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THE INTERACTIONS OF MATHEMATICS AND SOCIETY IN HISTORY SOME EXPLORATORY REMARKS

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Summaries

An exploration of the relations of mathematics and society in history, intended as a basis for general discussion. This includes an argument for doing social history of mathematics; a survey of themes, problems, and methods, structured by distinguishing different social forms of mathematics; and, finally, reference to some special themes.

Der Aufsatz will eine Basis liefern für eine allgemeine Diskussion des Verhältnisses von Mathematik und Gesellschaft in der Geschichte. Dazu wird im ersten Teil mit Verweis auf bestehende Traditionen, mit methodologischen Gründen und mit der Frage nach der Verantwortung des Mathematikhistorikers, für eine Sozialgeschichte der Mathematik argumentiert. Weiter wird der Versuch gemacht, das Problemfeld möglichst breit zu erkunden. Den Ausgangspunkt bildet dabei die Unterscheidung von verschiedenen sozialen Formen, in denen Mathematik auftritt. Die Wechselwirkungen zwischen Mathematik und Gesellschaft werden erörtert und Begriffe und Methoden zur Behandlung dieser Themen angegeben. Zum Abschluss werden einige spezielle Themen angesprochen und Literaturhinweise gegeben.

0. INTRODUCTION

This is a slightly revised English version of a paper [1] presented at the conference on the history of mathematics at Oberwolfach, Federal Republic of Germany, in December 1975. It was designed to serve as introduction to a general discussion on the interactions of mathematics and society in history.

In accordance with its special purpose, the paper has three aims. First it argues for the importance of the subject, which, we feel, deserves more attention from historians of mathematics than it receives at the moment. In part 1 we discuss three arguments for studying the relation of mathematics and society

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and for treating this theme in teaching. The second aim, treated in part 2, is to provide a preliminary exploration of the field. To do this we have introduced certain concepts to describe different roles which mathematics may play in society. These concepts are, of course, debatable, but we hope that they may be helpful in a further discussion. The third aim, the subject of part 3, is to mention a number of specific themes on which research on the relation of mathematics and society could focus, and to provide references to literature, in particular for a discussion of these themes in teaching.

For both of us, interest in the problems of mathematics and society arose mainly in connection with problems of the teaching of mathematics and the history of science in universities. This certainly determines the approach adopted in this paper. We hope that the paper will fulfill its purpose to incite discussion and interest in the social context of mathematics in history.

1. WHY SOCIAL HISTORY OF MATHEMATICS?

Study the historian before you begin to study the facts. [Carr 1961, 17].

1.0. Historiography is determined by the present. This gives meaning to history, but it also presents methodological problems and the general problem of the nature of history as a science. Our plea for a certain change of interest within history of mathematics, and the suggestions of new methods to be used, are motivated in large part by a concern for the relevance of our subject for the present day. Hence we feel that we should explicitly mention our motives in order that these too can be discussed.

There seems to be no danger yet of arguments on method and meaning becoming fashionable in historiography of mathematics. Even the well-known external-vs-internal and revolutionary-vs-evolutionary debates have left few traces indeed in the writings of historians of mathematics.

To argue for the necessity of a social history of mathematics we shall point to an existing tradition withing the historiography of science. Further we shall offer two arguments of methodological character, and finally we examine the role of the historian of mathematics in the social system of mathematics and discuss his social responsibility.

1.1. *The tradition.* Thinking about social history of mathematics and its methods, we find ourselves in an "internal" tradition of historiography of science. There are a number of papers on methodology of history of mathematics. For instance, Scriba [1967, 1968, 1970] describes nine different ways of doing history of mathematics, including the social history of mathematics. He points out very clearly the necessity of relevance for the

present. For example,

> *History of mathematics should, among other things, help to clarify, and to make people now living aware of the relations that exist between the structure of society at the turn of the eighteenth century and the ideas on culture and education which were developed then. It should teach us to understand that, and why, all mathematical education is also determined by the conditions of the time; it should make us more willing to seek new ways, appropriate for the present.* [Scriba 1968, 11] [2]

In historiography of science there has already been much more discussion of methodology, especially on the assessment of external factors in the development of science. The thirties were a fruitful period in this respect; the "classic" papers by Merton [1938] and Hessen [1931] were published then. Their new editions prove the present revival of interest in such problems. After World War II the conception of history of science as the history of scientific ideas became dominant under the far reaching influence of A. Koyré. Today the debate has proceeded so far that the ideas of historians of science about the nature of their subject are themselves seen in their historical context [Kuhn 1968, Thackray 1970, McGuire 1973]. We cannot go into this debate in detail but we should mention that one of the prime arguments is the relevance of historical studies for the problems of modern science.

