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Bolzano's Theory of Ground and Consequence

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Abstract The aim of the paper is to present and evaluate Bolzano's theory of *grounding*, that is, his theory of the concept expressed and the relation brought into play by 'because'. In the first part of the paper (Sections 1 – 4) the concept of grounding is distinguished from and related to three other concepts: the concept of an *epistemic reason*, the concept of *causality*, and the concept of *deducibility* (i.e., logical consequence). In its second part (Sections 5 – 7) Bolzano's positive account of grounding is reconstructed in axiomatic form and critically discussed.

1 Grounding

It is well known that Bolzano's theory of *deducibility* (*Ableitbarkeit*), which in fact is a theory of what nowadays is called "logical consequence," anticipated in many respects Tarski's much later work on that topic. It is far less well known that Bolzano also provides a theory of *grounding* (*Abfolge*), that is, a theory of the concept expressed and the relation brought into play by the connective 'because' (*weil*). His accounts of deducibility and grounding *together* form the core of Bolzano's logical theory. He declared that nobody would be able to understand him who is not acquainted with *both* of these concepts.[1] In this paper I shall consider Bolzano's widely unknown theory of the second concept acquaintance with which he himself took to be inevitable for understanding him.[2]

Bolzano's theory of grounding takes the form of a semantic theory of sentences of the form ⌜p because q⌝. It rests upon the basic assumption that sentences of the form

⌜p because q⌝

are literally true only if

GROUNDING (P, Q)

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where 'GROUNDING' stands for a certain relation—the *grounding-relation*—whereas 'P' and 'Q' are placeholders for singular terms denoting the propositions expressed by 'p' and 'q'.[3] Bolzano thus introduces grounding as a relation holding between propositions or—more specifically—between *true* propositions. Grounding, he says, is a relation between *truths* (*Wahrheiten an sich*). I shall take for granted Bolzano's basic semantic assumption throughout this paper.[4]

I shall also follow him in taking propositions—his famous *sentences in themselves (Sätze an sich)*—as mind and language independent entities which are the primary bearers of the properties of truth and falsity, have no causal powers, and figure as both contents of thought and as meanings of (utterances of) sentences in one sense of that term.[5] Truths, I am assuming with Bolzano, are nothing but true propositions.

Bolzano uses the terms '(objective) ground' (*Grund*) and '(objective) consequence' (*Folge*) for the terms of the grounding-relation. (He does *not* use 'consequence' in connection with deducibility, that is, his special conception of logical consequence!)[6] His theory of grounding therefore might also be called a theory of ground and consequence. He gives the following example.[7] The sentence

(1) Well-functioning thermometers stand higher in summer than in winter because it is warmer in summer than in winter

is clearly true whereas its converse

(2) It is warmer in summer than in winter *because* well-functioning thermometers stand higher in summer than in winter

is clearly false. (For simplicity, facts about pressure are ignored.) Using square brackets to denote the propositions expressed by the displayed sentences, we can say that, according to Bolzano's basic semantic assumption, (1) says that the truth [It is warmer in summer than in winter] is (part of) the *ground* of the truth [Well-functioning thermometers stand higher in summer than in winter], which itself is (part of) the *consequence* of the former. And this seems to be true. Due to the falsity of (2), the converse does not hold: the latter truth is not (part of) the ground of the former, and the former is not (part of) the consequence of the latter.

Bolzano's investigations into grounding are mainly included in Bolzano [5]. Here is an overview of Bolzano's theory of ground and consequence in [5] (on the right I refer to the sections of the present paper which deal with the relevant parts of Bolzano's theory). In the passages subsumed under (I), Bolzano tries to throw some light on the idea of grounding by comparing it with three other related ideas and working out the connections between them. I shall go through these comparisons in Sections 2 – 5. In Sections 6 and 7 then, I shall be concerned with Bolzano's positive account of grounding (II and III) which I shall put in terms of implicit definitions of two related concepts: immediate and mediate grounding. Bolzano's ideas about criteria for the obtaining of the grounding relation (IV) are discussed elsewhere.[8]

I.	Informal explanations and comparisons	
	The general idea (§§162, 168, 198)	(§1)
	Objective grounds versus epistemic reasons (§§177, 198)	(§2)
	Grounding versus causality (§§168.2, 201)	(§3)
	Grounding versus deducibility (§§162, 200)	(§4)
II.	Immediate grounding (§§203 – 13)	(§6)
III.	Mediate grounding (§§214 – 19)	(§7)
IV.	Criteria for grounding (§221)	

If Bolzano's assumption about the relational character of sentences of the form ⌜p because q⌝ is true, then 'because' not only brings into play the grounding-relation, it does so by expressing a certain *concept*, namely, [because].[9] (Assuming—with Bolzano—that concepts are constituents of propositions, I am here extending my use of angular brackets in an obvious way.) Bolzano's theory is about both, the concept [because] and the relation B. He does not spend many words on explicitly drawing the distinction again and again on any occasion it is relevant, but he is nevertheless aware of it.[10] It is clear in almost every passage whether he focuses in the first place on the concept or on the relation.

2 Objective Grounds versus Epistemic Reasons

Bolzano uses the word 'epistemic reason' (*Erkenntnisgrund*) in two (connected) senses. I will dub them the "psychological" and the "propositional" sense. A collection[11] of judgings (*Urteile*) x is the epistemic reason in the psychological sense of a certain judging y if and only if y is true and x is the complete cause of y (and x and y occur in the mind of one and the same person).[12] The word 'judging' in this definition, of course, has to be taken to denote particular mental events which occur in the mind of a certain person at a certain time. An early astronomer's judging that the earth is round might have been caused (in part) by his judging that the earth's shadow on the moon is round. In this case the latter judging would have been (part of) the epistemic reason for the former judging. One thing should be remarked with respect to the definition: By 'the collection of judgings x is the complete cause of the judging y' Bolzano merely means that there are no other judgings which are causally responsible for the occurrence of y. He does not mean that there is nothing more at all responsible for the occurrence of y.[13]

Objective grounds are clearly different from epistemic reasons in the psychological sense. Epistemic reasons in the psychological sense are (collections of) judgings and thus have causal powers. Objective grounds are (collections of) propositions and thus do not have causal powers.[14]

Epistemic reasons in the propositional sense are the contents of epistemic reasons in the psychological sense, or more precisely: A collection of propositions α is the epistemic reason in the propositional sense for the proposition β if and only if there is a collection of (particular) judgings x and a single (particular) judging y such that the propositions that are part of α are the contents of the judgings that are part of x (such that the parts of α and the parts of x are related one by one), β is the content of y, and x is the (psychological) epistemic reason of y.[15] Hence, if there has ever been an astronomer who has reasoned in the way I described in the last but one paragraph, the proposition that the earth's shadow on the moon is round is (part of) an epistemic reason for the proposition that the earth is round.

The aforementioned categorial difference between objective grounds and epistemic reasons in the psychological sense does not hold between objective grounds and epistemic reasons in the propositional sense since they both are (collections of) propositions. But the property of being an epistemic reason in the propositional sense is nevertheless different from the property of being an objective ground, and, mutatis mutandis, the grounding-relation is different from the relation which holds between a certain epistemic reason and the proposition it is an epistemic reason for. There are some different reasons for this. (I will use the word 'epistemic reason' in its propositional sense for the rest of this section.)

1. The proposition that the earth is round might be (part of) an epistemic reason of the proposition that the earth's shadow on the moon is round and vice versa. But, whereas the former proposition is (part of) the objective ground of the latter, this does not hold vice versa.[16] The same can be said, for example, with respect to the thermometer-example mentioned in Section 1.

