we have seen in Needham's "Author's note" examined above. Febvre's outline for this "history of humanity" represents, in my view, the cleverest scheme possible for a historiography striving to fulfil this task. According to this conception, Volume I (Febvre 1953: 959) should call upon the contributions of various fields: anthropology, psychology, linguistics, ethnology, and sociology, to address "the problem of the unity of the human race in its diversity." Analysing the differences between ethnic groups, between peoples, the origin, nature and extent of these differences, focusing on the historical importance of "racial interbreeding" in the perspective of showing that "There is no pure race in the human kind" and that "all existing groups are the products of multiple crossings", such were some of the goals set to these fields. General issues relating to the "paths followed" by the "development" of "the various human groups," to the comparison between these paths and to the possibility of implementing changes based on these observations were also high on the agenda.

Volumes II and III would precisely focus on circulations in general (Febvre 1953: 959, 961.) They would deal with the history of the means of communication and would also address how items related in particular to science and technology had "circulated from one group to the other…" The point was to highlight that "humanity" had been "in motion from its origins, constantly shifting about in an endless series of transcontinental migrations." Volumes IV and V would "take up" what each part of the world, conceived in geographical terms, has "received from other parts of the world and what they have given in exchange." As a result, such an inquiry should show how each part of the world is primarily constituted of contributions coming from the others. Moreover, it would highlight why it is impossible to consider one part of the world as separate from the others. This is precisely what Febvre sets as a goal: "From this picture would emerge the idea that the partitioning of the world is nothing but a fiction and that the earth has never ceased to change, to enrich itself, and to propagate itself, by a flood of peaceful interchanges." [26] We are at the antipodes of a historiography a la Spengler. Finally, Volume VI would present a synthesis from the

[24] [17]. The theme is taken up in the report on the *Scientific and cultural history of mankind.* Febvre, in his capacity of "promoter of the enterprise," presented in May 1949 to the International Council for Philosophy and Human Sciences. This document was published both in French and English as [19]. For this argument, see pp. 956-957.

[25] [19: 954-5]. In French: "Or, précisément, le livre dont je viens vous présenter une esquisse n'est pas un livre de science ordinaire. Il prétend agir sur les mentalités pour en extirper le mortel virus de la guerre. Sur les mentalités des hommes et des femmes, sans doute : mais avant tout sur celles des enfants". Quoted in [18: 21].

[26] (Febvre 1953: 959, 961), also quoted in [18: 21], which shows that Needham supported this scheme. Crouzet and Crouzet-Pavan quote the report at greater length in [20: 337-40], where they also discuss this type of historiography and elements of the history of the project.

viewpoint of a history of humanity, that is, a "recapitulation of the great phases of the historical development of humanity, of the great stages of interchange and borrowing."

In Febvre's idea, the main purpose of the intended volumes was the training of teachers responsible for teaching history. In 1949, turning his back on a traditional teaching of national histories, which would "never tend to reconcile the various peoples," Febvre was imagining "a new kind of teaching, (…) by definition, consecrated to peace." This was the aim ascribed to a "history of humanity."[27] However, still in the context of UNESCO, Febvre also developed the idea of reforming the teaching of national histories along similar lines.[28] In 1951, with François Crouzet, he presented the manuscript of a History of France whose title makes the project explicit. It reads: "International origins of a civilization".[29] The point was to show how France was, in every respect, constituted from elements and contributions coming from all over the world. The manuscript of that book was only published in 2012, under a title chosen by its editors and perfectly fitting the authors' intention. The title reads: *Nous sommes des sang-mêlés. We are of mixed blood.*[30] The completion of such a historical project was best suited to exposing the theoretical problems attached to considering a part of the world as separate. We recognize the similarity of inspiration with the project Febvre had formed for the *Scientific and cultural history of mankind*.

