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BEING AND TIME AND BROUWER'S INTUITIONISM

In the introduction to *Being and Time*, Heidegger links the crisis in the foundations of mathematics to the need to return to the question of Being.[1] But how is the relationship between Heidegger's fundamental ontology and the basic issues in the foundations of mathematics to be understood? In *Mathematische Existenz*, which was originally published in the same volume of the *Jahrbuch für Philosophie und phänomenologische Forschung* as *Being and Time*, Oscar Becker offers an interpretation of the intuitionism–formalism controversy that is based on Heidegger's ontology.[2] Becker's approach rests on the premise that Brouwer's choice sequences presuppose Heidegger's notion of finitude. In this paper, I examine Becker's approach and suggest an alternative understanding of the relationship between intuitionism and the ontology of *Being and Time*.

1 becker's *mathematische existenz*

Heidegger's philosophy plays two major roles in Becker's *Mathematische Existenz*. First, it provides a general framework for the book, in the sense that Becker interprets the debate over the foundations of mathematics in terms of the different meanings of "existence." Second, Heidegger's ontology provides a means of deciding which of the two positions in question, namely intuitionism and formalism, best captures the notion of existence in mathematics. Formalism considers a theory's consistency to be a sufficient condition for the existence of the entities of which it speaks; intuitionism deems this inadequate. Entities do not exist unless the means for constructing them can be provided (29).

Becker maintains that, from a Heideggerian point of view, intuitionism is the correct position. Essentially, he has two reasons for this. First, the centrality of the concrete lives of individuals in Heidegger's ontology is paralleled by the intuitionistic requirement of constructability: according to the intuitionist, the very possibility of intuiting mathematical objects rests on our ability to realize them concretely in the world. Concrete reality restricts the possibilities for mathematical construction (196). Second, intuitionism endorses a distinction between sequences generated by a rule, and choice sequences, which Becker interprets as paralleling the Heideggerian distinction between natural time and historical time. Let me explain.

An example of a rule-governed sequence is the series of the squares of natural numbers (1, 4, 9, 16 . . .). A choice sequence is not rule governed, and each member is chosen independently of the

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others; an example would be the series (8, 1, 92, 6 . . .) with no rule for continuation. Becker argues that this distinction is meaningful only on the assumption of a finite perspective. From a God's-eye point of view, there is no essential distinction between these two types of sequence (230). Following Heidegger, Becker conceives human finitude in temporal terms. He distinguishes between natural time and historical time (221). Natural time is based on the possibility of iteration; it is time as measured. Historical time, on the other hand, is best grasped by looking at the phenomenon of the passage from one generation to another, not in the biological but in the cultural sense, as in "the generation of 1914" or "Gen X." According to Becker, the emergence of a new generation is not determined by the previous one, that is, in historical time the transition from one period to another is not law governed.

Applying this distinction to the mathematical realm, it is easy to see that a rule-governed series such as that of natural numbers exemplifies natural time, while a choice sequence exemplifies historical time. Hence, fundamental elements of Brouwer's intuitionistic mathematics are clearly echoed in Heidegger's notions of time as interpreted by Becker.

Toward the end of the book, Becker develops a philosophical approach to mathematics that does not simply adopt the Heideggerian framework, but sets forth an alternative to it. According to Heidegger, the notions of natural time and historical time presuppose a more basic stratum of time, "reckoning" time (*Being and Time* 456). Becker, on the other hand, claims that there is an unresolved tension between nature and history that does not arise out of any more basic framework. This tension, he contends, is mirrored in the debate between formalism and intuitionism, both of which are rooted in classic philosophical approaches to mathematics. He situates Brouwer's position within the critical tradition that includes Aristotle and Kant (311), contrasting this to the approach taken by Plato and Leibniz, which he characterizes as the "symbolic" approach. In the final chapter, Becker goes even further, concluding that the meaning of being in mathematics is not only the ontology of concrete life, but also the overcoming of death (321). Mathematics evolves in the living world, and in this sense, he argues, the intuitionistic approach is correct, but eventually mathematics advances from the realm of history to that of nature. This conclusion leads Becker further away from Heidegger. In later years, he comes to endorse an irreducible distinction between essence (*Dawesen*) and existence (*Dasein*) that corresponds to the distinction between nature and history.[3]

