on the fundamental consanguineal numbers and their structural basis[1]

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introduction

Inherent in the Western folk notion of consanguinity—and intrinsic to all anthropological specializations of this folk concept—are certain *numbers*. My aim is to explicate the logico-syntactical basis of these "fundamental consanguineal numbers" by rooting them in a suitable structural formula for the general consanguineal relation. A second objective is to suggest the fruitfulness of this structural exegesis, by showing that it (1) yields a rigorous formal grounding for a variety of consanguineal concepts of historical and anthropological importance, (2) provides a productive analytic framework for investigating the formal interconnections among such concepts, and (3) leads to precise and useful statements of these connections in closed algebraic form.

Various essentially numerical kinship notions, including "generation," "degree of consanguinity," and the like, have been accorded explicit cultural recognition for at least 2,000 years of Western history. Figuring repeatedly in the discourse of such preanthropological kinship specialists as clerics, jurists, physicians, logicians, philosophers, genealogists, and advisors to kings, such concepts had been evolving for centuries before Morgan wrote. Morgan himself, by virtue of his legal training, understood the main features of this ancient legacy of quantitative kinship concepts and explicitly discussed certain aspects of the relevant Western tradition (Morgan 1871: especially 10-11, 17-26, and Plates I-III).

However, few of Morgan's successors in the field of kinship studies have shared his rather keen interest in the numerical side of his subject. In particular, there has been no concerted effort to seek a thoroughgoing *formal* understanding of that "natural system, numerical in its character" (Morgan 1871:11) that consanguinity creates wherever life breeds life. Yet, in the case of man, it is precisely this inherently numerical "natural system" which gives rise—through its conscious recognition and manipulation by humans

A rigorously grounded symbolic representation of the general consanguineal relation (a) yields succinct definitions for a number of historically important quantitative measures of consanguinity and (b) facilitates an examination of the algebraic relationships among these measures. Several formal devices—including standardized exponent variables, zero power factors, and a new concatenation operator called the "geneaproduct"—are introduced for their anticipated general usefulness in kinship analysis as well as for their specific utility in the explication of consanguineal numbers.

everywhere—to most of what we professionally talk about in the field that Morgan pioneered. Considering the impressive vigor, the diversity of perspectives, and the sheer man-hours that anthropologists have invested in kinship investigations over the past century, it is surprising that the specifically metrical aspects of kinship connections still remain so largely neglected. One of my purposes here is to stimulate a reversal of this long-standing neglect.

consanguineal syntax

In order to achieve a satisfactory explication of consanguineal measures, our analysis must be rooted in a suitably fundamental definition of consanguineal connection. What is required is an accurate, succinct, and perspicuous formulation of the special mathematical "shape" that is common to all instances of interpersonal relatedness of the consanguineal kind. Ideally, the formulation should capture just those features of the concept that are essential to our purpose while ignoring all other features.

These essential features are purely logico-syntactical, or structural, for numerical kinship concepts depend in no essential way on the "content" or "substance" of the elementary links of which chains of kinship connection are composed. In other words, it is from structural facts alone that kinship numbers flow. This is fortunate, for it means that it is possible to avoid here that vexing quagmire of ambiguity and disagreement from which anthropologists apparently find it difficult to extricate themselves when they argue about the "nature" of kinship, or ask what kinship "is" (e.g., Needham 1960; Barnes 1961, 1964; Beattie 1964; Schneider 1964, 1965).

Clearly, the notion of consanguinity depends on—and even in some sense "reduces" to—the notion of *parenthood.* Equally clearly, however, there exists no substantive definition of the term 'parent' that has attained general acceptance in anthropology—or that is capable of embracing in fact the wide variety of ways that this word is actually used in our discipline. The relationships denoted may be biological, quasi-biological, or wholly nonbiological; factual, jural, or fictive; "sociological," "ethnogenealogical," or "emic." Patently, the diversity of content among such relations is enormous. On the one hand, then, we have a congeries of highly varied empirical meanings that anthropologists and others have given to the term 'parental relationship,' and on the other hand we have an at least equally diverse set of meanings that has been given to the term 'consanguineal relationship.' Aside from mere faith that there must be some underlying unity and order, what in fact exists here that is reasonably constant, or that at least presents itself as sufficiently firm and intersubjective to serve as an adequate basis for the analysis to follow?

The answer lies, I believe, in (1) the largely unremarked constancies of logical syntax which characterize the otherwise rather heterogeneous totality of empirical relations that have been seen as somehow parental, and (2) a similar logico-syntactical constancy of the semantic linkage between any *given* notion of parenthood and its strictly *correlative* notion of consanguinity. In other words, the observed substantive variability of parental relationships does not carry over to the logical syntax of the relations so described. Further, if it be agreed that each of the individual links in a given chain of interpersonal connections does indeed qualify as somehow "parental" or "filial," then any remaining question as to whether the connection as a whole is "consanguineal" is solely a matter of how these individual links are arranged or ordered in the chain. The very feasibility of the task undertaken in this paper depends on the existence of these constancies in the syntax

of parental notions and in the form of the syntactical linkage between parenthood and consanguinity.

With regard to the concept of parenthood, all that needs to be assumed for our present purpose is: (1) that no person may be a parent of himself; (2) that no two distinct persons may be mutually parental, each to the other; (3) that if a given person is a parent of a parent (i.e., 'grandparent') of another, it is neither assured nor precluded that he is also a parent of the latter; and (4) that any given person may be a parent of more than one person, and vice versa. In short, it is assumed that the dyadic relation 'is a parent of' is *irreflexive, antisymmetric, nontransitive,* and *many-many*[2]. (Further postulates for the general class of parental and "parentaloid" relations have been discussed elsewhere; Atkins 1972.)

In order now to provide a brief intuitive basis for the development immediately to follow, let the statement

[1] 'x is a consanguine of z'

be understood to mean:

[2] 'Person x is an ith-remove descendant of some person y,
who in turn is a jth-remove ancestor of person z'

where the letters i and j represent nonnegative integers, and where the phrases 'is an ith-remove descendant of' and 'is a jth-remove ancestor of' each reduce to the phrase 'is identical to' when $i = 0$ or $j = 0$, respectively. In the limiting case when both $i = 0$ and $j = 0$, the original assertion 'x is a consanguine of z' may itself be replaced by the identity relation, yielding 'x is identical to z.' In all other cases, statement [1] may be expanded to read 'x is a *proper* consanguine of z.' Finally, in order to prohibit genealogical tracings that involve "doubling back," let it be stipulated that person y in [2] shall be a "nearest" common ancestor (not necessarily proper) of x and z, in the sense that [2] would become false if y were replaced by any proper descendant of y.

Now let the symbol P represent any dyadic, irreflexive, antisymmetric, nontransitive, and many-many relation for which the informal gloss 'is a parent of' is anthropologically acceptable, and let the symbol $\mathfrak{q}$ ("reverse-P") represent its converse, 'is a child of' (i.e., $\mathfrak{q} = \breve{\mathrm{P}}$, or P^{-1}, with the -1 exponent signifying a converse, not an inverse). The understandings established in the preceding paragraph can be shown to be consistent with interpreting the expressions in [1] and [2] above as informal verbal glosses for the following rigorously definable expression:

[3] $$x\ \mathfrak{q}^i\mathrm{P}^j\ z$$

where $\mathfrak{q}^i\mathrm{P}^j = \mathfrak{q}\mathfrak{q}\mathfrak{q}\ldots\mathfrak{q}\mathrm{P}\ldots\mathrm{PPP}$ (that is, i iterations of $\mathfrak{q}$, followed by j iterations of P). Immediate juxtapositions of relation-symbols, as in $\mathfrak{q}\mathfrak{q}\mathfrak{q}$ or $\mathfrak{q}\mathrm{P}$, are to be understood here as representing an operation of concatenation called the "geneaproduct," to be defined presently.

Since the geneaproduct is similar to, but not identical to, the conventional *relative product* of the logic of relations, let us consider the latter first. The traditional relative product, R|S, of any two dyadic relations R and S may be defined as follows:

[4] $$\mathrm{R}|\mathrm{S} =_{\mathrm{df}} \hat{x}\hat{z}\ [\,(\exists y) : x\mathrm{R}y\ \&\ y\mathrm{S}z\,]$$

—that is, 'R|S' labels the relation that links all ordered pairs x,z such that there exists (for

each such pair) at least one y, such that xRy and ySz. (For a brief guide to logical notation, see Church 1955.)

By contrast, for any n dyadic genearelations $R_1, R_2, \ldots, R_n$, $n \geqslant 2$, the *geneaproduct* $R_1, R_2 \ldots R_n$ is specified as the following modification of the traditional relative product:

[5] $$R_1 R_2 \ldots R_n = R_1 \mid R_2 \mid \ldots \mid R_n \;\&\; \{(R_i \Rightarrow J) \Rightarrow [(R_i \mid \ldots \mid R_k) \Rightarrow J]\}$$

where $\Rightarrow$ means 'implies' ('if . . . then'), J is the diversity relation ($\neq$), and $1 \leqslant i \leqslant k \leqslant n$. The effect of the clause enclosed in braces in [5] is that J, the normally nontransitive diversity relation, is made *transitive* within all relational strings employing the geneaproduct operator. *What this stipulation assures is that geneatracings involving "doubling back" are excluded as empirical interpretations of geneaproduct strings.*

The practical value of this distinguishing feature of the geneaproduct may be illustrated by considering the difference in denotation between the expression

[6] $$a\ q^3 P^3\ b \qquad (= a\ qqqPPP\ b)$$

and the expression

[7] $$a\ q^{(3)} \mid P^{(3)}\ b \qquad (= a\ q \mid q \mid q \mid P \mid P \mid P\ b)$$

(where parenthesized exponents distinguish traditional powers from geneaproduct powers, a convention I owe to Richard J. Greechie and Martin Ottenheimer, personal communication 1972). The latter expression, employing the traditional product, may represent any of the eight different genealogical tracings that are shown in Figure 1. What is asserted is that a is the same person as b, or a sibling of b, or a first cousin of b, or a second cousin of b. By contrast, the expression $a\ q^3 P^3\ b$ in [6] permits but one interpretation: namely, that a is a second cousin of b (i.e., situation H in Figure 1).

The possibility is not excluded that a and b are linked by other geneapaths—perhaps even paths that are shorter than that described by $q^3 P^3$. Rather, what [6] guarantees, but [7] cannot assure, is the existence of an a-to-b geneapath of the form required to establish second-cousinhood. In order to guarantee this meaning through use of the traditional product, it would be necessary to write:

[8] $$q^{(3)} \mid P^{(3)} \;\&\; \sim (q^{(2)} \mid P^{(2)}) \;\&\; \sim (q \mid P) \;\&\; \sim I$$

where $\sim$ means 'not,' and where I is the identity relation ($=\sim J$). Similar prolixity would arise in describing almost any determinate collateral consanguineal relationship if we were to restrict ourselves to the '|' operator in generating the definitions required. The geneaproduct avoids these difficulties. Moreover, it does not sacrifice efficiency in the description of nondeterminate relationships in so doing. (For example, in the unlikely case that a need should arise to describe the relation of [7] above, it may be represented as $q^j P^j$, with the restriction $j \leqslant 3$.)