1.2. *The methodological argument.* A methodological problem for history of mathematics lies in the necessity of the relation to the present. Unavoidably, present-day mathematics strongly influences the points of view, the choices of subject, and the value system applied. But this influence may obstruct the endeavour to treat, as far as possible, history objectively and to do justice to the mathematics of the past. Here the approach of social history may prove useful. It draws attention to the social position of mathematicians in a period, to their ways of communication, to their implicit aims, to the ways they justified their activities, etc.; and in this way it forces us to consider and judge the mathematical works in their contemporary context. It is true that social history is also in danger of proceeding implicitly teleologically. For instance the concepts of "mathematician" and "scientific community" have to be differentiated according to the historical period [Thackray/Shapin 1974, 1-4].

The assistance which a consideration of social aspects can give to the appropriate understanding of the past forms our first methodological argument. A second one is related to the internal-vs-external debate, i.e. the question, whether the consideration of social factors is necessary for a proper understanding of events in science. There is certainly more to the

relation science-society, but the debate tends to be reduced to this question of the necessity of external explanations. But mathematicians are social beings and the development of mathematics is a process of interaction between mathematicians. Hence, obviously there is always a social element in the history of mathematics. The important methodological question is how in individual cases these social elements should be valued and what they can contribute to the explanation of mathematical innovations. It is impossible to argue in general how such an evaluation should be performed. It will depend on the personal viewpoints of the historian, and even more on the framework within which the explanation functions. In general it can be argued that the more global the processes one wants to explain, the more important are social factors. In "local" cases, as in the explanation of mathematical details (e.g. Poncelet's introduction of imaginary elements into geometry) social factors usually have no significant role. In some such cases the social influences, though clearly present, are difficult to evaluate, as for example in Huygens' studies on curvature, which were motivated, through his studies of the pendulum clock, by the problem of longitude. But in the case of more general developments, such as the rise of a new mathematical discipline or school, it is more feasible to assess the role of social factors. In the development of descriptive geometry [cf. Taton 1951] or of Operations Research, social conditions (requirements of (military) architecture and of modern warfare, respectively) have played a considerable role. Finally it is hardly possible to give any other but social explanations in the case of completely global processes in the development of mathematics (e.g. the rise of a renewed mathematical tradition in the Renaissance and the scientific revolution; for the latter cf. Needham [1956]).

1.3. *Responsibility.* After these remarks on tradition and methodology, we come to the responsibility of the historian of mathematics. Not very much has been written on the meaning and motivation of work in the history of mathematics. Prefaces and introductions of histories of mathematics hardly give a general foundation or justification. Usually the group of readers for which the book is meant is specified. These are mostly mathematicians and students of mathematics whom the book intends to help to understand better and more fully grasp their science. Then there are the popular books which try to inform a more general public about mathematics by means of its history; these books, so to speak, serve as representatives for mathematics to the outer world. Often the attempt to prove the cultural value of mathematics is the motivation of such books. From a sociological point of view (and grossly speaking) history of mathematics appears to have the function of strengthening the self-assurance of the mathematicians, of giving their community more inner coherence, and of proving the value of mathematics to the outside

world. Characteristic for this function are the historical examples that serve to prove that internally developed parts of mathematics may later lead to fruitful applications. Every mathematician knows a number of such examples and produces them if asked for the use of his work.

The question arises whether under these conditions history of mathematics is able to function according to its responsibility towards its subject-matter, towards the community of mathematicians and towards society in general. If history conforms to the functions described above, it will become mainly, in Nietzsche's terms, "antiquarian" and "monumental" *(monumentalisch)*. It will exhibit to the present-day mathematician the monuments of the past in whose greatness he may participate and thereby be comforted. It nurtures and conserves the remnants of a past whose connection with the mathematics of today has become thin. The heroes and antiquities of this kind of historiography are determined by the purely cognitive structures of mathematics now, because it is there that the mathematician feels himself secure. Antiquarian and aesthetic interests are legitimate, but in themselves they remain meaningless for the understanding of either the past or the present.

The social context of science and its problems will fall outside the interest of a historiography which serves to confirm and encourage the mathematician in his traditional self-understanding. Thus it is not able to pass outside the confines and the language of the tradition of present-day mathematics and it cannot fulfill its task to master the problems of the present [3]. It masters the present only by offering retrospection and consolation.