However, in the end, the outline Febvre had prepared for the *Scientific and cultural history of mankind* was not adopted. A debate took place later in 1949, at the end of which the project had taken a significantly different direction.[31] Needham summarized the new version of the project in a letter written in the first days of 1950 to Cortesao, to whom the realization of the project had been entrusted.[32] Some of its added features are striking for my purpose in this article. Needham's summary shows that the same ideas as we saw earlier were still providing the main guidelines. The stress was still laid on "the mutual indebtedness of all peoples" and on the contribution of all peoples and cultures "to the total human patrimony." It was still asked to place emphasis "on the factors which have united mankind throughout history, rather than on those which have divided the various peoples." However, some new historiographical foci had crept in, and from my perspective they significantly modified the spirit of the project. Febvre's main idea of the hybrid character of every civilization had given

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[27] (Febvre 1953: 957). Petitjean and Domingues [18: 42-6] note the multiple projects of writing "general histories" of science, "universal histories" or "world histories" that flourished at the time. Among them, they mention George Sarton's *Introduction to the history of science,* 1927-48. Baltimore: Williams & Wilkins. We have quoted Needham's criticism of some presuppositions in one of them (see footnote 13). More generally, Petitjean and Domingues evoke the different options that they embody with respect to the history of science.

[28] Crouzet and Crouzet-Pavan [20: 318-25] outline the circumstances in which, in the context of his participation to the work carried out at UNESCO, Febvre adopted the idea of acting through the writing of textbooks.

[29] Part of the manuscript, put online at the following address: http://unesdoc.unesco.org/images/0014/001423/142305fb.pdf, was circulated in the context of UNESCO. Its first pages explain the circumstances in which this work was carried out and the goals assigned to it.

[30] [21]. UNESCO spread the manuscript, requiring that similar histories be written in different countries. I am not aware of any study of the future of this project.

[31] For an analysis of the minutes of this debate, I refer the reader to [18: 25-33]

[32] Needham's letter to Cortesao (14/01/1950), quoted in [17], reads as follows (the addition of paragraphs and the emphasis are mine, where the underlined words are in the original): "After an opening part, introducing certain fundamental knowledge about Man and the world in which he finds himself, there would be a second part describing the series of chronologically successive stages in the *progress* of *humanity* in social organisation and control over, and understanding of, Nature.

The third part will be concerned with exchanges and *transmissions* in all branches of human knowledge, practice and experience; *demonstrating the mutual indebtedness* of *all peoples*, and bringing out the fact that there is *no people* or *culture* which has *not contributed* elements of essential value to the total human patrimony.

The fourth part will outline the various *patterns of the great cultures and civilisations*, their *particular world-outlooks* which were *characteristic* of them, and which, *though not transmitted* in former times, are *now fusing* into the *world-picture of universal man*.

The fifth concluding part would be of a synthetic character. In so far as the attainment of *perfect historical objectivity* might be considered to be *impossible*, the committee felt that *emphasis* might well be placed on the factors which have *united mankind throughout history, rather than on those which have divided the various peoples*." Petitjean and Domingues [18] also describe the later further transformations of the project until its eventual completion in the 1960s.

way to a milder formulation. The notion of peoples' and cultures' "contributions" was more forcefully promoted than that of their debts to other parts of the world. Finally, specific cultures and civilizations, with their "particular world-outlooks," were re-entered the picture, even if with a proviso. The worm was back in the fruit.[33] As Petitjean's conclusion in [17] emphasizes, these were in the end the main foci of the books published.

In Needham's summary, these "particular world outlooks" were supposed not to have been "transmitted in former times" —note the idea that transmission trims knowledge and separates what can be shared from what is specific. They were also supposed to disappear in modern times through their "fusi[on] into the world-picture of universal man". We recognize, here and above, UNESCO's main doctrines in its early years, as well as Needham's key ideas in *Science and Civilisation in China*. More generally, this new sketch for the *Scientific and cultural history of mankind* outlines a framework closer to what *Science and Civilisation in China* embodies.

To achieve these aims, *Science and civilisation in China* also grants, in its way, pride of place to the comparative method in the historian's project. However, in this context, the method is of a completely different nature. Comparison aims, in this case, to identify sameness, and this is most often in this way that transmission of knowledge between two parts of the planet is argued for. In any event, this was how the "contributions", or the "firsts", of peoples or cultures were for the most part established in *Science and Civilisation in China*. Highlighting the transmission of knowledge and thereby peoples' contributions was supposed to highlight the unity of mankind and that of knowledge. For someone like Needham, who wanted to emphasize "factors that had united mankind", it is a bitter irony that, in practice, the race for contributions has proved rather divisive too. Such a problem would not have affected a history of the type Febvre was calling for.

To analyse more closely the problems attached to this form of comparative history from a historiographical viewpoint, I would like to return to a piece of research I did in the 1990s and that was in the vein as I just outlined. Criticizing my old self will make no one unhappy. Moreover, I can misrepresent myself at will, without running into any danger, at least I hope.

