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Brouwer and Nietzsche: Views about Life, Views about Logic

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Friedrich Nietzsche and Luitzen Egbertus Jan Brouwer had strong personalities and freely expressed unconventional opinions. In particular, they dared to challenge the traditional view that considered Aristotelian logic as being absolute and intrinsic to man. Although they formed this opinion in different ways and in different contexts (Nietzsche was born in Saxony and lived in the second half of the nineteenth century, beginning his career as a classical philologist before turning to philosophy; Brouwer was a Dutch mathematician who lived from 1881 to 1966), they both based it on a view of life that considered it as a struggle for power in which logic was a weapon. Therefore, it is interesting to carry out an in-depth analysis on the origins, the core of their opinions concerning logic and the consequences. As a matter of fact, a detailed analysis and a comparison of their contributions to the philosophy of logic have not yet been carried out. In this paper, I intend to present the results of my research on this subject performed by taking into account the studies conducted by E. Steinhart, O. Schutte and S.D. Hales.

1. Introduction

Both Nietzsche and Brouwer had strong personalities and freely expressed unconventional opinions. They even dared to challenge Aristotelian logic both basing their criticisms of it on a view of life that considered it as a struggle for power in which logic was a weapon. Therefore, it is interesting to carry out an in-depth analysis on the origins, the core of their opinions concerning logic and the consequences. The aim of this paper is to analyze these aspects to which little attention has been paid[1] and to compare these two authors. There is probably more familiarity with Brouwer than with Nietzsche as this is a framework of history of logic. For this reason, I will start each aspect of my comparison with Brouwer, even if he lived after Nietzsche.[2]

2. Life

2.1. Brouwer had a very gloomy and pessimistic view of life which he described in detail in his 1905 pamphlet entitled ‘Life, Art and Mysticism’: life is suffering, only suffering,

[1] Vladimir Tasić in his *Mathematics and the roots of post-modernist thought* carried out a short comparison between Brouwer and Nietzsche aimed at showing the partial similarities between mathematical and postmodern philosophical thought in the twentieth century, even if he stated (p. 47) that Brouwer appeared be closer to early Romanticism than Nietzsche since Brouwer claimed that it was impossible to be sure of transmitting unambiguous messages through words, which was similar to Humboldt’s view that no one means the same as anyone else. However, Tasić noticed that Brouwer and Nietzsche ‘share a penchant for being shocking’, and emphasized that both refer to ‘will’ and both consider the so-called real world as an apparition; only Brouwer believed that there can be space for creativity in mathematics (*Tasić 2001*, pp. 46–47). We will reconsider Tasić’s opinions at the end of this paper.

[2] Brouwer does not directly refer to Nietzsche. Only in Adama van Scheltema’s letter to Brouwer (6 August 1907), his friend described himself as ‘Apollinius’ and Brouwer as ‘Dionysus’ according to the terminology in Nietzsche’s ‘The birth of tragedy’. Brouwer replied (18 August 1907) that he would read the book but never mentioned the subject again in later letters (*Brouwer and Adama van Scheltema 1984*, p. 74).

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due to the desire and the will for power, which leads to the search for new targets, fatigue loads and concerns.

> It is part of the balance of the eternal and omnipresent life that everyone is called away from this earthly existence when one's time has come. Until then man suffers in mind and body as befits his evil mood of thrift, his lust for power, his vanity and fear. (LAM, p. 392)

It is therefore a source of frustration and disappointment:

> Life of the individual is an illusion, an anxious and laborious pursuit of ends—disillusionment. [. . .] Life of mankind as a whole is an arrogant tearing up and devouring its nest on this pure hearth, messing up its mothering growth, gnawing and mutilating her and making her rich creative power sterile, until all life has been swallowed up and the human cancer has withered on the barren planet. (LAM, pp. 392–393)

We can alleviate this suffering by returning to our 'self' and observing the illusory bubble of space and time disappear as well as the negative emotions that it produced:

> If you however succeed in overcoming all inertia and proceed, you will find that passion will be silenced, you will feel dead to the old world of perception, of time and space, and all other forms of plurality; and your eyes, no longer blindfolded, will be opened to a scene of joyful quiescence (LAM, p. 393) Your journey through this sad world will be a steady passage in a light and colorful cloud, full of love for all that is clear in it [. . .] for in your eyes it is no longer a reality separated from the self, but directed from within the self and with the self. You will no longer ask what to do: all will be clear of its own. (LAM, p. 394)

2.2. Nietzsche was well aware of the suffering in human life:

> Wenn Jemand mit einem 'seelischen Schmerz' nicht fertig wird, so liegt das, grob geredet, nicht an seiner 'Seele'; wahrscheinlicher noch an seinem Bauche (grob geredet, wie gesagt: womit noch keineswegs der Wunsch ausgedrückt ist, auch grob gehört, grob verstanden zu werden . . .) Ein starker und wohlgerathener Mensch verdaut seine Erlebnisse (Thaten, Unthaten eingerechnet) wie er seine Mahlzeiten verdaut, selbst wenn er harte Bissen zu verschlucken hat. (GM III, 16)

> Schmerz ist auch eine Lust, Fluch ist auch ein Segen, Nacht ist auch eine Sonne,—geht davon oder ihr lernt: ein Weiser ist auch ein Narr. Sagtet ihr jemals Ja zu Einer Lust? Oh, meine Freunde, so sagtet ihr Ja auch zu allem Wehe. Alle Dinge sind verkettet, verfädelt, verliebt. (Z IV, 10)

However, he declaimed and diffused a joyful and positive conception of life: he focused all his works on this theme with the aim of enhancing and defending the disruptive nature of life in its originality and creativity, from those who wanted to downgrade it to mere survival, scheme, rule.

> In media vita.—Nein! Das Leben hat mich nicht enttäuscht! Von Jahr zu Jahr finde ich es vielmehr wahrer, begehrenswerther und geheimnissvoller,—von jenem Tage an, wo der grosse Befreier über mich kam, jener Gedanke, dass das Leben ein Experiment des Erkennenden sein dürfe—und nicht eine Pflicht, nicht ein Verhängniss, nicht eine Betrügerei! (GS, 324)

For Nietzsche life is animated by the will to power, just as it will be considered by Brouwer.[3] Nevertheless, Brouwer will have a negative attitude and will believe every contact with the outside world to be morally negative. On the contrary, Nietzsche saw man as an explosion of will to power, that is, a will to expansion, which must be accepted and encouraged:

> Das Leben selbst gilt mir als Instinkt für Wachsthum, für Dauer, für Häufung von Kräften, für Macht: wo der Wille zur Macht fehlt, giebt es Niedergang. (A VI)

He observed science, mathematics and physics within this framework.

3. Science

3.1. In 1905 Brouwer placed the origin of science in the intellect which separates a 'subject' from a world of 'objects'—thus arousing desires and expectations:

> Intellect has made him forfeit the amazing independence and directness of his rambling images by connecting them with each other rather than with the self. In this way the intellect made him persist with apparent security in the conviction of a 'reality'. (LAM, p. 395)
>
> Bodily awareness has strayed so far away that it is all concentrated in the human head, ignoring and excluding the rest of the body. At the same time man becomes convinced of his own existence as an individual and that of a separate and independent world of perception. (LAM, p. 396)

Following which, intellect connects experiences in a cause–effect relationship:

> [. . .] In this life of lust and desire the intellect renders man the devilish service of linking two images of the imagination as means and end. Once in the grip of desire for one thing he is made to strive after another as a means to that end. (LAM, 395)

The need for science is due to the abandonment of the self (which knows all about past and future, what to do and where to go): without it, man became more and more anxious about the future and began to crave for the power to predict its course. Therefore, science came into being which had a specific purpose in the beginning and

> it may happen that a direction is pursued which not only deviates into other dimensions but opposes the direction of the original end and therefore counteracts it. (LAM, 395)

For instance, in the beginning, the purpose of industry was to create the most favorable conditions for human life in nature, yet by manufacturing products from nature's resources, it upsets the balance of nature itself and human conditions.

[3] The will to power had been a central theme in Schopenhauer's work; therefore, it may be interesting to compare these two authors to Schopenhauer. In the beginning, Nietzsche appreciated Schopenhauer for his reference to the will to power and for the role that he attributed to the arts. However, Nietzsche changed his mind definitively because of the negative value that Schopenhauer attributed to the will to power and for the 'solutions' that he had proposed to contrast this will: some form of compassion and asceticism. Brouwer did not mention Schopenhauer in the papers in which he described his *Weltanschauung* (except for a reference to his theory of colors as 'polar splitting by the human eye' LAM, p. 413), but we can see some similarities in Brouwer and Schopenhauer: they both inherited the idea that time, space and causality are a priori forms from Kant (although after 1907 Brouwer did not deem space to be a priori); they believed that the will to power is the motor of our life and the cause of our suffering (yet Brouwer considered will as being individual rather than unique—not as the 'noumenon' behind all phenomena); they considered asceticism as a solution to the will to power (however, Brouwer did not think that compassion could also be a solution). We could also add that the three authors shared misogyny (that prevented A. Heyting from including an integral version of LAM in Brouwer's Collected works—see CW I, p. 565). See also *Koetsier 2005* for a comparison between Brouwer and Schopenhauer.

Later, the means became an end in itself and science began to be practiced for its own sake; when it climbed too high it was almost completely shrouded in even greater isolation. The 'foundations' of its branches were investigated, and that soon became a new branch of science. 'As they climb higher and higher confusion grows until they are completely deranged' (LAM, 395).