2. A certain truth might have a multitude of different epistemic reasons. Different people at different times might recognize it in different ways. By contrast, Bolzano claims that it is impossible for a truth (or a collection of truths) to rest on different (complete) objective grounds. This, Bolzano goes on to say, does not prevent it being the case that a truth rests on different (collections of) propositions which figure as parts of its *one* objective ground. So he argues that in cases in which a certain truth seems to rest on different objective grounds, the latter should always be treated as different parts of the one complete objective ground.[17] (Bolzano here presupposes a very strict and narrow concept of ground. One might doubt that our ordinary use of 'because' is tied to *this* concept. Cf. my remarks at the end of Section 3.)

3. It is possible to know a certain truth via (the judging of) some epistemic reason without knowing its objective ground. Somebody might know that $E=mc^2$, or even that today it is warmer than it was yesterday for some epistemic reason (e.g., testimony might be involved) without knowing anything about the objective ground the truth in question rests upon.[18] Scientists very often know of a certain collection of propositions *that* they are true but not *why* they are true. The latter is often what they are after.

4. Objective grounds (and consequences) have to be (collections of) true propositions, whereas epistemic reasons might be false or might have parts that are false propositions.

5. It is possible (and probably the case) that there are objective grounds which neither have been nor will be the content of any particular judging(s). But it is not possible that there are epistemic reasons which neither have been nor will be the content of any particular judging(s).

Bolzano mentions only the first three of these reasons explicitly. But he surely would accept them all. Sometimes we use a sentence of the form ⌜p because q⌝ in order to state that Q is an epistemic reason for P. But these uses, Bolzano argues, are not literal uses. What plausibly might somebody mean who sincerely and on reflection utters a sentence such as 'It is warmer in summer than in winter *because* well-functioning thermometers stand higher in summer than in winter'? "[I]n saying this we speak only of the recognition of these truths, . . . we merely want to indicate that the recognition of one of these truths *causes* the recognition of the other" ([5], §198).[19] At the current stage of inquiry we have to confine ourselves to this plausibility-argument. But in the next section we will see that there is still more to say in favor of Bolzano's claim.

3 Grounding versus Causality

Another "concept closely akin to the concept of ground and consequence [i.e., the concept expressed by 'because'] is doubtless the concept of causality, that is, of the relation which holds between causes and effects. Perhaps we can use one of them to

analyze the other" ([5], §201). Let us ask first, Can [because] be analyzed (in part) via the concept of causality?

The most straightforward way in which this could be the case would be that [because] would be—as Bolzano puts it—subordinated to the concept of causality. (In Bolzano's terminology, a concept A is subordinated to the concept B (A *ist* B *untergeordnet*) if and only if everything which falls under A also falls under B, and not vice versa.[20] But this clearly cannot be the case. Grounds and consequences cannot be causes and effects since they are truths and thus have no causal powers.[21]

But even if [because] is not subordinated to [causes], perhaps the former can be analyzed in some other less straightforward way by use of the latter. Intuitively, it seems plausible to view the connection between the two concepts in the following way: "those truths which assert existence and attributes (*Beschaffenheiten*) of a cause are a *ground*, and those which deal with the existence and attributes of the effect are to be considered *consequences*" ([5], §201.3). The intuitive picture behind this idea is a *two level* picture: First, there is the level of concrete things "in the world" which are causally connected with each other. Then there is the abstract propositional level on which causal connections are "mirrored" by true propositions *about* things "in the world" and the causal relations among them. The truths mirroring the causal connections of things are themselves connected as objective grounds and consequences.

Bolzano does not reject the two level picture in general. But he stresses one particular point which, even though it has been neglected by the adherents of the two level picture, seems to be of great interest: *There are pairs of objective grounds and consequences which do not mirror causal relationships*.

> Thus, mathematical truths can be related as ground and consequence, although they do not deal with objects that have reality. The truth that in an equilateral triangle all angles are equal is a consequence of the truth that a scalene triangle has two equal angles. ([5], §201.3)

Bolzano's point seems to be correct not only for mathematical truths. It seems that we can perfectly well ask *why*-questions with respect to all kinds of propositions that do not state any effect and we can answer them by uttering sentences of the form ⌜q because p⌝ in their literal sense. Here are some prima facie candidates:

1. Either Bush or Gore won the election because Bush won it.
2. This glass contains mainly water-molecules because it contains mainly H_2O-molecules.
3. Peter is Paula's cousin because he is the son of a sibling of either her mother or her father.
4. Russell's paradox is deducible because Frege's fifth axiom is false.
5. A sentence of the form ⌜P and Q⌝ is true (if it is true) because the sentences P and Q are both true.
6. A sentence of the form ⌜It is true that p⌝ is true (if it is true) because the corresponding sentence P is true.
7. This apple is red because its skin is red.
8. He should be punished because he committed a crime.
9. There are no round squares because the concept of a round square involves a contradiction.
10. {Socrates} exists because Socrates exists.[22]

11. Socrates exemplifies paleness because he is pale.

It must be admitted that Bolzano himself would not have given the last two examples. He would have rejected the penultimate example because he does not have the concept of a set. The very last example he would have rejected because he would have thought that the two sentences connected by 'because' here express the very same proposition.[23]

We started this section with the suspicion that [because] might be subordinated to [causes]. We have not really reached the reverse of our initial suspicion but something similar to it. Although [causes] is not subordinated to [because] (since the objects which fall under these concepts belong to different "levels"), the grounding-relation also holds between propositions which do not mirror causes and effects. This might lead us to the suspicion that [causes] has to be explained (in part) in terms of [because]—and not the other way round, as we initially suggested.

Of course, it does not *follow* that [causes] has to be explained (in part) in terms of [because], as can easily be seen by other examples: Not every prime number is a natural number smaller than five. But [natural number smaller than five] does not have to be explained via [prime number].

Though this hardly comes as news to Bolzano, he nevertheless assumes that the case of [causes] and [because] is *not* of this kind. "[W]e have to derive the concepts of cause and effect from those of ground and consequence," he says.[24] His particular suggestion as to how this has to be done is this:

> 'X is the cause of Y' actually means 'the truth that X exists [*daß X sey*] is related to the truth that Y exists as ground (partial ground) to consequence (partial consequence)'. ([5], §168, cf. also Bolzano [9], vol. 1 16/2, p. 238)

One might think that this cannot be true for the following reason: It seems to be true that {Socrates} exists because Socrates exists but Socrates surely is not the cause of {Socrates}. But this objection fails. It rests on a misunderstanding of what Bolzano actually says in the quoted passage. In order to see this, one has to recognize that Bolzano uses 'existence' [*Dasein*] as synonym to 'actuality' [*Wirklichkeit*], by which he means the ability to act upon something [*Wirksamkeit*].[25] Hence the quotation should be paraphrased like this:

> [X is the cause of Y] = [the truth [X has the ability to act upon something] is a (partial) ground of the truth [Y has the ability to act upon something] as its (partial) consequence].

Given this reading, it is obvious that the right-hand side will not denote a truth for X = 'Socrates' and Y = '{Socrates}'. So the objection is blocked.

Let me put together some of the results of the last two sections. What does 'because' in its literal use express? Does it express (1) the concept of grounding or (2) the concept of causality or (3) the concept of something's being an epistemic reason for something else? If some causal theory of the mind is correct, then (3) has to be explained (in part) by means of (2). If, moreover, (something like) Bolzano's thesis about the link between grounds (and consequences) and causes (and effects) is true, then (2), in turn, has to be explained (in part) by means of (1). Given these assumptions, the concept of grounding comes out as "the most fundamental" of the three. Hence, in case 'because' has only one literal meaning, the concept of grounding seems to be first choice for that role.

Bolzano, after introducing the terms 'ground' and 'consequence' via intuitive examples like the thermometer-case, restricts himself to a very narrow, technical use of those terms: Mostly he uses 'ground' and 'consequence' for *immediate* and *complete* grounds and consequences. (Recall the doubts I expressed with respect to the second of Bolzano's arguments for the difference between objective grounds and epistemic reasons where he explicitly makes use of his strict concept of grounding in presupposing that (collections of) truths cannot have more than one ground.) I do not agree with Bolzano here. I think that a less strict concept that captures also *mediate* and *partial* grounds fits much better with our ordinary use of 'because'. I shall give arguments for that claim at the end of this paper.