IV. A critical glance at my early account of the history of algebraic equations

The pieces of work, to which I now return, deal with episodes in the early history of algebraic equations, and, more precisely, quadratic equations.[34] Here is a rough sketch of what I established at the time.

The earliest mathematical book handed down in China through the written tradition, *The Nine Chapters on Mathematical Procedures*, was probably completed in the first century of the Common Era.[35] Below, I shall abbreviate its title to *The Nine Chapters*. The book testifies to the existence, at the time in China, of a concept of equation —more precisely, of

[33] When, in 1953, the *Cahiers d'Histoire Mondiale* published two documents related to the history of the *Scientific and Cultural History of Mankind* project, including [19] quoted above, the introduction written by the editors, namely probably Febvre himself, makes clear that Febvre is aware of the dramatic changes brought to the nature of his project.

[34] Let us consider the simplest situation possible: for us here, an equation is a statement that involves data as well as an unknown (which we shall call x), and that is formulated in such a way that it should allow a practitioner to determine the value of the unknown. A quadratic equation is a statement in which only x and its square x^2 occur, and no higher power of the unknown. In [22], I outline the main idea and the research program to be executed in my view at the time. [23, 24, 25] carry out parts of the program. [26] sketches the realization of the last part. However, I intended to go back to that latter part in greater detail. In what follows, I only outline the new way in which I now approach the documents once dealt with, keeping for another publication the task of developing the argument with all needed details.

[35] [27] gives a critical edition and a French translation of the text of this Classic, as well as that of the ancient commentaries selected by the written tradition to be handed down with the Classic.

quadratic equation—, a concept which is quite specific.[36] In contrast to equations as we understand them in the first place, that is, as the statement of an equality, the type of equation to which *The Nine Chapters* attests is a numerical operation, like division. What do I mean by this?

An analogy with division will help me clarify my claim. Division can be approached in two ways. Either, it is the solution to a problem set in the form of an equality, such as "3 apples are worth 3000 dollars, what is the price of one apple?", or division is an operation which has two operands —a dividend and a divisor— and yields a quotient by a mathematical work applied to these operands. In the common conception of division today, we usually perceive the latter feature as being the most prominent. However, to understand the status of quadratic equations in ancient China, we should not forget about the former. Indeed, in *The Nine Chapters*, equation, I claimed at the time, was *not* perceived as the statement of an equality, as we emphasize today when, focusing on its statement, we identify the equation with an assertion such as: $a = x^2 + b\,x$ —where x is the unknown sought. Rather, in the context of *The Nine Chapters*, *exactly like division*, equation was perceived as the numerical operation solving the problem to which the statement corresponds. Moreover, this numerical operation had two operands, respectively called "dividend" (*a*) and "joined divisor" (*b*).

An equation of the kind encountered in *The Nine Chapters* thus presents analogies with division in several ways. First, it is a numerical operation that, like division, yields a result by a mathematical procedure applied to its operands. In this context, for both operations operands are written with respect to a place-value decimal position and the algorithms rely on these expansions to yield the result sought.

Secondly, equation is analogous to division because clearly its operands have names reminding us of those for a division. In fact, the execution of the operation-equation (this is how, in what follows, I shall refer to this type of equation) is also analogous to the computation of a division as it is attested to in *The Nine Chapters*.

Finally, the text of *The Nine Chapters* refers to a surface on which numbers were represented with counting rods and on which operations were executed. Prior to the 10th century, we have no illustration of what happened on this calculating surface. However, from the references to the computations we find in our sources, we can reconstruct the layouts for division and for that operation-equation used in relation to *The Nine Chapters*, and we see that the layouts for both operations were quite similar: the so-called "dividends" in both contexts were all placed in a middle row, whereas the "divisors" were put in lower rows, the result being obtained digit by digit and progressively put into upper rows (see table 1).

[36] See [27: 689-93, 732-5], for the Chinese text, its translation, and its interpretation.