Brouwer maintained this negative viewpoint throughout his lifetime but he refined it. In fact, in 1929 ('Mathematik, Wissenschaft und Sprache'), he described man's causal attitude in 'mathematical observation'. Mathematical observation was an act of our will *finalized to our self-preservation*, consisting of two phases: a temporal setting (zeitliche Einstellung) and a causal setting (kausale Einstellung):

> Erstere ist nichts anderes als das intellektuelle Urphaenomenon der Auseinanderfallung eines Lebensmomentes in zwei qualitative verschiedene Dinge, von denen man das eine als dem anderen weichend und trotzdem als durch den Erinnerungsakt behauptet empfindet. Dabei wird gleichzeitig das gespaltene Lebensmoment vom Ich getrennt und nach einer als Anschauungswelt zu bezeichnenden Welt fuer sich verlegt. [. . .] Nunmehr besteht die kausale Einstellung im Willensakt der „Identifizierung' verschiedener sich ueber Vergangenheit und Zukunft erstreckender zeitlicher Erscheinungsfolgen. Dabei entsteht ein als kausale Folge zu bezeichnendes gemeinsames Substrat dieser identifizierten Folgen. Als besonderer Fall der kausalen Einstellung tritt auf die gedankliche Bildung von Objekten, d.h. von beharrenden (einfachen oder zusammengesetzten) Dingen der Anschauungswelt, wodurch gleichzeitig die Anschauungswelt selbst stabilisiert wird. (CW I, p. 417)

'Mathematical observation' was followed by 'mathematical abstraction' (that emphasizes the common substrata, which could be the foundation of mathematics) that made it easier to manage causal sequences when they were projected onto subsystems of mathematics.

In 1948 ('Consciousness, Philosophy, and Mathematics'), he described the steps from the inner self to the sciences as follows:

> Consciousness in its deepest home seems to oscillate slowly, will-lessly, and reversibly between stillness and sensation. And it seems that only the status of sensation allows the initial phenomenon of the said transition. The initial phenomenon is a move of time. By a move of time a present sensation gives way to another present sensation in such a way that consciousness retains the former one as a past sensation. [. . .] Through this distinction between present and past, [consciousness] recedes from both and from stillness, and becomes *mind*. As mind it takes the function of a subject experiencing the present as well as the past sensation as object. [. . .] In measure of this estrangement, mind become disposed to desire and apprehension. [. . .] The free-will phenomenon of *causal attention* occurs. It performs identifications of different sensations . . . and in this way creates iterative complexes of sensations. An iterative complex of sensations, whose elements have an unvariable order of succession in time, whilst if one of its elements occurs, all following elements are expected to occur likewise, in the right order of succession, is called a causal sequence. There are iterative complexes of sensations whose elements are permutable in point of time. Those which are completely estranged from the subject are called *things* and individuals. The whole of egoic sensations indissolubly connected with an individual is called soul.

> Causal attention allows the development of the conative activity of the subject from spontaneous effort to forethinking enterprise by means of the free-will-phenomenon of cunning act. [. . .] mind, once having taken to causal attention, remains in a lasting *causal tension* [. . .] Systems of causal thinking underlying cooperative causal acts, are far more complicating than those inducing individual causal acts. Prominent among the former is scientific thinking, which is an economical and efficient way catalogues extensive groups of cooperative causal sequences. And this scientific thinking, in particular when concerned with technique, is based on mathematics. (CW I, pp. 481–482)

In other words, he introduced a distinction between simple consciousness and mind. Consciousness has perceptions in time; mind is a consciousness that separates itself as a subject against a 'world' and this separation arouses desires. Therefore, there is a new way of considering the world: causal attention, that is, the act of emphasizing a range of sensations. In this way, things, individuals and also causal sequences are *shaped*. The following step of mind is 'causal thinking', whose social and economic form is 'scientific thinking', which is based on mathematics.

Brouwer noticed that, by observing the similarities between one's own behavior and the behavior of others, people also attributed a mind to them. He warned not to believe in a plurality of minds by demonstrating how many contradictions a belief of this kind could generate:

> Mind. By the choice of this term the subject in its scientific thinking is induced to place in each individual a mind with free-will dependence on this individual, thus elevating itself to a mind of a second order experiencing incognizable alien consciousness as sensations. Quod non est. [. . .] This moreover would have the consequence that the mind of second order would causally think about the plurified mind of first order . . . and so on. Usque ad infinitum. And this non sense would still go further. In the group of individuals $I_1, I_2, \ldots$ not only primary minds of first order $M_1, M_2, \ldots$ occur, but also mind of second order M_{kl} (for every k and $l \leq n$) occurs, that is the sensations experienced by I_k as a mind of second order from I_1. Likewise, besides the primary minds of second order M_k, mind M_{rst} occurs (for every r, s and $t \leq n$), that is the sensation experienced by I_r as a mind of third order from M_{st}. And so on. Usque ad infinitum. Moreover, since the behavior from individual to individual differs only in degree and not in essence from the variation from individual to animal, so as a consequence of the plurality of mind, a mind would be assigned to animals as well.[4] (CW I, p. 485)

The consequence of this 'proof' of the impossibility of plurality of minds is the impossibility of exchange of thought through languages. 'Thoughts are inseparabily bound up with subject' (p. 485).

However, language accompanies intellect and is aimed at communication. This is an external activity, a form of interaction, and therefore it involves a clash of wills.

For these reasons, language can never guarantee success and there is no way of checking the result (except by making trivial commands together with visible gestures), because language would still be used for providing feedback. People start training themselves and their offspring with a crude form of sign language, painfully and with little success, because nobody has ever been able to communicate with others soul to soul. Language can only be the complement of an already existing mutual understanding. Even when two people

[4] And this appeared absurd to Brouwer.

share the same needs and aspirations, they will be in constant danger of being led by in different directions and drifting apart by their uncontrolled desires; they will suffer pain and anxiety in their struggle to keep together (LAM, p. 401). Only in very narrow domains of the imagination—as in the very abstract sciences that are completely separate from the world of perception and then skim the minimum of what is essentially human—only then mutual understanding can be maintained for some time:

> Even in the case of the most restricted sciences, logic and mathematics—a sharp distinction between these two is hardly possible—no two different people will have the same conception of the fundamental concepts on which these two sciences are constructed, and yet their wills are parallel and in both there is a small, unimportant part of the brain which forces their attention in the same way. (LAM, p. 401)

> When people are fighting against a common enemy, then the language will serve its purpose: it will keep separate persons on the same path. But the use of language is ridiculous when you try to express the subtle nuances of the will that are not part of the living reality of the people arguing: when, for example, so-called philosophers or metaphysicians argue among themselves of morality, God, consciousness, immortality, or of free will. These people do not even love each other, let alone share the same subtle movements of the soul; sometimes they even do not know each other personally. They either talk at cross-purposes or each builds his own little logical system which lacks any connection with reality. For logic is life in the human brain; it may accompany life outside the brain but it can never guide by virtue of its own power. (LAM, p. 401) It is hard for the attention to break away from the intellect so much that only the most extraordinarily events make an impression and get through to the central human emotions. By means of language as the slave of an illusion of reality one cannot reveal the truth. (LAM, p. 403)

There is no point in using language when there is an argument and people try to come to an agreement by means of reasoning. In this case, language, which presumes a harmony of will, may well be used to accompany strife and fighting. 'But they might just as well keep silent; they only play off their wills against each other and work on each other's desires and fears, and the strongest man wins' (LAM, p. 402).

The language of conversation is absurd and even comical between boys and girls. 'In their case, there is already a harmony of will, and language is completely superfluous; indeed its only purpose is to hide this harmony of will' (LAM, p. 402). There is only one sense of language in everyday life language which is to join together two people's already harmonious wills along the same path.

3.2. For Nietzsche, intellect is the everlasting source of metaphors which were originally a wonderful product of human creativity. However, intellect is also aimed at preserving the individual: as men do not have great physical tools (no horns or strong teeth to fight against one another), intellect reinforces metaphors,[5] in order to enable them to dominate the world. In this way, it provides the basis for ontology, science and logic. By reinforcing

[5] Hinman (*1982*) underlines that according to Nietzsche language is essentially metaphoric because a literal meaning would mean a complete coincidence between word and referent 'for he clearly thinks that we begin to enter the domain of the metaphorical when we 'leave out' elements of the original experience as well as when we carry over meaning from another experience' (p. 183). That is, Nietzsche completely agrees with the notion of a self-evident literality. Consequently, for him the metaphorical process begins with the act of perception itself, where (1) we 'carry over' from one sphere (the physiological sphere of the nervous stimulus) to another (the domain of imagination); (2) we consider to be identical things that only have some common traits. Secondly, the metaphorical process takes place in the transition from image to sound and finally in the transition from sound to word, where 'metaphorical' also means using similar experiences as if they were equal. On this topic see also *Schrift 1985*.

a few metaphors, the intellect sets a 'subject' before 'objects' and connects them to one another with causal links. The intellect does this by blanching two types of metaphors: the first type of metaphor is the nervous impulse that produces the image, and the second type is the sound expression of the image.

> [Der Sprachbildner] bezeichnet nur die Relationen der Dinge zu den Menschen und nimmt zu deren Ausdrucke die kühnsten Metaphern zu Hülfe. Ein Nervenreiz zuerst übertragen in ein Bild! erste Metapher. Das Bild wieder nachgeformt in einen Laut! Zweite Metapher. [. . .] Wir glauben etwas von den Dingen selbst zu wissen, wenn wir von Bäumen, Farben, Schnee und Blumen reden und besitzen doch nichts als Metaphern der Dinge, die den ursprünglichen Wesenheiten ganz und gar nicht entsprechen. (TL 1)

We can see the original metaphors as a higher level of creativity, aimed at the maximum of life (i.e. aimed at power), and the pale metaphors as a lower level of creativity, aimed at the minimum of life (i.e. aimed at mere survival).