Nevertheless, "Mathematics and Society" has become a slogan. There are many causes for this. One of the most obvious is the confrontation with marxist positions. These have caused a good deal of interest in the history of science, but they expect from history relevance for the present, theoretical background, and above all a consideration of social and economic connections. With these expectations students meet mathematicians and (perhaps less outspoken) historians of mathematics who, despite "computer age", "New math", "war gaming", and despite the fact that they educate future teachers and industrial mathematicians, refuse to discuss the social problems of science, of its application, and of science education. They keep to their view of mathematics as an esoteric, autonomous activity, based ultimately on aesthetic interests. One of the consequences of this situation is that insubstantial vulgar marxist interpretations of history flourish among students. But also those students who reject such points of view do not find the means or partners for a sensible discussion on these matters. Here lies a responsibility for history of mathematics, a responsibility to provide terminology, concepts and clear points of view that can serve in a discussion on

mathematics in modern society.

Through history, we can learn meaningfully to discuss certain themes, and to judge them with discernment. But we can also force things into certain frameworks and even completely remove them from view. To be concrete: if there is no language, no conceptual apparatus, to discuss the connections of mathematics and society, we are hardly in a position to observe the phenomena. We are then--in two meanings of the word--speechless if confronted with these phenomena. A certain training to see such problems and to judge them is necessary for a scientist to answer (again in two meanings) to his social responsibility.

As mathematics is a rather esoteric science, history of mathematics at universities has a special educational responsibility. Basically this is the responsibility for an aspect of the language and the ways of thinking with which we approach the present (and thus also the future), a responsibility which is also to be observed for neighbouring disciplines such as philosophy and the science of science. These considerations clearly imply that history of mathematics should encompass the social context.

These remarks are rather concise and possibly formulated somewhat strongly. Surely many things could be added to fill in necessary nuances. But there is a problem and we have tried to bring it out plainly.

2. EXPLORATION OF THEMES, PROBLEMS, CONCEPTS, AND METHODS

2.1. THEMES AND PROBLEMS

2.1.1. *The social forms of mathematics.* Striving towards a social history of mathematics presupposes that it is possible to define an entity "mathematics" which (at least in its important aspects) can be conceived as a social phenomenon. Traditional historiography of mathematics takes as its subject mathematical ways of thought, methods, results, and theories. In this definition of mathematics, no social categories are taken into account. It is clear, however, that mathematics develops in a social context, that it is affected by social factors, and that it has an influence on society. Trying to view mathematics in this way as a social phenomenon, one has to mark off a social entity "mathematics"--that is, to describe the social forms in which mathematics manifests itself. The study of these forms seems to be an appropriate approach to the historical examination of the mathematics-society interaction. Indeed the formation and maintenance of a mathematical tradition necessarily presupposes some social form of mathematics through which society exerts its most basic influence on mathematics. For instance, the social organization of mathematical activity defines the extent to which a free, unimpeded development of mathematical ideas is possible; and it determines which outside stimulations

can act upon mathematics. For these reasons the investigation of the social forms of mathematics and of the conditions for the emergence and presence of these social forms is a central element of the social history of mathematics.

In many cultures mathematical methods are used in the areas of material production, social organization and communication without being recognised and transmitted as an independent cognitive unity. We will call this the *dependent social form of mathematics*. This dependent form (characteristically: the use of elementary calculating and measuring methods) exists in every complex society, and here mathematics exerts its most immediate influence on society. Moreover, this dependent form is the basis for all other more independent social forms of mathematics.

We will discuss four such *independent social forms of mathematics*, namely: Mathematics as a subject matter of teaching; Mathematics as basic knowledge of certain professions or trades; Mathematics as a legitimate and accepted part of philosophical, religious or recreational activities; and Mathematics as an institutionally confirmed separate science.

The formation of independent social forms of mathematics in a society in which the dependent social form of mathematics is already in existence, can be considered as a result of the division of labour. Apart from more specific questions, social history of mathematics is interested in the socio-economic conditions for the origin and development of these social forms of mathematics and in the question whether special characteristics of mathematical style are induced by certain social forms.

Mathematics as a subject matter of teaching arises when a certain group of people needs mathematical skills and when the transmission of these skills is concentrated in a social institution, in an educational establishment. The ultimate aim of mathematics in this area is still the solution of problems arising in labour and organization in society. But in this social form mathematics may lose its direct dependence on practical problems and become of interest by itself (as is testified, e.g., by some completely unpractical problems found in Egyptian papyri). Also mathematics can be used in instruction and education to transmit attitudes and values not directly concerned with problem-solving; for instance the usefulness of mathematics may be seen in "teaching a man the importance of thinking things out for himself and so delivering him from the present dreadful yoke of authority..." [Griffiths and Howson 1974, 17]. The development and differentiation of institutions partly or fully dedicated to mathematical training is an important historical process. In the manifold educational institutions of the past aims and arguments for the teaching of mathematics have been many and most various. Educational institutions form one of the main supports for an independent tradition of mathematics.