Division On the calculating surface		Equation On the calculating surface	
quotient		(result-root)	
dividend	*a*	"dividend"	*a*
divisor	*b*	"joined divisor"	*b*

Table 1: The layouts of division and the operation-equation in The Nine Chapters

One could say —as I argued twenty years ago— that the equations to which Chinese sources testify until the 17th century are *all* of this kind. They were all given the identity of numerical operations. This identity stands in sharp contrast with what we find elsewhere in the world, and, to begin with, in al-Khwarizmi's *Book on algebra*, completed in the first half of the 9th century and devoted to quadratic equations.[37]

For al-Khwarizmi, the quadratic equation is primarily an equality, of the type "a square plus ten roots are equal to thirty-nine dirhams", where the term "root" refers to the unknown, and the term "square" to the square of that unknown [28: 100] This type of equation is thus closer, in this respect, to what, in modern terms, we write as $x^2 + b\,x = a$. This specificity of al-Khwarizmi's equation, through which it differs from what we saw in China, is closely related to how al-Khwarizmi works with equations. Indeed, his treatment relies in an essential way on equations as being statements of equality, primarily through the fact that he transforms them *qua* equalities. Moreover, al-Khwarizmi solves equations, by applying a sequence of operations to their coefficients (that is, the numbers *a* and *b* introduced above)— this is the so-called "solution by radicals." In brief, both the *nature* of the equation and the *way* of *solving* it stand in opposition to what Chinese documents attest to.

Several decades later in the Arabic world, al-Khayyam's (1048-1131) work testifies to a dramatic increase in the scope of equations dealt with and a key change with respect to al-Khwarizmi's approach to their solution. Most importantly, in addition to al-Khwarizmi's solution by radicals, al-Khayyam now seeks the solution of an equation in a geometric way, by the intersection of curves (more precisely, in fact, by the intersection of conic sections).[38] Such an approach presented a correlation with some ancient Greek writings, while remaining completely different from what the Chinese sources contain.

By contrast, in Sharaf al-Din al-Tusi's book *On equations*, completed at the end of the 12th century and recently published by Roshdi Rashed [30], we encounter a striking phenomenon. Al-Tusi combines al-Khwarizmi's and al-Khayyam's approaches to, and ways of working with, equations with a third approach that, as far as we can tell, is not documented earlier in Arabic sources and that powerfully evokes Chinese sources. Indeed, the layout of the equation is the same (see Figure 1): the constant term is in a middle row, the "divisors" are in lower rows, and the root of the equation appears gradually in the upper row. As in Chinese sources, the coefficients are written with a place-value decimal system and, as for a division, the root is obtained digit by digit. In both contexts, the same techniques are applied

[37] Recently al-Khwarizmi's *Kitab al-Jabr wa-al muqabala* was critically edited and translated in [28].
[38] Al-Khayyam's mathematical works were published in [29].

to the operands of the equation (the coefficients) in this layout to yield the root. Finally, the procedures used to solve the equation are similar.

In my earlier publications mentioned above, I emphasized that before al-Tusi, such ways of approaching and conceiving equations were attested *only* in Chinese sources. The comparison showing a similarity, along with this approach to equations as numerical operations appearing to be specific to China, led me to postulate, as I think Needham would have done, transmission in this respect from China into the Arabic world, and then, relying on Rashed's further study of transmission [31], even further westwards.

Such a comparison brought, in my view, interesting benefits, which I believe are still worth pondering. This has partly to do with the situation dealt with, and this is where I return to the idea that Needham might have envisaged the case of the circulation of mathematical knowledge as being simpler than it actually is.[39] Indeed, in the case outlined, algebraic equations were documented in *both* the Arabic world and China. However, interestingly enough, the concepts evidenced were *different*. The ways of working with equations were *also different* as were the ways of *solving* them. Evidence of conceptual diversity in the past, for what is for us one and the same concept, is an interesting by-product of the comparative approach, raising appealing philosophical questions.

Moreover, in this case, al-Tusi's book attests to the fact that these various concepts and approaches had been merged into a single whole. It evidences that a synthesis of former bodies of knowledge, and not a mere juxtaposition of them, had been carried out. Comparison thus brought to light the mathematical work that had been necessary to combine the concept and approach of equation seen as an arithmetic operation, which seemed to come from China, with other concepts and practices evidenced in the Arabic world. This episode suggests synthesis as a key operation deserving historians' attention. Such syntheses have hardly been studied as such, in the history of mathematics and yet they appear to constitute essential operations, in particular in the context of a world history. Comparison further showed that far from being by essence irreducible to each other, the various concepts identified before al-Tusi's synthesis could actually be combined with each other through the work of actors. Seen from another angle, comparison also showed that al-Tusi's approach to equations was the result of hybridization between several traditions, probably two coming from the Arabic world and one from China. It thereby called attention to the hybrid nature of our knowledge. It finally highlighted in particular the non-linear character of conceptual history, a phenomenon widely attested and yet still awaiting systematic inquiry.

