Jean Cavaillès

JEAN CAVAILLÈS (1903–44), in the course of the brief span of his professional career, wrote a number of short but rich tracts on the epistemological issues of mathematics, logic, and science in general, some of which, like this last work of his, were published posthumously. *On Logic and the Theory of Science* may be considered his most important philosophical work. The prefaces to the French editions included here supply the necessary clarifications on the circumstances and background of the book and its author, as well as a suggestion of its impact on the phenomenological movement.

It is hoped that the difficulties involved in reading the necessarily condensed style of this prison book will be somewhat alleviated by a study of some of the allied contributions in this collection. Especially related are the essays on Husserl and the selections by Suzanne Bachelard and Jean Ladrière, who have continued the work of a "phenomenology of rationality" along the lines suggested by Cavaillès. To complement the following overview of the more salient points of this rich text, which scarcely begins to exhaust its suggestive possibilities, the bibliography includes several studies on the work of Cavaillès. But the text itself stands in its own right as a remarkable document, whose pages, "stripped of all secondary thought, have that abstract beauty which has become a rarity in our times." *

* Gabrielle Ferrières, *Jean Cavaillès, philosophe et combattant* (Paris: Presses Universitaires de France, 1950), p. 232. This book is a complete biography of Cavaillès. The remark is quoted from a

The book is divided into three untitled sections, dealing respectively with Kantian philosophies of consciousness, logical positivism, and Husserl's phenomenology. The issue is the essence and destiny of the theory of science in view of recent developments in logic and new insights into the riddle of mathematical reality, where the problem of the relationship of mathematics to physics is a recurring theme. What a theory of science must do is to establish an equilibrium among the various aspects of science: its subjective and objective aspects, as both an ordered system and a perpetual progression of knowledge. The former antinomy is found in Kant's vacillation between psychology and logic and in Husserl's notion of intentionality. As Cavaillès sees it, neither of these philosophers of consciousness manages to achieve the necessary balance. The logicists have emphasized science as a system of demonstration to the neglect of everything else. That progress is constitutive of science is a thesis dear to Leon Brunschvicg, who had a profound influence on Cavaillès' early intellectual development. But Brunschvicg exalts the autonomous initiatives of the human mind to the point of derogating the determining norms and the established order within which it necessarily moves, and these are viewed as an inertia to be overcome by the creative mind. Science is thereby limited to the absolute interiority of a spirit which wars with the external forces of conservative pedagogies and consecrated authorities. It is against the background of these positions that Cavaillès attempts to shift the center of gravity of the theory of science from a focus on its conscious activity to a concern for its conceptual dialectic. This emphasis on the dynamics of science itself is also why *enchainement* has generally been translated here as "sequence," to suggest not only the usual "concatenation" or "connection" but also the step-by-step movement that enters into such a dialectic.

The first part begins with an account of the difficulties in Kant's theory of science. The basic difficulty is that Kant turns from a consideration of science in its own right and reduces it to consciousness. Science is only a product of the faculties of consciousness, which is absolute, irreducible, a priori. Logic, which reputedly directs consciousness, is ultimately defined in terms of

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brief study of Cavaillès' work by Gaston Bachelard, which is appended to this biography.

the a priori forms of consciousness. Such a transcendental logic is an empty and abstract logic, indifferent to the bearing of the object on its structures, having no reference to an ontology. It is a logic which exists even when every object is set aside, and therefore it hardly warrants being called a body of knowledge. The issue is compounded by Kant's uncritical adoption of traditional syllogistic logic, which resists integration into the transcendental logic envisaged by him, his indecision on the role of the mathematical organon in science, and his lack of a sense of the progression intrinsic to the life of science.

It was Bolzano, pursuing a philosophy of science along Kantian lines, who perhaps for the first time considered science as such and in its autonomous movement, without subordinating it to the absolute of consciousness. In regarding science exclusively as a demonstrative system, he paved the way for the logicists of the twentieth century, whose theory of science is the topic of the second part of Cavaillès' book. Cavaillès begins by establishing, in his own terms, the two aspects of demonstration that have been distinguished in the course of this formalistic approach to science: on the one hand, the paradigm, the posited meaning that results from the operation of demonstration, the primary formality of the formal system itself, whose structures are to be classified by the theory of science; on the other, the dynamics of the thematic, the positing meaning of the operation as operative, the more reflective formality which is the syntax of the formal system, whose rules of construction are to be developed. This distinction is not a separation, for the thematic can in turn be the occasion for paradigmatic considerations. The most recent logicists, in equating their theory of science with the general syntax of the syntaxes of all formal systems, in effect seek the comprehensive paradigm of thematization in general.

Cavaillès was one of the first Frenchmen to master the technicalities and intricacies of the logical positivism which was then radiating out of Vienna. In the second section, he concludes that its formalizing approach contributes immensely toward clarifying and specifying a theory of science but certainly does not constitute and found it, as the positivists claim. Despite its "principle of tolerance," it is the old dream of panlogism, and thus a totalitarian conception of science. The theory of science is not equivalent to logic. Other facets of science must be invoked to complete it.

Carnap's ideal of a general syntax is expressly the quest for a "logic without ontology." Its aim is to *prescribe* the use of scientific language and not to *describe* the world. His antimetaphysical predilection impels Carnap to forego even the correspondence theory of truth that the early Wittgenstein still maintained and to adopt a simple coherence theory. Even the so-called protocol sentences are ultimately not the result of direct observation but are based on the pragmatic choice of one's syntactical commitments. But Carnap's initially simplistic view of syntax was immediately countered by Tarski, who pointed out that formalization is not complete without a definition of the elemental objects which enter into the constructions. In analogy with the results of Gödel's theorem, syntactical formalization cannot complete itself but must ever remain open to another level, that of a reference to objects. For Cavaillès, this reference to an exteriority means that logic must ultimately be completed by an ontology. Even now, Carnap refuses to admit this, for when he speaks of an "ontology" it is still a matter of a question internal to the linguistic system, where the real is simply anything which can find its place within this system.** All external questions are "metaphysical" and therefore nonsensical. If one wishes to speak of a material beginning that precedes formalization, there is the system of signs which, if one wishes to go that far, are reducible to the marks on paper. However, the sign is not an object of the world, for it implies a reference to previous acts and sequences, whose very nature involves a reference to an actuality without which science would not be science. The difficulty is especially evident in the deficiencies of the logicist account of the relationship of mathematics and physics, whose coordination is necessarily preceded by a mathematical translation of the experimental object, a process that eludes all formalization and demands an ontological basis. A doctrine of science, which must come to terms with the complex site and sense that constitute the notion of a "physical theory," cannot pretend that natural science is not at once the science of nature.

Husserl's phenomenology (the subject of Part III), through its notion of intentionality, promises to restore the balance between the claims of logicism and a Kantian theory of consciousness and at the same time correct their deficiencies. The import

** "Empiricism, Semantics, and Ontology," *Revue internationale de philosophie*, IV (1950), 20–40.

of intentionality is already manifest in his conception of formal logic, which is centered on the equivalence of content and difference in direction of an apophantic and a formal ontology. Apophantic, the theory of judgment in view of its forms of meaning, implies a reference to the acts and operations of a judging consciousness. Formal ontology, which studies the forms of the meant object, is ultimately destined to refer to the individual in the real world. The mathesis is not a closed system of forms but necessarily involves an application to the world. Otherwise, it is merely an empty mental game. A formal logic thus finds its roots in a material logic, and both are based on a transcendental logic which justifies them in the constituting activity of an intentional subjectivity.

But Kant's dilemma is not thereby bypassed, for this subjectivity is itself subject to norms that call for a governing logic that is not a transcendental logic, which means that transcendental logic itself is not absolute. Moreover, the theses of logical empiricism still lurk beneath Husserl's formulations of the relationship between mathematics and physics. If a physical theory is simply an empty mathematical form applied to the invariant intuitive content of the lifeworld, if mathematization is thereby only a technique for idealizing and organizing predictive activity in this world, if it does not augment our knowledge of the world but only represents an abstract interlude and complication of our more direct concerns, if mathematics finds its justification only through its ultimate reference to this world, if an exaggeration of its role only promotes an obfuscation of intuitive meaning (*Sinnentleerung*)—then mathematics does not have its own content of knowledge and is merely a tautology. This positivistic thesis stemming from Wittgenstein is again evoked by Husserl's view of axiomatics as a nomology, whereby all the posterior deductions are reducible to the prior axioms. But Gödel has shown that such a tautological nomology holds only for manifolds that envelop relatively small domains. The truly mathematical theory, which reaches for the infinite, cannot be dominated in advance by consciousness in the act of positing axioms. What in fact is first posited is an operation or system of operations whose directive possibilities are unpredictable, whose results evoke further developments which in turn cannot be dominated. And as one of its deductive fields develops, its ramifications reverberate through the entire body of mathematics, which de-

velops accordingly. All this amounts to a new notion of deduction which a philosophy of consciousness cannot account for. For the consciousness now is not a fixed absolute; it also progresses along with the progression which is intrinsic to the definition of science. It is this secret life of science, whose intelligibility is dispersed in concatenated moments that preclude comprehension in one fell swoop, whose progression is at once creative and necessary, whose rhythm of expansion involves discontinuous steps that bring suppression as well as resumption, which makes Cavaillès conclude that the fulcrum of a theory of science is to be found in a dialectic of the concept rather than in the activity of consciousness. One can only guess how he might have developed the particulars of this dialectic. Whether such a dialectic is alien to Husserl, whose last manuscripts also probe into the essential historicity of science, is open to question.†

† Cf. Jacques Derrida's remarks on Cavaillès in the introduction to his translation of Husserl's *L'Origine de la géométrie* (Paris: Presses Universitaires de France, 1962), pp. 157–59.

15 / On Logic and the Theory of Science

Jean Cavaillès

Preface to the 1960 Edition

In spite of its difficulty, this book has come a long way. With the first edition long out of print, a second edition has become necessary. This second edition is the textual reproduction of the first, which was polished to perfection through the faithful efforts to Georges Canguilhem and Charles Ehresmann.

The contemporary logician deliberately considers as "prehistory" all the logical literature preceding the publication in 1910 of the *Principia Mathematica* by Whitehead and Russell. And since this publication, logical studies have developed at a rate so rapid in directions so diverse that it can be said, without yielding to skepticism, that any attempt at synthesis is doomed to "grow old fast." It might even be said that logic and current events are practically inseparable nowadays. Yet the book which Jean Cavaillès wrote nearly twenty years ago, in 1942, is still timely, for the logician of 1960 as well as for the philosopher. Thoroughly saturated in the new logical culture, the author of this book has in fact found his way back to the fundamental inquiries which a philosophical awareness necessarily forces on the logician. In a sense, modern logic shares the fate of all the sciences in being a discipline which can be constructed and perfected without endeavoring to inquire radically into its original meaning. Husserl

The following translation of the prefaces to Jean Cavaillès, *Sur la logique et la théorie de la science*, 2d ed. (Paris: Presses Universitaires de France, 1960), and pp. 1-78, has been made expressly for this book by Theodore J. Kisiel.

recognized that what makes science possible is fortunately not the reflection which penetrates to the essence of things but the scientific instinct. The scientist, in order to build, need not involve himself in the preliminaries of philosophical critique. But logic, insofar as it is a theory of science and not only a particular science, cannot be content with that "hidden reason" which acts implicitly in every science. What basically happens here, at the heart of a single discipline, more so than anywhere else, is the rivalry of the science which claims to rid itself of all philosophical tutelage and of a philosophy which sometimes disdains to inform itself. Can this problem of modern logic be formulated more directly and, if we may say so, more definitively than Cavaillès did when he wrote, "The reference to technique is a subterfuge," and, several lines later, in recalling Gödel's results on the theory of demonstration, "Here technique takes its revenge by subverting the constructions realized in an abstract realm that surpasses it"?

This book, in which an entire mathematical and logical culture is continually taken for granted, is nevertheless a book dear to the philosopher. To be sure, the reader will have to inform himself if he wishes Cavaillès' formulas, often rendered enigmatic by their conciseness, to yield their meaning. But once he has overcome his insensitivity to the problems of deductive science, the philosopher will find himself in familiar territory. He will never have to confront a history of naked facts or a garrulous interpretation. Nothing extrinsic or accidental will detain this inquiry into the essence and end of the theory of science. When he drafted this text, Cavaillès wrote to Albert Lautman: "I have tried to define myself in relation to Husserl, somewhat in opposition to him." Since the publication of the first edition, the studies in phenomenology and the French translations of Husserl's writings have become more numerous, perhaps inspired by Cavaillès' reflection. Hence the knowledge of Husserl's work will make the context of Cavaillès' reflection more evident. And in return this reflection will reveal the problems of Husserlian philosophy in an incomparable way.

Far from books, in the heroic solitude of a prison, Jean Cavaillès wrote this book. It is a book of meditation for the philosopher.

GASTON BACHELARD

EDITORS' PREFACE

SOME WORDS OF EXPLANATION appear necessary at the beginning of this work by Jean Cavaillès, his last.

After completing his philosophical theses in 1938, Cavaillès began preparations for a new step in the work of expressing his own thought. But one year later, the war designated him for other duties. Taken prisoner in May after a courageous campaign in the winter of 1939–40, Cavaillès escaped to Belgium while being transferred to Germany and in October of 1940 resumed his courses at the University of Strasbourg, then relocated at Clermont-Ferrand. It is also known that he was not satisfied simply to resume his professorial work, that he did not consider himself demobilized by the armistice, and that he was one of the four or five founders of the first Resistance movements in France. Cavaillès' work in this domain, until his second and final arrest in August of 1943, was staggering.

This task nevertheless did not detract him from his philosophical work. The two were not really separate for him. The return to reflection in the heat of action seemed to him indispensable in order to preserve his sanity. It was his intention, as he confided to us on several occasions, to write a *Treatise on Logic,* but he added laughingly: "I shall find time to write it only in prison." This is just what happened. Sailing for England one night in August of 1942, Cavaillès was arrested on a beach near Narbonne and taken to the military prison at Montpellier. In order to overcome the despair of captivity, it was then that he undertook to write the work whose project he had now brought to maturity.