Because of the special "J-clause" in the definition of the geneaproduct, products formed with its aid are not associative (in the technical mathematical sense); it follows that the insertion of parentheses in any geneaproduct containing three or more factors—unless the parentheses serve merely to mark the scope of an exponent—may

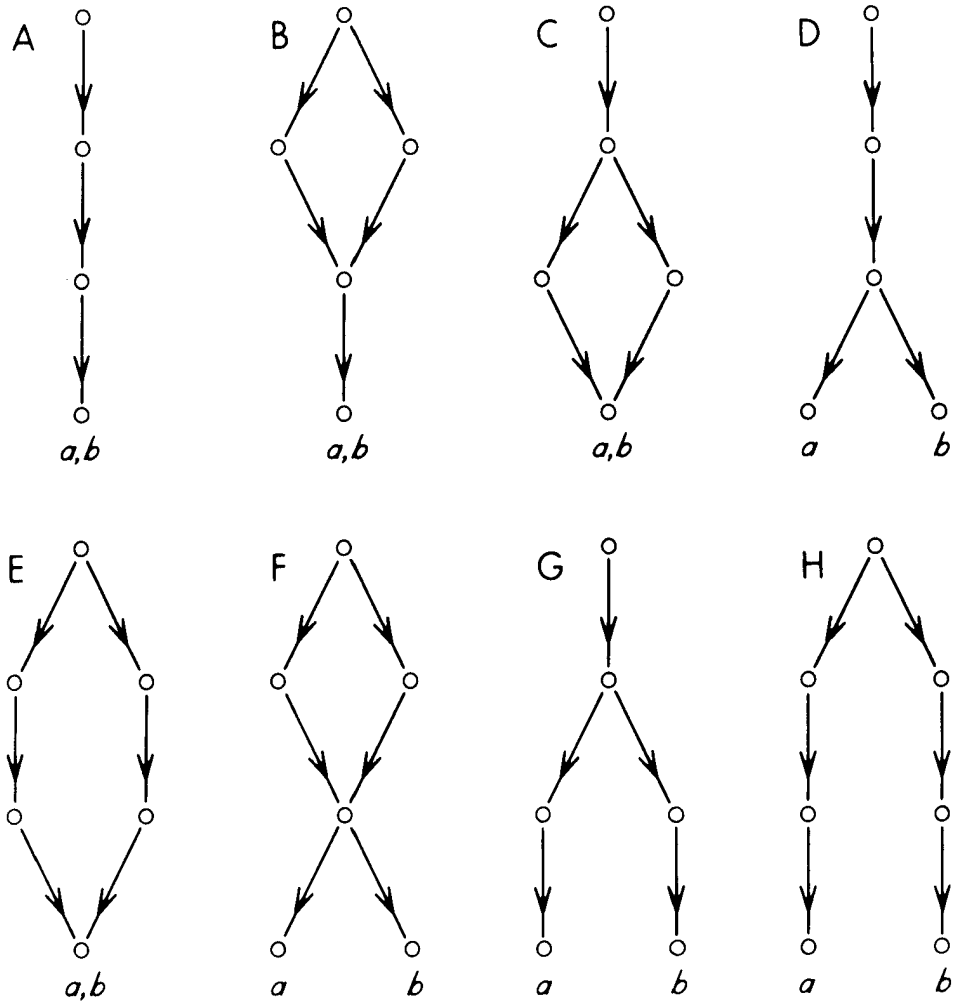


Figure 1. Eight genealogical tracings compatible with expression [7] in text. Nodes represent persons. Arrows may be read 'is a parent of' from tail to head, 'is a child of' from head to tail.

change the denotation of the expression. In effect, this is an enrichment of symbolic resources, since in general $(R_1 R_2)R_3$, $R_1(R_2 R_3)$, and $R_1 R_2 R_3$ will carry different meanings and hence are available to discriminate efficiently among the latter when such discriminations are needed.

In order to clarify further the meaning of the fundamental expression $\mathfrak{q}^i\mathfrak{p}^j$ for definite values of i and j, let now an *Egocentric geneaclass* be defined as any set or class of all persons who stand in a given (specified) relation to a person called Ego. The logical structure of this notion is that of a heterogeneous descriptional function (Reichenbach 1947:313). The compound incomplete symbol

[9] $\quad / \quad /\epsilon \qquad$ (Egocentric geneaclass marker)

is used here to represent this structure, with the epsilon (ϵ) standing for Ego. The marker is completed by inserting a relation-symbol (or a relation-defining string of symbols)

between its opening solidus and its closing solidus. Thus, if the relation-symbol P is inserted, the result is

[10] $/P/\epsilon$

with the meaning 'the set of all persons such that each stands in the relation *is a parent of* to Ego'—which reduces, without semantic loss, to simply 'the parents of Ego.' The effect of the marker, it will be noted, is summative.

Similarly, the string

[11] $/\mathfrak{q}P/\epsilon$

may be glossed 'the siblings of Ego' (i.e., 'the set of all persons such that each is a[nother] child of at least one parent of Ego').

Of special importance is the expression

[12] $/\mathfrak{q}^i P^j/\epsilon$

which denotes the class consisting of all consanguines of Ego. If now we take the form of this last designation as the structural model for every more specific consanguineal geneaclass, the sibling-class designation in [11] above may be rewritten as

[13] $/\mathfrak{q}^1 P^1/\epsilon$

(with 1's replacing the *i* and the *j* of [12]), since the first power of any relation is the relation itself (i.e., $\mathfrak{q}^1 = \mathfrak{q}$, and $P^1 = P$). Similarly, expression [10] may be rewritten as

[14] $/\mathfrak{q}^0 P^1/\epsilon$

since (1) the *zero-th* power of any relation is the *identity* relation, I, and (2) $IP = P$. (Persons identical to the parents of Ego are, of necessity, simply the parents of Ego.) From one point of view, the factor $\mathfrak{q}^0$ in [14] is entirely redundant, of course, and therefore expendable; its removal would produce the simplification already given in [10]. But more significantly, this factor may be seen as serving a vital "place-holding" function that establishes the status of the relation $\mathfrak{q}^0 P^1$ as a concrete realization of the more general relation $\mathfrak{q}^i P^j$. Identity factors created by zero exponents in geneaproducts are essential to the notation developed here, for they permit the latter to capitalize upon the powerful mathematical device of fully generalized exponentiation (i.e., the use of *variables* in exponent position).

We see, then, that relations such as $\mathfrak{q}P$ and P may be exhibited as determinate subrelations of the general consanguineal relation $\mathfrak{q}^i P^j$—obtained, in each case, by supplying specific numerical values for the variables *i* and *j*. The kinds of Egocentric designations that result are further illustrated in Table 1, which lists and glosses all determinate Egocentric geneaclasses for which the values of *i* and *j* do not exceed 3. The cutoff point of 3 is arbitrary, as no upper limits for *i* and *j* are specified in the formulation of the general consanguineal relation that is developed here. In other words, the outer boundary of the class of Ego's consanguines is left entirely indeterminate. On the other hand, a definite lower bound or "inner closure" to the notion of consanguinity (structurally viewed) *is* provided—in the person of Ego, who is the sole member of the class $/\mathfrak{q}^0 P^0/\epsilon$.

Certain pairs of geneaclasses listed in Table 1 are "reciprocals." Two Egocentric geneaclasses are mutually reciprocal when their defining relations are *mutually conversive*, and such a class is self-reciprocal when its defining relation is *self-conversive*. As with the traditional relative product of two or more relations, the *converse of a geneaproduct* is obtained by reversing the order of the factors and taking the converse of each: i.e.,

Table 1. Illustrative *i,j*-determinate consanguineal subclasses.

Designation	English gloss
$/\mathrm{q}^0\mathrm{P}^0/\epsilon$	Ego
$/\mathrm{q}^0\mathrm{P}^1/\epsilon$	the parents of Ego
$/\mathrm{q}^0\mathrm{P}^2/\epsilon$	the grandparents of Ego
$/\mathrm{q}^0\mathrm{P}^3/\epsilon$	the great-grandparents of Ego
$/\mathrm{q}^1\mathrm{P}^0/\epsilon$	the children of Ego
$/\mathrm{q}^1\mathrm{P}^1/\epsilon$	the siblings of Ego (half and full)
$/\mathrm{q}^1\mathrm{P}^2/\epsilon$	the consanguineal uncles and aunts of Ego
$/\mathrm{q}^1\mathrm{P}^3/\epsilon$	the consanguineal great- or granduncles/-aunts of Ego
$/\mathrm{q}^2\mathrm{P}^0/\epsilon$	the grandchildren of Ego
$/\mathrm{q}^2\mathrm{P}^1/\epsilon$	the consanguineal nephews and nieces of Ego
$/\mathrm{q}^2\mathrm{P}^2/\epsilon$	the first cousins of Ego
$/\mathrm{q}^2\mathrm{P}^3/\epsilon$	the senior-generation first cousins, once removed, of Ego
$/\mathrm{q}^3\mathrm{P}^0/\epsilon$	the great-grandchildren of Ego
$/\mathrm{q}^3\mathrm{P}^1/\epsilon$	the consanguineal great- or grandnephews/-nieces of Ego
$/\mathrm{q}^3\mathrm{P}^2/\epsilon$	the junior-generation first cousins, once removed, of Ego
$/\mathrm{q}^3\mathrm{P}^3/\epsilon$	the second cousins of Ego (narrowest sense)

$(RST)^{-1} = T^{-1}S^{-1}R^{-1}$, for any relations R,S,T. Given the mirror-image relationship between the physical shapes of the symbols P and q, the converse of a consanguineal geneaproduct such as qqqP is simply its physical right-to-left mirror reversal qPPP. Hence, $(\mathrm{q}^3\mathrm{P}^1)^{-1} = \mathrm{q}^1\mathrm{P}^3$, and, with full generality, we have:

[15] $$(\mathrm{q}^i\mathrm{P}^j)^{-1} = \mathrm{q}^j\mathrm{P}^i$$

This is a very convenient property.

simple *i,j* functions

Our stated quest is for certa consanguineal numbers" of anthropological and historical importance. The proposed structural formula for the general consanguineal relation contains two numerical variables that take integers as their values. Are these, then, the numbers that we seek?