Now, let us return to the topic of this article, that is, to the practice of comparative history that lay at the basis of my conclusion at the time. In a sense, this older piece of work of mine was much in the vein of the type of comparative history Needham advocated and practiced, when, from a comparison that revealed similarities between sources, he concluded that circulation had taken place. This was in general the type of reasoning on which basis he often claimed that knowledge elaborated in China had become part of human patrimony. However, as I shall now argue, my approach to these episodes in the history of algebraic equations had also embraced features of the first type of comparative history I have identified earlier.

Why do I feel the need today to review my earlier account in a critical way? My claim is that, twenty years ago, seeking to strengthen the conclusion that circulation had taken place, I was led to focus on specificities of approaches to equations in China and on their contrasts with other traditions. The logic of the argument I was making thus led me to overdo the uniformity of the concept of equation in China, without paying due attention to the differences between equations evidenced in different Chinese sources. In particular, I

[39] See Needham's statement quoted in footnote 11. In fact, the argument developed here has a validity that goes beyond the case of mathematics.

embraced uncritically the view of a single tradition with respect to equations in China. We recognize a disease typical of the first form of comparative history as exposed by Needham, which is to focus on what is similar among Chinese sources and what is different between them and sources produced elsewhere. My comparative approach led me to a form of historical account that basically amounted to an identification of "The Chinese equation" — that is, the equation specific to China— and to a survey of its specific development. These elements were tools required by my approach to the circulation of knowledge. One of the consequences was that the hypothesis I made in this respect was formulated in too coarse a way. How do I wish now to revise my earlier account?

V. Taking mathematical cultures into account

I suggest now that, if it remains true that equations as documented in Chinese sources were numerical operations, this account is too coarse and captures only part of the phenomena dealt with. To go beyond this first account, one needs to interpret Chinese documents in a finer way, and this can be only done through methodological work. The case of equations in China, as I shall now outline, illustrates *how* it is only through introducing *in a certain way* a context in which these documents were produced that one can offer a finer interpretation. The context in question will *not* be "China" as a whole, but rather, a context much closer to actual practice of the actors, that is, the "mathematical cultures" in relation to which the various sources attesting to work on equations in China were composed.

By the term "mathematical culture", to put it loosely, I refer to a "given way of practicing mathematics."[40] I claim that one can identify several different mathematical cultures in Chinese history. These various cultures present overlaps as well as breaks, which may explain why they have been so far overlooked. Without entering into the details of how this could be done, let me simply illustrate this claim by showing how differences between the writings composed in the context of these different cultures manifest differences between these contexts.

To begin with, let us return to *The Nine Chapters*, a book that was, as we have seen, probably completed in the 1st century of the Common Era and on which commentaries were composed and handed down through the written tradition. This set of sources attests to a concept of, and approach to, equation, some features of which I outlined earlier. Other features will appear if we consider the mathematical culture to which these sources testify and that can be perceived through a way of writing down mathematics. Figure 2 shows a page of *The Nine Chapters* with its commentaries, in a 15th century edition. We see that mathematical writing is purely discursive. On the other hand, the discourse refers to a calculating surface, which practitioners use while engaging with the text. As we have seen, numbers were materially represented with counting rods on this surface. The commentaries also refer to diagrams for plane geometry and blocks for space geometry. Analysis of the way in which commentators speak of artefacts for visualization had led me to argue that diagrams were material objects too, on which practitioners added colours, and which they divided into pieces and manipulated. In other words, practitioners in this context carried out mathematical activities with purely discursive writings and various types of material objects.

By contrast, the mathematical writings evidencing a work on equations in the second time period I distinguish have undergone a mutation. We can take Liu Yi 劉益's *Discussing the Source of the Ancient (methods)* 議古根源 (abbreviated below into *Discussing the Source*), probably composed in the 11th century, as illustrating both the practice of mathematics and the approach to equations in this second context. Apparently, now, the practice of mathematics

[40] I have presented in greater detail my approach to "mathematical cultures" in [32, 33].

has migrated onto paper, even though there are still material objects accompanying the text. In relation to this mutation, the type of mathematical writing to which our sources attest are of a radically different kind. As Figure 3 illustrates, these writings incorporate representations of the calculating surface as well as mathematical diagrams. The use of these elements presents similarities and differences with respect to what can be reconstructed for our first time period. For instance, diagrams still make use of colours. However, the use has undergone change and the meanings attached to them have been modified.