As a result of the division of labour special professions

and trades arise. In some of these the use of mathematical methods plays a characteristic role. Thus another social form of mathematics arises, namely, *mathematics as basic knowledge of certain professions or trades.* Mathematical activities of this kind are specifically connected with the relevant range of problems. Consequently, mathematics in this form is usually less clearly defined than in other social forms, and the mathematical character of a profession may mainly exist in its public image. Thus the mathematics of the mathematical practitioners of the 16th to 18th centuries was a very broad field without clear boundaries with, say, astronomy, mechanical technology or general professional skill. The example of the mathematical practitioners also shows that mathematical professions or trades may act as one of the bases of a mathematical tradition. Two further examples of this social form of mathematics are: the mathematical knowledge of the astrologer-astronomers in antiquity and the mathematics of the industrial mathematicians and computeroriented practitioners in the 20th century.

In the two forms discussed above, mathematics is narrowly linked with its practical function as a means of solving problems. Traditional historiography of mathematics, however, is mainly concerned with mathematics studied "for its own sake". This kind of science originated with the Greek and can--with respect to its social context--be characterised as *mathematics as a legitimate and accepted part of philosophical, religious, or recreational activities.* Another example is the mathematics of the "amateurs" of the 17th century. In this form mathematics is part of the social subsystem that produces and transmits ideology.

This form of mathematics is not determined by practical applications, so that the development of mathematical knowledge here depends mainly on internal factors and therefore proceeds in a way different from the development in the two previously mentioned forms.

The existence of this form of mathematics, however, does depend on a social system in which such activities are socially possible and legitimate. This legitimacy may contain restrictions which can be interpreted as an action of society on mathematics. Therefore also in this case purely internal historiography is insufficient.

We define as fourth social form *mathematics as an institutionally confirmed separate science.* The line between this and the other forms is necessarily vague. Mathematics here appears as an endeavour to gain knowledge, socially organized in private or public societies and academies, in universities or in research institutes. It is essential for this form of mathematics that its practical function (e.g., the teaching duties of university mathematicians in the 19th and 20th centuries), though important, influences its development only indirectly. Characteristic for this form is the existence of institutions with

certain social and organizational structures in which mathematics is studied. This implies an important role for institutional history in this connection, and usually it will not be possible to restrict oneself to the history of exclusively mathematical institutions.

In the course of history the various social forms of mathematics have simultaneously been in existence and have been interdependent in manifold ways. Their distinction highlights the complexity of the general question of the role of mathematics in society. The significance of the social forms is connected with the social function of mathematical activities. Grossly, we may distinguish two types of social function of a science: the practical and the ideological. In this connection we mean by "ideology" a thought-out and formulated system which combines interpretations of reality with values and attitudes. Mathematics, as a means to solve problems in society, has a practical function, which is most clearly evident in the mathematical professions or trades. Mathematics as a subject matter of teaching has both practical and ideological functions; on the one hand it serves to educate problem-solvers, on the other hand it helps to transmit attitudes and values (e.g. mathematical thinking valued as the "correct" way of thinking). Mathematics as part of religious thinking clearly has an ideological function (e.g. the mathematics of Nikolaus von Kues). The recent discussions on the "New Math" in the schools show how closely practical and ideological functions can be connected.

2.1.2. *Interactions of mathematics and society.* The occurence of such varied social forms of mathematics shows that the interrelations between mathematics and its social context are multiple and complex. To gain an orientation in this area we will first discuss in this section the influence of social factors on mathematics and then the influence of mathematics on society. This dichotomy only serves to get an overview of the subject matter; obviously these influences do occur not separately but simultaneously. So, for instance, the relation between mathematics and society in the period of the French Revolution can only be understood as a dialectical process.

2.1.2.1. *The influence of social factors on the development of mathematics.* In the discussion of the social forms of mathematics we have already touched upon several examples of social influences on mathematics. The most direct is through the origin of problems in society. This influence may act directly upon mathematics or it may be transmitted via other sciences, or, even more indirectly, it may assert itself through the establishment and organization of educational institutions for the training of problem-solvers. Examples are the requirements of commercial arithmetic in Italy in the late Middle Ages, the problems of physics, and in recent times the theory of optimization.

A still more indirect influence acts through the working conditions of the mathematicians. This applies globally, as in the existence or limitations of the freedom of research in academies, universities, and research institutes, and also locally, for example, in the working conditions of a mathematician in an industrial laboratory.

The social organization of mathematics is the product of outside social influences; simultaneously it is itself a social factor in the development of mathematical knowlege. Norms, which are formed within the group of mathematicians and by which their work is judged, are very important in this connection. As the education of mathematicians becomes more uniform the meaning of attitudes and values, acquired in this education, increases.