Unfortunately, the exponent variables *i* and *j* in the designation $/\mathrm{q}^i\mathrm{P}^j/\epsilon$ are virtually meaningless from an intuitive or conceptual point of view, when either of them is considered alone. (A minor exception for *j* is discussed shortly.) The first of these two variables, when evaluated, is simply a raw count of the number of child-to-parent links encountered in tracing a consanguineal connection from alter to Ego, while the second is a similar count of parent-to-child links. Considered in isolation from the other, neither number is very informative. There may even be pan-human agreement on this point, for none of the basic semantic contrasts so far identified in the world's known kinship terminologies can be equated with *i* or with *j*.

Nevertheless, there is a definite sense in which these exponent variables may be regarded as truly fundamental. When they are considered *jointly*—that is, when we examine some of the simplest possible algebraic functions in which i and j are *combined* in appropriate ways—they immediately yield a whole series of measures that have long been found meaningful in kinship description and analysis. Each of the algebraic functions in question employs i and j as *fundamental* measurements (in the strict, classic sense of Campbell 1920, 1928) to formulate a *derived* or *derivative* measurement that may be performed on any consanguineal chain.

The derived variables to be discussed here all take common integers as their values. Moreover, like i and j themselves, they are numerical quantitative variables in the strictest sense, for they use numerals—or occasionally, in the glosses, such number-*words* as 'once' or 'second' or 'tertiary'—in ways that are both "natural" and mathematically appropriate. Indeed, the scales marked off are in most cases not only well-ordered but in fact of the "strong" or ratio type (i.e., stepped in equal intervals from a nonarbitrary zero-point). Several families of such measures deserve consideration.

generation and its derivatives Anthropologists conventionally number the various distinct generations of Ego's consanguines Egocentrically, according to the scheme . . .,−3,−2,−1 (for "junior" or "descending" generations), 0 ("own" or "same" generation), +1,+2,+3, . . . ("senior" or "ascending" generations). Let the measure that produces these numbers as its values be represented by the letter γ (gamma). For all members of the class $/\mathfrak{q}^i\mathfrak{p}^j/\epsilon$, we have:

[16] $$\gamma = j - i \qquad \text{(Generation)}$$

This definition yields the conventional numbering scheme, assigning each "alter" among the consanguines of Ego to a specific generation (*vis-à-vis* Ego) on the basis of a particular geneapath of the form $\mathfrak{q}^i\mathfrak{p}^j$ that leads from alter (at the left) to Ego (at the right). If a relationship to Ego can be traced from a given alter through more than one geneapath, and if these paths involve different values for i,j, it is possible that [16] may assign alter to two or more different generations *vis-à-vis* Ego. Such assignments are therefore path-specific as well as Egocentric. (For similar reasons, virtually all of the measures discussed in this paper are path-specific.) Note that the generation of *Ego, vis-à-vis alter*, is given by $-\gamma$.

Among the more important γ-derivative variables are the following: (1) a measure that is best called *generational sign*, which yields the three-way contrast of "senior" vs. "own" vs. "junior" generations, without further subdivision of the first and last categories; (2) a measure to be referred to here as *generational removal*, in which generations are numbered according to their respective remoteness from Ego's own, without regard for whether they are "above" Ego or "below" him; (3) the contrast between *same* and *different* generations (i.e., between Ego's own generation and all others); and (4) the contrast between *odd* and *even* (i.e., alternating) generations. These derivative generation measures may be defined formally as follows:

[17] $$\sigma = \text{sgn}(j - i) \qquad \text{(Generational sign)}$$

[18] $$\delta = |j - i| \qquad \text{(Generational removal)}$$

[19] $$\text{sgn}\delta = \text{sgn}|j - i| \qquad \text{(Same/different contrast)}$$

[20] $$\delta \,(\text{mod } 2) = |j - i| \,(\text{mod } 2) \qquad \text{(Even/odd contrast)}$$

The symbol 'sgn' represents the conventional *signum* function, which assumes the values

+1, 0, or −1 according to whether the number on which it operates is respectively positive, zero, or negative. The stroke enclosures ('| |') convey an injunction to take the absolute (sign-free) value of the number enclosed. The symbol 'mod' means *modulo*, and the function $\delta(\text{mod } 2)$ in [20] transforms the sequence 0,1,2,... (of values for δ) into the sequence 0,1,0,1,... (with 0 for even-numbered generations and 1 for the odd). The letter σ (sigma) is chosen to suggest 'sign,' and the letter δ (delta) is used to suggest 'difference' (sign-free). All four of these measures are "truncations" of the fully fledged generation concept and hence may be regarded as informationally *degenerate*, in a peculiarly appropriate application of this term.

Certain alternative representations for the measures σ, δ, and γ follow from their definitions and are often useful. In particular, these include:

[21] $$\sigma = \operatorname{sgn} \gamma$$

[22] $$\delta = |\gamma|$$

[23] $$\gamma = \sigma\delta$$

What these equalities emphasize is the fact that the signed number γ, representing

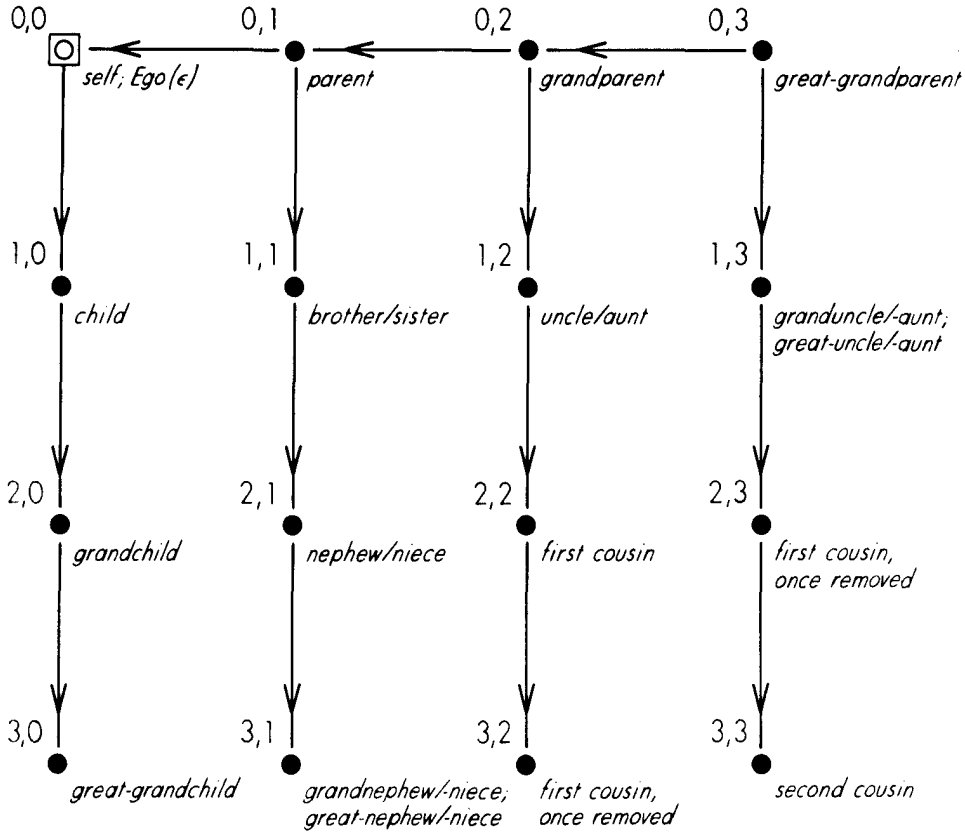


Figure 2. All unsexed consanguineal geneatypes through third canon degree, so arranged that values of *i* and *j* in [12] increase downward for *i* and to the right for *j*. Label of geneatype at each node shows specific pair of values for *i,j* and corresponding English kin term(s). Arrows may be interpreted as in Figure 1.

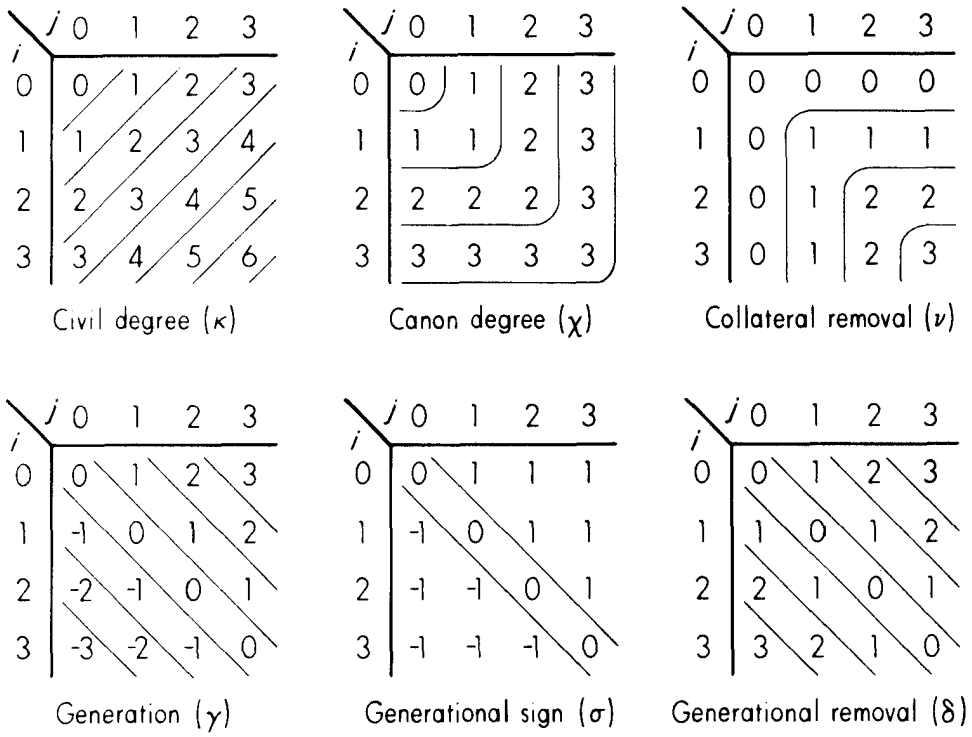


Figure 3. Numerical assignments of six consanguineal measures as functions of *i,j*. Each array is "overlay" on pattern established in Figure 2 and shows numerical value of indicated function for each node in that pattern.

generation, may be factored into its sign component (generational sign, or σ) and its unsigned numerical component (generational removal, or δ), and that jointly these two components exhaust the informational content of γ. The sigma and delta measures thus extract for separate attention two different aspects of the generation concept, which may then be put back together again, if desired, to reconstitute the original notion, as shown in [23].