One may well wonder how Cavaillès could draft in prison a text in which numerous references are made to less common works. At that time, General de Lattre de Tassigny was in command of the military region of Montpellier. He knew and had a high regard for Cavaillès. Through his authorization, friends could visit Cavaillès in prison and bring him books. Albert Lautman came from Toulouse to Montpellier to bring some highly specialized texts which he possessed. When in the month of November the Germans retaliated for the landings in Algeria by occupying the southern zone, there was some vacilla-

tion and lack of decision in the military command. General de Lattre de Tassigny was arrested, under well-known circumstances. The military tribunal of Montpellier closed the Cavaillès case because of insufficient evidence. But the prefect of Hérault issued a warrant for the arrest of Cavaillès. He was sent to the camp at Saint-Paul-d'Eyjaux near Limoges. There he remained, if we may put it thus, just long enough to finish his work, and then escaped with his manuscript in his pocket in the last days of the year 1942. It was only a matter, he said, of putting the references at the bottom of the pages before turning the manuscript over to the publisher.

It was this final work of touching up that circumstances prevented him from accomplishing. In March of 1943, Cavaillès managed to get to England. He returned in May. In June, he barely escaped arrest by the German police. In August, he was arrested by German counterintelligence forces. In the interim, he had as a precaution entrusted a typewritten copy of his work, corrected by him, to his sister, Mme. Ferrières. A second copy, of which some pages were found in his room on the rue Chardon-Lagache after the German search, was kept by him after the final revision.

Cavaillès on several occasions stated to his sister and to us that he had no illusions about the difficulty of his text and that it would need a long introduction. He intended to write it but never realized this project. This preface, of course, does not in the least pretend to supply that introduction. We have limited ourselves to the purely material work of definitively establishing the text and investigating the references, without which the manuscript could not be published. Cavaillès, in short, has left us a work which is complete but not completely polished. When the clarity of the text or of the citations seemed to us to require the addition here and there of some expletive, we have always placed it in brackets. All the notes at the bottom of the pages are ours, with the exception of three indicated as being by the author. With regard to the citations between quotation marks, it will often be observed that they are rather free though always faithful in spirit. This should not surprise us, given the conditions under which Cavaillès worked and his working method as revealed by his other works. The title of the work, *On Logic and the Theory of Science,* is ours. For it is clearly not a treatise on logic. And since this text represents a part of the complete work but lacks

the introduction projected by Cavaillès, there was no ground for considering a title standing for the whole. The title we have proposed is more a formality than a program and withholds all reference to the spirit of the doctrine it covers. It is simply the resumé of the reading, without premeditation or rethinking.

The decision to publish this text as it stands, without an introduction, was made by the family and a group of Cavaillès' friends. It seemed that an explanatory introduction made by anyone but himself could after all only be a commentary, bringing to light that level of philosophical and mathematical culture which Cavaillès precisely wished to be taken for granted, no doubt judging that those who did not make the effort to understand did not deserve to be enlightened.

We now present this text to the public with the assurance that it will raise, through its own importance and through comparison with the other works of our friend, all sorts of problems related to the profound intention and to the genuine significance of a work so tragically interrupted. That these problems cannot honestly receive their necessary solution will only render more striking the void left in philosophy by the death of him who rests in the cemetery at Arras, under the cross which identifies him as "Unknown No. 5." ‡

Strasbourg
May, 1946

GEORGES CANGUILHEM
CHARLES EHRESMANN

CAVAILLÈS' TEXT

IT IS SAID in the *Course on Logic* [of Kant] that to have recourse to psychology would be "as absurd as to draw morality from life. It is not a matter of contingent rules (how we think) but of necessary rules, which must be drawn from the necessary usage of the understanding, which we find in ourselves without psychology." [1] Yet the beginning of the *Course* unfortunately

‡ On November 11, 1946, the body of Jean Cavaillès was placed in the crypt of the chapel at the Sorbonne.

1. *Immanuel Kants Werke*, ed. Ernst Cassirer, 11 vols. (Berlin, 1912–18), Vol. VIII: *Logik*, p. 134. (References to the Cassirer edition are hereafter cited as "Cassirer.")

recalls that of Arnauld. "Everything in nature happens according to rules, including the exercise of our faculties. Likewise the understanding . . . Understanding is to be considered as the source of rules in general. But according to what rules does understanding itself proceed? We cannot think and use our understanding except in conformity to certain rules." [2] In other words, science is the product of certain faculties, understanding and reason. Logic can only be defined afterwards in relation to these faculties, even though it claims to direct them. An abstract definition of these faculties will no doubt be given. Understanding, the source of rules, is "the power of judgment," and all of the judgments are functions of the unity of our representations, while a function must be understood as "the unity of the action of ordering different representations under a common representation." [3] But it remains that the notions of action and of power, which make sense only by referring to a concrete consciousness, intervene here in a fundamental way, not to mention the notions of order and of unity which, far from defining logic, are posterior to it.

The necessity of rules, i.e., their unconditioned normative character, thus remains subordinate to the absolute of a consciousness whose presence and essential structure—which is consciousness in itself—are an irreducible which no rational content defines. There is nothing prior to consciousness. But what is important is what is being thought here, not directly attained or intuitively posited in an original act. What is important, for the definition of logic as well as for any discipline, is the initial term to which a development will be connected in an intelligible way. The test of the theory of consciousness as far as logic is concerned is the determination which it provides of the content of these apodictic rules.

Here again we discover the kinship to Port-Royal. Logic is no longer a matter of a reflection of the mind upon its acts. Instead, it is "knowledge through themselves of the understanding and of reason according to form." [4] Understanding and reason, no matter how abstractly they may be described, remain powers of the

2. *Ibid.*, p. 332.

3. *Ibid.*, p. 408. In fact, the definitions cited by Cavaillès are found exactly in this form not in the *Logic* but in the *Critique of Pure Reason,* Transcendental Analytic, bk. 1, chap. 1, sec. 1.

4. *Ibid.*, p. 333.

irreducible consciousness and as such are characterized by the property of an internal self-illumination. The act is present to itself and can thus determine itself. But what is an immediate determination which would not be realization, and how to separate it from its singularity? The problem of method (or of rule) is resolved here by recourse to the notion of form.

It is because the self-knowledge of understanding and of reason take place "according to form" that they are possible—inasmuch as they bear on an absolute whose form yields their results—and that they produce a necessity. By recourse to form, the philosophy of consciousness strengthens itself and likewise illuminates itself, at least the way Kant presents it. In fact, the ambiguity of the formulas in which the nature of the invocation of our thought intervenes then becomes inoffensive. The starting point is indeed the actual, the immediate experience of an actual consciousness with its empirical accidents, but the absolute finds itself isolated as a result of a double process of elimination.

In the first place, the contingent is expressly rejected from it insofar as it finds itself implicated in a concrete experience. This is the passage from the empirical to the pure or to the a priori. There are two movements here: a positive one in which the a priori reveals itself independent of all experiences and conditioning them, as in mathematics; a negative one insofar as the empirical manifests its essential fragility in the unpredictability of its characters, in its inclusion of that which to some degree is unlawful. One of the fundamental difficulties of Kantianism appears here, in the positing of something totally empirical which, radically heterogeneous to the concept, does not permit itself to be unified by it. If experience is the singularity of an instant, no imaginative synthesis can integrate it into the unity of consciousness. The application of the synthesis as an act upon a datum presupposes a preliminary definition of the datum, as any sort of possibility that can be considered independent of the act. It is thus already in consciousness, as an element which is positively perceived and in a certain relationship with the act—unless the act alone is considered. In other words, a negative positing of the empirical, even if it were only to eliminate it, is inadmissible. Since what is at stake through its rejection is the securing of pure consciousness, there must be a way of relating it to something else. But in essence this "something else" is not consciousness. It accordingly eludes all attempts to grasp it, and

the suspicion arises that this pseudo-empirical is only the consciousness once again, denying itself in a game in which it is the first to be deceived, leaving no way of distinguishing in itself degrees of absoluteness or of necessity except through the intelligible sequence of the contents themselves. But then it is these contents which must be attained; they are really what is essential in their movement, and the primordial pseudo-experience of the consciousness dissipates before the autonomous dynamism which they manifest and which no longer leaves room for anything but them. A purification takes place only in a setting where all the levels can be situated and be given meaning. If it were merely a nothing of intelligibility, the impure could not be deleted. But the doubt rebounds upon the positive movement. If the a priori is an a priori of condition, like the representation of space in relation to perception or mathematical thought in relation to physical thought, both condition and conditioned must be located within a system, or at least may be considered as such. Accordingly, either the kind of connection between them is already given—which presupposes a known logic to posit this absolute consciousness which is to reveal it and upon which it depends—or it is uniquely realized each time; but then it is the sequence itself which of necessity will take its course, and it will no longer be a question of purification.

The second elimination isolates the formal from the material. But the notion of matter is a limit notion, in itself devoid of meaning. This is what Hegel was obliged to affirm. "A matter (or content) without its concept is something extraconceptual, therefore without essence." [5] Here therefore we can no longer proceed positively. But a process of radical abstraction seems to provide the solution. "General logic abstracts from every content of knowledge, i.e., from every relation of knowledge to its objects." [6] Not only are the objective contents neglected but also the way in which they present themselves to knowledge. The formal coincides with the act of thinking in general, i.e., with the act of unifying diverse representations under a single one. It

5. *Greater Logic:* Introduction to the third book, *On the Concept in General,* ed. Lasson, vol. II (IV of the *Complete Works*), p. 232. We owe the communication of this reference to the kindness of André Kaan.

6. *Critique of Pure Reason,* Transcendental Logic, Introduction (Cassirer, III, 82.)

is not a question of thinking this empty thought or of carrying out the abstraction itself, but of having the assurance that logic exists even when every object is set aside, characterized as an internal armature not directly attainable by consciousness but which posits it as an original essence in the presence of the object. "Similarly universal grammar comprises the simple form of language in general," [7] without the words which belong to the language, where language in general equally eludes every concrete grasp. It remains to be seen whether the comparison is not lame, whether logic thus defined can actually be realized. First, the notion of a universal language raises serious difficulties. In particular, the words—or semantic system—certainly do not represent a matter in relation to the corresponding syntactic system, but both admit of elements of variation and permanent elements. Then, the process of abstraction is here applied to something actually concrete for which, on the one hand, the process of positive elimination can be realized (by the finite and actual comparison of different languages), and [for which] on the other hand there is a possibility of dissociating individually verifiable concrete totalities from the intelligible connections heterogeneous to them, inasmuch as these concatenations unify them. The abstraction which gives logic its radical character falls to the ground. This abstraction does not acquire a positive qualification either from the aspect of matter, since it bears on the indefinite "every object," or from that of form. How is a science possible if it has only the notions of unity, plurality, and representation at its disposal? What can be drawn from the requirement of the agreement of thought with itself except eternal repetition? In order for agreement to assume its full meaning, there must at least be a differentiation within thought, and the occasion for disagreement must already possess a content. But then logic would be transcendental or dialectical logic. Now general dialectic or a cathartic of the understanding is unlawfully grafted onto logic. The notion of negation is no more a part of the preceding definition of logic than the notions of concept, judgment, reasoning, and method. To be sure, we can repeat the radical operation for each of them, eliminating all objective content and regaining that pure form of unity. But the disturbing thing is that we always regain a selfsame unity and that

7. *Logik*, Cassirer, VIII, 332.

there is no way—which may not forfeit the abstract—to tie these four unities together. It is because the actual consciousness establishes definite relations between them that the four abstractions realized separately permit the supposition of the persistence of abstract relations. But can we indicate them by referring to nothing but the emptiness of logical identity? What is the subject and the predicate, what are the categories defining the judgment, the rational sequences or science in general, if we do not refer to an ontology, as Aristotle's two *Analytics* do or as another analytic after Leibniz outlined. There must be a substrate or substance in order to distinguish subject and predicate. The act of judgment must be acknowledged as an act that can be reached as such and linked with the act of reasoning. But a direct characterization presupposes knowledge, i.e., construction of the object of the concept (here judgment and reasoning) in intuition. Now Kant refuses to make logic into "an algebra by which hidden truths can be discovered." [8] He likewise refuses to identify general logic with transcendental logic.

Yet these were the only two ways to make logic into a body of knowledge, since in the second case "it has before it a manifold of the sensibility which the transcendental aesthetic offers to it in order to give a matter to the pure concept of the understanding." [9] For the rest, the Transcendental Aesthetic does not play a necessary role here, as the abstract part of the transcendental deduction shows by considering only the unifying activity of thought in the presence of a receptivity supplying the matter of its objects—without the intervention of the proper forms of this receptivity. What is important is the synthesis performed by the understanding upon a sense manifold, i.e., the primordial logical operation. "The same function which gives unity to different representations in a judgment also gives unity to the simple synthesis of different representations in an intuition, a unity which in general terms is called the pure concept of the understanding." [10] But what is this concept outside of the synthetic act? We clearly see that Kant wants to respect the possibility of an understanding which would not receive the manifold to be unified from any other faculty, but would produce

8. *Ibid.*, p. 339.

9. *Critique of Pure Reason,* Transcendental Analytic, bk. 1, chap. 1, sec. 3, § 10 (2d ed., 1787). (Cassirer, III, 96).

