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Bolzano's Deducibility and Tarski's Logical Consequence

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In this paper I argue that Bolzano's concept of deducibility and Tarski's concept of logical consequence differ with respect to their philosophical intent. I distinguish between epistemic and ontic approaches to logic, and argue that Bolzano's deducibility presupposes an epistemic approach, while Tarski's logical consequence presupposes an ontic approach.

1. Introduction

There has been a persistent growth of interest in the work of Bernard Bolzano since the beginning of the twentieth century. In the past thirty years a great deal of attention has centred on the comparison of Bolzano's writings on logic with those of Alfred Tarski. Some have argued that Bolzano gave the semantic definition of logical consequence in his *Wissenschaftslehre*, first published in 1834, hence predating Tarski's more celebrated discovery of the concept by nearly one hundred years. The term which Bolzano associates with his definition is not 'consequence' (*Folgerung*), but 'deducibility' (*Ableitbarkeit*). The strong claim of identity between the definitions of Bolzano and Tarski has been questioned on the ground that following Tarski logical consequence is a semantic relation in modern logic, while deducibility is a syntactic relation.[1] Perhaps the best argument for the strong claim to appear in English is the one offered by John Corcoran, who concludes: '. . . then Bolzano truly deserves credit for the explication of logical consequence, if Tarski does, because in my opinion Bolzano offered precisely the same idea' (Corcoran *1975*, 71).

This strong claim is criticized by Rolf George in the introduction to his translation and condensation of Bolzano's *Wissenschaftslehre*, and by Jan Berg in his book *Bolzano's logic*. George remarks that Bolzano's definition is different in the several respects noted below (in Section 3). George then takes the weaker position that Bolzano 'was the first to give a formal definition of the notion of consequence. It is akin to that given a century later by Tarski' (George *1972*, xxxiv). Note that George takes Bolzano's definition to be one of *consequence* rather than *deducibility*. Berg also points out several distinctions, including the problem of syntax and semantics above, and concludes with the similarly weaker claim that Bolzano's deducibility for propositions corresponds to a relation defined for formal objects which '. . . would

1 The bulk of this discussion appears in German journals with the strong claim being argued by H. Scholz, and the critique being offered by E. W. Beth. For a synopsis of the controversy, see Buhl *1961*.

suit Tarski's classical notion of logical consequence when extended in a natural way to sentential classes . . . ' (Berg *1962*, 117). Berg's goal in *Bolzano's logic* is a modernization of Bolzano in which the 19th-century logician's system is rendered in 20th-century terms. I take his claim to suggest that Bolzano's concept differs from Tarski's with respect to what might be called technical points, in virtue of the fact that the conceptual apparatus of symbolic logic and metalanguage was unavailable to Bolzano.

All of these authors assume that Bolzano intended to offer a definition of logical consequence like Tarski's: they disagree only as to the extent Bolzano succeeded. In fact, this conflation of Bolzano and Tarski obscures fundamental differences in intent. The thesis which I hope to establish is that Bolzano's deducibility is best understood as an epistemic relation, while Tarski's logical consequence is better understood as an ontic relation.[2]

2. The ontic and epistemic aspects of logic

Epistemology and ontology are so broadly conceived in the history of philosophy that a few remarks are necessary to indicate the senses to which I appeal in this paper. Epistemology is here understood as the theory of knowledge in which knowledge is taken to be 'the true account of the way things are' (where 'true' is used in a pragmatic sense, corresponding to the sense of correctness) rather than something like 'justified true belief' which lays stress upon the psychological possession of knowledge by some thinking being. Knowledge in this sense is identical with science, where 'science' is taken to mean the goal of scientists rather than their activity. Epistemology might therefore be understood as the theory of science. A science is a true account of the way things are, and as such will surely differ from an arbitrary list of true propositions. In science descriptive claims and general laws are bound together to yield explanatory or predictive force. This binding together introduces epistemic relations which must hold for the propositions of the science.