The most important variables of this section are those just discussed: γ, σ, and δ. The patterns followed by these measures in assigning numbers to familiar classes of consanguines are shown in the lower half of Figure 3. Each of the labeled arrays in Figure 3 is arranged according to a diagrammatic scheme that is explained in Figure 2 (q.v.), with consanguines of Ego's own generation occurring on a diagonal line that runs from upper left to lower right within each individual matrix.

measures of collaterality According to material presented by Morgan (1871:16-26, Plate I), the Roman civilians adopted a system for numbering collateral, or "transverse," lines of consanguines in which the *first* collateral line consists of the descendants of *parents* of Ego (other than Ego himself and his own descendants), while the *second* line consists of the descendants of *grandparents* of Ego (excluding Ego's parents and their descendants), and so forth.

It is not entirely clear whether the proper ancestors of Ego, from which such lines descend, were themselves to be included in these lines—that is, it is uncertain whether

'descendants' should be interpreted as '*proper* descendants,' or be given an inclusive, more general sense that better suits a mathematical point of view. On the latter option, the exponent variable j in the expression $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$ yields precisely the required numbering scheme for all proper collaterals of Ego (i.e., consanguines for which $i \neq 0$ and $j \neq 0$). The proper ancestors of Ego (for whom $i = 0$) are included as respective "limiting case" or founding members of the lines so numbered, whereas Ego and his proper descendants (for whom $j=0$) form a separate "zero-th line." This taxonomy exhausts the class $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$. Should a more restrictive interpretation of "transverse lines" be favored, it may be stipulated that the j-scheme is to be limited to classifying only those consanguines for whom $j \neq 0$ (to exclude Ego's line) and/or $i \neq 0$ (to exclude Ego's proper ancestors as well).

The Roman collateral numbering system, just discussed, is identical to that employed in the "Ancient," or pre-Han period, Chinese system, in the representation offered by Han-yi Fêng (1937:162, Diagram I). By contrast, the same author's interpretation of the mode of differentiation and numbering of collaterals in the second-millenium or "Modern" Chinese system (1937:163, Diagram II) exactly corresponds to a measure more recently introduced by Lounsbury (1956:168n) under the name "degree of collaterality."

Since it seems preferable to reserve the term 'degree' for measures of overall consanguineal removal or distance from Ego, I shall refer to the Fêng-Lounsbury measure as *collateral removal*. With reference to the basic designation $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$, this measure may be defined quite simply, as follows:

[24] $\qquad \nu = \min i,j \qquad$ (Collateral removal)

where 'min i,j' means the smaller of the two numbers i and j. (By standard convention, either number qualifies as "the smaller" if $i = j$.) The letter nu (ν) is chosen to suggest the n in mi*n*imum.

The numerical assignments of ν are shown in the upper right matrix of Figure 3. All proper lineals of Ego, together with Ego himself, are numbered 0; all siblings of ancestors and descendants of siblings of Ego are numbered 1; and all consanguines who are called *cousin* in standard English are identified as of 2nd or greater collateral remove.

As Lounsbury notes, his measure is a generalization of the familiar lineal/collateral contrast among consanguines. The lineals of Ego, including Ego himself[3], are those members of $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$ for whom $\nu = 0$. A measure that cleanly divides all (proper) collaterals, not otherwise distinguished, from all lineals is the following:

[25] $\qquad \mathrm{sgn}\nu = \mathrm{sgn}(\min i,j) \qquad$ (Lineal/collateral contrast)

This expression defines what mathematicians call a "characteristic function," for it is a binary measure which assigns the numeral 1 to all objects within its domain that possess a given characteristic or attribute of interest, while assigning 0 to the objects that lack this characteristic. It is the purpose of such a function to select, from a given collection of entities, those which exhibit a certain trait or feature. In the present case, the trait of interest is collateral status *vis-à-vis* Ego. As required, the function sgnν yields +1 (i.e., 1) for collaterals, 0 for lineals. (Note that the value −1 does not arise as a possibility here, since the values of ν are exclusively nonnegative integers.)

Both ν and sgnν classify Ego's siblings as collateral relatives. This conflicts with a rather peculiar and apparently *ad hoc* contrast between "direct" and "collateral" kin that is due to Romney and D'Andrade (1964:153). The "collateral" category of these authors is identical to the usual concept (i.e., consanguines for whom $\nu \neq 0$), except that Ego's siblings are excluded. The latter are classified rather as "direct kin," who consist

otherwise of lineal consanguines in the sense defined above, save for the noninclusion of Ego. It follows that Ego's "collaterals"—in the special meaning used by Romney and D'Andrade—may be selected from $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$ by the function

[26] $\quad \mathrm{sgn}(\nu + \mathrm{sgn}\delta - 1) \qquad$ (Direct/collateral contrast)

where δ, as previously defined, is generational removal. This function assigns the value -1 to Ego. If the condition $i + j \neq 0$ is imposed to exclude Ego (in replication of his omission by Romney and D'Andrade), the possible values for [26] are thereby limited to 0 and 1. It then becomes a characteristic function in the same sense as $\mathrm{sgn}\nu$ in [25] above.

As a final case to be considered in this section, consider the English-language sequence *first cousin, second cousin, third cousin,* . . . , for which the corresponding values of ν are 2, 3, 4, . . . If *every* consanguine of Ego were to be regarded as a "kth cousin," where

[27] $\quad k = \nu - \mathrm{sgn}\nu \qquad$ (English cousin numbering)

then "proper" cousins would be numbered correctly as 1st, 2nd, 3rd, and so forth, whereas all consanguines who are *not* cousins in English would be designated as *zero-th* cousins.

civil degree and its homologues One historically important motive for measuring consanguineal chains is the need to resolve legal disputes concerning such matters as marriageability, inheritance, and succession. When nearness or remoteness of consanguineal relationship is seen as a relevant factor in judging who may marry, succeed, or inherit from whom, the question of "how near is near" arises—and an objective decision criterion may be sought.

Not surprisingly, the criterion adopted is often grounded in the metrical properties of consanguineal chains, which offer a natural structural basis for defining concepts of 'degree,' or overall closeness or remoteness, of consanguineal connection. Various nonnumerical genealogical facts may also be regarded as relevant, or course, but it is the purely numerical notion (to which the phrase 'degree of consanguinity' usually refers) that is of special interest here.

One of the oldest Western measures belonging in this category was formulated by the Roman civilians approximately 2,000 years ago and remains in use today in several parts of the world. Often called degree of consanguinity "in the civil mode" or "according to the Roman calculation," this measure will be referred to here as *civil degree*, or κ (kappa). (The choice of the symbol κ is essentially arbitrary[4].)

For convenience, let the various Egocentric "grades" of consanguinity from the Roman codes be represented by Arabic numerals (rather than I, II, III, etc.), and let zero—missing from Roman arithmetic—be added as a lower bound for κ, to represent Ego's own position. The possible values for the measure then become 0, 1, 2, . . . Given the general consanguineal class $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$, the defining function may be expressed as

[28] $\quad \kappa = i + j \qquad$ (Civil degree)

As this definition shows, civil degree is simply a count of the total number of $\mathfrak{q}$-links and P-links in any geneapath of the form $\mathfrak{q}^i\mathrm{P}^j$. It follows that parents and children are mutually 1st-degree or *primary* relatives, but that siblings—either half or full—are *secondary*, and that first cousins (of Ego's own generation) are *quaternary*. The pattern followed by the measure κ in assigning numbers to familiar classes of consanguines is shown in the first matrix of Figure 3.

Other known measures may be called "homologues" of civil degree if they are distinguishable historically from the Roman measure, but create exactly the same ordering of consanguines. Given the simplicity of the defining function of κ, the existence of such homologues is to be expected. For example, the same scheme was employed by the ancient Hebrews, according to Morgan (1871:25, citing "Selden's Uxor Hebraica, I.c.4."). Within anthropology, a measure identical to κ was invented by Greenberg (1949:64), who called it "a useful index of kinship distance." It later was reinvented by Fischer (1960:183), who cited ethnographic examples and described the concept as degree in an "ascent-descent" or "pure generational" system. It seems that neither author was aware of the precedent set by the Roman jurists, although Morgan had devoted a full-page diagram and several pages of discussion to civil degree (1871:16-26, and Plate I).

It appears that there has been an unbroken historical continuity from Roman times to the present day in the practical use of this ancient measure in human affairs, for several countries and districts with legal systems that were modeled initially on the Roman codes still retain the civil mode of calculating degree of consanguinity.

canon degree and its homologues According to some accounts (e.g., Burtsell 1913), it was Isadore of Seville in the seventh century A.D. who was responsible for replacing civil degree—in affairs of the western branch of the Christian Church—by a quite different mode of calculation. The new measure is said to have been based on *genicula*, an ancient system of body-joint-counting that was used by Germanic peoples (Henry 1967:194). Whatever its earlier history, the measure apparently began to supplant civil degree in clerical contexts—chiefly in determining the marriageability of Christian couples—some twelve or thirteen centuries ago (although it has not yet been adopted by Eastern Orthodox Christians). It later became incorporated into English common law. I shall use the term *canon degree* to refer to this concept, although 'common degree' would be equally appropriate. The symbol χ (chi) will serve as a briefer representation.

If *a* and *b* are lineals, the value of χ agrees with the value of κ and corresponds to the total length (in number of $\mathfrak{q}$-links or P-links) of the consanguineal chain by which *a* and *b* are connected. On the other hand, if they are collaterals, the value of χ is given by the length of the longer of the two subchains into which the position of their common ancestor divides the whole. Thus the definition in terms of $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$ is simply:

[29] $$\chi = \max i,j \qquad \text{(Canon degree)}$$

where '*max i,j*' is the standard mathematical instruction to take the larger of the two numbers *i,j* as the value of the function (and where, again, either number qualifies if $i = j$). The symbol χ is chosen to suggest the letter *x* in maximum.

The numerical assignments of the measure χ for consanguines are illustrated in Figure 3, central array, top row. By contrast with civil degree, the canon calculation regards siblings (half or full) as *primary* relatives, like parents and children, while same-generation first cousins are of the *2nd*, rather than the 4th, degree. The latter are aligned by χ with Ego's grandparents, uncles and aunts, nephews and nieces, and grandchildren.