I mention all of this in order to stress the following point: Even if one thinks that 'because' in its literal use expresses *some* (nonepistemic, noncausal) concept of grounding it is not yet clear exactly *which* concept one thinks it expresses. There are different possibilities for fixing a concept that prima facie corresponds to the intuitions Bolzano is alluding to. Hence you can think that 'because' is univocal and that it expresses some (nonepistemic, noncausal) concept of grounding without thinking that it expresses the very strict concept of complete and immediate grounding.

4 Grounding versus Deducibility

In the famous §155 of [5] Bolzano gives the following definition:[26]

> I say that propositions M, N, O, . . . are *deducible* from propositions A, B, C, D, . . . with respect to variable parts i, j, . . . , if [and only if] every collection of ideas whose substitution for i, j, . . . makes all of A, B, C, D, . . . true, also makes all of M, N, O, . . . true. ([5], §155.2)[27]

Let me explain this using the following even more famous argument:

All humans are mortal.
Socrates is a human.
Therefore, Socrates is mortal.

The proposition expressed by the conclusion of this argument is deducible from the propositions expressed by its premises, for example, with respect to the idea [human]. Let the i^*/i-variation of a proposition P be that proposition P* which is the result of substituting the idea i^* for (all "occurrences" of) the idea i in P. For every idea i^* the following holds: If the i^*/[human] variants of [All humans are mortal] and of [Socrates is a human] are true, then the i^*/[human] variant of [Socrates is mortal] is true either. This means that [Socrates is mortal] is deducible from [All humans are mortal] and [Socrates is a human] with respect to the idea [human] (in Bolzano's sense of the phrase). And the same holds with respect to the ideas [mortal] and [Socrates]. Hence, in case of the famous argument, the proposition expressed by its conclusion is deducible from the propositions expressed by its premises with respect to all these three ideas.

In the passage quoted above, Bolzano defines the (conceptually basic) *three*-place predicate 'β is deducible from α with respect to i'. Now it is easy to define a further *two*-place predicate which will be useful for the present discussion. Let us say that the collection of propositions β is deducible from the collection of propositions α if and only if there is at least one idea i such that β is deducible from α with respect to i. Bolzano himself works with this derived two-place predicate when he compares the grounding-relation with the relation of deducibility,[28] and I will follow him in

this respect. Hence, by 'the relation of deducibility' I will refer to the two-place relation throughout this section.

It might be suggested that the grounding-relation is nothing else than the relation of deducibility. But this cannot be the case for an obvious reason: The grounding-relation only holds between truths whereas the relation of deducibility takes also false propositions as terms.[29] Hence, the most interesting comparison might be the one between the grounding-relation and *T-deducibility*, where this is defined as follows:[30] The collection of propositions β is *T*-deducible from the collection of propositions α if and only if β is deducible from α and all propositions that figure as (direct) parts of α are true.[31]

Throughout the rest of the paper 'α' and 'β' are used as variables ranging over (and sometimes as schematic letters for) collections of true propositions (truths). Is the following statement true?

$$\forall\alpha\forall\beta(\beta \text{ is } T\text{-deducible from } \alpha \rightarrow \alpha \text{ grounds } \beta). \tag{1}$$

No. The falsity of (1) can easily be seen with regard to some of the aforementioned examples. [It is warmer in summer than in winter] is *T*-deducible from [Well-functioning thermometers stand higher in summer than in winter]. Both propositions are true and the former is deducible from the latter with respect to the ideas [summer] and [winter]. But the latter proposition does not (even partly) ground the former.[32] The example also illustrates a further difference: The two propositions mentioned are *T*-deducible either way. But they certainly do not ground each other. And a third difference between the two relations is that every true proposition is *T*-deducible from itself. Nothing analogous holds for grounding.

If (1) is false, what about its converse?

$$\forall\alpha\forall\beta(\alpha \text{ grounds } \beta \rightarrow \beta \text{ is } T\text{-deducible from } \alpha). \tag{2}$$

Bolzano himself is undecided with respect to (2). "Though [the truth of (2)] seems quite probable to me," he says, "I do not know any proof that would justify such a view" ([5], §200). Even more, he goes on by giving an example which, as he says, "seems to prove" that (2) is *false*.[33]

I will not go into these matters here. I neglect the example not only because it is not easy to see how exactly it bears on (2) and because Bolzano himself has his doubts about it but mainly for the following reason: The question whether (2) is true or false does not necessarily bear on the question whether there is a conceptual connection between [because] and the concept of *T*-deducibility, and it also does not necessarily bear on the question whether the grounding-relation is a *special kind* of the relation of *T*-deducibility (as long as property-kindhood is not construed in a completely extensional way).[34] Even if (2) were true, it would not follow that there is a conceptual connection between the relevant concepts, nor would it follow that the grounding-relation is a special kind of the relation of deducibility.[35]

One might hold that we sometimes use sentences of the form ⌜p because q⌝ in order to say that P *follows from* Q (or, in case the concept of derivability captures our ordinary concept: that P is derivable from Q). Given that there are such uses, are they literal uses of those sentences? If so, 'because' would be equivocal after all. (Cf. the end of Section 3.) But, as we have seen, the meaning of 'P follows from Q' can hardly be a literal meaning of 'p because q' since the latter entails that both P and Q are true while the former does not. So the assumption needed for arguing for

the ambiguity of 'because' would have to be that there is a literal use of 'p because q' in which it means this: P follows from Q and P and Q are both true.

The comparisons of the foregoing sections should have thrown some light on the intuitive idea of grounding. But their upshot for the enterprise of giving a precise explanation of the concept is rather small. The only result which might be of interest for somebody trying to reduce the concept of grounding to other concepts is that perhaps the relation brought into play by 'because' is a special kind of the relation of T-deducibility. But even if it could be shown that this is the case, it would not give us much for an explanation of the concept. Thus, it seems appropriate to look for an independent explanation. And this is what Bolzano does.

5 Bolzano's Theory: Some Preliminaries

I now come to Bolzano's positive theory of grounding in which he distinguishes between two different concepts: *immediate* and *mediate* grounding. Immediate grounding is essentially a one-one relation. The intuitive idea of mediate grounding is that a collection of truths α is a mediate ground of another collection of truths β if and only if there is a "chain" of instances of the relation of (at least partial) immediate grounding from α via some other elements to β.[36]

Bolzano explicates the intuitive idea of grounding by invoking both concepts. What he actually does in giving his account of grounding, I think, is best seen as giving (partial) implicit definitions of the concepts of immediate and mediate grounding. In doing so, Bolzano states a whole range of propositions in order to characterize the concepts in question without much of a systematic ordering. These propositions, however, can be reduced to three implicitly defining clauses (two for the concept of immediate grounding, one for the concept of mediate grounding). Thus Bolzano's theory can be given by three axioms that have to be understood as giving two (connected) implicit definitions of the concepts of immediate and mediate grounding. The first two axioms (characterizing immediate grounding) will be given in Section 6, the third (characterizing mediate grounding) will be given in Section 7.

It has been suggested to me that it might be a mistake to reconstruct Bolzano's method as giving implicit definitions. An implicit definition, as usually considered, is given by stipulating that certain sentences are (should be) true. But, so the objection goes on, for Bolzano the primary bearers of truth and falsity are propositions, not sentences. So it seems that he would have to regard an implicit definition as a stipulation to the effect that certain propositions are (should be) true. But can you really stipulate of a proposition that it is (should be) true?