10. *Ibid.* (Cassirer, III, 97–98.)

it itself. "We do not have the slightest conception of such a power." [11] So we ask ourselves if such an understanding would have something in common with the one which the formalizing analysis isolates and whose legislation defines logic. "The same understanding . . . , through the same acts [by which] it established the logical form of a judgment in the concept [in availing itself] of the analytic unity also brings a transcendental content into its representations by means of the synthetic unity of the manifold in intuition in general, by reason of which they are called pure concepts of the understanding, applying themselves a priori to objects which general logic cannot realize." [12]

The formal priority of analytic unity over synthetic unity consequently becomes at least dubious. Where can this analysis truly be realized? Here we have reference not only to *consciousness* but to *a world* which it would think and whose plural structure would be preserved by the process of abstraction. This is actually what the *Course in Logic* makes explicit when the traditional definitions are considered: "A concept is a representation according to common or discursive notes." [13] It originates from three classical operations: *comparison* of three representations among themselves, in relation to the unity of consciousness, *reflection* on the possibility of understanding different representations in one consciousness, finally *abstraction* which suppresses everything in which the representations differ. In other words, the unity of the empirical concept must be borrowed from experience; it is the analysis of a datum. But if the abstraction which isolates the understanding retains this analytical character, it must maintain the notion of a determining datum. We then no longer see how to come back to the synthetic stage. If our interpretation of the reference to consciousness is correct, when the elimination of what is extrinsic makes the absolute nucleus appear, this cannot be the same understanding which was synthetic at a step of lesser abstraction and becomes analytic at its extremity—no more, for that matter, than the possibility of analysis being a foundation for synthesis. If the philosophy of consciousness can provide a logic, it must arrive at an irreducible act, the act of unification. From the transcendental deduction, we sense almost fully the malaise which comes

11. *Ibid.*, bk. 1, chap. 2, sec. 2, § 17 (*in fine*). (Cassirer, III, 119.)
12. *Ibid.*, bk. 1, chap. 1, sec. 3, § 10. (Cassirer, III, 98.)
13. *Logik*, Cassirer, VIII, 399.

from the utilization of general logic as a guiding clue and a preliminary ground, received ready-made without critique or justification. But the notions which it uses—quality, quantity, relation—either are of alien origin (and to be justified if they are retained afterwards) or are posterior to the act of synthesis. Besides, how can logic be called transcendental analysis if it does not replace the ancient theory of *logos* in an authenticity without division? The collaboration with traditional logic is impossible. In a philosophy of consciousness logic is transcendental or it is not.

But then it is an ontology at the same time. This is acknowledged clearly enough in the *Report on the Progress of Metaphysics since Leibniz and Wolff*, in order to subordinate it to metaphysics "for which it is only the vestibule." [14] It is a curious thing and a consequence of this dual inspiration that science is not directly subordinated to logic, in spite of the *Analytic of Principles*. In the first place, general logic, reduced to the role of *canon* in relation to science, juxtaposes, following the Port-Royal tradition, a chapter on method to chapters on the concept, the judgment and reasoning. "Just as the elementary doctrine of science has the elements and conditions for the achievement of a knowledge, so the general theory of method, as the other part of logic, deals with the form of a science in general or of the way to tie the manifold of knowledge into a science." [15] This definition represents a change in relation to the Cartesian definition by putting the accent on the formal notion of science, but it could not succeed because of the inadequacy of its theories of judgment and reasoning. Upon the distributive unity of understanding, reason superimposes the collective unity required by the ideas and which has no other instrument than the syllogism. Since its only procedure is subsumption, science would then fatally assume the Aristotelian uniform of a hierarchizing classification.

Now in the second place, on the contrary, for science the *Organon*, i.e., the discipline "which indicates how a particular knowledge must be put on its feet," is "mathematics . . . which contains the foundation of the extension of our knowledge in view of a certain usage of reason." [16] In the *Analytic*, the differ-

14. *Ibid.*, p. 238.
15. *Ibid.*, p. 443.
16. *Ibid.*, pp. 333–34.

ent principles direct or found the extension of our knowledge with the same right as the axioms of intuition. The principle of causality, essential source of questions, is extramathematical. Yet Kant previously declared that there is science only insofar as it is mathematical. Is it simply a matter of expressing relations which are actualized in intuition? But we cannot dissociate a meaning and an expression of a relation: to what extent does causality, tied to the hypothetical judgment, express itself mathematically?

The same difficulty is found in a magnified form in the *Metaphysical Foundations of Natural Science*, where the requirement of a scientific ideal based on the notion of demonstration leads, as a result of its indecision between the syllogism and mathematical construction, to the creation of that pure and necessary part of a science from which its development is deduced through the addition of empirical principles. It is well known how, in the execution of the program, pseudo a priori elements and empirical elements are joined in an inextricable fashion for phoronomy, dynamics, and mechanics. The notions of effective velocity (in a movement distinct from mathematical displacement), of inert force, of active force already presuppose a recourse to experience. But in what do they consist? Is it a matter of a process of incomplete abstraction? We do not see the limit which obliges him to stop, unless it is the extrascientific consideration of a physical universe. Neither do we see, as a consequence, the authority which can come from it. These are hardly simply unelaborated notions from the vernacular which serve as guides here. Now if the subsequent development of science is deductive, therefore analytic, all the empirical elements to come must already be found concentrated in the synthesis which produces these notions. A particular experience either offers no interest or overturns the edifice—unless the demonstration is of a mathematical order, through a construction of concepts. But then the physical concepts must be represented completely within mathematics, which Kant does not admit and which moreover raises the difficulty of the specific character of physical construction in relation to mathematical construction.

Finally, mathematical demonstration is hardly defined with clarity. The validity of the construction which permits one to start from the concept is based on the unity of the formal

intuition of space. But the extraintellectual character of this intuition renders illusory every effort to transform any science whatsoever into a deductive system, beginning with geometry. The Euclidean model itself—perhaps unknown to Kant—is impossible to maintain. In fact, geometry would consist of nothing more than findings connected to each other, with the connecting bond having no other authority than the autonomous affirmation of an act. Here again it is the recourse to consciousness which is indispensable. [But] can we speak of an irreducible element when it is no longer a matter of pure thought but of a sensible thought whose form seems as arbitrary as the spatial form? As for the unity of the latter form, it is a simple postulate which the uniformity of development of geometry could guarantee, if this unity existed and if it were legitimate to consider it within the Kantian system that destroys it.

The authority of method, as logical canon as well as mathematical organon, is thus reduced to the preeminence of a formality which does not even have an irreducible simplicity to guarantee it. Transcendental Analytic can indeed procure for science its framework, since "all of the empirical laws are only particular determinations of the pure laws of the understanding," [17] so that "understanding is not only a power of making rules for itself through the comparison of phenomena, but is itself the legislation for nature." [18] The type of science, its striving toward a classical mathematical type as well as its mathematical extension, completely neglects the bearing of the object on the structure of the theory. The theory is given in advance once and for all. The indifference to the object, the third characteristic of the Port-Royal Logic, is thus represented here by the subordination of matter to a form which tends to absorb it, even in the case of mathematization. There is no science as an autonomous reality and characterizable as such, but a rational unification, according to a fixed type, of a manifold already organized by the understanding, or a survey of a mass of evidence without plan or discovery.

Two possibilities are still open for the theory of science along Kantian lines, depending on whether the accent is put on the notion of demonstrative system or on that of mathematical orga-

17. *Critique of Pure Reason*, Transcendental Analytic, bk. 1, chap. 2, sec. 3 (*in fine*) (1st ed. 1781). (Cassirer, III, 627.)

18. *Ibid.*

non. To the first belongs the logical conception inaugurated by Bolzano and continued simultaneously and in different ways by the formalists and by Husserl. To the second belong the epistemological philosophies of immanence, as the philosophies of Leon Brunschvicg and Brouwer may be labeled.

It is perhaps preferable to forego the examination of the latter until our scientific epistemology has been systematically developed. The notions which they invoke are too closely tied to the development of science for it to be possible to specify them in the course of a simple classification of points of view. Besides, Brouwer's mathematical philosophy, arising from a technical problem, has been interpreted in different ways. Thus H. Weyl, in admitting that he is faithful to the orthodoxy of the school, wishes to relate himself to Husserl.[19] Yet it seems that the true thought of the master, expressed for example in *Mathematik, Wissenschaft und Sprache*,[20] or in the works of Heyting, is oriented in a different direction. In the first place, mathematics is an autonomous becoming, "more an act than a becoming," for which a definition at the origin is impossible but whose moments in their necessary interdependence betray an original essence. From the dyad to the elaborated theories, there is continuity and unpredictability. The only kinship with Husserl is the reference to an act. But this generative act and criterion of true mathematics must situate itself concretely, submit itself to conditions of actual accomplishment. If there is accordingly no method in the traditional sense, there is in fact a domination of the course of science by rules. They can in truth be modified at any time, according to the theory to which they are applied. Nevertheless, they have a common principle, the requirement of a positive constructibility, the refusal of an existential affirmation which refers to a collectivity merely posited in the abstract by virtue of an indefinite state [translating itself], when all is said and done, through a negation (or an indetermination): refusal of the excluded middle and above all of nonpredicative definitions. What relation this rule has to traditional logic, and to what extent it can replace or transform it, is what in part is specified by the formal developments of Heyting and above all by the role

19. On the same subject, see Cavaillès' note in *Méthode axiomatique et formalisme* (Paris: Hermann, 1938), p. 33.

20. *Monatsheft für Mathematik und Physik*, XXXVI (1929), 153–64.

played by the intuitionist doctrine in the problem of the foundations of mathematics. Formal ontology as well as a more detailed realization of mathematical epistemology must then intervene. We can only discuss two difficulties here. First of all, the reflection on the essence of mathematics—which founds the intuitionist norm—must be situated in relation to the spontaneous movement of its becoming, since it demands an amputation or rather a redress of classical mathematics. It is a question of knowing whether there is a reference to an absolute consciousness characterizable in other respects, to the content of concepts submitted to a dialectic which itself can also be apprehended, or finally to the irreducible specificity of the mathematical movement. But then the second question poses itself: how to distinguish the mathematical movement within the general advance of science or even of culture? "Mathematics," says Brouwer, "is the ordering of the world, rational thought of the world"—without submitting the terms "world" and "rationality" to any further critique. The relation to physics in particular remains vague. If every science is defined as a type according to Brouwer's rule, it is because here again mathematics serves as an organon, but to such an extent that it absorbs the rest. It has been observed that intuitionist mathematics is particularly useful in modern physics. Is it not because the intuitionist attitude is first of all the attitude of the physicist, because the representation of a world, in which the mathematical act is performed and which determines it by furnishing it with at least a point of departure and a point of application, remains a primordial and perhaps a dominating element? The independence of a direct theory of science thus finds itself rather gravely handicapped. However seductive it may be at first, and however supple its utilization in being thus linked with the advance of science, such an attitude leaves too many initial questions open, perhaps also too many vague ones at the end of the road, to be able to eliminate without further ado the other logical points of view.

The same remark applies to Brunschvicg's epistemology. In reaction against the logicism of Frege and Russell in which it saw a renewal of the Aristotelian tradition, it places thought, as a creation escaping every norm, in juxtaposition with its linguistic expression, which as a social phenomenon falls within the ambit of both the illusions of the city and the laws of nature. The requirement of communication on the one hand pushes to the

absolute that which was only a movement within a progression and which took on its true sense in the transition which it realizes and on the other hand transposes it into a system whose structure (e.g., the deductive syllogistic form) finds its justification within an organization or a social history which are themselves only parts of nature. The *idola tribus* is the revolution of the past, asleep and petrified; the presumed canon of science is a rule of pedagogy. Thus the disputes over the Cantorian infinite simply mark the resistance to a new creative thrust; likewise the periodic reappearances of the same paradoxes, like that of Epimenides, proof of the incapacity of language to express the indeterminate power of thought. But this indetermination itself remains a problem. The progress of consciousness is undoubtedly detected not externally through an increase in the volume of its results, but through an intensity by means of the spiritual enrichment that refined concepts attain or through a unification of disciplines previously separated. It remains that progress by its very essence appears [only] after it is realized, that as a consequence the science which as such detects it on the one hand locates itself with difficulty, and on the other cannot claim to dominate science. This is moreover not Leon Brunschvicg's wish: rational immanence eludes every grasp outside of the idea. But is there still room for a philosophy which is not a simple explicitation of the scientist's intentions? Can it even be asserted that the history of progress still does not remain external, a useful cathartic for the scientist to protect him from social temptations, but that only the actual sequence of intelligible contents is on the level of the spirit? Even that cannot be satisfying. The terms of spirituality and immanence presuppose the possibility of an asceticism or of a deepening of consciousness other than simply scientific comprehension. What is needed is either the absolute of intelligibility which legitimizes the Spinozist superposition of the idea of an idea, or the reference to a generative consciousness which has the property of grasping itself immediately in its authentic acts. In both cases an ontological analysis seems necessary. On the other hand, no answer is given to the questions of epistemology. Here again all the sciences are intertwined in the same movement. In particular, it is impossible to distinguish mathematics and the science of nature. The being of the world, of a world posited externally and which it is the vocation of consciousness to reduce to terms of interior-

ity, subsists as a determining condition of science. To know the world, to comprehend the world—here is a program which already represents an abandonment of creative autonomy, a surrender to a necessity which is related to nothing other than itself.

Emphasis on the necessary character of science is precisely Bolzano's overriding concern. Science is above all demonstrated theory. Bolzano in the *Rein Analytischer Beweis* [21] took issue with the overly facile recourse to evidence, an evidence that Descartes invoked only for the simple natures and which for Kant regulated all geometric reasoning. The crisis of growth of algebra then required a change in the type of evidence, as is shown in the *Stages of Mathematical Philosophy.*[22] For Bolzano, it was a matter of the radical transformation of verification into demonstration. On the mathematical level the inspiration is Cartesian, an achievement of intellectualization through analytical geometry, an achievement made necessary by infinitesimal calculus. The paradox of the actually infinite obliges one to reject the geometric continuum as a simple nature. Now, already the syncategorematic infinite of differential calculus claims a reference to a truly unique infinite as an absolute term. Thus in Leibniz, however much he rejects the infinite number—and does not see the possibility of a plurality of infinites—the actual division of matter at the infinite makes the arbitrarily small increment of a variable possible. Wherever we stop we hit upon an existing element that can serve as an extreme to the increment. The well-founded phenomenon of space has its intelligible base in the infinite extranumerical multiplicity of monads. Aside from other motives—philosophical or historical—Leibniz has a reason for relating mathematics to logic. Not only spatial intuition but number as well is relegated to the level of the imagination, inasmuch as it is by nature repugnant to the infinite, according to the classical antinomies reviewed by Leibniz' contemporaries. Through a revolutionary reversal, it is number which is driven out of the realm of perfect rationality and the infinite which comes into it. But it is no longer a question of the human intellect embracing a complete sequence in a single glance, even if this is only the system of the terms of a series.

21. (Prague, 1817). New ed. by B. Jourdain (Leipzig: Engelmann, 1905).