Ontology, on the other hand, will here be understood as the study of the being of things. An ontology of a given entity would consist of a description of its essential or intrinsic relations. One would generally attempt an ontology in this sense precisely because one would wish to exclude secondary ('as known', 'as employed', 'as perceived') properties and relations from consideration. An ontical relation between objects is thus one which obtains by virtue of the primary characteristics of those objects. The ontological approach thus precludes viewing objects as 'an account of' anything at all. Development of a science involves taking propositions which are already "there" in their own right and recasting them in a systematic fashion which yields the desired explanatory force. The propositions of any arbitrary list bear ontological relations to each other, but epistemic relations depend upon the possibility of a systematic arrangement of propositions into an account of the way things are. The truth or falsity of a proposition would be a primary characteristic, however. This

2 I developed my concept of epistemic and ontic with respect to logic as a student of John Corcoran, who makes such a distinction in his approach to logic. I do not know whether he would approve of the sense I have given the distinction in this paper.

follows from the fact that a proposition has its truth value whether or not it is 'known', or found in an account of the way things are.

Given this rough distinction between ontology and epistemology, logic can be construed in terms of its ontic and epistemic aspects. As the study of relations which constitute the possibility of systematizing propositions into a science, logic is epistemic. Such relations are epistemic relations. As the study of relations which are primary or hold without regard to propositions appearing in an account or science, logic is ontic. Such relations are ontic relations. Logical consequence may be characterized as an ontic relation, and deducibility may be characterized as an epistemic relation.[3]

A relation of deducibility is put forward to establish an advance in knowledge, where 'advance in knowledge' is understood to indicate the acceptance of some new claim on the basis of a deducibility relation from claims previously accepted in the science. Deducibility is usually established by doing a deduction which allows this advance in knowledge to be made. For example, the conclusion of an argument is said to be deducible from a given set of premises precisely if there is some chain of reasoning which makes evident that the conclusion is implied by the premises. In cases where the premises are not known to be true, they may be assumed or taken as hypotheses, and the conclusion thus deduced is then accorded the same degree of confidence as the assumed premises. In cases where the premises are known to be true, the chain of reasoning produces knowledge of the conclusion. In modern logic, rules for constructing such a chain are collected together into a deductive system; hence one speaks of deducibility in a given system of inference. This possibility of systematization suggests the characteristics of organization which distinguish an arbitrary list from an account or a scientific theory. The fact that in a scientific theory certain claims may be deduced from others is one of the principle distinguishing characteristics of such theories. Deducibility is thus an epistemic relation in that it is involved in the means for systematically arranging claims into a science.

Logical consequence is a relation which holds between propositions or sets of propositions irrespective of whether they have been framed into a scientific theory or systematized in any way. This relation may be described as ontic since it holds in virtue of some intrinsic characteristic of the propositions themselves. The logician's interest in ontology of language issues from the attitude that a proposition has certain properties without regard to whether it is known to be true. The study of logical form *per se* is not concerned with the uses of language in reasoning, inference, or within a scientific theory, but rather with the intrinsic properties of language itself. The concept of logical consequence falls within the scope of this enquiry.

Tarski's discussion of logical consequence uses the concept of a sentential function. The relation of consequence is then defined in terms of the semantic concept 'satisfaction of a sentential function'. For the purposes of this paper, the basic sense

3 Deducibility and logical consequence are the only relations which I discuss in this paper, and they are treated as relations holding among sentences or propositions. There is no obvious reason, other than simplicity of presentation, for confining the discussion of ontic and epistemic aspects to sentences or propositions.

of Tarski's definition is adequately described by the following necessary condition.[4] If X is a logical consequence of K, then 'If, in the sentences of the class K and in the sentence X, the constants—apart from purely logical constants—are replaced by any other constants (like signs being everywhere replaced by like signs), and if we denote the class of sentences thus obtained from K by "K'", and the sentence obtained from X by "X'", then the sentence X' must be true provided only that all the sentences of the class K' are true' (Tarski *1956*, 415). It is important to notice that this definition of logical consequence does not depend on any reference to the sentences under consideration being connected together by inferences or within some theory.[5] The definition depends upon observations on the ontology of the sentences, when ontology is conceived in the sense outlined above. In fact, logical consequence may hold when deducibility does not.