The answer to this objection, I think, is this: Of course, you cannot first fix a proposition (by whatever means) and then stipulate of that proposition that it is (should be) true. (It might turn out that the proposition does not obey.) But what you can do in order to give meaning to a term is this: You can stipulate of certain sentences containing that term that, whatever propositions they express, they (the propositions) are (should be) true. Thereby you can constrain the meaning of the term (more or less strongly). This picture, I think, captures what Bolzano is doing (at least if it is also taken into account that his stipulations are not arbitrary but are intended to capture our ordinary concept [because]).

In order to formulate Bolzano's theory we have to become clearer about the distinction between partial and complete grounds and consequences. Some general remarks about his use of 'collection' and 'part' are in order. Bolzano takes expressions

like 'a, b, and c' (where 'a', 'b' and 'c' are singular terms) to refer to collections. This, I think, is the central principle governing the meaning he attaches to 'collection'. If a, b, and c are all Fs, then the collection consisting of a, b, and c (and nothing more) is a collection of Fs. In the present context Bolzano speaks about collections of propositions many times.

Bolzano would say that a, b, and c are all *parts* of the collection of a, b, and c. But it is important to be aware of the fact that his use of the word 'part' is—perfectly natural, but—not in accord with modern mereology. This is so because the relation Bolzano denotes by 'part' is not transitive. "It is true only of collections of a special type that 'parts of a part are parts of the whole', as they say" ([5], §83.2). The king and his ministers—to mention one of Bolzano's examples—are part of the state, the arms of the king are part of the king, but the king's arms are not part of the state—in Bolzano's sense of 'part'.[37]

In the present context we are concerned with collections of truths (as terms of the grounding-relation(s)). At least for the sake of convenience it will be appropriate to assume that there are collections that have only a single part.[38] In accordance with Bolzano, I will also presuppose two principles holding for all collections of truths α, β, and γ: The collection of α and β = the collection of β and α. The collection of (α and β) and γ = the collection of α, β, and γ. The last of these principles implies that the parthood-relation is transitive as long as we restrict the domain of discourse to collections of truths. Thus the aforementioned difference between Bolzano's concept of part and that of standard mereology does not matter with respect to the domain I am mostly concerned with.

Bolzano's collections inherit their ontological status from their parts. The collection of my copies of the four volumes of [5], on the one hand, is as concrete as the single books are: its weight is the sum of the weights of the single volumes, its location is the location of the four single volumes (it is a "scattered object"), and so on. On the other hand, consider the object that might be called "Bolzano's *Wissenschaftslehre* in itself"—that is, a certain collection of propositions in a certain ordering. This collection is as abstract as any of the propositions that are part of it: it has neither spatial nor temporal properties.

Whereas these properties clearly distinguish Bolzano's collections from sets, they are shared by mereological sums or fusions.[39] So, are collections nothing but mereological sums? The fact that Bolzano does not mean the same by 'part' as mereologists mean by this word seems to speak against this possibility. On the other hand, it is important to notice that Bolzano's concept of a collection does not depend on the concept he expresses by 'part'. (This, by the way, contrasts with mereology where the concept of a sum is derivative on the concept of part.) The appropriate way of introducing Bolzano's concepts, it seems to me, is just the way I introduced them here: First, you introduce [collection] by means of appealing to form of words like 'a, b, and c', and then you introduce [part] by means of [collection]. Thus the difference in the concepts of part does not necessarily preclude collections from being mereological sums (though it does preclude the concept of a collection to be the mereological concept of a sum).[40]

Some symbols should be introduced. '$\alpha \Rightarrow \beta$' has to be read as 'α is the complete immediate objective ground of β, and β is the complete immediate objective consequence of α'. Notice that '$\Rightarrow$' connects singular terms and not sentences. In the way I put things here, the aim of Bolzano's theory (or at least one of them) is

to fix the meaning of '$\Rightarrow$' by some appropriate implicitly defining clauses in a way that captures as much as possible of the ordinary meaning of 'because'. It will be convenient to have also the following symbols.

Definition 5.1 ($_<\!\!\Rightarrow$) $\forall\alpha\forall\beta[\alpha \ {}_<\!\!\Rightarrow \beta$ iff $\exists\gamma(\gamma \Rightarrow \beta \ \& \ \alpha < \gamma)]$.

Definition 5.2 ($\Rightarrow_\ll$) $\forall\alpha\forall\beta[\alpha \Rightarrow_\ll \beta$ iff $\exists\gamma(\alpha \Rightarrow \gamma \ \& \ \beta \ll \gamma)]$.

'$\alpha < \beta$' abbreviates 'α is part of β or $\alpha = \beta$'. '$\alpha \ll \beta$' abbreviates 'α is part of β but $\alpha \neq \beta$'.[41] Thus '$\alpha \ {}_<\!\!\Rightarrow \beta$' has to be read as '$\alpha$ is a partial *or* the complete immediate ground of β', whereas '$\alpha \Rightarrow_\ll \beta$' has to be read as '$\beta$ is a partial *but not* the complete immediate consequence of α'. The sense of the asymmetry between these definitions will become clear later.

6 Immediate Grounding

The concept of immediate grounding, as characterized by Bolzano, might be given by two axioms. With respect to the proposition that serves as my first axiom, Bolzano explicitly appeals to Aristotle, who, in book Θ of the *Metaphysics*, says the following:[42]

> [A_1] It is not because we think truly that you are pale, that you are pale, but [A_2] because you are pale we who say this have the truth. ([1], book Θ 10: 1051 b 6–9)

Bolzano reads this as a thesis about a relation between propositions. He takes Aristotle to be saying this: (A_1) It is not the case that [It is true that you are pale] (partly) grounds [You are pale], but the other way round: (A_2) [You are pale] (partly) grounds [It is true that you are pale].[43] Bolzano's [5] contains generalized and more specific counterparts of both (A_1) and (A_2). His counterpart of (A_1) figures as premise in his argument for the asymmetry of immediate grounding (which will be quoted soon).[44] Here is Bolzano's way of putting (A_2):

> Whatever truth A may be: the truth "that the proposition A is true" is a genuine consequence of it; and this consequence needs as its ground surely no further truth beneath A, which therefore is its complete ground.[45] ([5], §205.1)

With the help of the symbolism introduced in Section 5 we can put Bolzano's version of Aristotle's claim as follows (remember that I use 'α', 'β', 'γ', ... as variables ranging over collections of truths):

Axiom 6.1 (First axiom for '$\Rightarrow$', Aristotle's principle)

1. $\forall\alpha\neg([\alpha \text{ is true}] \ {}_<\!\!\Rightarrow \alpha)$.
2. $\forall\alpha(\alpha \Rightarrow_\ll [\alpha \text{ is true}]$.[46]

The intuition behind clause 1 is quite clear: it seems unreasonable to assume that some collection of truths α could be grounded, in part or even completely, in the truth [α is true]. Intuitively, it seems to be equally clear that [α is true] always is a consequence of α. What might seem surprising is that Bolzano, moreover, took it as intuitively clear that [α is true] *never* is the *only* (partial) consequence of α. But he is assuming this quite explicitly:

> To the complete consequence of some truths A, B, C, D, ... belongs, among others, the truth "that all of the propositions A, B, C, D, ... are true." This apparently is a consequence (that is to say, a partial consequence) of no other collection [of truths] but this. ([5], §206)

Bolzano also explicitly mentions a theorem obviously resulting from clause 2 of Aristotle's principle saying that

> there are always several truths which collectively can be regarded as consequence. ([5], §205.2)

This can be expressed in symbols as the following.

Theorem 6.2 (Complexity of consequences)

$$\forall\alpha\forall\beta(\alpha \Rightarrow \beta \rightarrow \exists P\exists Q(P \ll \beta \;\&\; Q \ll \beta)).$$

Bolzano's idea seems to be that every truth or collection of truths α grounds, as one might say, "something substantial," and, *besides this*, the truth [α is true].