II the limitations of becker's interpretation

It seems to me that the insight that Heidegger's ontology should be understood in terms of its links to the foundations of mathematics is indeed sound. And I also grant that Heidegger's thought certainly has a greater affinity to the views of Brouwer than to those of Hilbert, and choice sequences are the appropriate place to demonstrate this affinity. Nevertheless, Becker's approach is problematic. The chief problem is that it is far from clear from his analysis why choice sequences presuppose Heidegger's notion of finitude and death, and not some other notion, or even the notion of potential infinity. On Becker's interpretation, the intuitionistic approach to mathematics presupposes a notion of time that has certain characteristics in common with Heidegger's notion of historical time, but it is not clear why it requires Heidegger's notion of primordial finite time. One possible response is that the "missing" connection between Brouwer's intuitionism and Heidegger's ontology is actually found in *Being and Time*, where Heidegger claims that "only because primordial time is *finite* can the 'derived' time temporalize itself as *infinite*" (*Being and Time* 331; emphasis in original). But in *Being and Time* this dependency thesis, that is, the dependence of the infinite on the finite, is not fully established. Its demonstration would require Heidegger to show both that given Heidegger's notion of finite time, a notion of infinite time can be arrived at, and that infinite time can be arrived at only via finite time. In other words, Heidegger must show that finite time is both a sufficient and a necessary condition for infinite time. In *Being and Time*, however, only the first part of the thesis is

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established: Heidegger shows how, despite Dasein's temporal finitude, Dasein can nonetheless interpret time as infinite. Heidegger claims that infinite time, which he interprets as an unending sequence of "now"s, results from Dasein's flight from death (*Being and Time* 477). Since this flight is a mode of relating to death, the conception of time as infinite can be arrived at from finitude and death. But the second direction of the dependency thesis is not addressed in *Being and Time*.

This limits the appeal of Becker's interpretation, since there remains a gap between his interpretation of the temporality of choice sequences and the finite primordial temporality of Dasein as Heidegger presents it in *Being and Time*. More importantly, the shortcomings of Becker's reading reveal a major problem in *Being and Time*. In that it lacks an argument for the dependence of all possible interpretations of time on Dasein's primordial temporality, *Being and Time* fails to fulfill one of its primary objectives, namely, to demonstrate that the meaning of Being can be understood only through Dasein.

In what follows, I will use results that emerge from Brouwer's intuitionism as the basis for an argument for the dependence of infinite time on Dasein's temporality. This argument recasts Becker's insight into the relationship between intuitionism and Heidegger's ontology, strengthening it and filling the aforementioned gaps.

III choice sequences and their limits

Choice sequences are usually described as having a definite beginning (our previous choices) and an open end. Choices subsequent to this beginning are determined neither by previous choices nor by any lawful rule. But careful analysis of the role played by choice sequences in Brouwer's intuitionistic approach to mathematics reveals a slight but highly illuminating modification of these characteristics. This new characterization is most readily discovered by examining one of the proofs Brouwer gives for the uniform continuity theorem, which states that every function defined on real numbers (i.e., every total function) is continuous. One version of this theorem states: "Any discontinuous real-valued function is not total."[4] Let us assume a discontinuity in the following way: $f(x) = 2$ for any $x < 1$ and $f(x) = 3$ for any $x \geq 1$. Then the value of the choice sequence $0.99999\ldots$ with the rule of free choice is not always determined. If at some point we choose a digit other than 9, then the value that the function assigns to the number will be 2, because no future choice will change the fact that the number is definitely less than 1. If we choose 9, the value of the function is not necessarily 3, since there is always the possibility that at a subsequent stage a different digit will be chosen, in which case the value of the function will be 2. To prove his theorem, Brouwer requires that the value of the choice sequence be indeterminate, as this will prove that if a function is not continuous, it is not completely defined. But this requirement can be met only if the choice made continues to be 9, though the option of choosing otherwise remains open. This result is paradoxical, since if 9 is indeed always chosen, then the value of the choice sequence is determined. Carl Posy describes this situation as "forced indeterminacy."[5] In a situation of forced indeterminacy, we must continue to choose a certain digit in order to maintain a series' indeterminacy. According to Posy, the state of forced indeterminacy captures a basic tension in the intuitionistic position. It characterizes Brouwer's position with respect to the notion of infinity,[6] and also lies at the heart of his approach to the continuum. For it is precisely the indeterminacy of the choice sequence that makes it possible for Brouwer to successfully steer clear of two outcomes he seeks to avoid: reduction of the continuum to a set of points, on the one hand, and restricting the arithmetization of the continuum to points that can be defined by rule-governed series and finite choice sequences, on the other.