As noted previously, modern logicians understand deducibility as a syntactic relation and logical consequence as a semantic relation. It does not follow, however, that a syntactic relation is epistemic and a semantic relation is ontic, at least when the logic under discussion is one of natural rather than formal language. For example, we might give the rule 'If B is true when A is true, then any account including A may also include B' or 'If B is a logical consequence of A, then B may be deduced from A'. Despite the fact that this formulation begs the completeness question for languages in which syntax and semantics have been carefully distinguished, it is quite capable of establishing a chain of reasoning which results in an advance in knowledge. If A is known and that B is a consequence of A is known, this rule can be used to deduce B. Never mind how it is known that B is a consequence of A, it is clear that this rule represents an epistemic relation in the sense indicated above. The characterization of epistemic and ontic aspects of logic is, in certain respects at least, quite distinct from the distinction of syntax and semantics.

Even a cursory reading of the history of logic reveals that many logicians before Frege conceived their subject as a branch of epistemology, often to the point of devoting a great deal of energy into the consideration of points which today are viewed as questions of psychology.[6] One might in fact argue the thesis that a true

4 One helpful reader has pointed out that Tarski's definition of logical consequence uses an interpretation semantics, but that the definition offered by Bolzano can at best be understood as a truth-value semantic definition. The crucial difference lies in the ontological import of the quantifiers. The interpretation semantics generalizes over domains of individuals, while the truth value semantics does not. My reader notes that my limitation of Tarski's definition to the necessary condition obscures this point. The distinction between interpretation and truth-value semantics, however, rests upon the way in which one conceives the ontic relation between language, individuals, and truth values. Since my principal point is to argue that Bolzano was interested in an epistemic relation, there is no purpose in belaboring the subtleties which arise when one interprets his remarks to have primarily ontic rather than epistemic import.

5 Clearly, this would be rather strange if by 'theory' one understands a set of sentences closed under logical consequence. I have used this term throughout in the rough and ready sense of 'scientific theory', which I hope has been made clear in the discussion of epistemology which heads this section.

6 Psychologistic theories of logic were widespread throughout the nineteenth century. Perhaps the best known of the psychologists is J.S. Mill. Bolzano should be noted for his unswerving adherence to a non-psychologistic vision of logic, but this does not entail that he would have also had a non-epistemic vision (given the restricted sense of epistemology we employ) as well. Perhaps the best critical review of psychologism remains in Husserl's Prolegomena to the *Logical investigations* (see Husserl *1900*, 55–266).

appreciation of the advance secured by Frege and his contemporaries requires an understanding of the way logic in the nineteenth century had been dominated by this epistemological perspective. In the following section I shall defend the view that Bolzano's concept of deducibility, despite its status as a clear precursor to the ontical concept of logical consequence, is firmly couched in the epistemological perspective, and that furthermore, Bolzano's epistemic approach to the concept accounts for some of the 'technical' differences.[7]

3. Bolzano's concept of deducibility

Bolzano's definition of deducibility is given in a single paragraph of the *Wissenschaftslehre*. The remaining paragraphs of the section on deducibility explore the ramifications of the definition. From Rolf George's translation of Bolzano (*1837*, art. 155.2):[8]

> Let us consider, first of all, the case that among the compatible propositions *A, B, C, D, ... M, N, O ...* the following relations obtain: all ideas whose substitution for the variable ideas *i, j, ...* turns a certain part of these propositions, namely *A, B, C, D, ...* into truths, also have the characteristic of making a certain other part of these propositions, namely *M, N, O, ...* true.
>
> This special relation which we think between propositions *A, B, C, D, ...* on the one hand and *M, N, O, ...* on the other is of special importance, since it puts us in a position to infer the truth of *M, N, O, ...*, once we have recognised the truth of *A, B, C, D, ...*
>
> I wish to give the name of *deducibility* [Ableitbarkeit] to this relation between propositions *A, B, C, D, ...* on one hand and *M, N, O, ...* on the other.
>
> Hence I say that propositions *M, N, O, ...* are *deducible* from propositions *A, B, C, D, ...* with respect to variable parts *i, j, ...* if every class of ideas whose substitution for *i, j, ...* makes all of *A, B, C, D, ...* true also makes all of *M, N, O, ...* true.