Bolzano takes both parts of Aristotle's principle for granted on several occasions.[47] However, [5] does not contain a special section dedicated to it (or even one of its parts). Bolzano does not give any argument at all for Aristotle's principle. Thus it seems that he would be prepared to accept my reconstruction which takes it as basic.

It should be mentioned that there is another possible interpretation of Aristotle. According to this second interpretation Aristotle in the above quoted passage does not express a thesis about a relation between propositions, but rather advocates a thesis about propositions and their "truth-makers"—where a truth-maker is not taken to be a proposition but something "in the world" (a state of affairs, an individual accident or whatever).[48] Bolzano neither attributes such a thesis about "truth-making" in the sense just ascribed to Aristotle nor does he himself endorse one. On the contrary, he thinks that there is no such relation of "truth-making":

> Does a certain thing, X, have the property x because the proposition, X has the property x, is true; or, conversely, is this proposition true because the thing X has this property? The right answer, in my opinion, is: neither the one nor the other. The ground why a proposition is true lies, if the proposition's truth has a ground, in another truth, not in the thing with which it deals. And even less can one say that the ground why X has the property x lies in the truth that X has the property x. If indeed X is an actual thing then there can be no ground why it has the property x, but there can be a cause why it has the property x, this cause lies in another thing. ([9], vol. 2 A 12/2, p. 60)

In denying that "the proposition, X has the property x, is true . . . because the thing X has this property" Bolzano does not contradict Aristotle's principle. This is because (as the continuation of the passage shows) Bolzano here apparently has the "truth-maker"-interpretation of this phrase in mind.

It is worth noticing that Bolzano here is at odds with many Austr(al)ian Realists who hold that there is a relation of truth-making between propositions and (some kind of) things "in the world." This is true even if Austr(al)ian Realists usually do not hold the particular view on truth-makers Bolzano considers in the quoted passage, that is, they do not identify a proposition's truth-maker with the thing the proposition is about (but with states of affairs, as in the case of Armstrong [2], or with individual moments, as in the case of Mulligan et al. [20]). This fact might be taken as particularly interesting because the tradition of Austr(al)ian Realism was founded in many respects by Bolzano himself.

Bolzano takes immediate grounding to be a *one-one* relation in the sense that every (complete) immediate ground has its unique (complete) immediate consequence, and every (complete) immediate consequence has its unique (complete) immediate ground. This can be regarded as a second axiom for immediate grounding.

Axiom 6.3 (Second axiom for '⇒', Uniqueness)
$\forall\alpha\forall\beta(\alpha \Rightarrow \beta \rightarrow \forall\gamma((\gamma \Rightarrow \beta \rightarrow \gamma = \alpha)\ \&\ (\alpha \Rightarrow \gamma \rightarrow \gamma = \beta)))$.

Bolzano argues that this is extremely plausible as soon as it is recognized that whenever it seems that there is more than one immediate ground (consequence) of a certain collection of truths these candidates might better be considered as different *parts* of *the one* immediate ground (consequence).[49]

However, it might be argued that the uniqueness of the immediate grounding-relation is intuitively inappropriate for the following reason: Consider a disjunctive proposition [P ∨ Q] the disjuncts of which are both true. One might be tempted to say that in this case both P and Q, independently of each other, are complete grounds for [P ∨ Q] (or some broader collection containing [P ∨ Q]) whereas Bolzano would have to say that [P ∨ Q] has only one ground, namely, the collection of P and Q.

One *might* be tempted to say this, but of course one does not *have* to consider things that way. Anyway, it would be good to have more than mere intuitions to rely upon with respect to the question of multiple grounds. And indeed, Bolzano has an argument for the conclusion that there are no cases of multiple grounds at all. Of course, it is a plausibility argument. (How else should one be arguing for an axiom?) In giving his argument, Bolzano reminds us that (complete) consequences never consist of a single truth (our Theorem 6.2)—contrary to what is suggested by the disjunction-case. With respect to examples analogous to the disjunction-case he writes:

> considered carefully, the examples show nothing else but that different grounds sometimes share *parts* of their consequences; they do not show that the *complete* consequence is the same. To the complete consequence of some given truths A, B, C, D, . . . belongs, among others, the truth "that the propositions A, B, C, D, . . . are all true." But this is a consequence (a partial one) which no other collection of truths has but this one. And so it becomes clear that every ground has a consequence containing at least one part which is not part of any other consequence. ([5], §206)

Applied to the disjunction-case, the idea is that

> the consequence of P—according to Aristotle's principle—embraces the truth [P is true], but—plausibly—it does not contain the truth [Q is true],

whereas

> the consequence of Q—according to Aristotle's principle—embraces the truth [Q is true], but—plausibly—it does not contain the truth [P is true].

P's (complete) consequence is therefore different from Q's (complete) consequence. Hence, it is not the case that [P ∨ Q] is grounded in both P and Q—at least not in the sense of "grounded" we are interested in here: neither [P ∨ Q] nor some collection of truths containing it is not *the complete* consequence of both P and Q, these both being its complete grounds. This is ruled out simply because P and Q have different (complete) consequences.

As I already said, this is a plausibility argument. It rests (besides Aristotle's principle) on the plausible assumption that there are no truths P and Q both being related as complete grounds to a single "big" consequence as *the one* complete consequence of the two, where such a "big" consequence would be one containing both the truth [P is true] and the truth [Q is true].

I shall now derive some more theorems. Immediate grounding is an asymmetrical relation: there are no collections of truths that immediately ground each other.

Theorem 6.4 (Asymmetry)

$$\forall\alpha\forall\beta(\alpha \Rightarrow \beta \rightarrow \neg\beta \Rightarrow \alpha).$$

The idea of an indirect proof of Theorem 6.4 is given in the following passage:

> That the truths A, B, C, . . . , if they are the complete ground of the truths M, N, O, . . . and the latter are the complete consequence of the former, are not also the consequence of those, can be seen from the fact that if M, N, O, . . . are the complete consequence of A, B, C, . . . then they will include the proposition that A, B, C, . . . are all true. Hence that proposition would have to be a partial ground of the truths A, B, C, . . . , which is absurd.[50] ([5], §209.2)

This can be made precise without much effort.[51]

Immediate grounding is also irreflexive: nothing is the complete ground (consequence) of itself.

Theorem 6.5 (Irreflexivity)

$$\forall\alpha\neg(\alpha \Rightarrow \alpha).$$

This again can be proved indirectly. Suppose that the immediate grounding-relation were not irreflexive. Then there would be a collection of truths A such that A $\Rightarrow$ A. But from this and Theorem 6.4 (asymmetry) follows a contradiction, namely, that A $\Rightarrow$ A & A $\Rightarrow$ A. Thus immediate grounding is irreflexive. Bolzano himself does not deduce irreflexivity from asymmetry. He takes irreflexivity to be a trivial feature of the concept of immediate grounding and gives no further reason for it.[52]

Bolzano mentions that many philosophers *seem* to deny the irreflexivity of the immediate grounding-relation. These philosophers say of certain propositions that "they are grounded completely in itself." Bolzano argues that these philosophers do not really deny that the immediate grounding-relation is irreflexive, but that they are merely talking in a loose way.[53] What is sometimes meant by these assertions, according to Bolzano, is that certain propositions do not have any ground at all (what is held by Bolzano as well, as we will see). In other cases, he says, it is meant that some propositions are grounded solely in the properties (*Beschaffenheiten*) of the concepts occurring in them. If we consider as grounds not these properties in themselves but true propositions describing them, this seems to be true. But, Bolzano continues, in this case the ground of the truth in question is not itself, but the collection of truths describing the relevant properties.

What about transitivity? It might sound surprising, but immediate grounding is clearly intransitive.