A further possible influence of society on mathematics is transmitted through the style of thinking. Political, economic and social circumstances are reflected in ways of thinking and worldviews, which in their turn contribute to the stock of ideas on which mathematics draws. This will, for instance, be evident in those cases where there is a certain unity of ways of thinking and ways of acting in one historical period, as for example in the Baroque.

Closely connected with the style of thought are the issues and values which prevail in the thinking of a historical period. These also may influence mathematics, for example, through priorities for certain research subjects. In such cases the results of research need not be directly socially relevant, and still the work of the mathematician is socially accepted as legitimate. Even the mathematician himself may not be aware of the fact that his subject matter is favoured by social values. Examples are the heightened interest in probability theory in connection with political arithmetic, the interest in problems of the infinite against the background of theological and philosophical themes, and the interest in systems theory in connection with the view that society should be seen as a functionally interacting system (and social problems as malfunctions of the system).

2.1.2.2. *Influences of mathematics on society.* The most obvious influences of mathematics on society are connected with the application of mathematical methods to practical problems in society. The significance of the influence of the dependent form of mathematics can be seen in the social importance of solved problems. These may be of great social import (as in government, taxes, finances, etc.). They may also be rather unimportant, or their effect on society may be hardly assessible (geometrical decorations, for example).

The social importance of mathematics as a subject matter of teaching and as basic knowledge of professions or trades can be gathered again from the problems treated, but even more from the social function and status of the educational institutions and

and the professions and trades in question.

The practical influence of mathematics as part of philosophical, religious, or recreational activities is only indirect, for instance, through the transmission of mathematical knowledge gained in this social form to other forms of mathematics. In particular mathematics as institutionalized, separate science takes over and elaborates this mathematical knowledge.

The practical influence of the institutionalized, separate science of mathematics shows mainly in the use of mathematical methods and ideas in the natural sciences and in technology. Clearly these are most important influences (e.g., in the late industrial revolution), but it is very difficult to assess in this case the special role of mathematics or of special mathematical theories. Only for a few fields and certain techniques may lines of influence be delineated (e.g., quaternions-vector analysis-theory of electricity-electric technology).

Along with the practical influences through natural science and technology, the last fifty years have witnessed a new, direct influence of mathematical science on society, namely the influence of mathematical theories concerning systems, organizations, and processes (Operations Research, Cybernetics, Game Theory, Programming). Of course mathematics has long been involved in problems of social organization (land measuring, tax systems); but the link of scientific mathematics with such problems is not so old. One could locate its beginning in Condorcet's projects [Condorcet 1974, Baker 1975]. In the middle of the 19th century statistics as used in demography and insurance was flowering, and in the beginning of the 20th century statistical theory followed. At the same time F. W. Taylor developed his principles of scientific management, and about half a century later there followed the theory of games, systems-theory, cybernetics, and the theories of linear and non-linear programming. These latter disciplines developed in close connection with the modern computers.

The essential difference of the two forms of practical influence of mathematics discussed above can be further characterised by the statement that in the first case the influence on society is through products (telephone, aircraft, etc.) and in the second through decisions (optimal strategies, price calculations, etc.).

In the same way as mathematics may be influenced by the style of thinking of a period, so, conversely, mathematics may influence the general way of thinking. Especially in times when mathematical thinking acquires high prestige, this ideological influence may be considerable. A clear example is the "mathematization" of jurisprudence in the 17th and 18th centuries [Stephanitz 1970], which had direct social consequences. We have already mentioned the ideological function of mathematics in the educational system. Mathematics may acquire the role of

a model of rational thinking, thus leaving less room or status to other ways of thinking. This may show itself in a fashion for mathematical concepts and terms, as is the case now with "set", "game", "structure", etc. It may also show in the attempted mathematization of fields whose content and development are not (yet) fit for mathematization. In such situations, the deductive rationality and the objectivity of mathematical methods, as symbolised in computers and in boards of scientific advisors, may serve to conceal interests of a different rationality.

2.1.2.3. *Concluding remark.* Historical research that sets itself themes as discussed above has itself both a practical and an ideological function. The social forms of mathematics and the influence of social factors on the development of mathematics are important for today's discussions on the organization and support of science and on science policy in general. The questions of the social effects of mathematics and its interactions with society are fundamental in the present day discussions on the social responsibility of the mathematician.

2.2. CONCEPTS AND METHODS

In the exploratory remarks on our problem so far we have already used a number of concepts that are characteristic for this theme and are in part based on methodological conceptions. A social history of mathematics must have at its disposal further specific methods and concepts. Some such concepts and methods--we will not draw a sharp distinction between these two--have already been worked out in other contexts than the history of mathematics. We mention a few of these in this section.

2.2.1. *Description of the social forms of mathematics.* In 2.1.1. we distinguished several social forms of mathematics. It seems necessary to work out further criteria for these distinctions. Their use in historiography should be tested, and, if necessary, they should be modified and supplemented.