22. By Leon Brunschvicg (Paris: Alcan, 1912).

The necessary bond seen between two terms guarantees the whole. The power of extension by iteration or combination is both a source of novelty and a caution of intelligibility. In the divine understanding, the rational homogeneity of the finite and the infinite is thus expressed in the universality of logic. What is necessary is what can be demonstrated. This is the source of Leibniz' idea of the *ars combinatoria* and of the universal outlines of science and of the *ars characteristica*. The poverty of the means employed should not obscure the sense and direction of the enterprise. Total science thus defined posits the absolute and represents the only way of attaining a part of it.

Bolzano considers—and fails to resolve—the same problems of mathematical legitimacy. After the difficulties with the principles of infinitesimal calculus with which the eighteenth century was embroiled, it was he who first correctly defined the limit and introduced the notion of set, from which a twofold philosophical enrichment of the Leibnizian vein emerged. First of all, the very being of science is submitted to critique, in order to determine both what constitutes a science as such and the motive for its development. If we put aside the imperfections due to the times, the idea is decisive for our problem. Perhaps for the first time, science is no longer considered as a simple intermediary between the human mind and being in itself, depending as much on one as on the other and not having its own reality. Now science is regarded as an object *sui generis*, original in its essence, autonomous in its movement. It is no more an absolute than an element in the system of existing things. If we can speak of a science of a country or of an epoch, we do not mean something which may be included among the natural realities, but affirmations localized in space and time within the sociological phenomenon and belonging to science. Hence science is not even situated within the universe of cultural objects, characterized by the participation in a value which such objects manifest. On the one hand, contrary to these, its mode of actualization is extrinsic to it and not so closely tied to value that this may, as with a work of art, essentially implicate it in the accidental exteriority of a sensory system. On the other hand, it requires unity, i.e., it cannot be reconciled with an actual multiplicity of singular realizations. There are not different sciences or different moments of one science, not even the immanence of a single science in various disciplines. But these disciplines are conditioned

among themselves in such a way that the results as well as the meaning of one science require, insofar as it is a science, the utilization of others or the common inclusion within a system.[23] A theory of science can only be a theory of the unity of science.

This unity is a movement. Since it is a question here not of a scientific ideal but of realized science, incompleteness and the requirement of progress are part of its definition. Simply an autonomous progress, a self-enclosed dynamism, without absolute beginning or end, science moves outside of time—if time signifies a reference to the lived experience of a consciousness. In it, change is growth in volume through the spontaneous generation of intelligible elements. On the one hand, every concept or system of concepts, through that which it poses to itself, is at once exclusion and need of the Other. In this sense, the representation of an absolutely simple infinitude of all knowledge is an image without any relation to the reality of militant science other than pushing to the limit a property of movement, namely, the absorption of the prior by the posterior which justifies it and to a certain extent suppresses it. But the road should not be abolished, if we want it to be followed. The true sense of a theory is not in an aspect understood by the scientist himself as essentially provisional, but in a conceptual becoming which cannot be stopped. On the other hand, even for the sciences of nature growth occurs without external borrowing. There is a break between sensation or right opinion and science. Experience, far from being insertion into nature, is on the contrary the incorporation of the world into the scientific universe. Even if its sense is not elucidated, even if it appears as an opaque body, an obstacle to theories actually thought, its value as experience lies at once in its detachment from a world of singularity and exteriority, where that which is does not have significance beyond its actual (and determined) existence, and in the virtual unification over which it must necessarily preside some day. Thus scientific autonomy is simultaneously expansion and closure, negative closure in its refusal to borrow from or terminate in what is external to it. If total knowledge does not make sense—there is a hiatus with an absolute consciousness as real as the one with opinion, so that there can be no question of either preparing for

23. Whence the idea of a hierarchy according to relations of dependence, with a subordinated science utilizing the results of the preceding ones as regulative principles. [Cavaillès' note.]

it or starting toward it—neither does radical extrascientific knowledge. In order for it to be posited as outside, it would have to be placed on the same plane and would therefore already be an experience. Science is a Riemannian volume which can at once be closed and without anything external to it. No method issuing from this theory of science can recover the Cartesian decision: *sub scientia non cadit.*

The difficulty soon appears not only in justifying and specifying these characters but in situating the discipline which posits them. The theory of science is also a claim to validity and to intelligibility. It wishes to be the science of science and therefore a part of itself. Accordingly, its assertions must not be constitutive of a particular development but must appear immediately in a self-illumination of the scientific movement, yet distinguishing themselves from this by their permanent emergence. Such is the role of structure. In defining a structure of science which is only a manifestation to itself of what it is, we specify and justify the preceding characteristics, not by an explication which would have its proper place and would in its turn be an object of reflection, but by a revelation which is not distinct from the revealed, present in its movement, a principle of its necessity. The structure speaks about itself. But it is only a way of imposing itself with an authority which borrows nothing from the outside; it is only a mode of unconditional affirmation, the demonstration. The structure of science not only is demonstration but is identified with demonstration. In it the essential traits are again encountered: unity, necessary and indefinite progression, finally closure upon itself. The internal rule which directs it posits each of its steps; it is entirely in them and impossible to grasp *ne varietur* in any of them. We are reminded of Descartes' difficulty with the connecting evidences—or simple natures. The toe holds can be multiplied, and if the demonstrating link consisted in these, others would have to be superimposed upon them indefinitely. There is in reality no essential distinction between the hardened rings which seem to mark the terms and the movement that traverses them. But the movement does not stop. Demonstration, simply by the fact that it posits the goal, extends and ramifies the domain created by means of combinations which it establishes as soon as they are possible. Finally it cannot be allied with the undemonstrated. We again meet the Platonic reserve against the *dianoia* which lends its hypothesis to

the visible world, an arrangement without justification from which nothing necessary can follow. True science does not depart from the demonstrated. The modern conception of hypothetico-deductive systems is subject to the same critique. How can a principle or a union of principles, which in their content and in their totality are not themselves intelligible, be the starting point for an intelligible development? The heterogeneous alliance of a verified pure concrete and a mode of rational sequence is a simple image without thought. If there is science, it is entirely demonstration, i.e., logic.

The problem which then arises is the apprehension of this principle in its generating movement, to find this structure again, not by description but apodictically, insofar as it elaborates itself and demonstrates itself. In other words, the theory of science is an a priori, not prior to science but the soul of science, not having external requisites but requiring science in its turn. There is a twofold difficulty with Bolzano's solution. If it avoids subordination to a historical existent or to the absolute of consciousness, it must itself posit the totality of what it attains, to discern afterwards, if it can, the permanent essential element of what is mobile through it. It is, on the one hand, a construction of a pure theory of rational sequences, and on the other a relationship to science as it has developed. Without the resolution of these problems, scientific epistemology cannot constitute itself directly as primary, which is its ambition. But it is posterior to the analytic which gives content to its object, and to ontology which brings the object into being.

II

THE TENDENCY of a faithful theory of science is such that the demonstrable is not given its proper due. But unity does not mean uniformity: within the identity of its essence, the demonstrative can assume many aspects. Two dimensions in particular can be distinguished: a realm of singularities where the demonstrated adheres to the demonstration to the point of being indiscernible from it and characterizing a single moment of science; and the set of sequences in the strict sense which, although they are indispensable only to the extent that they actually demonstrate something and as a result cannot be renewed in

their integrity, present a kinship of type according to groups, a mark of the unity of movements which can be manifested in the abstract. Thus emerges the notion of logical form indispensable to a theory of demonstration. The process of separation is twofold: longitudinal, coextensive with the demonstrative sequence; vertical, establishing a new system of conjunction that uses the old as a base from which to start, no longer as a stage traversed by a movement but as an object of reflection in its true bearing. In each case, a property constitutive of the essence of thought (or of intelligible sequences) manifests itself: the *paradigm* and the *thematic*.

The *paradigm* is characteristic of the actualization, not of an actualization *hic et nunc* in the lived experience from which the extrinsic is eliminated, in the simple seizure of thought by itself, but of an actualization required by the sense of what is posited, i.e., a relationship which as such is affirmed only in the singularity of realization of the sequence, but which calls for this singularity only in an indefinite way; it therefore suppresses it in positing it and thereby reveals an internal principle of variation. From arithmetical reasoning on the finite integer to the most abstract sequences, the same liberating dissociation of meaning occurs. "If *it is* is said of the one which is and of the *one* of being one, and if being and one are not the same thing but equally belong to this thing . . . does it not necessarily follow that the one which is is a whole and that one and being are its parts?" [24] The being of the relation consists in what it adds to its origin and is therefore different from the necessity which makes it one. In spite of this necessity, it affirms an independence which is translated into relative indifference which in turn is the source of plurality. "And so the one which is will be an infinite plurality." [25] And so already for the identical judgment of identity where the repetition neglects what is repeated; so for each of those mathematical moves where the connecting act, as soon as it is realized, becomes the connecting type; so for that indefinite flight toward meaning which, liberated as a result of the intelligible setting and fabricated from it, immediately escapes "as in a dream" and thereby posits its originality as well. This alone is what makes polymorphy in the single rational sequence possible, those suc-

24. Parmenides 142d. [Cavaillès' note.]
25. Parmenides 143a. [Cavaillès' note.]

cessive fractures of independence which on each occasion unleash upon what comes before the imperious profile of what necessarily comes after, in order to surpass it. The synthesis that Kant discloses within thought requires no furnished or different manifold, but simply itself, which is a multiplicity through its moments and its progress. What is unified is not given in advance as diverse, for how could it be given if not already within a synthesis? And to presuppose it for an analysis is to transform it in spatial imagination. What is unified is really the elaboration of the acts insofar as each of them, forgetting and realizing itself at once within a meaning, posits its own being only as an element of a set recognized as a plurality and at once as a basis from which to initiate new acts.

And so the synthesis is coextensive with the production of what is synthesized. The later developments will furnish examples of this. What is important here is the disengagement brought about with each suppression of singularity. This is what is represented in logical calculus by the rule of substitution, i.e., the possibility of replacing in the new element that from which it actually proceeds with something else equivalent to it from the new point of view.

This is the source of the distinction at each step between matter, the singularity as origin, and form, the actual meaning. This is likewise the source of a distinction which, when projected into the absolute, would extend from one end of the general sequence to the other, like a continuous flaw through the nodes of the particular distinctions. It is an image favored by language, sometimes the consequence of an ontology as well. The illegitimacy of it is evident, since it misunderstands what it generalizes, namely, the motive of the necessarily progressive passage from the act to its meaning. There is no meaning without an act, and no new act without the meaning which produces it. It remains that we can, for a certain length of the rational chain, by placing ourselves immediately at the last term, consider its meaning as the unique form all along the successions of acts. We proceed in this way in the systematic exposition of a mathematical theory, where the initial inventory of the proceedings springs both from the analysis of the final meaning of the sequences and from the artifice which consists in considering these chains as preparations for this meaning without intermediate or ultimate renewal. This is the *moment of the variable:* in

replacing the determinations of acts by the place vacated for a substitution, we progressively raise ourselves to a degree of abstraction which gives the illusion of an irreducible formality. This is what Leibniz tried in passing to the absolute through the mirage of an infinite whose simplicity makes conditions and conditioned simultaneous. Such are the attempts toward the algebraic or geometric art of characteristic, the theory of determinants, the symbolism of infinitesimal calculus, where in each instance the desire to preserve only what is essential manifests itself. Such finally is the reduction of all judgments to the predicative judgment, of all reasoning to repetition or to the distribution of a single and identical formula. Here the spatial image of juxtaposition, the utilization of an elementary combinatory, which preserves the simple characters of the finite as it reveals itself to itself (resigned to justifying every hiatus in an opaque way) within the infinite, are at once tied to the origin of the enterprise and a cause of its failure.

But formalization is realized only when the rules which regulate the structures are systematically superimposed on their outline. *Thematization* [26] takes its start in the sequence seized en route, a trajectory which moves toward meaning. Thought no longer goes toward the created term but starts from the modes of creating in order to provide its principle through an abstraction of the same nature as the other one, but directed transversely. The theory of the syllogism provides some rudimentary examples—*nota notae est nota rei ipsius* and other rules—where we see that it is a matter not of grinding out a general model under which all the species of syllogisms can be ordered but of a new intelligible development taking its start in the *vis probandi* of the argument. The works of Kant and Lachelier are in this direction, in contrast with the classifying tradition of forms.

But the most numerous examples appear in mathematics: theory of groups, theory of linear operations, of matrices, topology of topological transformations. If Leibniz was here again a precursor, the rules which he gives scarcely surpass the trivial, and we must wait for Hankel and especially Grassman for a systematic development. These efforts toward an effective theory of operations are the historical origin of the calculi of Boole,

26. Here begins the examination of the second of the processes of separation distinguished on p. 375.

Schröder, and Frege and the general formal systems. The process is in itself not conditioned solely by the structured character of the sequences whose principle is the morphological law but by the very dissociation between meaning and act which longitudinally gives the general principle. The preceding analysis was incomplete. In the moment which liberates the meaning, there appears a duality between *positing meaning* and *posited meaning*, between the signification of an operation as "operated" or *performed*—apart from the problem whose exigency created its active singularity and stripped of the accidental character of its starting point through the possible substitution—and its signification as *operating*, i.e., no longer the sense of the passage but of its *quomodo*, where the result is no longer considered in its essence, however pure it may have become, but in its simple position of a result which results, consequently an indication of the modes which have produced it and to which our interest returns. Thus addition, which the longitudinal process renders indifferent to numbers, with letters being added, becomes multiplication or abstract addition, for which the transversal process gives the laws of associativity and of commutativity. In the first case, the notion is purified following in some way the same lines, by the positing of increasingly abstract forms. In the second, the new form which is constituted by the principles of the first appears on another level.

The dual interplay of the two processes is evident. On the one hand, not only are both born from the same emergence of meaning, but also the abstraction of the first favors the second. This manifests itself in the descriptive definitions of certain very elementary notions (powers of irrational exponents, Lebesgue's integral) where it is important not to misunderstand that it is a question of the position of the new act, prolonging the previous acts in spite of the rupture, since it includes them as a particular case. The transversal process is indeed reinforced by their retroactive influence, as the theory of operations indicates. On the other hand, like all meaning the positing meaning as an act for longitudinal development, i.e., every positing meaning is at the same time a posited meaning of another act (which may recall the traditional principle that every idea has a formal reality). Thus, as soon as it is initiated, the movement of deepening makes way for a new sequence of the first species. The idea of the idea manifests its generative power on the level which it

defines without prejudice to an unlimited superposition. Henceforth the problem is posited equally for relations between longitudinal developments tied to two positing senses of acts situated at distinct points of the same inferior longitudinal development. Here we have an unlimited intelligible complication which at once shows how the image of the separation between matter and form in the rational sequence was simplistic and how a systematic study of forms is indispensable. There is no formalism without syntax, no syntax without another formalism which develops it.