Civil degree (κ) and canon degree agree in their numbering of Ego's lineals—and hence, at least implicitly, in their numbering of Ego himself (as zero). It follows that either χ or κ may be used to define the contrast between Ego and all *other* members of the class $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$. These other members are distinguished from Ego, in this paper, as Ego's *proper* consanguines (i.e., consanguines in the "normal" or ungeneralized anthropological sense). Either of the two signum functions $\mathrm{sgn}\chi$, $\mathrm{sgn}\kappa$ will serve equally well as a (binary)

characteristic function, performing the desired selection of proper consanguines by yielding 0 for Ego and 1 for all remaining members of the general consanguineal class. Hence we have:

[30] $\operatorname{sgn} \chi = \operatorname{sgn} (\max i, j)$ (Ego/proper contrast)

$= \operatorname{sgn} (i + j)$

$= \operatorname{sgn} \kappa$

Whether in a pastor's study or in a courtroom, an American today who encounters an explicitly formulated notion of consanguineal degree is likely to meet χ, the canon law/common law measure, rather than civil law κ. Moreover, informal research among Americans suggests that a concept resembling or identical to χ is rather close to the surface in the intuitive perceptions of many informants concerning the relative nearness or remoteness of consanguines. It is odd that anthropological literature is so curiously devoid of references to this old and still active element of what is, for most of us, our own cultural heritage. Once again, the exception is Morgan, who diagrammed and discussed the measure—and who also allowed his abiding admiration for the Roman jurists to prompt a strong negative opinion of it: "Our English ancestors," he wrote, "clearly made a mistake" in adopting the canon law computation, because "the civil law method . . . is the only one which is consistent and logical" (1871:26). As we shall see in later sections, the formal merits of canon degree are rather stronger than might be inferred from Morgan's judgment.

Earlier, Fêng's study of the Chinese system was cited in connection with the measures j and ν. An interesting homologue of canon degree occurs there also, for the Chinese classification of consanguines into "mourning grades" to determine the proper mourning period for a deceased relative (Fêng 1937:178-83) agrees precisely with the ordering provided by the measure χ.

a poset and two orthogonal bases

Let us now take stock. Ignoring several binary functions, a total of eight numerical variables have been considered thus far: $i, j, \gamma, \sigma, \delta, \nu, \kappa$, and χ. One of these variables, σ, is restricted to the values $-1,0,+1$. Another, γ, may take any integer as its value $(\ldots,-2,-1,0,+1,+2,\ldots)$. All of the rest range over the nonnegative integers $(0,1,2,\ldots)$.

Figure 4 shows illustrative calculations, for a particular geneapath, of the six nonbinary Greek-letter measures. The definitions and value ranges of these same six measures, together with several useful algebraic relations among them, are summarized in Table 2.

On the other hand, Figure 5 shows all eight nonbinary variables as a "partially ordered set," or *poset*, in which $\geqslant$ is the ordering relation. For any consanguineal chain segmented into two subchains of specific lengths i and j, we see that $\kappa \geqslant \chi$ (since the sum of two numbers cannot be smaller than its larger addend); that $\chi \geqslant i$, that $\chi \geqslant j$, and that $\chi \geqslant \delta$ (since the larger of two numbers cannot be smaller than the smaller, smaller than itself, nor smaller than the difference between the two); that $\delta \geqslant \gamma$ and that $\delta \geqslant \sigma$ (since no nonnegative number can be smaller than itself, smaller than its negative, or smaller than the signum of itself or its negative); and finally that $i \geqslant \nu$ and that $j \geqslant \nu$ (since neither of two numbers can be smaller than the smaller of these).

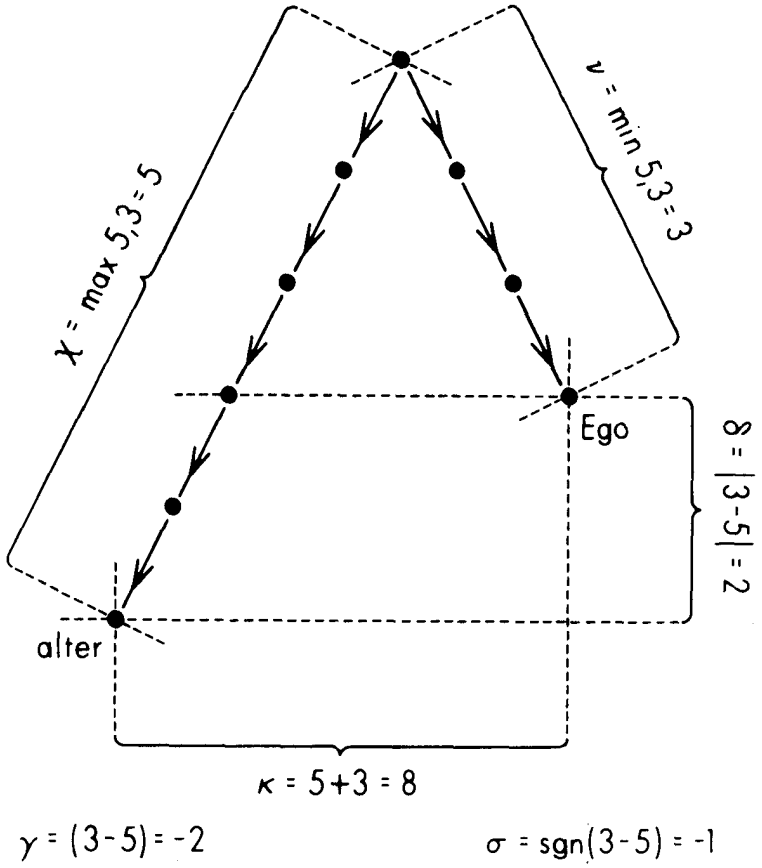


Figure 4. Example illustrating calculation of values of the six measures of Figure 3 for specific geneapath in which $i = 5, j = 3$.

It is obvious that a rather intimate mesh of algebraic interconnections obtains among the measures shown in Figure 5, as well as in the wider set consisting of all of the variables, binary or nonbinary, that have been discussed so far. *Is it possible—given the existence of these connections—to identify certain of the measures considered as mathematically more "basic," or more "fundamental," than the rest?*

It is useful to approach this question in terms of informational asymmetries. Consider, for example, the (Egocentric) distinction between odd and even generations, as given by the measure $\delta(\text{mod } 2) = 0,1,01,\ldots$ The relationship between this variable and the generational removal measure δ $(= 0,1,2,\ldots)$ is strictly one-way, for the transition to $\delta(\text{mod } 2)$ from δ involves a *loss of information*: we can obtain the former from the latter, but not vice versa. A similar informational loss occurs when generational removal, in its turn, is derived from generation through the relation $\delta = |\gamma|$, and occurs yet again when γ itself is found as the result of subtracting i from j. Does the existence of such sequences mean that the variables i and j are uniquely fundamental measures, with the others all merely derivative, or degenerate?

It must be granted that the order of exposition adopted in the preceding sections implies just such a conclusion. Nevertheless, it remains entirely possible that the set of

Table 2. Summary of definitions and algebraic relationships of principal measures derivable from i,j in formula $/q^i p^j/\epsilon$.

Measure	Definition in i,j	Value Range*	Useful Equalities
Generation (γ)	$\gamma = j - i$	$\ldots,-2,-1,$ $0,1,2,\ldots$	$\gamma = \sigma\delta$
Generational sign (σ)	$\sigma = \text{sgn}(j - i)$	$-1,0,1$	$\sigma = \text{sgn}\,\gamma$
Generational removal (δ)	$\delta = \lvert j - i \rvert$	$0,1,2,\ldots$	$\delta = \lvert\gamma\rvert$
			$\delta = \chi - \nu$
			$\delta = \kappa - 2\nu$
Collateral removal (ν)	$\nu = \min i,j$	$0,1,2,\ldots$	$\nu = \chi - \delta$
			$\nu = \kappa - \chi$
Canon degree (χ)	$\chi = \max i,j$	$0,1,2,\ldots$	$\chi = \delta + \nu$
			$\chi = \kappa - \nu$
Civil degree (κ)	$\kappa = i + j$	$0,1,2,\ldots$	$\kappa = \nu + \chi$
			$\kappa = \delta + 2\nu$

*Zero uniformly included for practical and theoretical completeness, whether traditional or not.

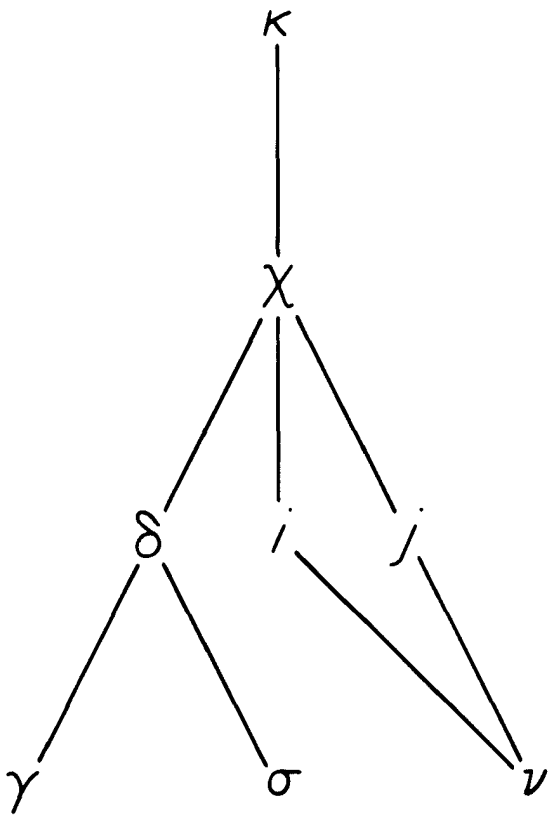


Figure 5. The variables of Figure 3, plus i and j, as an eight-element poset or partially ordered set. Connecting lines mean $\geq$, reading downward.

variables at hand (call it S) may contain *more* than one subset that is capable of serving as an "orthogonal basis"[5] for the set as a whole—i.e., as a basic subset, B, with the properties (1) that all variables in S but not in B can be derived from variables in B, and (2) that no variable in B can be derived from others in B. Hence it is possible that the subset consisting of i and j may not be uniquely fundamental.

However, the measures i and j clearly do form *one* such basis. The first requirement is satisfied by virtue of the fact that all of the consanguineal measures discussed so far may be defined as explicit functions of i,j. Table 2 shows such definitions for the major measures κ, χ, ν, γ, σ, δ. And definitions in i,j for the various binary variables considered—i.e., characteristic functions such as δ(mod 2) or sgnκ—are easily constructed. On the other hand, the second requirement is satisfied by virtue of the orthogonality or mutual independence of i and j themselves. That is, it is established by the fact that knowledge of the value of i (as descriptive of a given geneapath) provides no information whatever about the value of j (for that geneapath), and vice versa.

But is there another such basis? The poset diagram of Figure 5 is helpful here. Since inequality is inconsistent with independence, the various inequalities that are summarized in this diagram serve to disqualify many subsets of the variables that are shown in Figure 5. Among the possible subsets not eliminated from consideration in this way, there is in fact only one—besides the subset $\{i,j\}$, already discussed—that meets the criteria for an orthogonal basis, namely $\{\nu, \gamma\}$.

Orthogonality or mutual independence is satisfied by $\{\nu,\gamma\}$, since a consanguine in any generation (γ) may stand at any collateral remove (ν), and vice versa. In order to establish that the other requirement is satisfied as well, it must be shown that all of the remaining variables can be defined in terms of ν,γ alone. Since the various "characteristic functions" are unproblematic, it is sufficient to concentrate on the six nonbinary measures—which, aside from ν and γ themselves, are the following: i, j, δ, σ, χ, and κ.