As far as a group of mathematicians can be meaningfully distinguished within society, it may be studied with methods and concepts of the sociology of science such as Thomas Kuhn has supplied with his theory of scientific revolutions [1962, 1970a, 1970b]. One of us has discussed their applicability in the case of history of mathematics [Mehrtens 1976]. A very interesting study of the structure of the community of mathematicians is due to Fisher [1972]. The sociological concept of norms is of high importance here. Merton [1957] has summarized the ethos of science in four such norms: universalism, communism, disinterestedness, and organized scepticism. The significance of these norms for the sociology of science has later been questioned [Barnes and Dolby 1970]. In a case study, Mulkay [1969] has drawn attention to the normative function of the disciplinary

matrix (in Kuhn's sense). This implies that norms may be different in different sciences; knowledge of the discipline and attitudes toward the subject function as norms of the scientific community of the discipline. These norms are reflected in the criteria for judging the quality of scientists and scientific work (for instance, in letters of reference for job applicants [Bierman 1973]) and also in the conception of science itself [Mehrtens 1976a, 316 f.].

However, sociology of science until now has restricted itself almost completely to contemporary academic forms of science, so that its concepts will be helpful primarily in the social history of mathematics as an institutionally confirmed, separate science. For the other social forms of mathematics, concepts and methods from other disciplines within sociology (such as the sociology of professions, of education, and of knowledge) may be of use.

Closely connected with the social form of mathematics and its function in society is mathematical style. Here the traditional distinction of "pure" and "applied" mathematics comes to mind. In its style, the relation of mathematics to praxis becomes evident. Also, this style is an important factor in the development of mathematics. Hence it will be useful to introduce a further differentiation in this connection, especially since "applied mathematics" is too vague. A tentative further differentiation is given by H. J. M. Bos [1975]; he distinguishes autonomous, professional and practical mathematics and characterises these by their criteria of quality and by the sources of their problems. The study of differences of style in mathematics could also be useful in connection with the question of how far social conditions (within or outside mathematics) influence its cognitive content.

2.2.2. *The interaction of mathematics and society.* Society has a direct influence on science through the determination of the social form of science by socio-economic factors. Two obvious effects are the fluctuation of scientific productivity and the transfer of the centres of scientific activity, which, in course of history, settled in many different countries. Ben-David [1971] has studied this process from the sociological point of view. There are a number of similar studies, not directly concerned with mathematics, for instance, by Turner [1971]. The historiography of scientific institutions yields much important material in this connection.

The influence of society through problems (cf. 2.1.2.1.) involves a scale of possibilities. At one extreme there is the mathematician asked to solve certain problems considered socially important (e.g., in World War II). At the other, is the mathematician inspired by a problem outside mathematics to work out a mathematical theory (e.g., Kepler's studies on the volume of wine casks). In general the latter type of problem will lead

to a "pure" piece of mathematical research, while at the other end of the scale, "applied" mathematics dominates. Further differentiation, research, and assessment of the forms in which problems are suggested to mathematics will be an important starting point of the study of social influences on mathematics.

In the opposite direction there is the application of mathematics in problems occuring in social practice, that is, the practical influence of mathematics. This influence appears to be hardly assessible, precisely because it is so comprehensive and multiple. It is therefore important to find methodical starting points also in this area. One distinction that should be made concerns the question whether applications occur directly or indirectly (e.g., through physics or technology). This distinction is essential in connection with criteria of quality of mathematical work, since these criteria are different for the different applications.

Closely connected with the types of application is the development of mathematical professions. This includes the professionalization of mathematics as purely scientific work. But the more interesting questions are connected with professions and trades that use mathematics. Here again sociological concepts can be of use, such as division of labour, professionalization, bureaucratization, etc.

A further question concerns the significance of mathematics for the professions. These can be graduated from the clearly mathematical professions (mathematical practice, industrial mathematics, systems analysis) via professions where mathematics is one part of the basic knowledge (such as engineering) to those professions and trades that use elementary mathematical techniques (e.g., office work). To give a survey of these phenomena in a given historical period and to notice and assess changes (such as the disappearance of the mathematical practitioners at the beginning of the 19th century and the reappearance of practical mathematics of high cognitive content in the 20th century) certainly would be an important contribution to the study of the impact of mathematics on society.