The theory of rational sequences (judgments, reasoning, systems of reasoning) is thereby not only justified in its claims but situated and initiated in its development. If the accidental is excluded, if both necessary form and principles are brought to light, formalizing is a process of founding. The theory of science takes over the heritage of logic, surpassing even its ambitions. Insofar as it identifies itself with the system of all possible formalisms, it absorbs rather than canonizes the totality of formalizable demonstrations, and so the totality of science. This was the solution at the time of Frege and Dedekind. Mathematics is a part of logic, which here means that there is nothing more in mathematics than certain formal systems. And it neglects the problem of a demonstrative knowledge external to formalisms, so that their relation to formalisms, e.g., in physics, remained undetermined. The progress made from Frege to contemporary logicists like Carnap and Tarski coincides with the distinction of the two meanings: logical theory does not have its justification in their revelation but is the second meaning, inasmuch as this meaning is developed longitudinally in acts; logical theory is not the totality of formal systems but the totality of syntaxes of all the formal systems. Here is the source of the principle of tolerance, which Carnap placed at the head of his treatise: "In logic there is no canon but the unlimited possibility of choice among the canons." [27] Here is likewise the source of the

27. *Logische Syntax der Sprache* (Vienna: Springer, 1934), § 17. Cavaillès' quotation is not found exactly in this text of Carnap, no more than in *Abriss der Logistik* or in *Der Logische Aufbau der Welt.* On another occasion, in his article on the Vienna Circle at the Prague Congress (*Revue de métaphysique et de morale* [1935]), Cavaillès, in referring to this same principle of tolerance of syntax, quotes a text from page 45: "In logic there is no morality. Each person can construct his form of language as he understands it."

solution to the problem of the relation between mathematics and physics: mathematics is all of the formal systems, and physics is a certain privileged system resulting from the principle of choice that experience constitutes. There is a coordination between formal relation and sense phenomena, by means of an evidence *sui generis* which belongs to the other type of demonstration distinguished, where the demonstration adheres to the demonstrated. It is through this that physical theory is formally determined.

A precise examination of the logicist position requires the actual definition of syntaxes [and] therefore cannot be done here. But from the start two essential difficulties appear. In the first place syntactic formalization: Tarski is one of the first to have clearly dissociated the plane of primary formality and the plane of its syntax, which belongs to another formality. While, prior to this, syntax was defined in ordinary language without appealing to any definite type of demonstration for its deduced laws, its more profound study demands its formalization. But to define the formal system S_1 of the syntax S, we must define the syntax of S_1 which is realized within a system S_2, etc.—an apparently inoffensive regression, since it does not enlarge the base set of all the syntaxes. Although it is a matter of superposition, we find ourselves led to choose, in favor of the syntax of a system S_1 on the same level as it, e.g., among the systems $S_{1\text{-}1}$ previously defined. This initial system must undoubtedly still be posited. It is a result obtained in the actual act of formalism that every system containing arithmetic can formalize its own syntax. The secondary act for which the positing meaning of the primary act is a posited meaning then coincides with it. There is here a sort of return of thought upon itself which was impossible to predict before its realization and which assumes its true scope only in it.

The difficulty is elsewhere, in the question of knowing how to construct the system which is the basis of all syntaxes. What Carnap calls the general syntax is only a set of abstract rules whose entire précis is still borrowed from actually realized mathematics and from its syntax. The syntactic imagination seems to lose itself in the void of a radical abstraction. With the precisions that have been given, it could even be shown that the paths predicted in advance, where later science was to have busied itself, have remained barren. There is a more serious flaw here

than an error in prediction, a misunderstanding of the essentials of formal sequence, namely, that its progress is necessary and in every instance conditioned by the real. A position simultaneous with all those that are possible, even though nonrepresentable and as such simply admitted in order to serve as a basis, is in contradiction with the notion of a formal system whose signification demands a generation through successive thrusts and transcendences. That all may not take place in a single stroke has nothing to do with history but is the characteristic of the intelligible. To misunderstand it is to leave the security of its immediate presence for a projection whose internal emptiness as well as the lapse into the historical appears in connection with the actualizations of a time and their accidental veneer. There is no attainment of an absolute, but a hypostasis of systems and procedures which exist only as transitory. To abstract from the type is not to fix but to arrest the essence.

On the other hand, the formal definition of a system is not complete with the enunciation of the syntax. It is again to the credit of Tarski for having constituted in its originality semantics alongside syntactics as such. It is in fact a question of not only giving the form of statements invested with meaning (rules of *structure*) and the modes of passing from one group of propositions to another (rules of *consecutiveness*) but of defining the objects themselves, the elements and intervening components with their properties in the sequences: variables, functions, individuals, demonstrations, definitions—in short, introducing the concepts of the system. This role of semantics raises a problem which Carnap had already attacked elsewhere, in distinguishing descriptive syntaxes and strictly logical syntaxes. What does describing signify for an intelligible system? The very notion of a complete formal system is at stake here. For an incomplete formalization (i.e., the simple precision of a process of sequence) the separation between positing meaning and posited meaning is immediately manifested without requiring that the reference to its groups of acts to which the meanings are bound should culminate in the organization of these groups into exhaustively defined systems. In other words, the positing meaning of an act *A* appears as the posited meaning of another act *B* without *A* and *B* being expressly related to two distinguished sets, but rather as immediately acknowledged parts of the general group of intelligible sequences, i.e., justifying themselves

as such in their very realization. This applies to transitivity, the law of the syllogism of implication, to the properties of operations, to the general calculus of operations. The new operations which intervene bring the evidence of their fulfillment to bear upon the prior operations—or acts. Yet the intermingling of these operative groups of different essences, historically the confusions, the sources of paradoxes which they involve, in itself the internal appeal of their own unities, lead to their ordered reunion into separate systems which borrow nothing from the outside, which are closed, i.e., formal.

The definition of these systems is then external to them, i.e., the positing meaning of their constitutive acts is bound like the posited meaning to acts which are not only acknowledged as distinct from the first acts but situated on another plane. In fact, the thematic process is attached no longer to a particular act or a group of primary acts but to the total unity of a system, so that its own unity is articulated according to the organizing divisions of the system. In other words, the total character of the primary object—in admitting that it is authentic—the absence of bonds of mutual dependence with other objects, doubly transforms the positing meaning which on the one hand, in totally isolating it, permitted the representation of the passage as such and, on the other, owed its simplicity to the indeterminate utilization of prior relations. In the ordinary case, there is not really a description but a characterization through the very sequence of the secondary acts. Here the absolute of the beginning and of the end in some way obliges one to institute a basis for the setting, to the creation *ex nihilo* of an intelligible universe. Is that possible? Does it even possess a meaning? The logicists believe they have escaped the question by identifying the primary acts with their sensible representation, determinable objects for which, if not the totality, at least the modes of construction seem exhaustively definable. The starting point is a set of signs or of species of signs, the acts of regulated means for organizing them into groups of a determinate form. Semantics and syntax are, respectively, a description of initial structures and of results of operations and a regulation of operational procedures. The sign restores to secondary sequences the concrete base of certain primary sequences, in particular the familiarity of ordinary mathematical procedures which elaborate [themselves] in an

analogous way, except for the absolute starting point. Formal logic becomes symbolic logic, i.e., mathematical logic.

In actual fact, the problem of description is not resolved to the same extent. The sign is not an object of the world, but if it does not refer to another thing that it would represent, it refers to acts which utilize it, the indefinite character of the regression being essential here. All the comparisons of mathematics with a spatial manipulation run afoul of this fundamental character of the mathematical symbol, cipher, figure, even a stroke of the pen, of simply being there as an integrating part or basis of application for an activity which is already mathematical. The symbol is internal to the act; it can neither be its starting point nor its actual result (which is a production of other acts). The definition of a complete formalism therefore cannot claim such a sensory barrier. What it assumes to be an absolute beginning is only the surreptitious recall of prior acts and sequences. Semantics must be explicitly recapitulated from their syntax insofar as the latter is organized into rules of employment, which are the only things that are significant for the system to be defined. The priority, the quasi-simultaneity of what this syntax brings is sufficient to justify a separation from the syntax which provides the rules of later sequences for progressive development. But then the totality of the formal system is not attained. The presence of a semantics which plays an actual role certifies the failure of the enterprise as an integral definition. Thus completed, the secondary system cannot therefore be complete. If it cannot discern its object, it cannot a fortiori close upon itself. Or rather the reference to the object here recovers the aspect of the mathematical thematization in which the properties of the primary act are elements in another process of sequence. But we cannot call this a description that would be a regression toward an immobile model. There is superposition for the acquisition of novelties, but not for the situation at the bottom. Since the secondary acts are not of an order differing from the primary acts, it is not a question of manifesting the essence of mathematics through them. As formalized, they are themselves mathematical. The theory of science can be clarified and specified through formalizations, but it is not constituted by them.

This is confirmed by the second difficulty of logicism, the problem posed by physics. The notion of coordination is not in

fact more directly usable than that of description, but rather presupposes it. We coordinate only what by the same token is part of a higher system. There is a coordination of physics with mathematics only after a mathematization of physics, i.e., a descriptive effort that logicism is powerless to define. The "protocol sentences" invented by its naïve realism presuppose what is in question, namely mathematical relations which are a translation or reduction of physical experience. But the physical sequence, no more than the mathematical one, does not know an absolute beginning. On the one hand, as mathematical relations, the assertions assume their meaning only as elements in a system already posited and possessing an experimental meaning in a more or less precise fashion. On the other hand, the experiment itself as a system of acts is internally organized in a way in which it is impossible for its elaboration to be interrupted, except by a superficial abstraction. The experimental acts produce new ones by a sequence *sui generis* which, at least as such, is independent of the mathematical sequence, since it is of a different essence. The physical relation appears at the intersection of these two processes. From their more or less complete coordination, physical theory is born, for which the doctrine of science must determine the site and provide the meaning. But this a fortiori escapes all formalization. To say that "on such a page of such a book, a sentence composed of such and such signs is found on line *n*" has no relation with a physical proposition, since the book was designated as a definite copy situated in space and time and the other elements that enter in (line, sentence, composition) are cultural objects that no physical experiment can claim to reach. It is likewise totally absurd to "coordinate" physical experiment with the phrase materially written in an astronomer's notebook, "at such an hour such a star passed the zenith." The true experimental process is elsewhere, in the intentions, the utilizations, and actual constructions of instruments, the entire cosmic-technical system in which its meaning is revealed and whose unity as well as relation with the autonomous mathematical elaboration pose the fundamental problem of physical epistemology.

The traditional image of the internal emptiness of formalisms which an experimental matter comes to fill, thus alone giving them the solidity of true science, is not absent in the logicist solution of a choice imposed by physical experiment

between parallel mathematical systems. The protocol sentences, by being placed in a definite way within the system privileged by their presence imparts to it as it unfolds the fullness of meaning that they draw from their sensory origin. We find the twofold prejudice that the mathematical theories, on the one hand, are juxtaposed systems without any necessary connection between them and, on the other, are not self-sufficient but assume a meaning only as an instrument for the knowledge of the world. But the possibility of justifying this subordination through the insertion of mathematics into the imaginative construction of experiments finds itself barred by the abstract character that formalism bestows upon them. It then hesitates between the naïve realism of Wittgenstein and the pragmatism of Schlick. The problem of the object poses itself in its full acuteness in every way. It is the world which is in question, the world that physics proposes to describe whose articulations are already given by logic. "The world is what is . . . logic is the architecture of the world." This reference to the object is significant for a theory whose initial intention was the exhaustive definition of the operational processes which were to be sufficient unto themselves. The object is an ideal pole toward which they can be said to converge, but it in no way intervenes in their elaboration and all of its reality is in them. But the descriptive intention of physics transforms everything: the object becomes not only determining, the point of departure for the theories, the basis of reference for their results, but also (in virtue of the subordination of mathematics) like an orienting instrument for the formal sequences themselves. The internal necessity of their development certainly stands, and the possibility of a choice among them gives them the same independence in relation to existing things as the understanding in the Leibnizian divinity had in relation to the will. But it is still a matter of virtual existence, therefore of a necessary affinity between these sequences and the characters of the existing object. With this, we are but one step away from positing that mathematics describes an object which is more general than the realized object, that syntax determines its conditions and therefore is itself also descriptive —and Wittgenstein takes this step. Even if we would want to reject his realism, the problem remains and through it the problem of the relations between mathematics and physics [is] transformed, as well as, by rebound, the problem of a theory of

demonstration. The notion of object, an immanent necessity for the physical sequences, is *ipso facto* imposed on those of mathematics insofar as the latter at the same time constitute the framework of the former, and must be distinguished from them, i.e., situated on the same plane and polarized in a comparable fashion.

And so the attempt to found a theory of demonstration directly seems equally illusory. The difficulties of the relation between syntax and semantics as well as the system of sequences which they determine indeed show that the mere notions of intelligible sequence, of meaning dissociating itself into positing and posited sense, cannot suffice. The representation of the object, banished from one region, reappears elsewhere, even if this be only the object formal system or the object operational act, object in the sense of a reality sufficient unto itself and manifesting in some way or other a duality with the pure act which is immersed in the sequence which it develops. As soon as the direct theory of the sciences or of science refers to the theory of demonstration, the latter requires an ontology, a theory of objects that finally fixes the relative position of the authentic meanings and of the independent beings which either claim to found them or to which they are not related.

III

HUSSERL'S PHENOMENOLOGY claims to safeguard us from both acts and objects as irreducible elements. Thanks to the discovery, prepared by Brentano, of the intentionality of consciousness, because of the correlation that it establishes between noetic acts and noematic contents, it claims to be capable of simultaneously assuring the reciprocal independence between objects and the processes by which they are attained, and the higher unity in which both secure their source and signification. And so the doctrines for which phenomenology represents the deepening synthesis, logicism and the theory of consciousness—by a kind of reconciliation imitating Leibniz and Kant—would find their partial justification by being restored to their proper place.