One might rephrase the sense of this definition as follows: 'Let *A* and *B* be compatible sets of propositions and let *S* be a set of concepts occurring in *A* and *B*. *B* is *deducible* from *A* if every uniform substitution for occurrences of members of *S* in the propositions of *A* and *B* which makes every member of *A* true also makes every member of *B* true'. Bolzano also offers several special cases of deducibility which he discusses at various points throughout the text. One of these special cases is that of 'logical deducibility', the concept specifically cited by Corcoran (*1975*, 71) in his comparison of Bolzano and Tarski. This special case involves the stipulation to the above definition that *all* non-logical ideas must be considered as variable (Bolzano

7 Readers interested in Bolzano's epistemology will want to note his concept of *Abfolge*, a relation which turns upon the explanatory force of causal claims or natural laws. A careful discussion of this relation is needed, but is beyond the scope of this paper.

8 In the original text, these sentences follow each other in the usual manner. I have blocked them separately to facilitate discussion and to avoid any possible ambiguity which might arise regarding the use of periods both to indicate 'and so on' and to end sentences.

1837, art. 223), a stipulation which clearly moves Bolzano even closer to Tarski. Whether these special cases represent revisions of the concept or merely restrictions designed to call attention to special uses is discussed by Buhl (*1961*, 19–21), who decides in favor of the latter alternative.

The similarity of Bolzano's definition to the statement of a necessary condition for logical consequence quoted from Tarski in section 2 above, is obvious and undeniable. Bolzano seems to share the same insight into the nature of language and truth which enabled Tarski to formulate the semantic definition of logical consequence. The passage from the *Wissenschaftslehre* displays an ambivalence regarding the question at hand, however. The first sentence seems to indicate a relation descriptively noted as a property of certain classes of propositions, that is, an ontic relation. The second sentence seems to contain indications of an epistemic relation, calling specific attention to the role of the relation in the establishment of an inference. In my opinion, this ambivalence is irresolvable. Clearly Bolzano must establish certain facts about propositions under a given set of conditions. In doing so he describes properties of propositions and has recourse to an important ontical relation. There is no dispute that this is essentially the same ontic relation which serves Tarski in his definition of logical consequence, but there is an important sense in which Bolzano never intended to offer an ontic relation in this section at all. The argument for this claim rests on several points.

First, Bolzano has stipulated that all the propositions under consideration must be compatible, that is, there must be some set of ideas which makes all the propositions true when uniformly substituted throughout. This stipulation is unnecessary for logical consequence. Second, he thought that the set S of concepts occurring in both A and B could not be empty, that is, that deducible propositions must share common ideas (*1837*, art. 155.21). He held this to be obvious, yet for logical consequence such a claim is not only unobvious; it is incorrect. If one of the propositions in the antecedent (the set K for Tarski, A for our paraphrase) is self-contradictory, then no assignment will make every member true, hence every assignment which makes all of A true also makes all of B true. In addition, if the consequent is a logical truth, the relation of logical consequence similarly holds trivially and without regard to whether antecedent and consequent share any common ideas. If we assume that Bolzano intended to describe the ontic relation of logical consequence, how are we to account for these discrepancies? The strategy of previous authors has been to regard them as oversights in Bolzano's pioneering definition, oversights to which Bolzano would seem entitled. If we abandon the assumption that Bolzano intended to describe Tarski's relation, however, the discrepancies can be understood as an integral part of Bolzano's entire logical project.