Besides social forms and practical problems there is the interaction of mathematics and society through ideas. It is difficult to find appropriate methods to study this. A first approach, which does not yet attempt to assess mutual influences, is the search for structural similarities in the ideas prevailing in a certain historical period and place. A. Nitschke's studies [1967, 1969] are an example of this approach. He defined his method as *historische Verhaltensforschung* (historical ethology) [1974]. An example of such a study that touches on mathematics has been given by Eichberg [1975], who tried to comprehend geometry as a behavioural norm of the baroque period. Two examples from the historiography of the natural sciences should also be mentioned. L. Feuer [1971] has attempted to explain certain

aspects of the early works of Einstein from the socio-cultural background, using the concept of "iso-emotional lines"; and P. Forman [1971] has used the concept of the "intellectual environment" of German physicists and mathematicians in the period 1918-1927 in a study of the development of quantum theory.

The influence in the opposite direction, the effect of mathematical ideas and ways of thought on the ideology of society is equally difficult to grasp. It seems that the most promising starting point for research here lies in mathematics as a subject of teaching. After determining the levels at which each kind of mathematics are taught, one may ask further for the explicit and implicit motives of the various forms of education in connection with the ideological functions of education. Starting points may be found in Griffiths and Howson [1974, 7-26] and, on a marxist basis, in Neander [1974].

2.2.3. *Further concepts and methods.* From the realm of philosophy of science there are further conceptions to be mentioned besides those of Kuhn, such as Lakatos' model of "research programmes" [1973] and the ideas of Toulmin [1967], who describes the development of science in analogy to biological evolution, explicitly taking external factors into account.

Thackray and Shapin [1974] have proposed the method of "prosopography" to grasp the social context of science in history while avoiding "presentism". L. Stone [1971, 48] has characterized this method as "the investigation of the common background characteristics of a group of actors in history by means of collective study of their lives." An example of a study using this method (in a restricted sense) is Fisher's second study on invariant theory [1967].

Wilder's "anthropological" conception of mathematics within general cultural history may prove a valuable methodological tool for the historian of mathematics [Wilder 1968, 1974]. Finally quantitative methods in historiography should be mentioned: Rashevsky [1968], and, for mathematics, Folta and Novy [1965], May [1968], and Crane [1972].

3. SOME THEMES AND REFERENCES

There are only a few studies specifically treating the interaction of mathematics and society. Still there is ample material in the literature of the history of mathematics and of science in general that can be used in research and teaching. We have not attempted a complete survey of the literature; rather we give some examples of subjects which can be used in teaching, with relevant literature.

3.1. *General.* A stimulating discussion of the general problem with some references can be found in papers by Struik [1942] and Chapelon [1971], both of which survey social influences on the development of mathematics.

Materials of varied nature are included in collections, e.g., Kline [1968], LeLionnais [1971], and Newman [1961]. The periodical *Materialien zur Analyse der Berufspraxis des Mathematikers* includes historical discussions of the mathematics-society relation (cf. *HM* 2, 403).

The best history of mathematics regarding the social context of the development is still Struik's *Concise History* [1948].

For bibliographies we refer to the abstracts of *Historia Mathematica*, May [1973], *ISIS, Technology and Culture*, and to the the comprehensive bibliography on the science of science by Spiegel-Rösing [1973]. For the history of technology a survey of research programmes and a bibliography is included in Rammert [1975].

A collection of esssays on science as a cultural phenomenon, very suggestive in the choice of topics as well as in methodology, has been edited by Thackray and Mendelsohn [1974].

3.2. *The rise of Greek mathematics.* The question of the causes of the rise and the special character of Greek mathematics is an almost "classical" problem of the science-society relation. Explanations range from the tautological "instincts of the race" (Heath) to the view of mathematics as the outcome of the way of thinking attained by abstraction from the forms of commodity exchange [Sohn-Rethel 1972, 92-115].

In this case an explanation disregarding the social context is clearly impossible. It is equally obvious that stressing only one social factor (slavery, invention of coins, practical necessities) is also unsatisfactory. The standard histories of Greek mathematics combined with the social history of antiquity and with the manifold materials in philological studies (for instance the marxist study of Thomson [1955]) should give a basis for a discussion of Greek mathematics in its social context. The social background of the rise of Greek science and philosophy is treated in papers by Nieby1 [1974], Harig [1958], and Farrington [1953], who stresses the role of mathematics.

Although the special character of Greek mathematics is discussed in those essays, there is little material on its relation to Greek society. Characteristic of the disregard of the social aspects is a study by Pedersen [1974] treating the role of logistics in Greek mathematics. He considers only internal causes and reaches no conclusion. For the time being it is equally plausible to argue that the "class-character" of Greek mathematics determined its special form [Høyrup 1973].

For comparative studies the mathematics of pre-Greek and non-European cultures should be taken into account. For China we are able to refer to the outstanding work of Joseph Needham [1954, 1969].