Theorem 6.6 (Intransitivity)

$$\forall\alpha\forall\beta\forall\gamma((\alpha \Rightarrow \beta \;\&\; \beta \Rightarrow \gamma) \rightarrow \neg\alpha \Rightarrow \gamma).$$

Bolzano's argument for this is simple and convincing.[54] Essentially, it is this: If immediate grounding would not be intransitive there would be collections of truths A, B, and C such that A $\Rightarrow$ B & B $\Rightarrow$ C & A $\Rightarrow$ C. But this cannot be the case, since C would have two different (complete) immediate grounds, which is precluded by Axiom 6.3 (uniqueness).

Bolzano says that the intransitivity of immediate grounding might be found counterintuitive at least for somebody who—like himself—accepts that there is an intimate connection between the grounding and the relation of causality of the kind the

"two level picture" suggests.[55] According to this picture, causal facts are mirrored by the obtaining of the immediate grounding-relation between true propositions stating the relevant causes and effects. If the two level picture is correct, it seems that if the immediate grounding-relation is intransitive, the relation of causality has to be intransitive too.[56] But intuitively, this is not the case. In ordinary language, there seems to be nothing problematic in calling the cause of a cause of x a 'cause of x'. Suppose that a house has been destroyed by a rockfall which itself was caused by Max throwing a certain stone. In this case, the statement that Max's throwing of the stone has caused the destruction of the house seems to be as true as the statement that the rockfall has caused the destruction of the house.

Bolzano concedes the point.[57] However, I think that he is much too defensive here. It is important to be clear about the fact that we are dealing only with the concept of immediate grounding at the moment. The fact that the relation of immediate grounding is intransitive does not prevent the mediate relation from behaving otherwise. And, in fact, mediate grounding is transitive. This is one of the major differences between the two relations. Hence Bolzano could have easily referred to the concept of mediate grounding as interpretation of (the present aspect of) our usual way of using 'because'— or so I shall argue in Section 7.

From Aristotle's principle (and Definition 5.2) obviously follows that every collection of propositions is an immediate ground of something else.

Theorem 6.7

$$\forall\alpha\exists\beta(\alpha \Rightarrow \beta).^{58}$$

Notice that, despite this, it is not derivable from the two axioms that every collection of truths is a *consequence* (of something else) either. And in fact, in §214 of [5], Bolzano *denies* that every collection of truths is a consequence. His arguments for the existence of basic truths are not based on further characteristics of the concept of grounding (not captured by the axioms), but completely independent.[59] This is mirrored by our two axioms which neither preclude nor imply the existence of basic truths.

A further result of Bolzano's discussion of immediate grounding is that there are collections of truths that are both (partial or complete) immediate grounds *and* (partial or complete) immediate consequences.[60] From our basic implicitly defining clauses we can derive the following theorem.

Theorem 6.8

$$\forall\alpha\exists\beta\exists\gamma(\alpha \Rightarrow \beta \;\&\; \beta \Rightarrow \gamma).^{61}$$

In order to derive that there in fact are collections of truths β and γ such that $(\alpha \Rightarrow \beta \;\&\; \beta \Rightarrow \gamma)$ we have to assume that there are true propositions. For Bolzano, this assumption is unproblematic: it is a theorem of his *Theory of Fundamentals*, the first and most fundamental part of [5].[62]

Hence there might be collections of truths A, B, C, D . . . such that A is the immediate ground of B, B is the immediate ground of C, C is the immediate ground of D, and so on. Now, since the immediate grounding-relation is intransitive, in this case A is neither the complete nor a partial immediate ground of D. But A is a *mediate* ground of D. In §216 of [5] Bolzano broadens the perspective of his investigations by turning his attention to the relation of mediate grounding.

7 Mediate Grounding

> If somebody asks for the ground of a certain truth M, and then, if he found it in the single truth L or the collection of truths I, K, L, . . . , goes on to ask for the single ground or collection of grounds that one or some of the these truths have . . . , then I call this the *ascent from consequence to ground*.[63] ([5], §216, Bolzano's emphasis)

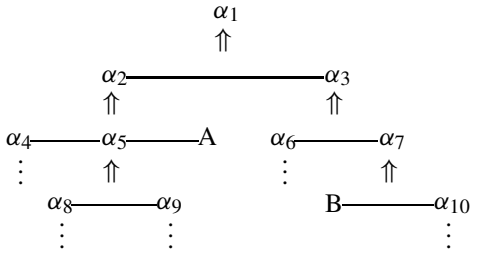
The mediate grounds of a certain truth are the truths and collections of truths one is led to by the ascent from consequence to ground. The intuitive point (as I already said in Section 5) is this: A collection of truths α is a mediate ground of another collection of truths β if and only if there is a 'chain' of instances of the relation of (complete or partial) immediate grounding from α via some other elements to β.[64] In order to state this in a more precise way let us use '$\alpha \rhd \beta$' as an abbreviation for 'α is a mediate ground of β' or 'β is immediately grounded in α'. Against this background we can give voice to the idea of the 'chain' by stipulating the relation of mediate grounding to be the transitive closure of partial immediate grounding (i.e., the relation denoted by '$_{<}\Rightarrow$'). Formally, this might be spelled out as follows.

Axiom 7.1 (Axiom for '$\rhd$')

$$\alpha \rhd \beta \leftrightarrow \alpha \neq \beta \;\&\; \exists n \exists \gamma_1\text{-}\gamma_n[\gamma_1 = \alpha \;\&\; \gamma_n = \beta \;\&\; \forall i < n(\gamma_{i\,<} \Rightarrow \gamma_{i+1})].$$

Notice that, strictly speaking, '$\alpha \rhd \beta$' abbreviates 'α is a *complete* or *partial* mediate ground of β'.[65]

The way the parts of a given collection of truths are connected by the relation of mediate grounding Bolzano calls the *objective connection* among those truths. It can be represented by tree-structures as follows:[66]



Here the immediate grounding-relation is represented by '⇑'. The horizontal lines indicate that the connected (collections of) truths together—as a collection—ground some other collection of truths. The dots indicate that the relevant collections of truths are further grounded. The single truths A and B are *basic* truths, that is, they have no further grounds. (In the next part of the paper I shall discuss Bolzano's argument for the existence of basic truths.)

If things are represented in this way, it can easily be seen that a collection of propositions α is a mediate ground of another collection of propositions β if and only if there is a 'chain' of immediately related items from a collection of propositions α or some (proper or improper) part of α to β. Thus, in the above example, α_9 mediately grounds α_5, α_2, and α_1. α_1 is also grounded by α_2, α_4, α_5, and A, and also by the collection of, say, α_4 and α_5, or α_4 and A. Suppose that α_9 is the collection of the truths C and D. Then C and D mediately ground everything which is mediately grounded by α_9 itself.

The axiom for mediate grounding implies that mediate grounding includes immediate grounding as well as the relation between a partial ground and its complete consequence as *special cases*, that is, that the obtaining of these relations are sufficient for mediate grounding.

Theorem 7.2 (Inclusion)

$$\forall\alpha\forall\beta((\alpha \Rightarrow \beta \vee \alpha \;{}_{<}\!\Rightarrow \beta) \rightarrow \alpha \rhd \beta).$$

(If $\alpha \Rightarrow \beta$ or $\alpha \;{}_{<}\!\Rightarrow \beta$, then $n = 2$ makes the right-hand side of the axiom true.) This consequence is in harmony with what Bolzano says. In the beginning of §217, for example, he explicitly introduces a term for *all* (collections of) truths which are involved in what he calls the "ascent from consequence to ground." Nevertheless it might be considered as a drawback of terminology. The natural use of 'mediate' and 'immediate' suggests that being *only* a mediate ground excludes being an immediate ground. Unfortunately, I don't know of any terminological alternative which does not have any drawbacks.

From the axiom for mediate grounding it obviously follows that the relation is irreflexive.[67]

Theorem 7.3 (Irreflexivity)

$$\forall\alpha\neg(\alpha \rhd \alpha).$$

It also obviously implies that mediate grounding is transitive.