Bolzano himself was remarkably explicit with regard to his own view of the end to which the *Wissenschaftslehre* was written. In his opening remarks he characterizes a science as an aggregate of truths which are compiled together in a treatise, a treatise being 'Any book which appears to have been written with definite intent of recording all known and important truths of a science in a manner that makes it easy to comprehend and accept them with conviction . . . ' (Bolzano *1837*, art. 1.1). He then goes on to describe the task which he has attempted to complete in the *Wissenschaftslehre* (art. 1.4):

> It should be possible through some reflection to find the rules which we must follow in dividing the total domain of truth into individual sciences and which must govern the writing of the respective treatises. There can also be no doubt that the sum of these rules itself deserves to be called a science, since it is clearly worthwhile to collect the most important part of them in a special book, and to order them and provide proofs for them so that everyone can understand and accept them with conviction. I allow myself to call it the *theory of science* [*Wissenschaftslehre*], since it is the science which teaches us to represent other sciences (actually only their treatises). Thus, by theory of science I mean the aggregate of rules which we must follow when we divide the total domain of truths into individual sciences, and represent them in their respective treatises, if we want to do a competent piece of work.

Bolzano is also clear that by the term 'logic' he intends precisely this theory of science (see art. 6).

Bolzano thus takes logic to be the aggregate of rules which must be followed in organising and representing the various sciences. A science is clearly an account of reality, rather than the activity of scientists; hence his use of 'science' is the one used above (see section 2) in describing the epistemic aspect of logic. He characterizes logic as the theory of science, a theory which gives the rules for collecting and presenting facts and thus focusing a set of claims in such a way that it may be understood as a system of knowledge. For Bolzano logic is therefore confined precisely to the area of investigation described above as epistemological. Given this predisposition for giving rules which bind knowledge claims together, we would expect Bolzano to concentrate upon epistemic relations, and not to limit himself to purely descriptive properties of language which have been characterized as ontical aspects of logic.

Furthermore, Bolzano's choice of the word 'deducibility' (*Ableitbarkeit*) to indicate the relation in question is itself further evidence that it was the rule-governed possibility of linking propositions together in chains of reasoning that he had in mind. His remarks indicate that he was aware of the connotative value of his term, and actually chose this name precisely because the word suggested the epistemic activity of making an advance in knowledge. Following his discussion of deducibility he offers an explanation of his reasons for calling the relation by this name:[9]

> Despite the fact that it is such a long word, and sometimes is understood with a totally different meaning, to the extent that one says that one proposition is deducible from another when it is formed in some way from that other proposition, I know of no other word which is so well suited to indicate the relation that obtains between premises and conclusions when they are not considered as thought propositions or propositions held to be true, but rather as propositions in themselves [*an sich*]. One such collection [*Inbegriff*] of propositions can be called

9 This passage is omitted from the English abridgements of the *Wissenschaftslehre*. This is my translation of a passage found in the 1929 German edition of Bolzano's work at vol. 2, 132, and is from note 2 of art. 155.

> a conclusion according to the prevalent grammatical usage of the word, as when it is said that someone uses his consideration of the relations obtaining between them as a tool to arrive at the knowledge of the conclusion from knowing (or also of only supposing) the truth of the premises.

This passage itself is rife with ambiguity. The most important unsettled point is with regard to Bolzano's concept of the *an sich*. In referring to propositions in themselves, Bolzano might be aiming at precisely the type of ontical characterization of language typified by genuine logical consequence. At the same time, Bolzano's most typical usage of this concept, the *an sich*, is to distinguish between a psychologistic view of propositions, in which they are considered only as actual instances of belief or opinion in the mind of some thinking thing, and what might be characterized as the logistic view, in which such contingencies are not to the point. In the quoted passage Bolzano seems to indicate that his choice of a name must involve psychological overtones, but at the same time he does not abandon the dynamic characteristic of true epistemic deducibility, the inferential movement from premises to conclusion.