Language plays an important role in the sciences. According to Brouwer, language is used to maintain two wills oriented in a single direction, while for Nietzsche, language (as a faded metaphor) creates and maintains a useful yet fictitious 'reality' in daily life.[6] In fact, according to Nietzsche, everything flows or rather what we perceive is only some flowing. Causality is appearance. Our belief in causality is due to our faith in the strength that we experience when we do something, yet force moves nothing; the force we feel does not 'put our muscles in motion'. Causality is only created by thought, which introduces the constraint in the process of succession.

The scientist considers the content of his discipline to be true. On the contrary, Nietzsche criticizes the alleged truth of science: it is, in fact, based on a world of fictitious entities interrelated in a fictional way for the sake of survival, while truth would be the absolute, the non-perspective, the non-anthropocentric:[7]

> Das 'Ding an sich' (das würde eben die reine folgenlose Wahrheit sein) ist auch dem Sprachbildner ganz unfasslich und ganz und gar nicht erstrebenswerth. (TL 1)

> Die bestgeglaubten a priorischen 'Wahrheiten' sind für mich—Annahmen bis auf Weiteres z.B. das Gesetz der *Causalität* sehr gut eingeübte Gewöhnungen des Glaubens, so einverleibt, daß nicht daran glauben das Geschlecht zu Grunde richten würde. Aber sind es deswegen Wahrheiten? Welcher Schluß! Als ob die Wahrheit damit bewiesen würde, damit die Menschen bestehn bleibt! (NF-1884, 26[12])

For him, the content of science is an epistemological lie regulated by society; such knowledge is finalized, perspective, just like mathematics and logic. In its claim to truth, science is a faith, and, as such, it can be likened to religion. Nonetheless, Nietzsche realized and appreciated the fact that science was a danger to the priests, who wanted to keep the world of human needs and fears under their control:

> Hat man eigentlich die berühmte Geschichte verstanden, die am Anfang der Bibel steht,—von der Höllenangst Gottes vor der Wissenschaft? . . . Man hat sie nicht verstanden. Dies Priester-Buch par excellence beginnt, wie billig, mit der grossen inneren Schwierigkeit des Priesters: er hat nur Eine grosse Gefahr, folglich hat 'Gott' nur Eine grosse Gefahr. (A 48) Der Anfang der Bibel enthält die ganze Psychologie des Priesters.—Der Priester kennt nur eine grosse Gefahr: das ist die

[6] About language in Nietzsche see also *Schrift 1985*, *Sonderegger 1973*, and *Thurner 1980*.
[7] On this topic see *Stack 1981*, *Ungeheuer 1983*, *Walters 1982* and *Warnock 1978*.

> Wissenschaft—der gesunde Begriff von Ursache und Wirkung. [. . .]Wenn die natürlichen Folgen einer That nicht mehr 'natürlich' sind, sondern durch Begriffs-Gespenster des Aberglaubens, durch 'Gott', durch 'Geister', durch 'Seelen' bewirkt gedacht werden, als bloss 'moralische' Consequenzen, als Lohn, Strafe, Wink, Erziehungsmittel, so ist die Voraussetzung zur Erkenntniss zerstört,—so hat man das grösste Verbrechen an der Menschheit begangen.—Die Sünde, nochmals gesagt, diese Selbstschändungs-Form des Menschen par excellence, ist erfunden, um Wissenschaft, um Cultur, um jede Erhöhung und Vornehmheit des Menschen unmöglich zu machen; der Priester herrscht durch die Erfindung der Sünde.[8] (A 49)

4. Mathematics

4.1. It is from his negative stance toward life, embedded in a mystical framework that referred especially to eastern tradition[9] that Brouwer set his vision of mathematics and, consequently, of logic.

At the very beginning, in his *Life, Art and Mysticism*, he considered mathematics and logic as 'sharply indistinguishable' and sinful. In fact he wrote about 'sinful bonds of science, of belief in 'reality' and of logical thinking'. Furthermore, as van Dalen notes in his *Mystic, Geometer and Intuitionist: The Life of L.E.J. Brouwer*, there are undated notes which possibly belong to that period, where he expressed contrasting opinions concerning mathematics:

> One could see as the goal of one's life: Abolition and delivery from all mathematics. [. . .] That mathematics and its application are sinful follows from the intuition of time, which is immediately felt as sinful. [. . .] Mathematics justifies itself, needs no deeper ground than moral mysticism. (*van Dalen 1999b*, pp. 83–84)

Van Dalen interprets them as a reportage of (successful) temptation: Brouwer's mystical views led him to avoid carrying out studies/actions or pursuing a career directed toward dominating nature, even if his desires and abilities made him tend toward carrying out academical research in mathematics (*van Dalen 1999b*, p. 83). He found a solution to his paradoxical situation by implementing a new way of developing mathematics that could reduce mathematical sinfulness to a minimum by quietly accepting his own 'karma'. In 1906–1907, he described his dissertation as a duty,[10] he separated mathematics from logic[11] and started a research project aimed at constructing mathematics 'in an inner way'.[12] In his 1907 dissertation, when searching for something inner on which mathematics could be based, he considered the Kantian aprioris—space and time—by emphasizing

[8] It is important to note Brouwer's and Nietzsche's opinions concerning 'sin'. According to Brouwer, sin is the cause of mankind's unhappiness in life, since happiness can be found only in the Self and sin is attempting to exit the inner self: Brouwer's opinion came from his mystical framework and he maintained it as he was convinced that man could experience the fact that *sin produces pain now*. On the contrary, Nietzsche's moralistic definition of sin came from the Christian context in which he lived: sin is the transgression of any of the regulations established by priests (who boast to have received them from God); *sin is pleasure*, it is enjoyment, which priests forbid and declare that we will suffer from our sins after death. Therefore, Nietzsche refused the idea of sin since it is a repression of our vital instincts.

[9] See his quotes from the Bhagavad Gita in his 'Life, art and mysticism' (LAM, p. 421, pp. 423–424) and also in 1948 (CW I, p. 486).

[10] See letter to Scheltema 7 September 1906 (van Dalen 2011, p. 23). Keeping his Hinduistic references to the Bhagavad-Gita as a framework, he may have accepted his fate, probably due to the clear attitude he had toward this discipline.

[11] This can be seen in detail in the following paragraph. Please note this quote: 'While thus mathematics is independent of logic, logic does depend upon mathematics' (CW I, p. 73).

[12] When G. Mannoury obtained his honorary degree of Doctor of Sciences in 1946, in his speech, Brouwer thanked Mannoury for having opened his eyes to a different kind of mathematics which was full of life and therefore beautiful:

the two meanings of 'apriority' (i.e. existence independent of experience and necessary condition for the possibility of science), abandoned the first meaning (according to which all mathematics would be a priori—non-Euclidean geometries as well as Euclidean geometry) and stated that, although we accept that we have no experience except in an empirical space where Euclidean geometry holds, we could admit that human intellect could place the concept of an external world in other receptacles without this being realized in practice. 'Human intellect disengages itself very well from three-dimensional Euclidean geometry' (CW I, p. 69). On the contrary,

> proper to man is a faculty which accompanies all his interactions with nature, namely the faculty of taking a mathematical view of his life, of observing in the world repetitions of sequences of events, i.e. of causal systems in time. The basic phenomenon therein is the simple *intuition of time*. (CW I, p. 53)

Therefore, time is an a priori element in the sense that a scientific (causal) study of our experiences requires the intuition of time, provided that time is intended as intuitive rather than measured time, while time intuition does not require an external experience just as an external experience does not require time.

Did Brouwer still believe that time is 'immediately felt' as sinful? Presumably, he did. In fact in 1908 he wrote:

> In *religious truth*, i.e. wisdom, which abolishes the discernment between the subject and something different, and where *the perception of time is no longer admitted*, there is no mathematical understanding, let alone reliability of logic. (CW I, p. 108)

Nevertheless, and despite the fact that time aprioricity was obtained by (so to speak) 'extracting' it from sinful science (and letting it develop without applicational aims), Brouwer accepted time intuition because (at least) it was granted as being inner due to its independence of experience.

In 1929 and in 1948, he refined his presentation of man's steps toward science. In particular, in 1948 he introduced[13] the distinction between consciousness and mind. The intuition of time takes place in consciousness, before it becomes mind, that is, *before the sinful separation* of a 'subject' from an 'object': in this way, mathematics has a source which is situated in a 'less sinful' place. Then, mathematics could be (sinfully) applied to the world by mind, yet it can also be kept pure by practicing it only for its own sake, for enjoying the pleasure of its beauty:

> The fullest constructional beauty is the *introspective beauty of mathematics*, where the basic intuition of mathematics is left to free unfolding. This unfolding is not

> After two or three years of university, I still could see the figure of the mathematician only as a servant of natural sciences or as a collector of truths: truths fascinating by their immovability, but horrifying by their lifelessness, like stones from barren mountains of disconsolate infinity [. . .] There I saw a man . . . I was captivated . . . mathematics had acquired a new character . . . These are harmonies I desired to realize. (CW I, pp. 474–475)

[13] The introduction of this distinction better explains what he had written in a footnote in 1907:

> Strictly speaking the construction of intuitive mathematics in itself is an action and not a science; it only becomes a science, i.e. a totality of causal sequences, repeatable in time, i.e. a mathematics of the second order, which consists of the mathematical consideration of mathematics or of the language of mathematics. (CW I, p. 61)

> bound to the exterior world, and thereby to finiteness and responsibility; consequently its introspective harmonies can attain any degree of richness and clearness. (CW I, p. 484)

Hence, we can suppose that this distinction represented a further step toward the description of a not-sinful mathematics. It seems to me that his 1948 remark supports this interpretation:

> Searching for wisdom, we may find it in knowing that causal thinking or acting is non-beautiful and hard to justify, and that in the long run it brings disappointment. And in knowing that the exterior world with its innumerable individuals and with its hypertrophied cooperation is wedded to mind, its disharmonies reflecting mind's free-will-guilt.[14] (CW I, p. 485)

Here all that is negative is attributed to mind and to causal thinking/acting. Therefore, we can presume that the source of time is in 'a sin-free area'.