IV choice sequences and heidegger's primordial temporality

What is the connection between the notion of forced indeterminacy and Heidegger's thought? My claim is that the relation between the notion of infinite time, viz., time composed of an infinite series of "now"s, and primordial temporality, should be interpreted in terms of the relation

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between the arithmetization of the continuum and forced indeterminacy.

An important element of this thesis is the idea that Heidegger's notion of death exemplifies the situation of forced indeterminacy. To establish the link between Brouwer and Heidegger, we must identify what is common to the problems confronting them. And indeed, both are preoccupied by the notion of the continuum, more specifically the question of reducing the continuum to a set of points. For Heidegger, this question arises in the context of his discussion of time. Heidegger explicitly acknowledges the importance of the notion of the continuum, inasmuch as it underlies the ordinary notion of time.[7] Although Heidegger mentions some of the modern literature on the notion of the continuum (Georg Cantor, Hermann Weil, and Bertrand Russell), his own approach is influenced chiefly by Aristotle. The relation between the point and the continuous line plays an important role in Aristotle's rejection of the conception that time is a continuum of "now"s.[8] For example, on the basis of the analogy between "now"s and points, Aristotle rejects the conception that a "now" has an immediate successor. One point cannot immediately succeed another on a continuous medium, since "things are called continuous when the touching limits of each become one and the same."[9] But in the case of points, the limit is identical to the point itself. Therefore, on a continuous medium there are no successive points.

Brouwer also rejects the idea that the continuum can be reduced to a set of points, although his argument is a little different. Brouwer's main concern is individuating points in terms of their numerical characterization. Since only points that can be constructed are acceptable, and these points cannot generate the notion of the mathematical continuum, where every point is determinate, Brouwer maintains that the continuum is irreducible to a set of determinate points. More recently, Michael Dummett has put forward a Brouwerian approach to the question of the reducibility of time to a set of instants.[10] Dummett begins by rejecting the possibility of discontinuous change. Instead of discontinuous change, he proposes a conception of time on which instants cannot be separated from their surrounding interval, a conception he calls "fuzzy realism." Dummett then offers an intuitionistic model of this conception of time that is based on the notion of choice sequences. By recourse to choice sequences, we are able to narrow the intervals, but we cannot eliminate them completely.

Now it seems to me that Dummett's approach to the question of the reducibility of continuous time to a set of instants can be applied to the interpretation of Heidegger's position. Dummett's approach has the advantage of providing grounds for the position that time cannot be construed as composed of instants without previously assuming the structures of Heidegger's primordial temporality.

The argument can be taken a step further. For Brouwer's case, as we saw, the characterization of an infinite choice sequence mandated the notion of forced indeterminacy. My contention is that this is also true of Heidegger's account. Heidegger claims that infinite time presupposes Dasein's finite time. And, I contend, Heidegger's notion of finite time likewise exemplifies forced indeterminacy. To show this, let me begin by characterizing Heidegger's notion of finitude. In *Being and Time* the elucidation of finitude is carried out by invoking death. Death is not understood from a vantage point beyond death, but, rather, from within its meaning for Dasein. Death is certain (*gewiss*), and yet, indeterminate (*unbestimmt*) (*Being and Time* 303). It is certain that we will die some day, but we do not know when this will transpire. The finitude that is linked to death is incomplete; it is being towards Being-a-whole without ever being complete. Death, as the end of Dasein, is the uttermost possibility. Death is not the end of a road, nor is it the most mature stage of development, but rather, an impossible possibility. That is, from Dasein's perspective, death is always a possibility. The realization of this possibility, however, marks the end of Dasein, and hence, Dasein cannot realize it; in this sense it is impossible.

It is thus not difficult to see that Heidegger's notion of death exemplifies a structure similar to the state of forced indeterminacy. For Heidegger, it is death that opens the space of possibilities, and without which there are no possibilities at all. In Brouwer's case, the possibility of choosing the

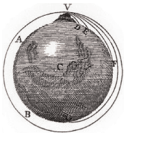
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sequence and maintaining its indeterminacy is created by the forced choice. The indeterminacy of the series' limit preserves the possibility of a further free choice. The two notions are not, of course, identical. Heidegger conceives of death as finite, while forced indeterminacy is not necessarily finite. As we saw above, it applies to infinite series as well. But this difference is not crucial, since Heidegger's notion of finitude is not finitude in the sense of a finite quantity, but in the sense of having a limit. The profound affinity between the two notions, namely Heidegger's notion of death and the notion of forced indeterminacy, is rooted, in my opinion, in the almost identical roles they play in their respective philosophies. Both Heidegger and Brouwer are seeking to delineate a limit where there is no possibility of characterizing it from the outside, that is, both require an immanent notion of limit.