Finally, a third mathematical culture in relation to work with equations can be identified in the 13th century. It is illustrated by part of the work by Li Ye, as shown in Figure 4. His *Sea Mirror of Circle Measurements*, completed in 1248, no longer uses diagrams typical of the previous time period and instead uses symbolic notations, inspired by work on the computing surface.[41]

How can the description of the different mathematical cultures in the context of which equations have been researched in China benefit our account of the history of algebraic equations in China and beyond? To begin with, the description of the mathematical culture, in relation to the equation attested to in *The Nine Chapters* and its commentaries, allows us to capture features of this mathematical concept more fully than was done before. To put it bluntly —I leave the development of the argument for another publication—, the description of several practices with artefacts used in the context of this mathematical culture, that is, the description of how practitioners used the calculating surface, how they dealt with algorithms, how they established the correctness of algorithms, how they handled diagrams, etc., reveals that, at the time, the equation-operation had in fact two facets, one of which I overlooked in my earlier work. In addition to being a numerical operation, as I have outlined above, it also had a diagrammatic facet, each being the support of part of the work on equations.[42] In fact, the equation was established and stated geometrically, in the form of a rectangle, having a given area (the "dividend," that is, the constant term) and being composed of two sub-rectangles, i.e., the square of the unknown and a sub-rectangle, a side of which was the unknown and the other side of which corresponded to the "joined divisor."[43] This structure explains how the two facets of the equation —the numerical facet and the geometrical— were related to each other. The description of the mathematical culture sketched above explains why neither of these facets appeared at the time in the pages of the writings, except through hints that the discourse makes to them. This is the main reason why the description of the way the mathematics was practised, in relation to which a writing was composed, is essential to carry out conceptual history.

Now, bringing these two facets to light allows us to perceive in a new way the continuity between the equation-operation, as dealt with in *The Nine Chapters*, and what we find in Liu Yi's writing, in the 11th century. Figure 3 shows how in the 11th century, the two facets were inserted into the pages of writings. Further, the description of the mathematical culture in relation to which Liu Yi worked on equations enables us to understand that, as in the earlier context, far from being mere illustrations, the diagrams played a key part in the book. I now refer to them as "graphic formulas", writing down the equation. They are combined with what I call "written diagrams," which record the operation-equation and allow practitioners to work with equations numerically. In particular, Liu Yi uses these "graphic

[41] Li Ye's case is quite interesting in that his second book, compiled in 1259 and entitled *Deploying the pieces for the* [Collection] augmenting the ancient (methods) (*Yigu yanduan* 益古演段), can be read as the establishment of a bridge between these two mathematical cultures. This book was analysed and translated into English in [34].

[42] See my introduction to Chapter 9, [27: 671--2, 89-93] as well as the footnotes associated to the French translation. [35] was an important step towards a better appreciation of the importance of this facet in the history of equations in China.

[43] The facts that the diagrams are to be read as assertions and that a mathematical work is carried out on statements made in this way illustrates, on the one hand, specificities of a given mathematical culture, and, on the other, the importance of taking these specificities into account to interpret the sources.

formulas" to state equations, to establish them and to prove the correctness of procedures solving them. Features like colours, which were used in relation to *The Nine Chapters*, are taken up in this context, but with new purposes, like expressing negative coefficients. The book also evidences other important transformations of the concept of equation.

Establishing the two facets of the equations in *The Nine Chapters* also allows us to perceive the dramatic shift, to which the 13th century *Sea Mirror of the Circle Measurement* attests. Now, as illustrated in Figure 4, the geometrical facet of the equation has been entirely erased, whereas work on the equation relies only on numerical inscriptions: its "operation" facet has been promoted to the fore and expanded. In correlation with these features, Li Ye now considers equations of any degree in addition to the fact that they can have positive and negative coefficients. The shift characterized by the disappearance of the diagrammatic aspect would have remained invisible, if its earlier significance had not been brought to light.

This richer account, which focuses not only on the concept of equation, but also on work practices with equations, reveals at least two traditions in China. Both traditions appear to be grounded in an approach to equations of the type illustrated by *The Nine Chapters.* In the first tradition, the concept of quadratic equation evidenced in *The Nine Chapters* is reworked, being reshaped and generalized with respect to its terms, while remaining in the same material and conceptual framework. By contrast, in the second tradition, the numerical facet of the operation-equation evidenced in *The Nine Chapters* is reworked anew, giving rise to a much more general, entirely numerical concept of equation, worked out only through written diagrams, while the diagrammatic facet disappears from the mathematical work. Bringing to light the existence of these two traditions in China helps us understand that the latter vanished, whereas in the 17th century only the former was still alive.