Now, how does mathematics develop from the intuition of time before being applied to the natural world?

In his Ph.D. thesis, Brouwer wrote: 'Mathematics develops out of its basic intuition in a self-multiplication guided by an entirely free choice' (CW I, p. 70).

A development of this kind in its appeal to 'free choice' was very similar to the description that he had given in 1905 regarding what occurs if man seeks for interiority:

> If you succeed in overcoming all inertia and proceed, you will find that passions will be silenced, you will feel dead to the old world of perception, of time and space, and all other forms of plurality [. . .] then you will understand the content of all your previous awareness and [. . .] you accept it as self-evident. [. . .] At the same time you are also aware of a wealth of phantasies, a mixture of all kinds of worlds [. . .] and in this confluent sea of colors, without separation, without firmness and yet about movement, this chaos without disorder, you see a direction, which you follow automatically, yet not of necessity. You will recognize your *free will*, free insofar that it can withdraw from the world of causality and remain free; it is only then that your will find a definite direction, which it follows freely and reversibly. (LAM, pp. 393–394)

More specifically, time is the source of natural numbers (and accordingly the source of rational numbers) with its scan instants—therefore, it is also called 'intuition of the two-ity'; furthermore, he affirmed that time could give the intuition of the continuum, even if only as a whole and not of each of its points.[15] In 1914, he made it clear[16] that he had formed a new opinion on this matter which he expressed in 1918 when he stated that the

[14] It is important to note that from this quote it appears that 'free will' is not always positively evaluated by Brouwer: it is only positive when it has its origin in the self!

[15] This was in line with the French tradition of intuitionism: in particular, with the following excerpt from the lecture given by Borel at the International Congress of Mathematics in Rome in 1908:

> I consider this notion [of the continuum] as obtained from the geometric intuition: it is known that the concept of full arithmetic continuum requires that one admits the legitimacy of an infinite countable series of choices. This seems to me highly questionable legitimacy, but nevertheless one should distinguish between legality and legitimacy of this more than countable infinity of choices (simultaneous or successive). (*Troelstra 1982*, p. 467)

This quote explains why it is possible to describe Brouwer's initial position in terms of acceptance of the geometric continuum rather than the arithmetic continuum. In addition, it sheds light on the origin of the name (later also adopted by Brouwer) of 'free choice sequences' for the sequences in question.

[16] In his review of Schoenflies and Hahn's book *Die Entwickelung der Mengenlehre und ihrer Anwendungen* (CW I, p.140).

intuition of time also provides the framework for constructing ever-branching trees: if an already acquired mathematical entity (e.g. a rational number) is associated with each node, one obtains the representation of the set of all real numbers, that is the continuum. The branches of the continuum-tree were called 'free-choice sequences' by Brouwer because they could be completely free (lawless).[17] The construction of the tree—in its generic shape—was described as follows:

> Eine Menge ist ein Gesetz, auf Grund dessen, wenn immer ein willkürlicher Zifferncomplex der Folge 1,2,3 ... gewählt wird, jede dieser Wahlen entweder ein bestimmtes Zeichen oder nichts erzeugt oder aber die Hemmung des Prozesses und die definitive Vernichtung seines Resultates herbeiführt, wobei für jedes $n > 1$ nach jeder ungehemmten Folge von *n-1* Wahlen wenigstens ein Ziffernkomplex gewählt wird, nicht die Hemmung des Prozesses herbeiführt. (CW I, p. 150)

It is initially called 'Menge' because it is the first type of set that Brouwer could point out (he would later call it 'spread'). The other type, also presented in 1918, was that of a group[18] of entities (already available), called 'Species':

> Unter einer *Species erster Ordnung* verstehen wir eine Eigenschaft, welche nur eine mathematische Entitaet besitzen kann[19]; unter einer *Spezies zweiter Ordnung* verstehen wir eine Eigenschaft, welche nur eine mathematische Entitaet oder Species erster Ordnung besitzen kann. In analoger Weise definieren wir *Species n-ter Ordnung*, wo *n* ein beliebiges Element von A repraesentiert.[20] (CW I, 150–151)

Brouwer's description of 'Menge' remained unchanged (CW I, p. 301) until the period 1946–1951, when Griss criticized him for considering hypothetical constructions which cannot be finished as part of mathematics.[21] From then on, Brouwer avoided sterilization and described the tree as having:

(1) As nodes of order 1 either all the natural numbers or only those not larger than a given m.
(2) As nodes of order $n + 1$ (for all n's) either the immediate descendants of the node p of order n, or only those descendants whose $(n + 1)$th component added to p's components is not larger than a given number m_p.

In order to obtain the spread, he attached either terms or nothing to each node.

During the construction of the tree, there was freedom of choice because each row represented a free-choice sequence. Brouwer pondered on the possibility of choice over and over again. In 1929, he added to all the copies of his 1925 paper regarding sets[22] (concerning free-choice sequences): 'Inklusive des Charakters ihrer Fortsetzbarkeitsfreiheit, welche sich nach jeder Wahl beliebig eventuell bis zur völligen Bestimmtheit, oder auch einem Mengengesetze entsprechend verengern kann' (CW I, p. 590).

[17] On this topic (with all its historical and philosophical details that go beyond the domain of the present paper), see *van Atten 2003*, *2004*, *2007*, *van Dalen 1999a*, *1999b*, *Niekus 1987*, *2005*, *2011*, *van Stigt 1990*, *Troelstra 1982*. Here a specification is required concerning the expression 'creating *subject*' which appears in Brouwer's last writings in his counterexamples to logical principles, meaning the mathematician that produces real numbers according to certain conditions. Concerning the comparison between Brouwer and Nietzsche, it is important to note that Brouwer intended to describe a mathematician's *activity* as his living construction of free choice sequences (and the result of this activity). There is no proof in Brouwer's papers of his intention to present the new 'mathematical subject' (see also *Franchella 2008*, pp. 132–134).

[18] Brouwer gave always an intentional definition of set.

[19] 'Mengen und Elemente von Mengen werden *Mathematische Entitaeten* genannt' (CW I, p. 150).

[20] A is the set of natural numbers.

[21] On this topic, see *Franchella (1994)*.

[22] *Troelstra* (*1982*, p. 473) recalls that there are some similarities in Brouwer's notes and his Berliner Lectures.

In 1946, he changed his mind and became definitively convinced that:

> In former publications I have sometimes admitted restrictions of freedom with regard also to future restrictions of freedom. However this admission is not justified by close introspection and moreover would endanger the simplicity and rigor of further developments. (CW I, p. 511)

In his later writings, he stated that intuitionistic mathematics was composed of two acts:

> First act of intuitionism
>
> Completely separating mathematics from mathematical language and hence from the phenomena of language described by theoretical logic, recognizing that intuitionistic mathematics is an essentially languageless activity of the mind having its origin in the perception of a move of time. This perception of a move of time may be described as the falling apart of a life moment into two distinct things, one of which gives way to the other, but is retained by memory. If the twoity thus born is divested of all quality, it passes into the empty form of the common substratum of all twoities. And it is this common substratum, this empty form, which is the basic intuition of mathematics.
>
> Second act of intuitionism
>
> Admitting two ways of creating new mathematical entities: firstly in the shape of more or less freely proceeding infinite sequences of mathematical entities previously acquired; secondly, in the shape of mathematical species, i.e. properties supposable for mathematical entities previously acquired, satisfying the condition that, if they hold for a certain mathematical entity, they also hold for all mathematical entities which have been defined to be 'equal' to it, definitions of equality having to satisfy the conditions of symmetry, reflexivity and transitivity. (CL 4–8)

From these two 'acts', mathematics must be developed by the two-ity through successive mental steps that must pass from evidence to evidence. The reason for this reference to evidence is that '*truth* is only in *reality*, i.e. in the present and past experience of consciousness' (CW I, p. 488).[23] There are no fixed rules of evidence, but rather new ways of moving from insight to insight, experiencing a vivid sense of beauty:

> The fullest constructional beauty is the introspective beauty of mathematics, where instead of elements of playful causal thinking, the basic intuition of time is left to free unfolding. This unfolding is not bound to the exterior world, and thereby to finiteness and responsibility; consequently its introspective harmonies can attain any degree of richness and clearness. (CW I p. 484)

Since mathematics does not follow fixed rules, logic must undergo a rather strong rearrangement. It becomes the collection of regularities in the expressions of mathematics.

4.2. Nietzsche does not devote much attention to mathematics. For him, arithmetic was based on the concept of 'Ego' (which is configured as a unity) *and* on the existence of identical things. Therefore, arithmetic was based on a series of lies or 'partially true statements' which are useful for survival, such as logic and science. (We must be able to count units: this does not mean that these units exist.)

[23] Here we recall that two particular 'true' cases were immanent truth ('truth which in this world points to the inevitability of the karma of that world') and transcendental truth ('truth which guides man in this world toward a personal life, free from the shackles of fear and desire' LAM, pp. 392–393).