Let me recapitulate my argument. Given the reasons for rejecting the conception of time as a continuum of instants, as argued by Dummett, among others, and the fact that Brouwer's choice sequences are invoked in describing the process by which instants of time are determined, the notion of forced indeterminacy plays a crucial role in explaining why the very possibility of formulating the notion of an infinite time composed of instants is conditioned on Heidegger's notion of finite temporality.

This interpretation of the relationship between *Being and Time* and intuitionism has several advantages. First, it provides a clearer sense of the relation between Brouwer's intuitionism and Heidegger's ontology than the account offered by Becker. Second, it provides an argument for Heidegger's claim that every notion of time presupposes primordial temporality. Third, in recent years philosophers and historians of mathematics have demonstrated the relevance of Husserl's phenomenology to positions taken in the 1920s debate over the nature of the continuum. My interpretation brings to light the fact that Heidegger too put forward ideas of particular salience to the question of the continuum, ideas that merit incorporation in the emerging understanding of the different positions in this debate.

But the argument I have outlined in this paper also shows that the Heidegger–Becker account of the relation between philosophy and mathematics needs to be revised. Both Heidegger and Becker maintain that mathematics can be grounded in fundamental ontology, but in this paper I have argued that in some sense the reverse is true: it is thanks to the mathematics of Brouwer's intuitionism that we can better understand the claims of *Being and Time*.



notes

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1 M. Heidegger, *Being and Time*, trans. J. Macquarrie and E. Robinson (Oxford: Basil Blackwell, 1962) 29–30. All references to *Being and Time* are to this edition.

2 O. Becker, *Mathematische Existenz. Untersuchungen zur Logik und Ontologie mathematischer Phänomene* (Halle: Niemeyer, 1927). Becker's discussion is based largely on Heidegger's 1924 lecture on the concept of time and on attending Heidegger's Freiburg courses, but not on the text of *Being and Time*, which came out together with *Mathematische Existenz*. All page references to *Mathematische Existenz* in this paper, however, refer to the self-standing edition.

3 O. Becker, "Para-Existenz. Menschlishes Dasein und Dawesen" in *Dasein und Dawesen. Gesamelte Philosophische Aufsätze* (Pfullingen: Neske, 1963) 67–102. This distinction also lies at the heart of Becker's adoption of Nazi ideology. See H. Sluga, *Heidegger's Crisis: Philosophy and Politics in Nazi Germany* (Cambridge, MA: Harvard UP, 1993) 219–23.

4 This proof appears in L.E.J. Brouwer, *Begruendung der Mengenlehre unabhaengig vom logischen Satz vom ausgeschlossenen Dritten. Zweiter Teil, Theorie der Punktmengen* (Amsterdam: Verh, Koninklijke akademie van Wetenschappen te Amsterdam) 3–33. Both my reconstruction and the conclusions drawn from it are based on C. Posy, "Epistemology, Ontology, and the Continuum" in E. Grosholz and H. Berger, *The Growth of Mathematical Knowledge* (Dordrecht: Kluwer, 2000) 199–219.

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5 Posy, "Epistemology, Ontology, and the Continuum" 202. Posy uses an analysis of the Hangman paradox to help articulate this notion.

6 Ibid. 206–07.

7 Heidegger, *Die Grundbegriffe der Antiken Philosophie*, in *Gesamtausgabe*, vol. 22 (Frankfurt am Main: Klostermann, 1993) 75–76.

8 Aristotle, *Physics*, Book IV, chapter 10, 218a 10–21.

9 Aristotle, *Physics*, Book V, chapter 3, 227a 10–15. Heidegger discusses this definition in *Plato's "Sophist,"* trans. R. Rojcewicz and A. Shuwer (Bloomington: Indiana UP, 1997) 77–83.

10 M. Dummett, "Is Time a Continuum of Instants?," *Philosophy* 75 (2000): 497–515.

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