In fact the restrictions discussed above make the epistemological import of Bolzano's relation more apparent. He takes it as obvious that propositions with no ideas in common cannot be deduced one from another. For Tarski's logical consequence, however, this is not the case; in particular, any formally true or analytic proposition is a logical consequence of any proposition irrespective of whether the propositions share any common ideas. Bolzano was aware of the properties of formal truths. It would therefore be odd that he should not have thought of this possibility if in fact he had intended to take the ontical perspective. If, on the other hand, he was describing the epistemic relation of premises and conclusion in a proof, then he was quite correct in noting the obviousness of this fact. One does not reason from A to B when A and B have no concepts in common. In attempting to describe a theory of science, it is quite reasonable that Bolzano would have taken the process of reasoning to parallel the structuring of claims into knowledge. A formally true proposition would be introduced into a science simply on the ground that it is formally true. One would not need to deduce it from previously accepted claims. If Bolzano was trying to describe the epistemic connection of one set of propositions to another within a theory, he would have been quite correct in excluding cases in which the truth of the conclusion is in no way dependent upon the truth or falsity of the premises.

Similarly, Bolzano's stipulation that deducibility applies to compatible propositions only makes more sense for an epistemic relation than for an ontic relation like logical consequence. For example, suppose a set A of propositions contains formally contradictory (hence incompatible) propositions. Any proposition B is a logical consequence of A since B is satisfied whenever A is satisfied (which is never). Bolzano does not even *define* deducibility for incompatible propositions, however. It seems odd that he would have bothered to make such a stipulation if he had only had an ontical relation in mind. On the other hand, one does not include contradictory premises in a proof. If one is attempting to describe the relationships of claims made within a theory, it is quite reasonable to stipulate that these claims should, above all, be compatible with one another. In fact, Bolzano introduces his concept of deducibility

as a *special case* of his compatibility relation (Bolzano *1837*, art. 155.1). This also suggests that he was primarily interested in how sentences might stand together in an account, or, in his terms, a treatise.

These stipulations are quite reasonable given Bolzano's stated objective of giving the rules for writing a treatise, that is, presenting the claims of a science. In giving such a rule, one would naturally describe the circumstances under which a deducibility instance might be found. Clearly *these* circumstances differ from the purely ontical circumstances which would be described if one were interested in characterizing logical consequence. Bolzano is above all interested in showing why the claims of a science form a theory and an arbitrary aggregation of propositions do not. His primary intentions are focused upon these essentially epistemological aspects of logic, and the discrepancies between his concept of deducibility and Tarski's logical consequence reflect this orientation.

4. Conclusion

In conclusion, it must be admitted that there is a genuine ambivalence in Bolzano's writings on logic. His concept of the *an sich* represents a basic move away from a psychologistic concept of logic. Those who understand epistemology strictly as philosophical psychology will find little help in Bolzano's *Wissenschaftslehre*. Bolzano provides an approach to logic which is not 'epistemological' in this psychologistic sense, which takes knowledge strictly in terms of its possession by a thinking being. It is quite clear, however, from Bolzano's overall statement of purpose that he hoped to achieve a 'theory of knowledge' in the restricted sense introduced above in section 2. The restrictions Bolzano built into his concept of deducibility reflect the aims he had established at the outset. In fact, he chose the name 'deducibility' wisely, since the concept he wished to define is one by which the various threads of a science might be woven together.

It is in this light that the fundamental distinction between Bolzano's logic and Tarski's can be made clear. Bolzano never clearly undertook an inquiry into language which distinguished a proposition from its appearance (or potential appearance) in a treatise. For him the description of essentially ontic relations and essentially epistemic relations was always relative to his overall project, the writing of a treatise which sets forth the rules for writing treatises. The essentially epistemic bent of this project undoubtedly obscured Bolzano's perception of ontic relations. It is the ability to firmly grasp the ontical aspect of language and logic that is at the very heart of formal logic. This grasp was the advance secured by Frege and exploited so brilliantly by Tarski. Bolzano's great historical contribution is his move away from psychologism. This move begins to set the stage for Frege's work, and as such Bolzano can truly be understood as a forerunner of modern formal logic.

Bolzano was a great logician and an important element in the movement of thought which led to the revolutionary logical developments of the early 20th century. He was not, however, a 20th-century mind lost in the 19th century, and his problems were those of his time rather than ours. His work and the extent of his contribution can only be appreciated when we approach his legacy from his intended perspective.

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