> Wir haben den Begriff der *Einheit* entlehnt von unserem 'Ich' begriff,—unserem ältesten Glaubensartikel. (NF 1888, 14[79])

> Die Zahl.—Die Erfindung der Gesetze der Zahlen ist auf Grund des ursprünglich schon herrschenden Irrthums gemacht, dass es mehrere gleiche Dinge gebe (aber thatsächlich giebt es nichts Gleiches), mindestens dass es Dinge gebe (aber es giebt kein 'Ding'). Die Annahme der Vielheit setzt immer voraus, dass es Etwas gebe, das vielfach vorkommt: aber gerade hier schon waltet der Irrthum, schon da fingiren wir Wesen, Einheiten, die es nicht giebt.—Unsere Empfindungen von Raum und Zeit sind falsch, denn sie führen, consequent geprüft, auf logische Widersprüche. Bei allen wissenschaftlichen Feststellungen rechnen wir unvermeidlich immer mit einigen falschen Grössen: aber weil diese Grössen wenigstens constant sind, wie zum Beispiel unsere Zeit- und Raumempfindung, so bekommen die Resultate der Wissenschaft doch eine vollkommene Strenge und Sicherheit in ihrem Zusammenhange mit einander; man kann auf ihnen fortbauen—bis an jenes letzte Ende, wo die irrthümliche Grundannahme, jene constanten Fehler, in Widerspruch mit den Resultaten treten, zum Beispiel in der Atomenlehre. (HH I, 19)

Geometry is also based on an untruthful leveling of sensory experiences: we never perceive straight lines, circles, dots; we introduce them into reality to make it static, predictable and controllable so that it becomes reassuring.

> Ebenso steht es mit der Mathematik, welche gewiss nicht entstanden wäre, wenn man von Anfang an gewusst hätte, dass es in der Natur keine exact gerade Linie, keinen wirklichen Kreis, kein absolutes Grössenmaass gebe. (HH I, 11)

We see that Nietzsche did not refer to Kant to explain the foundation of mathematics, as Brouwer did later when he proposed time intuition as its source. However, like Brouwer Nietzsche referred to the question of the morality of mathematics (as part of the general question of the morality of human knowledge), but only to explain why society did not judge mathematics, the sciences and logic negatively while it did for lies in general. The answer was that mathematics (like sciences and logic) are useful lies which are regulated by society: that is why it is so culturally acceptable to transmit them as truths.

Despite the lack of reference to mathematics observed in Nietzsche's works, Steinhart (*1999*, p. 25) set up a parallel between Nietzsche's thought and intuitionism. First, he suggested that there was a link between mathematics and morality in the fact that the 'strict honesty' required for practicing mathematics instills in us a strict discipline in introspection, and makes us sufficiently critical to reject superstition.

Steinhart then stated that Brouwer endowed mathematics with an artistic nature that Nietzsche would have appreciated. The artistic nature of mathematics consisted in the fact that mathematics was *built* according to Brouwer—hence it is an 'architectural', artistic activity—and free, 'conditioned by the sole bond of non-contradiction'. Indeed, Brouwer used the metaphor of 'building' throughout his life, from the beginning to the end of his work. In his dissertation, he wrote:

> the only possible foundations of mathematics must be sought in this construction under the obligation carefully to watch which construction intuition allows and which not, and why any other attempt at such a foundation is condemned to failure. (CWI, p. 52)

and in his 1949 paper 'Consciousness, Philosophy and Mathematics', he stated: '[. . .] intuitionistic mathematics is inner architecture' (CW I, p. 494). For Brouwer mathematics was a (joyful and) free construction, but 'free' meant 'not finalized' and 'lawless'; 'free'

did not mean 'non-contradictory': the condition of non-contradictoriness is absolutely far from Brouwer. In fact, he sees non-contradiction as *a result* of doing mathematics in a constructive way, not as a criterion of doing mathematics. Non-contradiction was required for Hilbert's finitism, which conceived the various branches of mathematics as axiomatic systems expressed in a symbolic and formal way: coherence was required for such formal systems as a guarantee for their content. In fact, any formula can be derived as a result from a system that admits an internal contradiction including a statement and its negation and this makes the system completely useless. Brouwer always criticized this point of view, which he considered merely 'linguistic' and not based on a mental content. In 1912, Brouwer wrote in 'Intuitionism and Formalism':

> The intuitionist can never feel assured of the exactness of a mathematical theory by such guarantees as the proof of its being non-contradictory, the possibility of defining its concepts by a finite number of words, or the practical certainty that it will never lead to a misunderstanding in human relations. (CW I, p. 128)

In 1928, he reiterated this opinion in the text 'Intuitionistische Betrachtungen über den Formalismus':

> Die Erkenntnis dass die (inhaltliche) Rechtfertigung der formalistischen Mathematik durch den Beweis ihrer Widerspruchslosigkeit einen circulus vitiosus enthält, weil diese Rechtfertigung auf der (inhaltichen) Richtigkeit der Aussage, dass aus der Widerspruchslosigkeit eines Satzes die Richtigkeit dieses Satzes folge, d.h. auf der (inhaltlichen) Richtigkeit des Satzen vom ausgeschlossenen Dritten beruht. (CW I, p. 410)

Therefore at no moment of his life did Brouwer accept non-contradiction as a 'guarantee' for mathematics. He preserved his original opinion that he had expressed in *Life, Art and Mysticism*:

> A language that does not derive its certainty from the human will but which claims to live on in a pure concept is an absurdity. It is indeed a great skill to be able to go on speaking without being caught in contradiction or without making silent presumptions rooted in the will [. . .] but a kind of skill one admires in an acrobat. (LAM, p. 403)

Therefore, with Steinhart we can share the idea that Nietzsche would have appreciated Brouwer's artistic vision of mathematics, insofar as Nietzsche encouraged every form of creativity, but we cannot accept that Brouwer placed the condition of being non-contradictory on mathematics.

Finally, Steinhart proposes an analogy in the birth of mathematics according to the two authors. For both, it would arise from the division of time: according to Nietzsche, it would occur by dividing the temporal continuum into equal parts, and according to Brouwer, it would come from the scheme of the 'two-ity' (duo-unit) in the flow of perceptions. However, it is important to note that Nietzsche did not insist on the aspect of temporality (which was important for Brouwer and explicitly stressed by him), but on identity and in particular on the specific identity of the 'Ego' as a foundation for unity.

5. Logic

5.1. For Brouwer, logic does not express human creative ability but is a second-level sedimentation. The proof of theorems is an inner, creative experience, because by performing them man tries following different paths to see where they lead. Man does not follow rules, and this resembles what Brouwer had written about the self in 1905: 'you see

a direction, which you follow automatically, yet not of necessity. You will recognize your *free will*' (LAM, p. 394).

Mathematics, which is all interior, does not require language, which is used to communicate and, therefore, does not require going sinfully out of the self.[24] Language occurs later, to communicate its results. What is logic? If we look at the linguistic expression of the mathematical experience and collect the regularities that we find, then we obtain logic. If we changed the linguistic expression, logic would change too.[25] Logic certifies ethnological data not psychological data.

> People try by means of sounds and symbols to originate in other people copies of mathematical constructions and reasonings which they have made themselves; by the same means they try to aid their own memory. In this way the mathematical language comes into being and is a special case of the language of logical reasoning. [. . .] given the same organization of the human intellect and consequently the same mathematics, a different language would have been formed, into which the language of logical reasoning, well known to us, would not fit. [. . .] We infer that theoretical logic as well as logistic are empirical sciences and that they apply mathematics; consequently they can yield no information whatsoever on the organization of human intellect; there would be better reasons to reckon them under ethnography than under psychology. (CW I, 73–74)

Initially, logical laws appeared trivial to Brouwer. They either expressed an identity (law of identity, non-contradiction and the excluded middle[26]) or an inclusion (the law of syllogism). Brouwer did not criticize the principle of identity nor, more fundamentally, the concept of identity, because he accepted everything which could be found in the interior, in the Self. He rejected the distinction between subject and object, because it turned man away from the Self but he did not deny the law of identity, because identification could take place within the Self.

Furthermore, he believed that 'the mathematics of whole and part' (i.e. syllogistic) was useless in the empirical world:

> The mathematics of whole and part [= syllogistic] does not teach us through its theory anything new for application. Once the system is applied to a part of the observational world, even a very mediocre intellect can immediately read off all the consequences: no intermediary logical reasoning is necessary. (Letter from Brouwer to Korteweg 23 January 1907, *van Dalen 2011*, p. 39)

And that:

> One knows well that the world is not a logical system and one cannot argue logically about it. (Letter from Brouwer to Korteweg 23 January 1907, *van Dalen 2011*, p. 39)

[24] Brouwer later realized that human memory can fail and briefly mentioned this. In fact in his 1932 Vienna lecture 'Willen, weten, spreken' ('Volition, knowledge and talk'), he spoke of 'an ideal mathematician without limitation of memory, which could do without the language' (CW I, p. 443): this proves that Brouwer realized that it is unreal to speak of a mathematical construction realized without the support of language. Language should be given a role in mathematics in order to support the weak human memory. However, he did not develop this aspect and never reopened discussion on the topic which his student Arend Heyting focused on. Therefore, mathematics remained an a-linguistic activity for Brouwer.

[25] Also see *van Dalen 1999c* on the relationships between logic and language in Brouwer.

[26] In his dissertation, Brouwer also presented the principle of excluded middle as a statement of identity. In fact he wrote: '"A function which is either differentiable or not differentiable" says *nothing*; it expresses the same as the following: "If a function is not differentiable, then it is not differentiable"' (CW I, p. 75). He was to change his mind the following year as we will see below.