The first of these may be expressed as

$$[31] \qquad i = \nu + \frac{|\gamma| - \gamma}{2}$$

and the second as

$$[32] \qquad j = \nu + \frac{|\gamma| + \gamma}{2}$$

As required, the only two variables occurring on the right-hand side of each of these formulae are ν and γ.

The third and fourth measures, δ and σ, simply extract different kinds of information from the generation measure, γ. In [21] and [22], they were shown to be expressible in terms of γ alone, as $|\gamma|$ and sgn γ, respectively.

The two remaining measures are canon degree (χ) and civil degree (κ). The former was defined as max i,j. Since the larger of any two numbers—here, i and j—is necessarily equal to the smaller (i.e., ν) plus their difference (i.e., δ), we have:

$$[33] \qquad \chi = \nu + \delta = \nu + |\gamma|$$

On the other hand, κ was defined as $i + j$. Since the sum of any two numbers necessarily is also the sum of the smaller (i.e., ν) and the larger (i.e., χ), it follows that:

$$[34] \qquad \kappa = \nu + \chi$$

—and in view of [33], this becomes:

[35] $$\kappa = \nu + (\nu + |\gamma|) = 2\nu + |\gamma|$$

The results shown in [33] and [35] are of exceptional interest. Since $\nu = |\nu|$ and $\delta = |\gamma|$, these results may be rewritten as follows:

[36] $$\chi = \nu + \delta = 1\,|\nu| + 1\,|\gamma|$$

[37] $$\kappa = 2\nu + \delta = 2\,|\nu| + 1\,|\gamma|$$

What is demonstrated here is that *both canon degree and civil degree, the two classical Western measures of degree of consanguinity, may be interpreted as simple sums of absolute coordinate values in an orthogonal two-dimensional space in which the coordinates are generation (γ) and collateral removal (ν). The sole difference between the two measures, when so represented, lies in the weighting factors by which the (absolute) coordinate values are multiplied.* Civil degree weights removal from Ego of the collateral sort twice as heavily as it does removal of the generational sort, whereas both of these two components of overall consanguineal removal are weighted equally by canon degree.

four additional degree measures

In order to provide a further test of the logical centrality of $\{i,j\}$ and $\{\nu,\gamma\}$ among kinship numbers, let us consider four additional measures of overall consanguineal distance or degree, selected from the literature. Each of these differs in some way from both κ and χ.

a further English measure? Petrovits (1919), in a source often cited in family law, wrote briefly of an "English mode of calculation" that is neither civil nor canon degree. As in all such computations, the numbers chosen to indicate relative remoteness increase systematically, outward from Ego, as a function of both generational removal and collateral removal. However, this new measure is unusual in assigning an implicit *one*, rather than zero, to the degree of remoteness of Ego from himself. This is inferred from the fact that measure classifies parents, siblings, and children all as *secondary* relatives—thus leaving only Ego to be primary.

The first array of Figure 6 shows the overall pattern of numerical assignments for this measure, along with the tentative label *"English" degree*, and the symbol E. For all collateral kin, the numerical assignments agree with those of civil degree (cf. Figure 3), but for all lineals they are higher, by one, than κ.

It follows that the simplest defining function for E is $\kappa - \mathrm{sgn}\nu + 1$. Translating this expression into absolute coordinate values for the two orthogonal reference systems $\{i,j\}$ and $\{\nu,\gamma\}$, and eliminating superfluous absolute value signs, the following pair of definitions results:

[38] $$E = i + j - \mathrm{sgn}(\min i,j) + 1 \qquad \text{("English" degree)}$$

[38′] $$= \delta + 2\nu - \mathrm{sgn}\nu + 1$$

Unfortunately, the origin and social contexts of usage for this "English mode of calculation" were not indicated by Petrovits, and further research on these points is needed. It would be particularly interesting to know more about the ethnohistorical interrelations of E, κ, and χ in England, with special attention to connections with the old *genicula* system, the Roman occupation, Anglican church history, and the development of English common law.

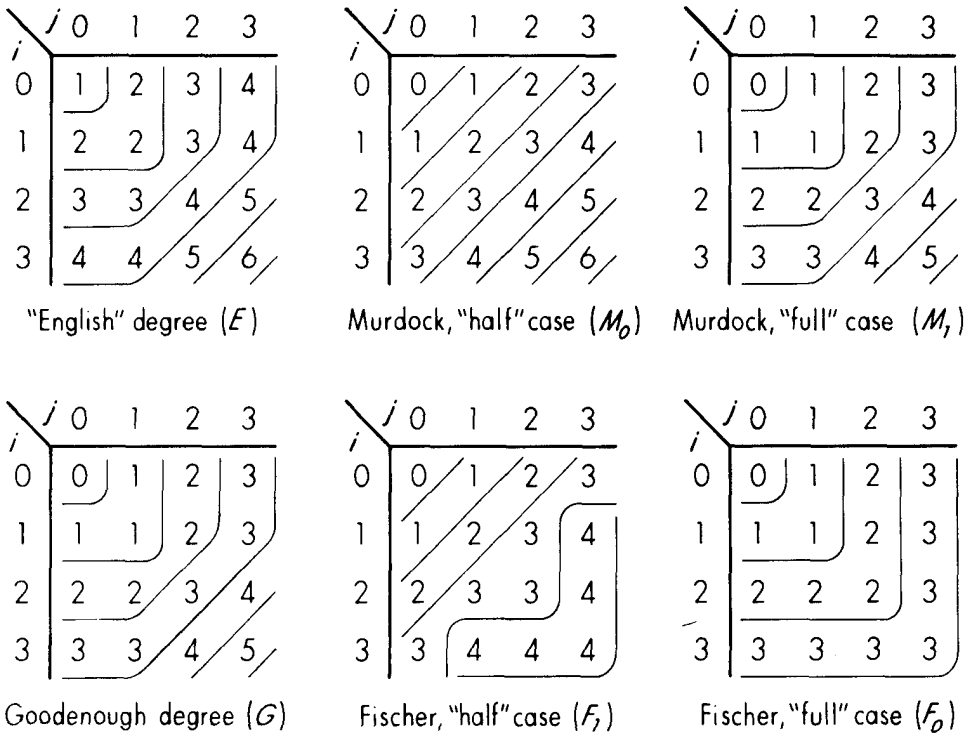


Figure 6. Numerical assignments for four additional consanguineal measures as functions of *i,j*. For explanation, see text.

Goodenough degree Closely related mathematically to E is the concept of "genealogical distance" that Goodenough (1965:273) introduced in his analysis of Yankee kinship terminology and that he revisited in his Morgan lectures (Goodenough 1970:93-95). Except for an erroneous comparison with a measure yet to be discussed, Goodenough makes no attempt to relate his distance concept to similar measures previously proposed in anthropology, nor does he mention any of the old Western modes of calculating such distances. It is therefore not entirely clear what advantages the new measure is thought to possess. (One of its attractions, apparently—at least in relation to the Yankee analysis—is the fact that the measure assigns to *siblings* the same distance number as it assigns to parents and children, thus "accounting for" the Yankee terms of Group 1 [Goodenough 1965:270]. But this feature is hardly novel, as it is also true of both χ and E. Moreover, in a qualified sense it is true as well of two measures yet to be discussed.)

The scheme of numerical assignments produced by G, or *Goodenough degree*, is illustrated in the lower left matrix of Figure 6. Comparison with the E matrix, just above it, shows that the two schemes are identical save for a scale factor of unity: that is, $G = E - 1$. Hence translation into the two familiar sets of orthogonal components gives the following analysis:

[39] $G = i + j - \text{sgn}(\min i,j)$ (Goodenough degree)

[39'] $= \delta + 2\nu - \text{sgn}\nu$

These expressions, in turn, can be shown to be equivalent to $\kappa - \text{sgn}\nu$. Hence Goodenough

degree matches κ, or civil degree, for all lineals, while for all collaterals it yields numbers that are smaller, by one, than κ.

Murdock degree A third measure of interest here appears to have been due initially to Radcliffe-Brown (1941:2) who spoke of various "orders" of Ego's relatives and was explicit in defining this term. The concept was adopted by Murdock in his *Social Structure* (1949:94-95), where it is called the "degree" of a kin type. Despite the historical priority in favor of Radcliffe-Brown, I shall refer to this measure as *Murdock degree*, or *M*, since the wide currency it has achieved in subsequent kinship literature seems to be due mainly to the influence of Murdock's book.

Murdock degree has been called the "nuclear family system" of distance measurement (Fischer 1960:183) in harmony with one of two different ways in which it was presented in *Social Structure*, and in consonance also with Murdock's later view that the concept might better have been called "familial removal," rather than "degree," on the ground that it was "not really intended as a genealogical measure" (personal communication 1961). Nevertheless, the notion originally was operationalized (and is most typically interpreted) as a length-of-chain or link-count measure—under the constraint that the chains be segmented according to the division scheme that is built into Murdockian kin-type notation. Thus 'MoBr' is to be regarded as a secondary relative, 'MoBrDa' as a tertiary relative, 'MoMoSoDa' as a quaternary relative, and so forth. Whatever the intention, it remains a fact that it is possible both to define and to calculate the measure *M* on a purely genealogical basis (which is to say that it "is"—among other things—a genealogical measure).

Fischer degree Analysis of *M* may be deferred and combined with the analysis of a fourth measure, due to Fischer, called "distance . . . in a multi-dimensional or . . . full collateral system" and recommended by Fischer as "the most suitable [basis] for selecting a core of close relatives for comparative kinship study" (Fischer 1960:187). This final measure to be discussed here will be referred to as *Fischer degree*, or *F*.

Unlike all previously treated variables, both *M* and *F* depend (in classifying consanguines) on information that goes beyond the data contained in single relational chains of the form $\mathfrak{q}^i\mathrm{P}^j$. That is, the numerical values of these measures cannot, in general, be determined by attending solely to facts about the linear structure of any *one* such chain. However, the additional data needed are just as strictly quantitative (and as fully genealogical) as the information required to compute the various "pure" measures of linear structure to which our attention so far has been restricted. What is different about *M* and *F* is that they both ask a rather special supplementary question about any collateral consanguineal relationship—one that is ignored in calculating all of the other measures considered here. The additional question is as follows: given that *a* and *b* are collateral kin, is the connection between them traced through a half-sibling bond or a full sibling bond? Both measures assign a larger number (i.e., a higher degree) to a connection of the first kind than to one of the second.