3.3. *The mathematical practitioners.* An important topic in the social history of mathematics from the Renaissance to

the Industrial Revolution is the group of mathematical practitioners. Here we find a specific social form of mathematics, professsions with the common characteristic of being mathematical. These practitioners and their forerunners played their role in the scientific revolution [Zilsel 1942], and their history mirrors the development of the status of mathematics in society. The literature concerning single persons, regions, etc. is extensive. For more general works we may refer to Folkerts [1974], Schneider [1970a], and Taylor [1954; 1966]. Further topics related to the mathematical practitioners are, e.g., the connection of mathematical technologies and the idea of progress in the 16th century [Keller 1972], the relations of mathematical practice and education [Turner 1973], the "philomaths" [Pedersen 1963, Wallis 1972], the mathematical societies (e.g., the *Hamburger Mathematische Gesellschaft* founded in 1690 [Schimank 1941]), and, at the end of the period, the societies of the arts [Hahn 1963].

3.4. *The academies.* In the same period the main institutions sustaining a high-level mathematical activity were the academies and scientific societies. The history of those institutions, which has found new interest in recent years, is of great importance to the social history of mathematics. For general surveys we refer to Brown [1934], McKie [1948a], Ornstein [1928], Yates [1911]; special subjects are treated by Crosland [1967], Hahn [1971], Hoppen [1967], Purver [1967]. These materials should be combined with general social history and biographies of mathematicians. Further connected topics of interest are the development of scientific journals [McKie 1948b] and the ways mathematicians earned their living (for an interesting case study cf. Duveen and Hahn [1957]).

3.5. *Mathematicians as university teachers.* With the Industrial Revolution emerged a new social basis of mathematics. It was institutionalized in the universities, which were designed for the education of a new elite, and the university teachers attained the status of a scientific elite. The process started with the French Revolution and was carried further in Prussia [Ben-David 1971; Turner 1971]. The development of mathematical education at German universities and technical colleges in the 19th century has been described by Lorey [1916] and Stäckel [1915]. The increasingly strong tendency toward abstract pure mathematics led to a countermovement in the 1890's, one of whose leaders was Felix Klein [Manegold 1968, 1969]. These developments, which are part of the more general process of the professionalization of science, should be studied in connection with social, economic and educational history. As examples of materials related to mathematics we may mention the history of mathematics and mathematicians at the university of Berlin [Biermann 1973] and the story of the political activities of

Jacobi [Ahrens 1906/7]. A first attempt to work these materials into a piece of social history of mathematics has been made by Mehrtens [1976b].

3.6. *The Scientific Revolution.* We should not omit one of the most prominent topics of the internal-external debate--the Scientific Revolution. Studies focussing on the (important) role of mathematics are rare. Main contributions to the general debate are the orthodox-marxist position of Hessen [1931], the sociological study of Merton [1938], and the internalist standpoint of Hall [1962, 1963]. Among shorter essays on the origins of the scientific revolution, is first of all Needham [1956], who is explicitly concerned with mathematics. We may further refer to Kemsley [1968], Lilley [1953], Mathias [1972b], Nieby1 [1972], and Rattansi [1972]. The papers of E. Zilsel on the social origins of modern science [e.g. 1942] have recently been collected and edited together with commentary and comprehensive bibliography in German [Zilsel 1976].

3.7. *Further topics.* Finally we point to some topics that could be used in courses on mathematics and society. Courses of such character (which are rare) are usually of a more or less experimental character. For some examples in disciplines other than mathematics we refer to Schrooer [1972] (physics), Arbeitsgemeinschaft [1974] (chemistry), and to the SISCON-units [4] which encompass manifold subjects.

Some possible topics in mathematics are:

The social role and function of mathematics education [e.g., Neander 1974; Griffiths and Howson 1974].

History of modern applied mathematics.

Development of the profession of the industrial (or insurance) mathematician [e.g., Fry 1941, 1964].

The role of mathematics in the social sciences [Baker 1975, Freudenthal 1966, Stephanitz 1970].

Professional self-conception and organization of mathematicians.

NOTES

1. The original German version may be obtained from the authors on request.

2. The original German: "Mathematikgeschichte sollte, unter anderem, dazu beitragen, jene Zusammenhänge zu klären und sie den heute Lebenden bewusst zu machen, die zwischen der Zusammensetzung der Gesellschaft um die Wende des 18. Jahrunderts und den Bildungs- und Ausbildungsvorstellungen bestehen, welche damals entwickelt wurden. Sie sollte uns lehren zu verstehen, dass und weshalb auch alle mathematische Ausbildung zeitgebunden ist; sie sollte unsere Bereitschaft, nach neuen, zeitgemässen Wegen zu suchen, erhöhen."

3. "To enable man to understand the society of the past, and to increase his mastery over the society of the present, is the dual function of history." [Carr 1961, 55]

4. Science in a Social Context Project, Project-Co-ordinator W. F. Williams, University of Leeds.

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