Formale und transzendentale Logik begins with the problems of the relations between formal ontology and apophantic.

Apart from the logic of truth which terminates in the actual attainment of the object, the theory of apophansis in its fullest sense is divided into three branches. First comes a study of *forms* (or a purely logical grammar) which describes all the structures, the possible architectures of judgments, and finally their modalization. The classification is accomplished by the subsumption of particular forms under a primitive general form (*"Sp is q"* subordinated to *"S is p"*) as well as by relating complications obtained by modifications to the initial simple form (*"If S is p"* related to *"S is p"*). This description acquires unity and necessity thanks to the notion of an operation subject to laws. Not only the modalizations but the transitions from categorial form to hypothetical form, the disjunctions and the conjunctions, even the fundamental form *S is p* "as a determination of a substratum of determination *S*,"[28] can be considered the result of operations whose combinations and laws can be fixed a priori. Moreover, these operations are universally subject to the law of iteration, by which the indefinite extension of the system of forms is guaranteed. And so the Leibnizian idea of a combinatory as uniform source of the entire instrumental apophantic reappears in an enriched form. But Husserl in fact does not concern himself with the explicit demonstration either of the internal justification for the unity of the notion of operation thus extended, or of a principle of diversification which brings together under this single notion aspects as heterogenous as those of simple apophantic combinations (conjunction and disjunction) and of modalization. The very notion of logical form determined from this notion thereby remains with no precise content or justification of its delimitation other than the permanent possibility of referring to examples.

The *analytic of noncontradiction* comes next. Its task is to study the relations of inclusion, exclusion, and indifference relative to the judgments whose structure and combinations are already fixed by the theory of forms. The evidence which the latter theory claims is distinction, i.e., separation into articulations whose apophantic elements are correctly arranged, without necessarily procuring thereby a guarantee of noncontradiction ("Every *A* is *B*, among which some are not *B*" is a distinct

28. *FTL*, p. 46. Cf. the French translation by Suzanne Bachelard, *Logique formelle et logique transcendentale* (Paris: Presses Universitaires de France, 1957), p. 75.

judgment). Thus the notion of rational sequence surpasses the pure and simple description of structures. Contradiction and consequence, although they can be influenced by the form of judgments and are thus amenable to grammatical analysis, represent an advance in the direction of indicating a possible truth. No matter how radically the separation between logic of truth and analytic may be maintained, the development of analytic is a condition for the logic of truth, inasmuch as the sanction that it provides can be exacted from all of the elements susceptible to an examination of truth. But it is a one-way relation. The privilege conferred upon noncontradictory statements is not justified by an intrinsic property of these statements, and the analytic remains purely formal. In other words, it limits itself to isolating a particular species among all of the possible structures of the theory of forms, and its characterization is immediately realizable by the presence of negations and implications combined in a definite way. The fundamental entity here is the judgment in the broad sense, that is to say, the integrating parts which never appear as isolated, like the hypothesis in the hypothetical judgment, as well as the architectures of already complex judgments compounded into a single affirmation. "The question of pure analytic is . . . : when are any judgments, as judgments and in their simple form, possible within the unity of one judgment, and according to what relations?" [29]

Finally the *theory of systems* or theory of theories considers the groups of judgments which are characterized by a certain unity of sequence. Historically the works of Hankel and Riemann provide the initial notion of a *theory of manifolds* whose deductive closure permits a field of indeterminate objects to be defined exhaustively. The notion of formalism is rigorously maintained. The addition then posited is no longer a concrete operation but a relation characterized by the properties fixed in the axioms. These axioms do not possess a realizable content but are properly only forms of axioms, premises for forms of reasoning leading to forms of conclusions. The only things that matter are the behavior and the modalities of sequence between judgments, according to the properties which the two prior theories can recognize in them. But then the notion of theory assumes its full meaning only when it represents the system of all the judg-

29. *FTL*, § 18, p. 57.

ments which can be deduced from these initial axioms, i.e., when it is definite (or complete). The theory of theories is truly itself only as a nomology, as a determination of the types of theories for which it could first decide whether they are definite, that is, such that every judgment (form of judgment) constructed in a purely logico-grammatical way from the concepts (forms of concepts) which appear in it, is either true, i.e., an analytic consequence of the axioms, or false, i.e., an analytic contradiction: *Tertium non datur.*

The edifice of a formal apophantic is thus completed. From the elementary grammatical inflections to the architectures of axiomatized systems, it fixes everything which is constitutive and determining for a valid statement through form alone, prior to any relation to a concrete object. Accordingly, since the very knowledge of the objects is expressed in judgments, the apophantic already enunciates the indispensable preface to all knowledge; it is first ontology. Yet the connection is not easy to establish. On the one hand, what is aimed at here is not the object, but the judgment about the object. And since it is only a question of its structure, there is nothing to prove that we obtain anything but the extrinsic conditions of expression or at least of thought, which determines nothing in itself but the accidental character of its actualization in a consciousness; the proper theme is and irreducibly remains the pure apophantic entity. On the other hand, independent of all logical development, there exists a spontaneous general ontology, formal mathematics, expanded into a *mathesis universalis* by Leibniz, in which what is directly aimed at are the properties of any object whatsoever. What in fact is number, what are the higher order operations, however complicated their superposition may be, if not those determinations to which the modes of appearance of any object whatsoever are necessarily subject, with a necessity which is related to the very necessity of mathematical development? What does the mathematician do if not describe or fix that which concerns every object, as an abstract element of a manifold? True, the abstraction is generally specialized, in correlation with the operational system of a theory. But, on the other hand, the object thus obtained is not the indeterminate X which would represent the individual as an irreducible individual of the physical world, stripped of every particular qualification, but what Husserl calls a *categorial entity,* whose range of complications

includes variable, function, set, and operation. Yet in and through the superposition of various levels, there is always the reference to "the universal empty *Object* or *Something in general*" [30] with its "derivative forms."

> From this is born the universal idea . . . of a formal mathematics in the full sense, whose domain is determined with precision as the range of the superior concept, *Object in general* . . . , with all the derivative forms which can be generated a priori in this field, which through construction always yields new forms. Besides the forms of set and number, these derivative forms are combination, relation, series, connection, whole and part, etc. This mathematics in its entirety is an ontology.[31]

Nevertheless, the "thematic separation" between *formal apophantic* and *mathesis universalis* should not mask their "actual solidarity." [32] Here an essential element in the Husserlian theory of judgment intervenes: the judgment is an expression of a "state of things" (*Sachverhalt*).[33] In other words, the role and the justification of the judgment is found not in the intelligible relation which it is in its immediacy, but in a relation, external and prior to it, which it is destined to express but which is directed through it toward another place, that of the world. The *Satz*, the position of any content whatsoever insofar as it is a judgment, dissociates itself into a position of judgment and a position of objects, of the *Sachverhalt* intended by the judgment and independent of it. The primacy of the *Sachverhalt* is a primacy of the object ("we are oriented toward the object"), and this manifests itself in all the stages of knowledge, even in that theory of knowledge which has the judgment as its term. "The syntactic transformations which are accomplished in the act of judging change nothing here," [34] in particular the operation of *nominalization* by which what was a property or an integral part of an asserted relation (therefore escaping the thematic field) becomes the theme of a higher reflection. For example, even plurality and singularity (the property of plural and singular

30. *FTL*, p. 68.

31. *Ibid.* [Husserl here actually describes the construction more fully, as "an ever new iterative construction."—Trans.]

32. All this is related to § 25: *Thematische Untersehiedenheit* . . . , *FTL*, pp. 68 ff.

33. *Ibid.*

34. *FTL*, § 42b, p. 100.

judgments) do not suppress the permanent and determining objective polarization through all the thematic levels. "No matter how many intermediate determinations of nominalized substrates there may be at the various stages, what matters ultimately are the lowest and primary substrates, in the sciences the objects of their domain; it is their determination which is intended through all the intermediate levels." [35] What emerges here is a kind of *principle of reducibility* by which, on the one hand, the scope and the true significance of every judgment are led back to a relation between primary objects and, on the other, homogeneity is reestablished between judgments from various levels. In fact "the syntactic operations have a double function": [36] they produce the categorial entities of whatever degree, and they introduce these entities to a syntactic role (subject, predicate) within new syntaxes. And so, due to nominalization, for example, a syntactic structure becomes a categorial object and what is intended and attained in it is always the primary object toward which every effort of knowledge tends. The categorial entities procured through syntax, "substrate, property, relation, species," [37] are qualifications of objects, and, since the superpositions performed here are necessarily actualized in the rational evidence of authentic knowledge, they are definitive for this knowledge and therefore for the object. They are essential elements of ontology.

Hence there is an equivalence of content between apophantic and formal ontology. Every element of the first is an element of the second and vice versa, since every object or every formal relation between objects is expressed by a judgment. There is nonetheless a difference of orientation between them. What apophantic fundamentally attains is in fact the judgment as such, with the property constitutive of its essence of being able to be true or false, or even suspended between truth and falsity and nonetheless retaining all of its characteristics as a judgment. To put it differently, it is the meaning of *Sachverhalt* which is in question here, a meaning in the sense of a relation whose actual validity is presupposed and which has import because it is presupposed and because it is actually true in reality. What is

35. *FTL*, pp. 100–101. [Husserl's phrase here is actually "the lowest and primary thematic substrates."—Trans.]

36. *FTL*, § 42d, p. 101.

37. *Ibid.*

important in the case of the formal judgment are thus its articulations, its fractions, the architectures into which it is inserted considered in themselves, although their signification requires that they be directed toward a possible object, without which determination, actually present at the moment when they are examined, they would be nothing. But there are still only possibilities of determinations here, and possibilities and determinations are the only two elements which are themes. By contrast, ontology must discern the object itself in its most general structure, the a priori conditions of its actual attainment. We can even proceed directly. Instead of following the natural line of the developments of abstract mathematics, we can immediately pose the problem of the properties of any object whatsoever. "What can we affirm within the empty region of the object in general? In this formal generality, the syntactic forms are placed at our disposal in a purely a priori way . . . the entire formal *mathēsis* springs from it." [38] But science as science, the system of the meaning of the relations between objects, is not in the thematic field. The *mathesis formalis* gives the determination of the possibilities of objects, while the apophantic gives the possibilities of the determination of objects.

And so the problem posed by the theory of science is resolved without sacrificing either the goal of objects whose being is presupposed independently of their attainment, or the autonomy of rational sequences. Moreover, the authority of logic over physics is thereby likewise explained. In fact, it is one and the same movement which, through mathematics, extends itself even to the realities of the world. There is no knowledge which can be halted in its course toward the self-enclosed intelligibility of a rational system. To know has only one meaning, and that is to attain the real world.

> A mathematics constituting itself into a special science as an end in itself need not concern itself with the fact that it is a logic and a logical method, that it has to exercise a function of knowledge, that its products are called upon to serve as formal laws of indeterminate cognitive sequences for cognitive domains which also remain indeterminate. It need not bother with the fact that the relation to a possible application, indeterminate and always open, belongs to its proper formal-logical signification. . . . But the

38. *FTL*, § 54c, p. 132.

> philosophical logician must concern himself with these matters. He cannot sanction a mathematics conceived *kata mēdemian symplokēn*, a mathematics which frees itself from the idea of possible applications, and becomes a subtle mental game.[39]

If applications are excluded, there is no such thing as mathematical knowledge. Mathematics, conscious of its original meaning, of what it truly is, divides itself into two parts: applied mathematics which is physics, and formal mathematics which is logic. It is only because he has forgotten his vocation that the mathematician can claim to be self-sufficient. "Formal mathematics becomes an art . . . one works with letters according to the rules of the game, as in cards or chess. The original thought which gives this technical procedure its proper sense is here obscured." [40] In fact, the application to the world dominates everything and unifies science, since formal logic in its ontological correlation gives the most general laws according to which things arrange themselves. "Formal mathematics is originally logical analytic, so that to its proper logical meaning and based on the cognitive interest, there belongs a range of cognitive function, i.e., of possible applications, which in all of their indeterminacy still belong to the mathematical meaning." [41]

In fact, the *principle of reducibility* guarantees that the most physical physics is never out of sight. "If we trace its syntaxes, every real and possible judgment leads back to an ultimate nucleus. . . . It is a syntactical structure of elementary nuclei which no longer contain syntaxes." [42] The reduction leads us to "ultimate substrates, absolute subjects (which are no longer nominalized predicates or nominalized relations), to ultimate predicates (no longer predicates of predicates, etc.), ultimate universals, ultimate relations." [43] As a result, "every conceivable judgment finally has . . . a relation to individual objects . . .

39. *FTL*, § 40, p. 97.

40. *H* VI, p. 46. See also the review published in Belgrade in *Philosophia* by Arthur Liebert, Vol. I, pt. 1 (1936), p. 121. We are indebted to Jean Hering for having been able to consult this article, and we thank him for his kindness and for the suggestions which he offered. Cf. also the French translation by Edmond Gerrer in *Etudes philosophiques*, Vol. IV, nos. 3–4 (July–December, 1949), pp. 242–43.

41. *FTL*, § 40, p. 97.

42. *FTL*, § 82, p. 180.

43. *Ibid.*

therefore a relation to a real universe, a world or a region of the world for which it is valid." [44] Formal logic, *mathesis universalis*, or a fortiori apophantic, absorbed in the complications of its own evolution, can forget this relation. The logic of truth, crown of the edifice, for which the rest is but a necessary though not sufficient preparation, reestablishes these perspectives. It is in arriving at knowledge that all the previous work is illuminated and that the respective values of the preliminary efforts—each tempted to take itself as an end in itself—assume their authentic sense relative to the one end being pursued. Formal ontology or apophantic logic represents the indispensable infrastructure in the architecture of science, one determining the most general categories of the object and the other the form of the meaning superimposed, in the sequence in which the object in general is pursued. But the object must actually be attained in order for there to be knowledge, mathematics in the end comes to completion in physics, there is only one movement and one knowledge for which the unity of the whole guarantees the validity of each of the moments and of their relations of subordination. Necessity is everywhere present and founded because it is unique.