To represent this situation, let the binary variables *f* and *h* be introduced. For any relation of the form $\mathfrak{q}^i\mathrm{P}^j$, the variable *f* is to assume the values 1 or 0, according to whether the relation respectively can or cannot be traced through an instance of full siblinghood (*f* for 'full'). Similarly, the variable *h* is to assume the values 1 or 0, according to whether the tracing respectively does or does not encounter an instance of half-siblinghood (*h* for 'half'). The Murdock and Fischer measures may then be defined

in terms of elementary orthogonal components—insofar as this is possible in these cases—as follows:

[40] $M = i + j - f$ (Murdock degree)

[40′] $= \delta + 2\nu - f$

[41] $F = \max i,j + h$ (Fischer degree)

[41′] $= \delta + \nu + h$

In view of [28] and [29], we therefore have:

[42] $M = \kappa - f$

[43] $F = \chi + h$

The measures M and F are clearly different, but as these two equivalences show, they parallel one another in an interesting way. Murdock degree may be interpreted as a variant of *civil* degree (κ), in which the assigned value is *reduced* for relationships involving a *full* sibling connection. By contrast, Fischer degree may be interpreted as a variant of *canon* degree (χ), in which the value is *increased* for relationships involving a *half*-sibling connection.

The consanguineal numerical assignments for M and F are shown in Figure 6, with two different arrays for each measure. M_0 is the Murdock pattern for $f = 0$, whereas M_1 is the Murdock pattern for $f = 1$. On the other hand, F_0 and F_1 show the respective Fischer patterns for $h = 0$ and $h = 1$. It will be noted that the array labeled M_0 is identical to that for κ in Figure 3, and that the array for F_0 is identical to that for χ in Figure 3.

It is to the measure M that Goodenough relates his own measure, discussed earlier. In his most recent discussion of genealogical distance, he writes that it is common for kinship systems "to count the sibling relationship (having a parent in common) as a unit of lateral distance, making siblings and parents equally one unit away [from Ego]" (Goodenough 1970:93). He goes on to say that "Murdock did this in defining his 'primary' and 'secondary' relatives" and that "this indeed is how it works in English terminology, as I know it . . . ." (Goodenough 1970:93). This statement agrees well with his earlier Yankee analysis, where the phrase "with a genitor in common" (Goodenough 1965:273) parallels the phrase "having a parent in common," just cited. But the assertion is incorrect in its reference to Murdock, who is quite as clear in distinguishing full from half-siblings as respectively *primary* and *secondary* (Murdock 1949: 94-95) as Goodenough is clear in regarding them both as *primary* (Goodenough 1965:280). It is in fact Goodenough's self-consistency on this point—and hence his disagreement with Murdock in practice, if not in words—that allowed us to define G in [39] above entirely in terms of the linear-structure variables i and j, with no need for f or h.

cardinality Further commentary is needed on the fact that the Murdock and Fischer measures, alone among all those considered here, discriminate consanguineal degrees on the basis of the half-sibling/full sibling contrast. In my view, this contrast is most reasonably interpreted as a familiar but rather special example of the general phenomenon of single versus multiple paths of kinship connection, or what I have called the "cardinality" of kin relationships (Atkins 1959). The cardinality of any kinship relation K of specified form F, linking a particular pair of persons a and b, may be defined as the total number of specific routes or paths of genealogical connection that (1) start at a, (2) end at b, (3) are all of the specified form F, and (4) are distinct one from

another, in the sense that each path differs from every other in the identity of at least one linking kinsman. In general, an *a*-to-*b* connection of the form $\mathfrak{q}^i\mathrm{P}^j$ may be *cardinally plural* or *multiple* whenever $i + j \geqslant 2$.

Variations in genearelational cardinality is a large and complex subject, and it therefore cannot be treated fully here. However, a single collateral example may be cited for its pertinence to a deeper understanding of the Murdock and Fischer measures. Consider, for this purpose, the relation between first cousins; i.e., suppose that $a\ \mathfrak{q}^2\mathrm{P}^2\ b$. If we assume biparentalism and ignore connections that involve violations of the incest taboo, there may exist 1, 2, 3, or 4 distinct geneapaths, all of the form $\mathfrak{q}^2\mathrm{P}^2$, that link *a* and *b* in the required manner. Thus the cardinality of the relation $\mathfrak{q}^2\mathrm{P}^2$ for *a* and *b* ranges from 1 to 4, corresponding (in the case at hand) to the number of shared grandparents.

Person *a*, for example, may be the child of a paternal half-brother of the father of *b* and may also (simultaneously) be the child of a full sister of the mother of *b*. In this case, $a\ \mathfrak{q}^2\mathrm{P}^2\ b$ through *three* distinct routes (through his paternal grandfather, maternal grandfather, and maternal grandmother) and hence stands to *b* in the first-cousin relationship—i.e., the relation $\mathfrak{q}^2\mathrm{P}^2$—with a cardinality of 3. What, here, is the "degree" of *a vis-à-vis b*?

In effect, Murdock and Fischer would handle this example by saying that *a* occupies two different kin-type slots in relation to *b*, namely 'FaFaSoCh' and 'MoSiCh,' and that he therefore happens to be both a quaternary relative and a tertiary relative of *b* (for Murdock) or both tertiary and secondary (for Fischer). Despite the fact that the general form of all of the cited geneapaths remains constant at $\mathfrak{q}^2\mathrm{P}^2$, the *a*-*b* connection is to be seen not as *one* relationship (namely $\mathfrak{q}^2\mathrm{P}^2$), nor as *three* relationships (namely the three cited geneapaths of the form $\mathfrak{q}^2\mathrm{P}^2$), but as *two* relationships. Similarly, the "remoteness" of a first cousin who is both 'FaPaChCh' and 'MoPaChCh' (cardinality 2) or else both 'FaSbCh' and 'MoSbCh' (cardinality 4) is to be determined by *tracing through each parent separately*, so that the latter cousin, for example, is secondary for Fischer (or tertiary for Murdock) in two distinct "ways."

The important point here is that no rule is provided by either author for *combining* the two or more numbers arrived at, through different "ways" of tracing, in order to obtain a single degree assignment that would reflect the total concrete relationship. (As already indicated, the sole exception to this statement is the *M* and *F* treatment of the sibling relation, $\mathfrak{q}\mathrm{P}$, and paths traced therethrough.)

The relevance of the foregoing discussion to an assessment of the Murdock and Fischer measures is that it allows us to see just how partial—and inconsistent—is their recognition of the cardinality variable. So limited a recognition of cardinality leads to anomalies, especially when it is accorded through a measure that responds otherwise to exclusively path-structure criteria. Moreover, experimentation suggests that there is no simple, satisfactory way in which either *M* or *F* may be modified in order to reflect, systematically, the full range of cardinal variation.

In my view, the whole question of multiple kinship connections is an important one, and the structural and behavioral implications of this exceedingly pervasive feature of ethnogenealogical networks deserve much more study than they have received to date. But adequate characterization of the linear structure of individual geneapaths—"connections in series"—is one problem, and the measurement of the multiplicity of paths—"connections in parallel"—is quite another. Until we know more about how to cope with the latter problem adequately, it would seem wise to keep these two kinds of measurement clearly separated in our minds and in our measures. If there is, indeed, no

straightforward way to modify M or F so that *both* serial and parallel aspects of relational structure are well handled, it then becomes tempting to suggest that these measures either be (1) abandoned, or (2) divested of what little reflection of cardinality they now do incorporate. Following the latter course would not, of course, create much that is "new." For without the components f and h, the measures M and F are simply 2,000-year-old κ and 1,300-year-old χ, respectively.

the six degree measures compared In the foregoing paragraphs, four measures of overall consanguineal removal from Ego—"degree" of consanguinity—have been reviewed for comparison with civil degree (κ) and canon degree (χ), treated earlier. Each has been expressed, insofar as possible, in terms of the two orthogonal consanguineal bases $\{i,j\}$ and $\{\nu,\gamma\}$. The ν,δ or $|\nu|,|\gamma|$ reductions for all four measures considered in this section, plus κ and χ, are reproduced in Table 3 (q.v.). This table permits all six measures of degree of consanguinity to be compared directly, on the $\{\nu,\gamma\}$ basis.

It will be noted from Table 3 that *the weighting attached to generational removal (δ) is identical in all six cases.* Indeed, except for the scale-shifting constant, +1, in E and the cardinality components in M and F, the six measures differ from one another only in their treatment of *collateral* removal (ν). The ν-weightings range from 1 (for χ and F) to 2 (for κ and M), with the function $2\nu - \text{sgn}\nu$ (in E and G) producing values that are intermediate, when different. (Curiously, this "intermediate" weighting, $2\nu - \text{sgn}\nu$, may be expressed alternatively as $\nu + k$, where k is the English cousin-numbering scheme defined earlier in [27].)

The definitions in Table 3 are listed in an order determined by the values the measures assume when $\delta = 0$, as shown in the final column. Except for the anomalous position of F when $h = 1$, each measure yields values as great or greater than all measures listed below it. The ordering is therefore partial (see Figure 7A). However, if the measure F is split into F_0 and F_1 (as it was in Figure 6) and if M is split similarly, the resulting eight functions are no longer merely a poset; they map onto a (five-node) *complete* ordering, which is shown in Figure 7B.

Table 3. Generational, collateral, and other components in six measures of overall consanguineal distance or degree.

Measure		Generational Component	Collateral Component	Other Components	Values when δ=0
E	=	δ	$+ 2\nu - \text{sgn}\nu$	+1	1,2,4,6,...
κ	=	δ	$+ 2\nu$		0,2,4,6,...
M	=	δ	$+ 2\nu$	$-f$	0,2,4,6,... / 0,1,3,5,...
G	=	δ	$+ 2\nu - \text{sgn}\nu$		0,1,3,5,...
F	=	δ	$+ 1\nu$	$+h$	0,2,3,4,... / 0,1,2,3,...
χ	=	δ	$+ 1\nu$		0,1,2,3,...

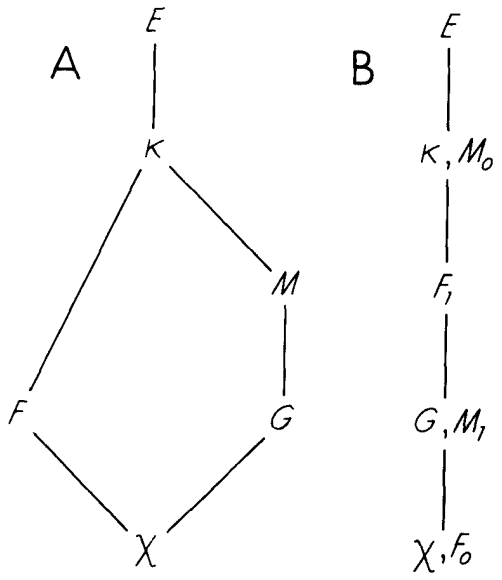


Figure 7. Orderings by magnitude of six measures of degree of consanguinity. A: the basic poset. B: complete ordering obtained by splitting *M* and *F* as in Figure 6.