Theorem 7.4 (Transitivity)

$$\forall\alpha\forall\beta\forall\gamma((\alpha \rhd \beta \;\&\; \beta \rhd \gamma) \rightarrow \alpha \rhd \gamma).$$

With this we can also easily prove the following theorem.

Theorem 7.5 (Asymmetry)

$$\forall\alpha\forall\beta((\alpha \rhd \beta) \rightarrow \neg\beta \rhd \alpha).$$

Suppose that mediate grounding were not asymmetric. Then there would be collections of truths A and B such that A $\rhd$ B & B $\rhd$ A. But this implies in connection with Theorem 7.4 that A $\rhd$ A, which contradicts Theorem 7.3.

In Section 6 I mentioned that the concept of immediate grounding, because of its intransitivity, did not seem to be an appropriate means for interpreting 'because' in sentences mirroring causes and effects. Since mediate grounding, unlike immediate grounding, is transitive, it gives us a means to interpret these sentences in accordance with our ordinary use of them. Also the examples listed in Section 3, I think, show that we do not always use 'because' in the strict sense of immediate grounding.

There is a further reason to think that 'because' in ordinary language expresses the concept of mediate grounding, and not that of immediate grounding. The reason is that we usually do not know whether two collections of truths are related as complete ground and complete consequence. Even with the routine of a philosopher, it is extremely hard to find even a few remotely plausible examples. If 'because' would express the concept of immediate grounding, then it would be hard to see how we could ever be justified in making a 'because'-statement in an adequate number of cases. The concept of mediate grounding, on the other hand, does not have such high standards for its application. It suffices for its application that one collection of truths is a *partial* mediate ground of another one.

I therefore conclude that if Bolzano's basic assumption, that 'because', in its ordinary language use, brings into play a relation between truths, is true at all, then the concept of mediate grounding is the best candidate for its meaning. However, it should be clear that Bolzano's pioneering on the idea of grounding can hardly be taken as a complete and satisfying theory. Bolzano himself was very clear about this. In the introductory remarks preceding his discussion of grounding in [5] he says:

> Since the investigations which will concern me now are very difficult, and since there is almost no preceding work which I could have used in this area: so I request the reader in advance, regarding this part even more than regarding the foregoing ones, not to expect anything perfect. Almost everything I have to say is, for myself, interwoven with uncertainties, with respect to some things I do not risk any final judgement, and in the best case my investigations are only fragments and hints which have served their purpose in case they stimulate others to think about these matters. ([5] §195)

Notes

1. Cf. Bolzano [11], p. 40.

2. The intimate connection between Bolzano's account of grounding and Gentzen's theory of normal proofs (cf. Gentzen [15]) is explored in some detail in Tatzel [24].

3. Cf. [5], §§168, 177. I generally use 'P', 'Q', . . . as placeholders for terms denoting propositions, and I generally use 'p', 'q',. . . as placeholders for declarative sentences expressing these propositions.

4. Notice that Bolzano is not saying that 'because' isn't really a sentence-connective but a two-place predicate connecting two singular terms. For this reason it is, strictly speaking, false to say that 'because', according to Bolzano, denotes a relation. Nevertheless, Bolzano needs to assume that sentences of the form ⌜GROUNDING (P, Q)⌝ have the same truth conditions as sentences of the form ⌜p because q⌝. And, moreover, he also seems to make the stronger assumption that (something like) sentences of the form ⌜GROUNDING (P, Q)⌝ display the meaning of sentences of the form ⌜p because q⌝ (cf. [5], §177, but see also §127). Against this background we might say that, even if Bolzano is aware of the fact that 'because' is a connective which does not denote anything, it corresponds closely to the (artificial) predicate 'GROUNDING' (which will be given some sense in the present paper, and) which in fact denotes a relation. I shall summarize this in what follows by saying that 'because' brings into play the relation of grounding.

5. For Bolzano's conception of propositions cf. [5], §§19 ff., Textor [25], p. 9–28, and Künne [18]. For his account of truth cf. [5] §§25 ff.

6. More about deducibility and its relation to grounding in §4.

7. Cf. [5], §162.

8. Cf. [24].

9. Unfortunately, Bolzano sometimes uses the verb 'to denote' (*bezeichnen*) for the relation between an expression and the concept it expresses (cf. [5], §285). I shall not follow him is this usage.

10. Cf. [5], §162 where Bolzano introduces the grounding-relation, and [5], §198 where he introduces the *concept* expressed by 'because'.

11. A *collection* (*Inbegriff*, cf. [5], §82), roughly, is a whole consisting of *parts*. I shall say a bit more about collections in Section 5.

12. Bolzano officially introduces the term 'Erkenntnisgrund' in this sense in [5], §313. I added the bracketed part of the definition to Bolzano's official formulation. Though he does not mention this condition explicitly it is quite obvious that he is implicitly presupposing it.

13. Cf. [5], §300.2.

14. Bolzano mentions this difference in [5] §§194.3, 198 and in Bolzano [7], p. 81.

15. The term 'Erkenntnisgrund' is used by Bolzano in the propositional sense in [5] §§177, 194.3, 198, [7], p. 68, and Bolzano [4], vol. I, p. 6.

16. Cf. [5] §209.

17. Cf. [5], §206, [7], p. 68. I will mention a possible doubt against this in Section 6.

18. Cf. [7], p. 77, [4], vol. I, p. 6.

19. Unfortunately, not much of Bolzano's main work has been translated into English yet. There is not even a complete translation of [5]. In general, I follow George's translations [16]—in the case there is one. But with respect to some words I resist: I translate Bolzano's "Inbegriff" by "collection" (and not by "class"). My reasons for this will become clear in Section 5. Furthermore, I translate "Erklärung" by "analysis" (and not by "definition") because Bolzano's intent in giving what he calls "Erklärung" is to explain (more or less) ordinary concepts we already have and use and not to stipulate "new" ones. Bolzano holds a mereological view of concepts according to which concepts have parts. These parts might be other concepts as well as intuitions (cf. [5] §§72, 73). Analysis then is a special kind of explanation: To analyze a given concept is to give an explanation of it by pointing out its parts and the way they are combined. For Bolzano's concept of analysis cf. [5], §554.

20. Cf. [5], §97.

21. Cf. [5], §201.2.

22. An account of the relation which holds between Socrates and {Socrates} just in case the proposition expressed by this sentence is true has been given by Correia (cf. [13]). This relation of 'ontological grounding' is, of course, closely related to the 'propositional' grounding-relation we are considering here. As far as I can see, Correia's account of ontological grounding fits quite well with Bolzano's account of propositional grounding.

23. Cf. [5], §127.

24. [5], §201.4.

25. Cf. Bolzano [6], p. 85—which also shows that he takes this way of speaking to be justified by ordinary language-use. Cf. also [18], p. 204 where Bolzano's "Wirklichkeit" is also translated by "actuality" (and not by "reality" which prima facie seems to be the best candidate).

26. The most precise and extensive work on the concept of deducibility is Siebel [21].

27. The addition of '[and only if]' is justified by the context of the quoted passage. Cf. [21], p. 14. Ideas (*Vorstellungen an sich*) are those parts of propositions which are not themselves propositions. Cf. [5], §48. Bolzano investigates the connection between the grounding-relation and deducibility mainly in [5], §§162, 200. The propositions concerning the grounding-relation which he states in [5], §§203–21 are formulated as comparisons between the grounding-relation and deducibility. The topic of this section is also treated in Buhl [12], p. 25–28.

28. Cf. [5], §200.

29. Cf. [5], §203, Bolzano [10], p. 22.

30. Also Bolzano himself considers this to be the central point of interest. He raises the question of how these relations relate to each other mainly in §200 of his [5].