Let me sketch how I envisage that these conclusions might affect the comparison with Arabic sources.

To begin with, the analysis outlined above raises new comparative issues. The first tradition identified in China, characterized by its recourse to diagrammatic means to state and establish equations, now appears to present features similar to those we had observed in al-Khwarizmi's algebra. Our perception of these similarities derives from the fact that we now understand the means this tradition developed to state equations as equalities. However, whereas al-Khwarizmi stated, and worked on, equations as discursive statements, our new historical account reveals that in the context of *The Nine Chapters*, the equality that a quadratic equation states was formulated through, and reworked with, a "graphic formula." On the other hand, it now also appears that al-Khwarizmi and his followers, as well as Chinese authors working within this tradition, all gave rectangles decomposed into sub-rectangles a key role in their mathematical work with equations. Which consequences derive from the differences between the practices of stating equalities and working with them, or between the practices with rectangles? These are new questions awaiting research. In fact, a similar question arises for Liu Yi and Li Ye: their material inscriptions for dealing with equations differ. What are the consequences for the work on equations? One need not have sources in different languages for comparative method to be useful.

If we turn to al-Tusi's book, the issue of circulation is now seen in a different light. First, on the basis of this more detailed account, we see that none of the Chinese sources available to us attests to exactly, and only, the equation-operation, as it appears in Sharaf al-din al-Tusi's account. However, Tusi's numerical approach to the equation presents more important similarities with the first tradition in China. Did transmission occur? I don't know. But if it did, the knowledge gained on the sources raises new questions. For example, how can we assume circulation occurred? Was it based on written documents, and in that case, on which type of documents? Or did it occur orally, and in that case, through which type of ephemera for the concrete engagement with inscriptions?

Finally, from a conceptual viewpoint, if transmission occurred in relation to this first tradition, it went along with a key transformation, since it eliminated the diagrammatic facet of the equation. This remark points to a general issue: the description of the transformations in the concepts of equation that might have been synthesized in that work appears to be a task as delicate as it is needed to gain a better understanding of the work required by the synthesis. Such are now newly open questions.

No definitive answer can be reached, except that Sharaf al-Din al-Tusi's book *On Equations* certainly encompasses approaches to equations that present both similarities and differences with distinct Chinese practices as well as with distinct traditions from the Arabic world. More importantly, however, the new account allows us to understand more clearly the non-uniformity of Chinese sources. Thanks to a comparative method, conducted on Chinese sources themselves, we now see different cultures and different concepts of equations in China, with overlaps and breaks. Perhaps overlaps and breaks, conceived in this broad way, might provide interesting tools to ponder, if we are to write a world history acknowledging similarities without denying differences, whether sources be written in the same or in different languages?

In conclusion, whether documents be written in Chinese or in Arabic, any two sets among them present a pattern of similarities and differences, both essential to better understand each document taken separately, from a conceptual viewpoint as well as from the viewpoint of the mathematical practice to which it adheres. The fact that Chinese documents appear connected through the overlaps they present is no smaller wonder than the fact that Chinese and Arabic documents show similarities. Perhaps the overlaps and breaks that a systematic practice of a comparative method reveals can help us understand how humanity is indeed piecewise connected while still showing local, very local, differences.

Captions

Figure 1: The numerical writing of the quadratic equation $112992 = x^2 + 31x$ in Sharaf al-Din al-Tusi's *On equations* [30: vol. 1, 26].

Figure 2: A Typical page in our early source material —A page of *The Nine Chapters*, with its ancient commentaries, dealing with square root extraction. Edition included in the 15th century imperial encyclopaedia *Grand Classic of the Yongle period* (永樂大典 *Yongle dadian*).

Figure 3: A Typical page in our source material of the second period. Liu Yi 劉益's *Discussing the Source of the Ancient (methods)* (議古根源, *Yigu genyuan*, abbreviated below to *Discussing the Source*) 11th c., quoted by Yang Hui, Korean print, 1433.

Figure 4: A Typical page in our source material of a third period. Li Ye 李冶: *Sea Mirror of Circle Measurements* (測圓海鏡, *Ceyuan haijing*), 1248.

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