> It serves only lawyers and demagogues, not to instruct other people but to deceive them, and that is because the vulgar herd unconsciously reasons: the language with its logical figures is there, so it will be useful and so they meekly let themselves be deceived; just as I heard several people defend their habit of gin drinking with the words: 'What else is gin for?' Whoever has the illusion to improve the world, may just as well agitate against the language of logical reasoning as against alcohol. (Letter from Brouwer to Korteweg 23 January 1907, *van Dalen 2011*, p. 39)

However, in 1908, immediately after the discussion of his Ph.D. thesis, Brouwer criticized the principle of the excluded middle, because, if logic has to be an expression of mathematics, it would have to speak of mathematical reality. Therefore, affirming P meant giving the mental construction of P, and negating P meant showing the failure of the construction of P:

> The words of your mathematical demonstration merely accompany a mathematical construction that is effected without words. At the point where you enounce the contradiction, I simply perceive that the construction no longer goes, that the required structure cannot be imbedded in the given basic structure. (CW I, p. 73)

Therefore, the principle of the excluded middle meant that, for any property and for any entity, we could always either demonstrate that the entity possesses the property or be capable of leading any attempt to demonstrate this to failure. In other words, the principle of the excluded middle became the assertion of the possession of the solution of all mathematical questions that could ever be asked (CW I, p. 109). This was much more than Aristotle intended to say with the same principle and it could not be accepted as a logical law. In fact, only one single unresolved mathematical problem is enough to suspend the validity of the law. As a side effect of his vision of life, in the end Brouwer created an alternative to the classical sort of logic.[27]

5.2. Nietzsche referred to Aristotelian logic and believed that it enables us to posit things as equal, even if we only perceive continuous change.

> Die Logik ist geknüpft an die Bedingung: gesetzt, es giebt identische Fälle. Thatsächlich, damit logisch gedacht und geschlossen werde, muß diese Bedingung erst als erfüllt fingirt werden. Das heißt: der Wille zur logischen Wahrheit kann erst sich vollziehen, nachdem eine grundsätzliche Fälschung alles Geschehens vorgenommen ist. Woraus sich ergiebt, daß hier ein Trieb waltet, der beider Mittel fähig ist, zuerst der Fälschung und dann der Durchführung Eines Gesichtspunktes: die Logik stammt nicht aus dem Willen zur Wahrheit. (NF 1885, 40[13])

Logic does not emerge from the objective data in our possession but it is a pattern that we sneak into the continuous flow of perceptions in order to control it and is therefore non-anxiogenic: 'die Logik beruhigt, giebt Vertrauen' (GS 370).

Logic, like science and mathematics, is a useful simplification to avoid drowning in uncertainty or anxiety, but it is not 'true' in the profound sense of the term. This simplification is not intrinsic to man but is the product of the evolution of the human species and has been preserved due to its survival function: those who began to see the differences less clearly, observed a uniformity in perceptions and started to distinguish objects and classify them (leaves, food), and were therefore capable of finding food and defending themselves. In this way, they survived longer than the others, were able to reproduce and pass this trait onto their offspring.

[27] On this topic see *Franchella 1995*.

> Herkunft des Logischen.—Woher ist die Logik im menschlichen Kopfe entstanden? Gewiss aus der Unlogik, deren Reich ursprünglich ungeheuer gewesen sein muss. Aber unzählig viele Wesen, welche anders schlossen, als wir jetzt schliessen, giengen zu Grunde: es könnte immer noch wahrer gewesen sein! Wer zum Beispiel das 'Gleiche' nicht oft genug aufzufinden wusste, in Betreff der Nahrung oder in Betreff der ihm feindlichen Thiere, wer also zu langsam subsumirte, zu vorsichtig in der Subsumption war, hatte nur geringere Wahrscheinlichkeit des Fortlebens als Der, welcher bei allem Aehnlichen sofort auf Gleichheit rieth. Der überwiegende Hang aber, das Aehnliche als gleich zu behandeln, ein unlogischer Hang—denn es giebt an sich nichts Gleiches—,hat erst alle Grundlage der Logik geschaffen. Ebenso musste, damit der Begriff der Substanz entstehe, der unentbehrlich für die Logik ist, ob ihm gleich im strengsten Sinne nichts Wirkliches entspricht,—lange Zeit das Wechselnde an den Dingen nicht gesehen, nicht empfunden worden sein; die nicht genau sehenden Wesen hatten einen Vorsprung vor denen, welche Alles 'im Flusse' sahen. An und für sich ist schon jeder hohe Grad von Vorsicht im Schliessen, jeder skeptische Hang eine grosse Gefahr für das Leben. Es würden keine lebenden Wesen erhalten sein, wenn nicht der entgegengesetzte Hang, lieber zu bejahen als das Urtheil auszusetzen, lieber zu irren und zu dichten als abzuwarten, lieber zuzustimmen als zu verneinen, lieber zu urtheilen als gerecht zu sein—ausserordentlich stark angezüchtet worden wäre.—Der Verlauf logischer Gedanken und Schlüsse in unserem jetzigen Gehirne entspricht einem Processe und Kampfe von Trieben, die an sich einzeln alle sehr unlogisch und ungerecht sind; wir erfahren gewöhnlich nur das Resultat des Kampfes: so schnell und so versteckt spielt sich jetzt dieser uralte Mechanismus in uns ab. (GS 111)

This type of tool is now consolidated so as not to appear to be a tool based on perception but intrinsic to an external 'reality' and to a static 'subject':

> Was den Aberglauben der Logiker betrifft: so will ich nicht müde werden, eine kleine kurze Thatsache immer wieder zu unterstreichen, welche von diesen Abergläubischen ungern zugestanden wird,—nämlich, dass ein Gedanke kommt, wenn 'er' will, und nicht wenn 'ich' will; so dass es eine Fälschung des Thatbestandes ist, zu sagen: das Subjekt 'ich' ist die Bedingung des Prädikats 'denke'. Es denkt: aber dass dies 'es' gerade jenes alte berühmte 'Ich' sei, ist, milde geredet, nur eine Annahme, eine Behauptung, vor Allem keine 'unmittelbare Gewissheit'. Zuletzt ist schon mit diesem 'es denkt' zu viel gethan: schon dies 'es' enthält eine Auslegung des Vorgangs und gehört nicht zum Vorgange selbst. Man schliesst hier nach der grammatischen Gewohnheit 'Denken ist eine Thätigkeit, zu jeder Thätigkeit gehört Einer, der thätig ist, folglich—'. (BGE 17)

According to Nietzsche, the classification according to logic also has moral consequences (the formation of the moral of the herd):

> Der Calcul 'thue nichts, was dir selber nicht angethan werden soll' verbietet Handlungen um ihrer schädlichen Folgen willen: der Hintergedanke ist, daß eine Handlung immer vergolten wird. [. . .] Dagegen ist der Spruch werthvoll, weil er einen Typus Mensch verräth: es ist der Instinkt der Heerde, der sich mit ihm formulirt—man ist gleich, man nimmt sich gleich: wie ich dir, so du mir—Hier wird wirklich an eine Äquivalenz der Handlungen geglaubt, die, in allen realen Verhältnissen, einfach nicht vorkommt. Es kann nicht jede Handlung zurückgegeben werden: zwischen wirklichen 'Individuen' giebt es keine gleiche Handlung, folglich auch keine 'Vergeltung' . . . Wenn ich etwas thue, so liegt mir der Gedanke

> vollkommen fern, daß überhaupt dergleichen irgend einem Menschen möglich sei: es gehört mir … Man kann mir Nichts zurückzahlen, man würde immer eine 'andere' Handlung gegen mich begehen. (NF 1888, 22[1])

This may demonstrate Nietzsche's attitude of hostility toward logic. However, he also emphasized the importance of learning how to think at school:

> Die Schule hat keine wichtigere Aufgabe, als strenges Denken, vorsichtiges Urtheilen, consequentes Schliessen zu lehren: desshalb hat sie von allen Dingen abzusehen, die nicht für diese Operationen tauglich sind, zum Beispiel von der Religion. (HH 1, 265)

He expressed some appreciation to the Jews for their subtlety and perfection in deduction (which may be due to need of finding ways of convincing their opponents over the centuries):

> Ein Jude umgekehrt ist, gemäss dem Geschäftskreis und der Vergangenheit seines Volks, gerade daran—dass man ihm glaubt—am wenigsten gewöhnt: man sehe sich darauf die jüdischen Gelehrten an,—sie Alle halten grosse Stücke auf die Logik, das heisst auf das Erzwingen der Zustimmung durch Gründe; sie wissen, dass sie mit ihr siegen müssen, selbst wo Rassen- und Classen-Widerwille gegen sie vorhanden ist, wo man ihnen ungern glaubt. (GS 348)

and he criticized the *causa sui* as a logical violation:

> Die *causa sui* ist der beste Selbst-Widerspruch, der bisher ausgedacht worden ist, eine Art logischer Nothzucht und Unnatur: aber der ausschweifende Stolz des Menschen hat es dahin gebracht, sich tief und schrecklich gerade mit diesem Unsinn zu verstricken. (BGE 21)

What was really Nietzsche's evaluation of logic[28]?

6. Opinions about Nietzsche in comparison

There is not much material about Nietzsche's logic in the literature: there seems to be a general opinion that Nietzsche criticized logic. This reminds me of Stephen Hales' and Ophelia Schutte's viewpoints because they devoted much attention to the subject and offered meaningful contributions.

First, Hales stressed two facts:

(1) Nietzsche did not know anything about the logical stars of the XIX century—Boole and Frege, in particular (*Hales 1996*, p. 820);
(2) When Nietzsche discussed the connections between logic and ontology, he referred to that aspect that we now call semantics (*Hales 1996*, p. 823).