31. In these cases, the relation of deducibility between α and β, of course, ensures that the propositions which figure as (direct) parts of β are also true. What is a *direct* part of a collection of propositions? Consider the collection of [The earth is round] and [The earth is round $\rightarrow$ The moon is round]. [The earth is round] is a direct part of this collection. [The moon is round] is a part of it, but it is no direct part of it. Cf. Bolzano [8], p. 77.

32. Cf. [5], §162. Siebel has pointed out to me that there is a second reason for the falsity of (1). Since the relation of deducibility can hold between collections of propositions and an idea i even in cases in which i does not occur in some or even all of those propositions, it follows that every truth is T-deducible from any collection of truths with respect to some idea i which occurs neither in the premise(s) nor in the conclusion. Perhaps it would be possible to get rid of this additional problem by working with a notion which puts some appropriate constraints on the relevant ideas. But the argument against (1) which I have given in the main text, it seems, would also work against an analogous principle which might be constructed with such a refined notion.

33. Cf. the end of §200. A further prima facie reason (not mentioned by Bolzano) why (2) might probably be false is this: However the link between grounding and causality has to be specified in detail, it does not seem very probable that (collections of) propositions that "mirror" effects should in all cases be deducible from (collections of) propositions "mirroring" their causes.

34. According to a purely extensional conception of property-kindhood the fact that everything satisfying the n-place property F also satisfies the n-place property G would not be sufficient for the property F to be a special find of the property G. There has to obtain

a certain stronger connection. A not completely extensional conception of property-kindhood does not fit with Bolzano's use of the predicate 'property F is a special kind of property G' (e.g., in [5], §200). However, I think, it fits well with our ordinary talk about properties.

35. Cf. the example of [natural number smaller than five] and [prime number] given in Section 3.

36. The terms 'mediate' and 'immediate grounds', I think, bring out the main point of the relation between the two concepts better than any other pair of terms. Despite this, Bolzano does not use these terms. His reason is that he thinks that what I call 'mediate grounds (consequences)' are not really grounds (consequences) in the proper sense of that word (cf. §§213, 217). I don't think that Bolzano is right in thinking so, as long as our understanding of 'ground' and 'consequence' is derived from our ordinary use of 'because'. I shall make this point more explicit at the end of this paper.

37. Cf. [8], p. 77.

38. This is not in accordance with Bolzano's official introduction of the term 'collection' in [5], §82, where he suggests that every collection of Fs has at least two Fs as its parts. Nevertheless it is in accordance with Bolzano's own usage of his term on many occasions. To use the word in the strict sense of the official introduction would complicate many formulations but it would not change anything substantial.

39. Cf. Simons [23].

40. Krickel [17] argues that Bolzano's collections are best seen as mereological sums. Simons [23] argues against this claim, whereas Behboud [3] tries to defend a version of it.

41. The relation between what is denoted by '$<$' and '$\ll$' in mereology is similar to the relation between what is denoted by '$=$' ('*is less than or identical with*') and '$<$' ('*is less than*') in arithmetic. Cf. also Simons [22], p. 10 f.

42. Cf. [5], §198, note.

43. The acceptance of what is said by this interpretation of Aristotle, of course, presupposes that one rejects Frege's famous claim that 'P' expresses the same proposition as 'It is true that p' (cf. Geach and Black [14], p. 64). Bolzano does so explicitly: "If the proposition 'A is B' is true, then the assertion 'The proposition that A is B is true' must undoubtedly also be a true proposition, and since the latter has different parts, it is a different proposition from 'A is B' and thus a second different truth" ([5], §32, note). This contrast has been pointed out by Künne in his detailed comparison of Bolzano's and Frege's doctrines about propositions ([18], p. 229).

44. Cf. [5], §209.2.

45. [5], §206 contains a version of (A_2) that applies to collections of propositions.

46. Assume that α is the collection of propositions A, B, and C. Then '[α is true]' abbreviates '[A, B, and C are true]'. Cf. Bolzano's own notation in [5], §206. One might complain

at this point that it is odd to offer as axiom for a symbol a statement which, strictly speaking, does not contain that symbol at all (but rather two others which are defined in terms of it). My justification for putting things in this way is that it is very close to what Bolzano himself is doing: on the one hand, he takes the concept of complete immediate grounding to be central. It is this concept he wants to characterize in the first place. On the other hand, one of the most central claims he makes in order to characterize it deals not strictly speaking with that very concept, but with two other concepts which are defined in terms of it. The axiom simply mirrors this.

47. Cf. [5], §§205.1, 206, 209, 212, 214.

48. For the concept of a truth-maker cf. [20]. The difference as well as the relations between these two possible interpretations of Aristotle is explained in detail in Künne [19], Ch. 3.5.

49. Cf. [5], §206.

50. As already mentioned, this passage contains Bolzano's echo of the first part of Aristotle's principle, that is, (A_1). Notice that also (A_2) figures as a premise here.

51. Suppose immediate grounding were not asymmetrical, that is, that there are collections of truths A and B such that (1) A $\Rightarrow$ B & B $\Rightarrow$ A holds. This trivially implies that A is a collection of truths. Thus it follows by clause 2 of Aristotle's principle that (2) A $\Rightarrow_<$ [A is true]. From (1), (2), and Definition 5.2 it follows that (3) [A is true]<B. On the other hand, now, it follows from (1) and clause 1 of Aristotle's principle that (4) $\neg$([A is true] $_<\!\!\Rightarrow$ A). From (1), (4), Definition 5.1, and Axiom 6.3, it follows that (5) $\neg$([A is true]<B). But (5) is inconsistent with (3) against the background of the logic of parthood which says that $\forall x, y(x \ll y \rightarrow x < y)$. Since (1) is the only assumption we have made (besides Aristotle's principle), it has to be rejected: (6) $\neg$(A $\Rightarrow$ B & B $\Rightarrow$ A). We made no further assumptions about A and B, and so are justified in generalizing (6) into (7) $\forall\alpha\forall\beta\neg(\alpha \Rightarrow \beta \;\&\; \beta \Rightarrow \alpha)$ which is tantamount to Theorem 6.4.

52. Cf. [5], §204.

53. Cf. [5], §204, note.

54. Cf. [5], §213.

55. I gave a sketch of such a picture in Section 3.

56. Notice that Bolzano's particular thesis about the meaning of 'causes', which I mentioned in Section 3, does not have to be presupposed for this implication to hold.

57. Cf. [5] §213, note.

58. Cf. [5], §214.

59. Cf. [5], §§214, 221.3.

60. Cf. [5], §208.

61. Let α be an arbitrary collection of truths. According to Theorem 6.7, there is a further collection of truths β such that $\alpha \Rightarrow \beta$. The same holds for β itself: There has to be a γ such that $\beta \Rightarrow \gamma$. Thus the existence of an arbitrary collection of truths α ensures that $\forall\beta\forall\gamma(\alpha \Rightarrow \beta \;\&\; \beta \Rightarrow \gamma)$. Thus Theorem 6.8 follows.

62. Cf. [5], §31.

63. Notice that Bolzano does not precisely distinguish between complete and partial grounds in this passage.

64. The relation of mediate grounding is introduced in §§216 and 217 of [5]. At some points in §§216–19 of [5] he seems to suggests that the relation of mediate grounding he has in mind obtains between collections of truths "on the left-hand side" and a single truth "on the right-hand side." But he cannot really intend this since such a relation would not be transitive (what he clearly intends, as we shall see in a minute).

65. Bolzano uses the word 'dependence' (*Abhängigkeit*) for the relation of mediate grounding. He says also that the parts of β are "supporting" or "helping" truths of α (*Hülfswahrheiten*) (cf. [5], §217). I am not using this terminology since it does not mirror the main point of the distinction between mediate and immediate grounding (the idea of a 'chain' I have already mentioned twice).

66. As Bolzano himself shows in §220 of [5].

67. Cf. [5], §218.

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