It is thus the reference to the primacy of consciousness which in the end permits the difficulties to be overcome. As far as their being in relation to consciousness is concerned, the independence of the objects is not an affirmation of a heterogeneity which would involve subordination and, because of their diversity, a polymorphism of the corresponding knowledge. But consciousness is the totality of being; what it affirms is only because it affirms it, if it is truly what it affirms in full self-assurance. Since transcendental phenomenology is an analysis of all the claims and all the acts of knowledge, as in general of all the contents aimed at or intended by consciousness and of all the operations by which this aim is accomplished, it has to find in this correlation of the noetic and the noematic the means for resolving all the problems which are not false problems. There is no need searching elsewhere, since everything is only in its very life. And because its essence is truly conscious of itself, it permits an analysis that is an explanation. "Absolute being is being in the form of an intentional life which, whatever else it may be conscious of, is at the same time self-consciousness. The possi-

44. *FTL*, § 83, p. 181.

bility of self-reflection belongs to its essence, a reflection which discovers the original reality of its being under the veils of vague thought." [45] Habits, repetitions not actually justified of thoughts finding their meaning in their initial embellishments, thoughts muffled because they are rapid and confused—it is enough to raise these masks in order to bring to light a certitude of consciousness which does not admit of a beyond. It is the intentionality of consciousness, i.e., "the experience of having something in consciousness," [46] which explains and insures the duality between the intended object and the act which intends it, but the object is nothing but the pole of these acts and the system of contents, the pole of noeses, the unity of noemas. With its independence thus assured, knowledge is permitted autonomy of development and homogeneity of essence. In fact, since being is unique, there can be no unbridgeable gap between two domains of knowing. Sense experience or physical experiment are particular modes of evidence, but even if the diversity remains well-founded, it does not bring a barrier with it. Between the rational evidence of a mathematical demonstration and the sensory evidence of the historical perception of an object, there is the profound homogeneity of their both being the full insight of the same consciousness, so that relations of mutual conditioning are possible and justifiable through an analysis of the acts which secure both. Both the legitimacy of the relations and the means for their discovery are found in a coming to consciousness through the very consciousness of what it accomplishes. Truth is one in its many aspects because there is fundamentally only one knowledge, and this is consciousness. But there is no denying the variety of its manifestations insofar as it withstands the elimination of all that is confused and factitious. The role of transcendental analysis here is to recognize the genuine diversities and to establish their relationships. "It is only by a decisive return to subjectivity, to the subjectivity insofar as it establishes all the valid affirmations of the world with their content and in all prescientific and scientific modalities, as well as by a return to the What and How of the [productions] of reason that the objective truth can be rendered intelligible and that the ultimate meaning of the world can be reached." [47] The problem posed by

45. *FTL*, § 103, p. 241.
46. *FTL*, § 60, p. 143.
47. *H* VI, p. 70.

logic in particular will thus be resolved by bringing to light the role played by developments of formal ontology in the actual knowledge of the world. It is not a matter of asking how a priori categories can be applied to the real, nor how the form of physics, nomological or otherwise, can be described in an autonomous way by the theory of systems. It is sufficient to examine the point that logic and the theory of systems are in fact the indispensable preface for the actual thought of real objects. It is sufficient to describe their intervention and to bring to light the profound relations which make this necessary. The problem of phenomenology is a problem of *constitution*. In other words, it is a matter of "knowing, by scrutinizing all the strata and levels, the complete system of the configurations of consciousness constituting the original presentation of all the objectivities, and thereby making intelligible the equivalent in consciousness of the type of 'reality' under consideration." [48] The authority of logic has its foundation in its relation to the life of the transcendental subjectivity, which develops itself in internal necessity. It is here that the rules which logic asserts, as well as the general object which it considers and the products that it finally engenders, find their place as well as their relation with each other and with everything else. But it is not enough to have their assurance by decree. We must investigate and justify each of these by actually connecting them with the acts and the conditions of this life. Formal logic and the logic of truth find their conclusion and foundation only in transcendental logic.

Here we see the opposition to Kant, for whom the pure logic of the tradition retained an authority beyond critique, to the point of dominating it for the sake of guiding it. He in fact posed the critical problem too narrowly and from the outset too loftily. At first, it is the discovery of logical entities as independent objects possessing their own reality and still produced by the consciousness which is to initiate transcendental analysis. "How do ideal objectivities, which originate purely in our subjective activity of judgment and knowledge, and have original existence only as a manifestation of our spontaneity in our field of consciousness, acquire the sense and the being of objects [which exist] in themselves relative to the contingency of the acts and of the subjects?" [49] The answer comes close to asserting that the

48. *Ideen I*, § 152, p. 319. Cf. *Ideas I*, p. 390.
49. *FTL*, § 100, p. 233.

sense and the being of all objects can really be conferred only by a subjectivity which is their source and foundation. The problem posed by logic is transformed into a problem of the transcendental constitution of objective entities. It is only then, and parallel to it, that the problem of a nature attained and defined by science can be fruitfully examined. Even here the objects are tied in their definition to the transcendental subjectivity, and in a way more radical than with Kant, since the operation is already finished with regard to that which pertains to logic. But on the other hand, it is necessary for the same question to have been resolved in a preliminary way for prescientific intuitive nature. "It is only after the transcendental philosophy of nature, at first limited to intuitive nature, has been educed, that it will be suitable, through an incorporation of ideal realities, for motivating the development of a transcendental logic." [50] The abstract object of science has no more primacy than the object of sense intuition. The unity of knowledge demands a homogenization and a hierarchic sequence in the investigations which bring these transcendental dependencies to light. And so we once again discover the two driving motifs inherited from empiricist logicism and the philosophy of consciousness.

In the first place, experience remains the master of syntactic sequences, in the peculiar doctrine of the *relevance of nuclei.* "The syntactic materials which appear in the unity of a judgment must have something to do with each other materially," [51] through the internal interdependence of the contents which they join together. We cannot juxtapose just any syntaxes for a true judgment, simply limiting ourselves to following the laws of distinction and of noncontradiction. "This stems from the fact that the original genetic mode of judging (which is an eidetic genesis and not a psychophysical genesis) is the evident judgment in its most rudimentary stage [which is based upon] experience. Prior to every action of judging lies the universal ground of experience which is constantly presupposed as a harmonious unity of possible experience." [52] Whence the variety of a transcendental philosophy which starts directly from pure logic. The source and the justification of the apophantic unity must be

50. *FTL,* § 100, p. 235.

51. *Ibid.* On the relevance of nuclei, cf. § 87, p. 189; on the function of syntactic materials, cf. § 89b, p. 194.

52. *FTL,* p. 194.

sought in a preliminary affinity of the experiential contents which it organizes. It is thus important to examine this affinity first. "And so in its content every original act of judging," i.e., inasmuch as it no longer resorts to other judgments upon which to base itself, "has coherence through the coherence of things in the synthetic unity of experience, which is the terrain upon which it stands." [53] We see here once again the intervention of the *principle of reducibility*. For the categorial superstructures do not have definitive efficacy. In order to explicate the transcendental meaning of the judgment as it is really and truly posited (without allusion or symbolization), we must in the end go through them to reach the individuals, the brute singularity of the experiential ground. Hence the full value of evidence cannot be obtained on the categorial level. From this two consequences follow: on the one hand, the necessity of a transcendental aesthetic (in a larger sense than Kant's) which "treats the problem of a possible world in general as a world of pure experience" [54] prior to every categorial activity and defines the a priori conditions of unity *sui generis* which can be attributed to it and which will serve as a basis for the apophantic sequences studied by logic proper; on the other hand, the irremediable insufficiency of a universal analytic developing itself in autonomous fashion or, if you will, the superiority of "material" doctrines of science over the formal doctrine. Not only are they indispensable for the determination of the meaning of a positive science, fixing by means of eidetic variation (hence expressly without recourse to general formal ontology) the ontological categories of the objects which they attain, but also they have for their crown "the highest and most encompassing of all, the logic of transcendental phenomenological philosophy itself," [55] upon which both analytical logic and material logic depend. Now this second relation compromises, or at least transforms the very notion of logic.

Secondly, in effect, if traditional objective logic appears, like every science prior to transcendental analysis, as "a sort of philosophical infancy," [56] if transcendental logic is not a second logic but "the radical and concrete logic which can grow only at

53. *FTL*, § 89b, p. 194.
54. *FTL*, Conclusion, p. 256.
55. *Ibid.*
56. *FTL*, Introduction, p. 12.

the heart of phenomenological investigations," [57] it will not be developed within the empty abstraction of analytical logic but will itself be a material logic of phenomenological science. From the dissociation of this latter logic into superimposed branches comes the stratification of several transcendental logics. What characterizes phenomenological analysis is its "irremediable relativity, a provisional character" which comes from the fact that "at every stage it surmounts a naïveté but carries with it the naïveté" belonging to the stage to which it moves, making necessary in its turn a new deepening and the passage to the higher stage.[58] Now each stage is characterized by its own problems of evidence. Evidence cannot be prescribed once and for all but reveals itself each time in a mode of radical novelty through its immediate presence. Logic, the norm of truth, i.e., of evidence, is also new each time. If the logic of the positive sciences is oriented toward the world, "how does it stand with that logic under whose norms the transcendental investigations are realized? We forge concepts, we express judgments by drawing them from transcendental experience (from that which is given in the *ego cogito*) . . . we attain truths by adequation, we make inductions." [59] But these operations have only a homonymy with the corresponding operations of positive science, and the problem is to know to what new norms they must be subjected as conditions for the proper structure of their evidence. And so for the stages of egological phenomenology (through which *Ideen I* moves) "the notion of truth in itself no longer has a normal sense," [60] since there is no longer a relation to a "no matter who" but to the one possible subject. Truth is solipsistic here. We see "a peculiar transcendental discipline being developed . . . with essential truths, theories which are valid exclusively for me, the Ego [i.e., valid once and for all], without being able to vindicate a relationship with other subjects. Thus is posed the question of a subjective logic whose a priori has only a solipsistic value." [61] But there are higher stages, those of intersubjective phenomenology, where the truth and therefore the corresponding logic are defined differently. The subordination of analytic logic or the doc-

57. *FTL*, Conclusion, p. 256.
58. *FTL*, § 102, p. 239.
59. *FTL*, § 102, p. 238.
60. *Ibid.*
61. *Ibid.*

trine of the positive sciences to the transcendental logic which founds them is therefore not accomplished in a single stroke. In fact, "formal ontology as constituted by transcendental phenomenology is only a moment depending upon another formal ontology which is related to every being in every sense, to being as transcendental subjectivity and to everything that constitutes itself in it." [62] What is therefore needed is a logic to give norms not only for the constitution of the constituted being, but also for the constitution of the constituting being. Is this the same? Do the successive enlargements of ontologies corresponding to successive stages of phenomenological investigation come to the unique domain of an absolute formal ontology which absorbs and totally realizes the prior investigations? This is at least what Husserl affirms. But here it seems as if the course of the questioning has been arrested by a kind of violent truncation. One of the problems announced—and not examined—is to know how "it is possible to actualize the most general idea of a formal logic as formal ontology and formal apophantic on the absolute terrain where it constitutes itself within the framework of the absolute and ultimate universal science, transcendental phenomenology, as a necessary integral [layer]." [63] But [if] the absolute and ultimate science also requires a doctrine which governs it—ontology [and] apophantic—it cannot include this as part of itself. Perhaps we abuse the singularity of the absolute by withholding from it the coincidence between constituting moment and constituted moment. Besides, there is not really a coincidence but an insertion of the first in the second, since the norms of the constituting constitution are only one portion among the constituted constitutions. Now it seems that such an identification of planes is especially difficult to admit for phenomenology, where the motive of research and the foundation of objectivities are precisely the relation to a creative subjectivity. If this subjectivity is in its turn subject to norms, a new transcendental investigation would be needed in order to relate its norms to a higher subjectivity, since no content but rather consciousness alone has the authority to posit itself in itself. If transcendental logic really founds logic, there is no absolute logic (that is, governing the absolute subjective activity). If there is an abso-

62. *FTL*, § 102, p. 239.
63. *Ibid.*

lute logic, it can draw its authority only from itself, and then it is not transcendental.

Perhaps further phenomenological investigations will at least permit us to challenge a dilemma so bluntly posed. Yet it seems fundamental for the point of view of a philosophy of consciousness. If the *epochē*, in separating [the] transcendental consciousness from a consciousness situated in the world, removes the naïve aspect and somewhat scandalous aggressiveness of logical empiricism and psychologism, they still lurk beneath the surface of phenomenological development. We can recognize them by their fruits, especially in the two essential problems of logic and epistemology: the relation between mathematics and physics, and the role and the definition of logical entities.

We have already seen that Husserl divided mathematics in two: a formal part identified with general ontology, and an applied part integrated into physics. The formal part, in determining the general features of being, puts itself at the service of the knowledge which is nothing other than the attainment of the beings of the world. But can we say that physics itself is qualified for this end? Here again the internal exigency of the phenomenological attitude intervenes. On the one hand, mathematical physics in fact only represents a technical advance in relation to sense experience. It is an illusion dating from Galileo that the system of symbols and concepts of mathematical physics constitutes a cosmos more real than the world apprehended directly in intuition. "Through geometrical mathematization, we tailor for the vital world . . . a close-fitting garment of ideas." [64] But this is only a mask, a disguise under which the carnal body of things is alone important. There is no physical progress in the sense of a transformation of the landscape through the forward march of science. "The intuitive real world . . . remains as it is, invariable in its proper essential structures, in its concrete causal style." [65] Being is posed before science in the act which intends it; it is a constitutive character of its essence that both its reality and its internal structure are independent of the improvements and complications of this act. Since things are not in themselves but are founded as realities in the intentionality of consciousness, a

64. *H* VI, p. 51.
65. *Ibid.*

privilege must be granted to the primitive as well as permanent acts through which it manifests itself. The existing stable world demands perception just as perception demands the stable world, since in the end, even in a laboratory experiment, everything begins through it and everything ends in it. Physical theories are only an abstract interim, and their succession a play of shadows upon its immobile mass, always ready to intervene.