[28] Clark (*1991*, p. 105) changed her opinion of Nietzsche's logic after observing the following expression: 'Oder Formal-Wissenschaft, Zeichenlehre: wie die Logik und jene angewandte Logik, die Mathematik. In ihnen kommt die Wirklichkeit gar nicht vor, nicht einmal als Problem; ebensowenig als die Frage, welchen Werth überhaupt eine solche Zeichen-Convention, wie die Logik ist, hat' (TI III 3). The change of opinion would consist of not seeing reference to reality in logic any longer. Hales (*1996*) replied to this interpretation firstly by suggesting to recognize that the author style would consist in general of making a breaking statement, denying it and then taking it partially; secondly by explaining the supposed 'change' of Nietzsche's opinion about logic through the distinction between syntax and semantics. When Nietzsche referred to logic as a 'doctrine of signs', he had in mind what today is called syntax. Therefore, it makes sense that he said this type of logic did not 'touch' ontological—metaphysical issues. However, when Nietzsche discussed the connections between logic and ontology he referred to that aspect of logic that we call semantics. In other words, Nietzsche referred to two different aspects of logic when he said that logic does not touch ontological questions and when he said that it does.

Second, Hales focused on the central core of Nietzsche's criticism which consisted in the fact that logic assumes the identity of objects, and then he pointed out the aspects implied or assumed by such criticism, namely (1) logic presupposes the existence of objects; (2) things are only fictions invented by humans; (3) logic presupposes an identity of objects that endure over time, (4) logic assumes the identity of objects to a certain instant, (5) there is identity through time; (6) nothing is self-identical, or self-identical objects are only fictions (*Hales 1996*, p. 824).

Finally, Hales affirmed that semantics presupposes the existence of objects, but not necessarily in a realistic way. Semantics refers to a domain but this can be composed of fictitious objects. Therefore, according to Hales, Nietzsche's position on logic is not destructive: we can agree with Nietzsche about the fictitious character of the objects of logic without necessarily putting logic aside, because when semantics refers to objects, it does so without realistic pre-conditions (*Hales 1996*, p. 835).

Schutte (*1984*, pp. 28–36) gave her interpretation of Nietzsche's philosophy of logic by referring to the contrast between the Apollonian and Dionysian, expressed by the author in *The Birth of Tragedy*. He had noticed that two types of gods had emerged in ancient Greece: Dionysus, god of vitality, whose unbridled instinct might strike terror, and Apollo, god of the order of rational calm. At first there were the Dionysian cults that had come from the East. The Greeks knew of them and feared unbridled joy and pain, and began to worship Apollo, specifically designed to stem the invasion of the Dionysians. The interaction between the two types of deities gave rise to tragedy, which produced a balance between Dionysian instinct and Apollonian calm. Balance was shattered by Socrates who unbalanced the scales in favor of rationality. An application to logic could originate from this perspective. In fact, Schutte suggested an opposition between logic and the Dionysian view of life: logic makes things static, while Dionysus expressed the dynamism of life through dance and music. Schutte proposed a way to recover logic within a Nietzschean perspective, by being aware that there is a specific purpose to logic (survival). This recovery can be done in Dionysian terms: the process must be mediated by a Dionysian acceptance of life so that man can be free both from the bondage of the exclusive authority of reason and from a dualistic view of life (that would be formed by the contrast between life and non-life).

Bearing in mind Hales' and Schutte's contributions, we can now try to form an opinion on the matter. First, we share Hales' suggestion to consider inadequate Nietzsche's criticism of logic as presupposing the existence both of objects and of a subject. It should be noted that Nietzsche believed that logic shares the subject–object structure of ordinary language, while, from Frege onwards, the way in which logic expresses the sentences of ordinary language is in terms of function—argument, borrowed from mathematics and certainly neutral from an ontological point of view. However, Nietzsche did not know Frege's work. It is a well-known fact that in the nineteenth century traditional formal logic was substantially modified by some *mathematicians* in Britain and in Germany.[29] In fact, in Germany, the mathematician Ernst Schröder carried out a transformation of logic, by

[29] Concerning the interplay between philosophy and mathematics in shaping logic in the nineteenth century, see *Peckhaus 1999*. At the time, in Germany, the philosophical discussions around logic concerned Hegel's variant of logic, according to which logic coincided with metaphysics. After Hegel's death, 'the logical question' (formulated by Trendelenburg) was 'Is Hegel's dialectical method of pure reasoning a scientific procedure?' (see *Peckhaus 1999*, p. 438). As for the British philosophical framework, the nineteenth century saw the development of inductive logic (Mill, Bain) and some work regarding the problem of trying to express the quantification of predicates within syllogisms, after Whately's book *Elements of logic* (*Whately 1826*) had recalled attention to formal logic. The birth of the algebra of logic, which represented the first step toward the astonishing renewal of formal logic, took place both in Germany and Britain quite independently of the philosophical debate around logic (even if Boole refers to it—see *Peckhaus 1999*, p. 436).

establishing an abstract view of *mathematics* which focused on symbolic operations with arbitrary objects rather than mathematical objects. The result was algebra of logic, which developed parallel to (and independent of) its British counterpart, whose main author was George Boole. Frege made his contribution in a framework of this kind. He was also a mathematician who focused on a rigorous foundation of arithmetic. In this perspective, he shaped a new symbolic language that was able to supersede the monopolistic deductive structure of syllogism and offer an adequate axiomatization of arithmetics on a pure logical basis. However, his 'ideography' (*Begriffsschrift*) was not a success even among mathematicians. Therefore, it is understandable that the philosopher Nietzsche did not know of it and he certainly could not have known of Alfred Tarski's contributions to semantics which were written 30 years after his death (see *Tarski 1933, 1944, 1983*). *Now* we can say that logic can no longer claim to describe reality and that it simply proposes tools for dealing with identity and diversity. *Now* we can say that logic does not necessarily preserve the subject–predicate structure of everyday language, but, as much as Nietzsche knew of logic who actually meant to describe reality in terms of subject/predicate. Therefore, as much as Nietzsche knew of logic, that is, as long as we remain in the framework of Aristotelian logic, his two criticisms were centered.

Furthermore, he was certainly revolutionary for the following reasons. (1) He did not present logic as something absolute but as something perspective (i.e. aimed at some specific purpose—in particular, material survival). (2) He considered logic as a result of natural selection, emerging due to its ability to provide food thus enabling people to reproduce more easily.

As regards his 'evaluation' of logic, Schutte's suggestion can be accepted and we can even say that it was directly proposed by Nietzsche himself. It is my opinion that Nietzsche considered that it was important for man to take into account *both* irrationality and rationality. Nietzsche recognized that the authentic form of life is the Dionysian, but he was also aware of the risks that can arise if man *totally* indulges in it. We can see a kind of opening to the Apollonian under the form of a calibrated use of logic as a tool for survival providing that it leaves space for the expansion of creativity. That was the situation in Greece at the time of the tragedy, before the advent of Socrates, a time of balancing between the Apollonian and the Dionysian that Nietzsche appreciated. In fact, he negatively described Euripides' changes in tragedy, which echoed the prevalence of the Apollonian attitude after the golden period of the Sophoclean tragedy, according to a 'Socratic esthetics' which destroyed art:

> verharren wir hier einen Augenblick, um uns jenen früher geschilderten Eindruck des Zwiespältigen und Incommensurabeln im Wesen der aeschyleischen Tragödie selbst in's Gedächtniss zurückzurufen. Denken wir an unsere eigene Befremdung dem Chore und dem tragischen Helden jener Tragödie gegenüber, die wir beide mit unseren Gewohnheiten ebensowenig wie mit der Ueberlieferung zu reimen wussten—bis wir jene Doppelheit selbst als Ursprung und Wesen der griechischen Tragödie wiederfanden, als den Ausdruck zweier in einander gewobenen Kunsttriebe, des Apollinischen und des Dionysischen. [. . .] Jenes ursprüngliche und allmächtige dionysische Element aus der Tragödie auszuscheiden und sie rein und neu auf undionysischer Kunst, Sitte und Weltbetrachtung aufzubauen—dies ist die jetzt in heller Beleuchtung sich uns enthüllende Tendenz des Euripides.[. . .] Dionysus war bereits von der tragischen Bühne verscheucht und zwar durch eine aus Euripides redende dämonische Macht. [. . .] so werden wir jetzt dem Wesen des aesthetischen Sokratismus schon näher treten dürfen; dessen oberstes Gesetz

> ungefähr so lautet: 'alles muss verständig sein, um schön zu sein'; als Parallelsatz zu dem sokratischen 'nur der Wissende ist tugendhaft'. Mit diesem Kanon in der Hand maass Euripides alles Einzelne und rectificirte es gemäss diesem Princip: die Sprache, die Charaktere, den dramaturgischen Aufbau, die Chormusik. (GT 12) Hier überwächst der philosophische Gedanke die Kunst und zwingt sie zu einem engen Sich-Anklammern an den Stamm der Dialektik. In dem logischen Schematismus hat sich die apollinische Tendenz verpuppt. (GT 14)

Socrates, the 'inspirer' of such changes, represents the 'logical man' who Nietzsche blames for having repressed vitality, by developing (and teaching us to develop) only one of our brains:

> Angesichts dieses praktischen Pessimismus ist Sokrates das Urbild des theoretischen Optimisten, der in dem bezeichneten Glauben an die Ergründlichkeit der Natur der Dinge dem Wissen und der Erkenntniss die Kraft einer Universalmedizin beilegt und im Irrthum das Uebel an sich begreift. In jene Gründe einzudringen und die wahre Erkenntniss vom Schein und vom Irrthum zu sondern, dünkte dem sokratischen Menschen der edelste, selbst der einzige wahrhaft menschliche Beruf zu sein: so wie jener Mechanismus der Begriffe, Urtheile und Schlüsse von Sokrates ab als höchste Bethätigung und bewunderungswürdigste Gabe der Natur über alle anderen Fähigkeiten geschätzt wurde. (GT 16)

Yet, like science, practiced to its extreme, discovers its limitations: 'Nun aber eilt die Wissenschaft, von ihrem kräftigen Wahne angespornt, unaufhaltsam bis zu ihren Grenzen, an denen ihr im Wesen der Logik verborgener Optimismus scheitert' (GT 15), analogously, Socrates, the logical man, feels his second brain, his Dionysian instinct knocking at the door:

> Jener despotische Logiker hatte nämlich hier und da der Kunst gegenüber das Gefühl einer Lücke, einer Leere, eines halben Vorwurfs, einer vielleicht versäumten Pflicht. Oefters kam ihm, wie er im Gefängniss seinen Freunden erzählt, ein und dieselbe Traumerscheinung, die immer dasselbe sagte: 'Sokrates, treibe Musik!' [. . .] Jenes Wort der sokratischen Traumerscheinung ist das einzige Zeichen einer Bedenklichkeit über die Grenzen der logischen Natur: vielleicht—so musste er sich fragen—ist das mir Nichtverständliche doch nicht auch sofort das Unverständige? Vielleicht giebt es ein Reich der Weisheit, aus dem der Logiker verbannt ist? Vielleicht ist die Kunst sogar ein nothwendiges Correlativum und Supplement der Wissenschaft? (GT 14)

Nietzsche outlined the strengths and limitations of both sides of man, and called for a collaboration:

> Desshalb muss eine höhere Cultur dem Menschen ein Doppelgehirn, gleichsam zwei Hirnkammern geben, einmal um Wissenschaft, sodann um Nicht-Wissenschaft zu empfinden: neben einander liegend, ohne Verwirrung, trennbar, abschliessbar; es ist diess eine Forderung der Gesundheit. Im einen Bereiche liegt die Kraftquelle, im anderen der Regulator: mit Illusionen, Einseitigkeiten, Leidenschaften muss geheizt werden, mit Hülfe der erkennenden Wissenschaft muss den bösartigen und gefährlichen Folgen einer Ueberheizung vorgebeugt werden. (HH I, 251)

Therefore, we can say that Nietzsche's criticism of logic was not aimed at discarding it but only consisted in de-mythologizing it mainly in two ways, by stressing that:

(1) it works but it is not true: it does not describe reality—because there is no reality;
(2) besides logic, there is human creativity, which cannot be overshadowed, or relegated as less important than it.

This explains his apparent 'change of mind' or oscillation regarding logic: he simply considered the positive *and* negative consequences of the logical viewpoint.

7. A final comparison

We have seen that in Brouwer and Nietzsche, there is a path that leads directly from the vision of life to their opinions on science, mathematics and logic.[30]

Nietzsche and Brouwer partly agree about science. For both, it consists in projecting the pattern of causality onto the world. Brouwer sees it as a sinful exercise of the will of man over nature, which triggers a series of research studies joined by causal links. Man may even forget the initial goal and may produce an opposite effect to that originally intended. Nietzsche does not consider it as sinful; nevertheless, he judges science negatively, because it goes against life in its creativity and effervescence. Science is part of the tools that man has invented in order to feel reassured in the universe of perceptions and change. Such tools are used for framing and categorizing in order to keep everything under control which is precisely the opposite to what favors the booming of the vital, the individual, the unique, the special.

As for mathematics, Brouwer distinguishes it from science and he therefore only 'saves' it on condition that he redefines the boundaries of a constructivist mathematics, which is not aimed at being applicative. It must be carried out as an expansion of the intuition of time: therefore, mathematics is creative and free from preconceived methods. Therefore, it is also free from logic. This is not applied in mathematics, but opposingly, it comes after mathematics as an extrapolation of the regularities that can only be found in retrospect in the expression of mathematics. Mathematics, in itself, is something a-linguistic (because language, according to Brouwer, is only used to communicate). Nietzsche shares the meaning and appreciation of creativity with Brouwer but he includes mathematics in the same field of science and logic; he considers it as something schematic and repetitive, based on the forced distinction between subject and object: for him, the ego is the source reference, the prototype for the production of units for the calculation.

Nietzsche and Brouwer reject the view of a unique logic intrinsic to man and both consider logic as a human product. According to Brouwer, logic is not a list of correct reasoning-patterns applied when proving new theorems: logic comes after mathematics, as a kind of statistics in its accompanying language. Therefore, it is conditioned by culture and, consequently, by ethnicity just like any other discipline that refers to language.

[30] We can reconsider here Tasić's short comparison between Brouwer and Nietzsche. We have seen that Brouwer and Nietzsche do agree about the concept of 'will', but—we should add—only partially. 'Will' is for both of them 'will to power', but Nietzsche considered it very positively, as the source of energy in life, while Brouwer considered it negatively, as the source of dissatisfaction and, hence, of pain in life. For both of them, the 'reality' offered by science is only appearance, but Nietzsche believed that there was no reality at all, while Brouwer believed in a transcendent, undefinable reality to which man realizes to belong only after his death (a reality to which he alluded through references to Boehme: 'When all images are removed from the soul, and she beholds the Only One, then the naked essence of the soul finds the naked formless essence of the Divine Unity, the presence of the superior being waiting in the self' LAM, p. 393) Surely, only Brouwer sees creativity in mathematics, while Nietzsche sees creativity in the arts. How would Nietzsche have reacted to Brouwer's view of mathematics? Obviously, we cannot be sure of his reaction, but we can imagine that Nietzsche would have found some difficulties in appreciating a mathematics that was so linked to the 'intuition of time' (i.e. a capacity of *individuating* instants) of the *subject*. This link would have also prevented Nietzsche from considering intuitionistic mathematics as 'true': such mathematics was clearly perspective (due to a *specific* capability!); therefore, it could not be absolute, as required by Nietzsche to consider something as true.

Furthermore, its set of laws cannot remain the same because logic must express a mental content, that is, an evidence of mind. Even negation must express some evidence: the evidence of the fact that the construction of a proof fails. Therefore, the principle of the excluded third is to affirm the holding of the resolution of all mathematical problems and it therefore becomes and remains unreliable as long as an open mathematical problem still exists. In fact, despite his lack of interest in logic, Brouwer had obtained a new logic as a 'side effect' of a program for a new foundation of mathematics. The details of such intuitionistic logic would later be developed by his student Arend Heyting. In any case, as far as Brouwer was concerned, Aristotelian logic was not intrinsic to man, it was not something that man has in his own frame of mind: it could be replaced by another logic.

For Nietzsche—who referred to an Aristotelian framework—logic serves to identify, to draw boundaries, to stabilize, while we experience a continuous flow of perceptions. Logic is a product of human evolution, finalized at our survival: it is not something absolute, but it is aim-oriented. Even if for Brouwer, there may have only been truth in logic when it describes a mathematical reality, for Nietzsche, there can never be truth in logic. Still, it was not only negativity that Nietzsche attributed to logic: he appreciated the use of logic to argue and persuade, and complained of the poor exercise of deduction in German schools and universities. He simply wanted to determine the limits of logic exactly, in order to stress that it does not have an absolute value. The two sides of the human, the rational and the irrational (the two brains, as Nietzsche called them)—coexist and should be made to interact without penalizing either. The rational side smooths, allowing for survival. The irrational side innovates, creates, leaves out the strong emotions, the uniqueness, the individuality of each person.

With an eye to the twenty-first century, we point out that logic is a tool: it offers its laws to treat the concept of identity, but it does not require the real existence of identical objects. Logic could even exist as a mere intellectual exercise. Furthermore, identity can also be fictitious. It is not necessary that it be 'true' (i.e. non-perspective). However, this is a typical viewpoint of the twenty-first century: this awareness was only reached a hundred years after Nietzsche. Such awareness was obtained by ignoring Nietzsche's contribution, perhaps because in the nineteenth century logic had moved from a philosophical framework toward a mathematical framework. On the contrary, Brouwer directly contributed to such awareness, by providing a new logic that differed from traditional logic. Other non-classical logics were later introduced (modal, fuzzy, quantum logic for example), welcoming the (Nietzschean) opinion that logic is a human product, driven by certain needs and aimed at very specific purposes in well-defined areas, providing a broader outlook of the existence of a pluralistic range of logics.

Note: Nietzsche's quotations in this article come from Friedrich Nietzsche, *Digital critical edition of the complete works and letters*, based on the critical text by G. Colli and M. Montinari, Berlin/New York, de Gruyter 1967—edited by Paolo D'Iorio. Wherever it is not expressly stated that it mentions the page, the reference refers to the aphorism number.

Abbreviations

A = Der Antichrist (The Antichrist)
BGE = Jenseits von Gut und Böse (Beyond Good and Evil)
GM = Zur Genealogie der Moral (On the Genealogy of Morals)
GS = Die fröhliche Wissenschaft (The Gay Science)
HH = Menschliches, Allzumenschliches (Human, All Too Human)

NF = Nachgelassene Fragmente (Posthumous Fragments). They are followed by the number of the year in which they are supposed to have been written.
TI = Götzen-Dämmerung (Twilight of the Idols)
TL = Über Wahrheit und Lüge im aussermoralischen Sinne (On Truth and Lies in an Extra-moral Sense)
Z = Also sprach Zarathustra (Thus Spoke Zarathustra)

Brouwer's Abbreviations
CL = *Brouwer 1981*
CW I = *Brouwer 1975*
LAM = *Brouwer 1905*

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