Also, on the other hand, their role from the start was exclusively technical. Only the sedimentations of traditions, the process of forgetting the primitive meaning of initiated habits could transform and inflate their importance. Physics is not a science but an art. "What we really do with it is prediction, a prediction extended to infinity. All life is based on prediction." [66] It is here that we slip toward the intuition of the vital. Physics is a speculation which is not alien to life but a moment of this life, a part of industrial technical activity, a part which can be isolated through abstraction alone or for the sake of a division of labor. It is only in this life that it finds its true meaning and its motive of progress. To predict is not to see in advance, to deny the event as radical novelty, to reduce it to something already seen like a regular manifestation of a permanent essence. The dialectic of prediction is that of regulated action. It involves both the refusal to abandon itself to time as dominant and situating itself within the rhythm of this time through which something takes place, across a necessary thickness of duration independent of conscious duration. It presupposes that the movement is irreducible, which therefore risks departing from itself in an adventure toward the Other, at once already there and not yet there, deceptive in anticipation and wont to move at its own pace. Its modality is probability and not necessity. Mathematization intervenes only as a coordination of spontaneous predictions, particularly in eliminating what is useless among them, through a process of infinite idealization. What does it mean to idealize, if not to smooth over the extrinsic, that which for the scientific mind is tied to the facticity (i.e., arising from the fact) of its present actualization and can make sense only in relation to it. Hence the invention of straight lines and perfect planes, the positing of an infinite and homogeneous space. The infinite is elimination of the arbitrary, the arbitrary in the choice of an example, in the

66. *Ibid.*

reputed obstacle to research and to construction. Measurement, a consequence of the postulate of homogeneity, is the domination of lived space through coordination and iteration of its actual apprehensions. Their synthesis, the exactness of relations characterizing Galilean physics, is simply the indefinite convergence of real operations and real contacts toward an ideal pole which before long will be substituted as their representative and conceived as more real than they are. Rather than genuine speculative moments, there is in all of this a methodic organization of a self-conscious and predictive action whose intermediaries and instruments can become thematic and take themselves as ends. If mathematics is in the service of knowledge, it is a service which accepts several masters.

But it is not a service which reveals itself only in its realization. We have seen that the categorial entities of ontology merely represent complications which can be suppressed at any time. But throughout this ontology, there remains the one object which is always intended, whose being is irrevocably posed, namely, the individual in the real world. And so we find again the thesis dear to empiricist logicism that mathematics does not have a proper content of knowledge. It is an organization or combination of what is already there, which alone matters, and we can undo and retie their interlacings without augmenting or diminishing the reality known. "What do the propositions of logic say? They all say the same thing, namely, nothing." [67]

Mathematics, the logicists of the neopositivist school repeat, is merely a tautology. Hence the short circuit from the idea of an abstract ontology to actual knowledge. We can ignore the progressive rational development and "pose the problem of a formal ontology to ourselves directly, without passing through the idea of a doctrine of science. . . . It is evident that the entire [formal] *mathēsis* will then emerge," [68] since such a *mathēsis* is a pure and simple combination which can readily be taken as complete. This is really the idea of a universal syntax as Carnap tried to describe it in a single stroke. We have seen what came of it. The mathematical sequence possesses an internal coherence which cannot be treated brusquely. Its progressive character is essen-

67. Wittgenstein, *Tractatus Logico-Philosophicus* (London: Routledge & Kegan Paul, 1922), proposition 5.43.

68. *FTL*, § 54c, p. 132.

tial and the decisions which neglect this lose themselves in the void.

A particularly clear example is Husserl's notion of *nomology*. It in fact characterizes for him the axiomatized theories of mathematics and mathematical physics. Only a *nomological* theory permits the univocal definition of the system of objects which constitute its domain. On the other hand, it can be incorporated into a larger theory without risk of contradiction and in preserving the permanent faculty of eliminating new elements in the results. With regard to the introduction of imaginary numbers, Husserl posed the following question: "When can we be sure that deductions which are obtained in working with the new concepts, but which yield statements in which these do not figure, are in fact . . . correct consequences of the axioms which define them? . . ." The answer: "If the systems are nomological the calculation with imaginary concepts can never lead to contradictions." [69] The advantage of a tautological conception of mathematics becomes apparent here. First, the entire content of knowledge is assembled in the enunciation of the axioms, so that the later deductions, the proper movement of the theory, merely make explicit what was already posited in advance as an independent and indispensable introduction to the mathematical labor. The progressive expansion of the theories is realized by juxtaposition, without any internal necessity or dependence of meaning between the restricted theory and the generalized theory. It can be halted at any stage. The domination by the thought which traverses and posits them is always present, since in instituting these systems of categorial entities what it is concerned with is only a technical apparatus which is to disappear, either at a single stroke or partially, once the statement relevant to real beings is obtained. It is indeed evident that for saturated theories, this condition is always fulfilled. Every proposition in terms of this theory is either demonstrable or refutable on the basis of the axioms. But then it is the utility of the detour which now poses [a] problem. Why is a demonstration simpler (unless the only one possible) because of the addition of ideal elements en route, which in the end disappear, e.g., complex numbers in ordinary algebra and real numbers in the arithmetic of whole

69. *FTL*, § 31, p. 85. [Husserl's word here is not "nomological" but "definite."—Trans.]

numbers? The reference to technique is a subterfuge. The success and value of simplification must be founded on an essential property. And so in the cases cited, the answer is not in doubt. Neither the theory of real numbers nor the theory of whole numbers are nomologies. The condition posited by Husserl for the expansion of a theory is indeed sufficient but not necessary. In the quasi-totality of the case, it is not realized.

The result of Gödel's work is well-known: every theory containing the arithmetic of whole numbers, i.e., practically every mathematical theory, is necessarily nonsaturated. A proposition can be asserted within them which is neither the consequence of the axioms nor in contradiction with them: *tertium datur*. On the other hand, the previous investigations have indicated the difference between the closure of the field of objects and the closure (or saturation) of its conceptual system. Abstractly considered, the two appear linked. In defining saturation, Husserl evokes the *axiom of inextensibility* introduced by Hilbert "for motives which go . . . essentially in the same direction, since it was in fact a question of closing the field of point-objects satisfying the axioms of geometry." [70] But the system of these axioms is not saturated despite the axiom of inextensibility (which for that matter has another end, to guarantee the continuity of the field: every convergent series of points defines a new point which by virtue of the axiom belongs to the field). Other simpler examples of nonsaturated systems and for an inextensible field have been given. Saturation evidently involves inextensibility but not vice versa. Here technique takes its revenge by subverting the constructions realized in an abstract realm that surpasses it. This surprising turn of events is particularly serious for the Husserlian conception of logic and mathematics. In the first place, the very notion of a theory which can be dominated and isolated cannot be maintained. If the nomologies are only the exception, it is impossible for the rest of the mathematical texture to isolate the prefaces (which are extramathematical) and to indicate the ruptures of dependence. Only the theories smaller than arithmetic, that is, the theories which may be called quasi-finite, can be nomological. Their development is indeed of a combinatory order, and their domination through the sole consideration of axioms is truly effective. But genuine mathematics

70. *Ibid.*

begins with the infinite. The incorporation of a theory into a larger theory is clearly subject to the sole condition of noncontradiction, but by virtue of the same result of Gödel, the noncontradiction of a theory can be demonstrated only within a more powerful theory. The demonstration retains its interest, which is to concentrate on a single procedure or a canonic system of procedures, the doubts being distributed over an indeterminate polymorphy of intertwined procedures. But there is no longer that apodictic assurance at the start. One must commit oneself to the canonic procedure, to the indefinite iteration of its employment. Thus the deductive sequence is essentially the creator of the contents that it attains. The possibility of assembling some privileged assertions at the outset is a source of illusion if we forget the operational rules which alone give them a meaning. Concrete axiomatics, like those of Hilbert for geometry, are in part responsible for the error by their reference to well-known notions. If we take away the indeterminate notions of point, straight line, plane, between, etc., the axioms still have a meaning, an aspect of assertion which would suggest the idea of exhaustive definition. Abstract axiomatics brings out clearly what is valid for both types, namely, that the only reality posited initially and constituting the unity of the system is the operation or the system of operations which are destined to simultaneously fix both assertions and rules. But here again the definition is not exhaustive, for it presupposes the constitutive operations of previous systems—with their characteristics and their results—and appeals as well to other operations which establish other theoretical unities. The body of a theory is a certain operative homogeneity—which the axiomatic presentation describes—but when the theory involves the infinite, the iteration and the complications furnish results and an intelligible system of contents which are impossible to dominate, and an internal necessity obliges it to surpass itself through an expansion otherwise unpredictable and which appears as an expansion only after the fact. There is no juxtaposition any more than there is an initial fixation. It is the entire body of mathematics which develops itself through the steps and in a variety of forms, and it is likewise this which in its entirety, technical devices included, accomplishes or does not accomplish the very function of knowledge. Knowledge, if secured by mathematics, can only be deduction. Second, the failure of an essential point renders the method doubtful. Neither

objective logic—the analysis or combination of the formations already produced by spontaneous logic and presented for communication in a systematic aspect which can be extrinsic, because as a system it wants to be definitive and not what it is in reality, namely, a simple movement of the actual development —nor subjective logic—foundation of the first inasmuch as it relates its products to the activity of absolute consciousness—can account for either the actual progress or the structures and entities which mark it off. We have already seen this with regard to the pure theory of forms and the theory of noncontradiction. The content of the decisive notions of implication, deduction, negation is not questioned but simply presupposed for the definition of noncontradiction, which then remains an external consideration of forms. The same holds for the relation between noncontradiction and possible truth. It would no doubt be the role of subjective logic to examine all of this closely. But with the phenomenological method and its point of view, subjective logic is limited to analyzing the constitutive acts and intentions of the transcendental subjectivity, i.e., to unraveling the entanglement of motivations and elementary actions, but without questioning the logical entity itself. It is evident that it cannot be questioned since no consciousness is a witness of the production of its content by an act, since the phenomenological analysis will always move only in the world of acts or, for the corresponding noemas, be able to dissociate architectures of contents. But in both instances, it will stop before the simple elements, i.e., the realities of consciousness which refer to nothing which is other. And even more, in order to qualify them, it will have to utilize those same fundamental notions. Thus for implication: "he who has the thought of a judgment and . . . sees an analytic consequence of it . . . cannot do otherwise than to judge in this way." [71] And for noncontradiction: "of two contradictory judgments, only one can be valid if it is fully or distinctly actualized." [72] A lived impossibility or a distinct actualization are the ultimate instances for phenomenological analysis. And the meaning of consequence or of analytic contradiction is not made into a problem. Otherwise stated, what can be obtained will only be a distribution of realized sequences, with the critical purifica-

71. *FTL*, § 75, p. 168.
72. *Ibid.*

tion bearing exclusively on the alterations produced by the condensed or symbolic employment of a process which finds itself again only in its primitive profusion, or through ruptures between a method [and] the setting or the end which give it its meaning, so that it is no more than a habit or a technique. This is why historical investigations are important. History reveals authentic meaning to the extent that it permits us to rediscover lost links, first to identify automatisms and sedimentations as such, then to revitalize them by thrusting them back into their conscious actuality. It is a double necessity of time that the immediate act prolongs itself in habit and that the new act is weighted down and aided by the system of muffled traces of the past. But every simple position in its freshness is irrefutable. The return to the origin is a return to the original. We have seen that to make comprehensible in the phenomenological sense is not to change a plan or to reduce a content into something other than itself but to dissociate the entanglements, to pursue the referential indicators in order to bring out into the open the polished system of acts which "no longer refer to anything." In this sense, says Fink, phenomenology should be called an archeology. On the one hand, there is nothing to question beyond the act or the content in their immediate presence. On the other, the higher authority is this very presence or rather the impossibility of dissociating a single part or character from it through variation without losing everything. The foundation of all necessity is this "I cannot do otherwise" of eidetic variation which, however legitimate it may be, is an abdication of thought.[73]

We can see the difficulty for our problem: neither the illegitimacy for contents is reduced, nor the progress justified as such. If empirical history is used to reveal essential sequences, it is on the wrong side, not as a forward movement, but through the myth of the return of the past. "Before the geometry of ideal objects came the practical art of land measurement. . . . Its pre-geometrical results are a foundation of meaning for geometry."[74] But in what way? Is this not an arbitrary decree which adapts the origin of consciousness to a temporal pseudo-debut which appears to us as such only by virtue of the sequel as

73. *Ibid.*: "er kann nicht anders als so zu urteilen . . ."

74. *H* VI, p. 49. [*Leistung*, here translated as *résultat* by Cavaillès, is usually translated nowadays as *performance, production, work.*— Trans.]

well as by approximation? Husserl also speaks of an immanent teleology for the history of philosophy, for example, which illuminates through "a harmony finally full of meaning . . . the hidden unity of intentional interiority." [75] Here there is no longer a return to the origin, but an orientation according to the flux of a becoming which presents itself as such only by the intelligible enrichment of its terms. To be sure, because of its indefinite plasticity, phenomenology justifies everything even to the point of a hierarchy among the types of evidence. Yet the authority of these has only one source. At least the evidence which justifies the transcendental analysis is necessarily unique: if there is a consciousness of progress, there is not a progress of the consciousness. Now one of the essential problems of the doctrine of science is that progress itself may not be augmentation of volume by juxtaposition, in which the prior subsists with the new, but a continual revision of contents by deepening and eradication. What comes after is more than what existed before, not because it contains it or even because it prolongs it but because it departs from it and carries in its content the mark of its superiority, unique every time, with more consciousness in it—and not the same consciousness. The term "consciousness" does not admit of univocity of application—no more than does the thing, as the unity which can be isolated. There is no consciousness which generates its products or is simply immanent to them. In each instance it dwells in the immediacy of the idea, lost in it and losing itself with it, binding itself to other consciousnesses (which one would be tempted to call other moments of consciousness) only through the internal bonds of the ideas to which these belong. The progress is material or between singular essences, and its driving force is the need to surpass each of them. It is not a philosophy of consciousness but a philosophy of the concept which can provide a theory of science. The generating necessity is not the necessity of an activity, but the necessity of a dialectic.

75. *H* VI, p. 74 [The break in the quotation has been introduced by the translator—Trans.]