Combining this complete ordering of 7B with the poset of Figure 4, and adding *f*, *h*, sgnν, sgnδ, and $-\gamma$, we obtain the complex sixteen-node ordering that is shown in Figure 8.

an application: non-*i,j*-determinate subclasses of consanguines

Let us now use some of the results at hand by applying them to the problem of specifying, in an efficient manner, various broad subclasses of Ego's consanguines—classes smaller than $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$ itself, yet more comprehensive than the *i,j*-determinate classes of Table 1. There are rich possibilities for subdividing $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$ on the basis of cardinality, ordinality (e.g., birth order and relative age), and sex or other geneaproperties of individuals, but these possibilities will continue to be ignored in favor of selections based solely on $\mathfrak{q}$- and P-counting, our constant theme. Here, however, the classes to be defined are those for which *i,j* values are not fully *determined*, as in Table 1, but rather merely *constrained*[6].

The fully *i,j*-determinate subclasses of consanguines may be called "basal" classes. If *q* and *r* are taken as the respective upper limits of the value ranges of *i* and *j* (0, 1, . . . , *q*; 0, 1, . . ., *r*), there are $k = (q + 1)(r + 1)$ basal classes. If these *k* classes are then taken as the elements of a set *C*, there are $2^k - 2 - k$ distinct subsets of *C* that (1) consist of two or more of the basal building blocks, and (2) are smaller than $/\mathfrak{q}^i\mathrm{P}^j/\epsilon$ itself. Clearly, however, a great many of these subsets will be utterly devoid of practical interest, by virtue of consisting of collections of Ego's kin that the people of no society anywhere would treat as distinctive, unitary groupings (e.g., the subset consisting exclusively of $/\mathfrak{q}^{1\,2}\mathrm{P}^7/\epsilon$ and $/\mathfrak{q}^3\mathrm{P}^{1\,3\,2}/\epsilon$). But also included among the proper plural subsets of *C* are many groupings that are so regular in their basal composition as to have enjoyed repeated cognitive, terminological, or behavioral recognition by anthropologists, by their informants, and by other kinship specialists.

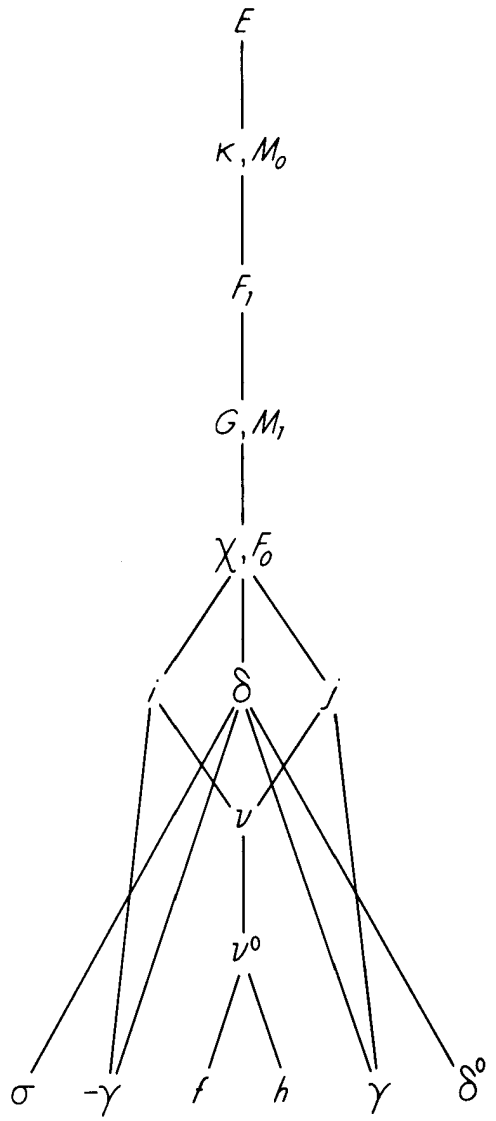


Figure 8. Poset of fifteen selected consanguineal measures, plus M and F split as in Figure 6. Symbols defined in Tables 2, 3 and text. Ordering of sixteen nodes shown here summarizes eighty-four pairwise inequalities of magnitude, for fixed values of i,j.

And it is precisely here—that is, in defining broad but "meaningful" selections from $/q^iP^j/\epsilon$—that the various quantitative measures under discussion in this paper come clearly into their own. In order for a nonelemental subset of C to be capable of exciting human interest and widely shared intellectual comprehension, it must exhibit a marked regularity of composition of the sort that results from imposing *simple* constraints on i and j. This is just the kind of constraint that measures such as χ or $\mathrm{sgn}\nu$ or $\delta(\mathrm{mod}\ 2)$ impose. To stipulate that $\delta = 0$, for example, or $\delta \leqslant 2$, is to allow both i and j to range freely over their respective value ranges (thereby assuring nonelementality or "breadth" to the class so defined), while at the same time constraining the *joint* variation of i and j in such a way that their absolute difference remains constant (or, in the second case, does not

exceed a certain number). These are constraints of a very simple type, and all of the Greek-letter functions discussed above operate in just this way.

It follows that one method of formally specifying "meaningful" consanguineal subclasses would be to supplement the designation $/\mathrm{q}^{i}\mathrm{P}^{j}/\epsilon$ by the addition of appropriate side-restrictions, involving the Greek variables or their equivalents in *i,j*. For example, the class consisting of 'all second-ascending-generation consanguines of Ego' might be denoted by

[44] $$/\mathrm{q}^{i}\mathrm{P}^{j}/\epsilon, \quad \gamma = 2$$

or

[45] $$/\mathrm{q}^{i}\mathrm{P}^{j}/\epsilon, \quad (j - i) = 2$$

Although adequate, designations of this general type are neither elegant nor optimally efficient. On both counts, the equivalent but briefer designation

[46] $$/\mathrm{q}^{i}\mathrm{P}^{\mu+2}/\epsilon$$

is more appealing. The contrast in this particular case raises a wider question: does a *general strategy* exist for constructing optimally compact, efficient designations of important nonbasal subclasses of consanguines?

It appears so. The strategy depends on the existence of some rather remarkable equivalences that were implicit earlier—especially in Table 2—but that deserve now to be stated explicitly. It turns out that the general consanguineal relation $\mathrm{q}^{i}\mathrm{P}^{j}$ may be rewritten in several equivalent ways that flow directly from our findings concerning fundamental consanguineal measures. Among these equivalent ways of rewriting $\mathrm{q}^{i}\mathrm{P}^{j}$ are the following four:

[47] $$\mathrm{q}^{i}\mathrm{P}^{j} = (\mathrm{q}^{\kappa-\chi}\mathrm{P}^{\nu+\delta})^{\phi}$$

[48] $$\mathrm{q}^{i}\mathrm{P}^{j} = (\mathrm{q}^{\kappa-\chi}\mathrm{P}^{\chi})^{\phi}$$

[49] $$\mathrm{q}^{i}\mathrm{P}^{j} = (\mathrm{q}^{\nu}\mathrm{P}^{\nu+\delta})^{\phi}$$

[50] $$\mathrm{q}^{i}\mathrm{P}^{j} = (\mathrm{q}^{\nu}\mathrm{P}^{\chi})^{\phi}$$

In each of these cases, the "conceptually weak" exponents i and j disappear, in favor of two or more of the intuitively and historically important measures κ, χ, ν, δ. Only ϕ (phi), which may be called the *conversion exponent*, is new. The conversion exponent ϕ takes the values $-1,+1$ (i.e., $\phi = \pm 1$) and may be defined formally as follows:

[51] $$\phi = \mathrm{sgn}\,(j - i) - |\mathrm{sgn}\,(j-i)|+1$$

$$= \sigma - |\sigma| + 1 \qquad \text{(Conversion exponent)}$$

If $a\ \mathrm{q}^{i}\mathrm{P}^{j}\ b$, it is also and equally the case that $a\ (\mathrm{q}^{\nu}\mathrm{P}^{\chi})^{\phi}\ b$, and similarly for the remaining rewritings suggested by [47], [48], and [49]. For example, if $i = 3$ and $j = 2$, we have

[52] $$a\ \mathrm{q}^{3}\mathrm{P}^{2}\ b = a(\mathrm{q}^{5-3}\mathrm{P}^{2+1})^{-1}\ b$$

[53] $$a\mathrm{q}^{3}\mathrm{P}^{2}\ b = a\,(\mathrm{q}^{5-3}\mathrm{P}^{3})^{-1}\ b$$

[54] $$a\ \mathrm{q}^{3}\mathrm{P}^{2}\ b = a\,(\mathrm{q}^{2}\mathrm{P}^{2+1})^{-1}\ b$$

[55] $$a\ \mathrm{q}^{3}\mathrm{P}^{2}\ b = a(\mathrm{q}^{2}\mathrm{P}^{3})^{-1}\ b$$

in a sequence that corresponds to [47] through [50].

Table 4. Illustrative non-i,j-determinate subclasses.

Expression in ν,δ,ϕ	Simplification	English gloss
$/(\mathfrak{q}^{\nu}\mathfrak{p}^{\nu+\delta})^{\phi}/\epsilon$	$/\mathfrak{q}^{j}\mathfrak{p}^{j}/\epsilon$	consanguines of Ego
$/(\mathfrak{q}^{0}\mathfrak{p}^{0+i})^{-1}/\epsilon$	$/\mathfrak{q}^{i}/\epsilon$	generic descendants of Ego
$/(\mathfrak{q}^{0}\mathfrak{p}^{0+m})^{-1}/\epsilon$	$/\mathfrak{q}^{m}/\epsilon$	proper descendants of Ego
$/(\mathfrak{q}^{0}\mathfrak{p}^{0+i})^{+1}/\epsilon$	$/\mathfrak{p}^{i}/\epsilon$	generic ancestors of Ego
$/(\mathfrak{q}^{0}\mathfrak{p}^{0+m})^{+1}/\epsilon$	$/\mathfrak{p}^{m}/\epsilon$	proper ancestors of Ego
$/(\mathfrak{q}^{0}\mathfrak{p}^{0+i})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{p}^{i}/\epsilon$	generic lineals of Ego
$/(\mathfrak{q}^{0}\mathfrak{p}^{0+m})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{p}^{m}/\epsilon$	proper lineals of Ego
$/(\mathfrak{q}^{i}\mathfrak{p}^{i+0})^{\phi}/\epsilon$	$/\mathfrak{q}^{i}\mathfrak{p}^{i}/\epsilon$	own-generation consanguines of Ego
$/(\mathfrak{q}^{i}\mathfrak{p}^{i+j})^{-1}/\epsilon$	$/\mathfrak{q}^{i+j}\mathfrak{p}^{i}/\epsilon$	own- or junior-generation consanguines of Ego
$/(\mathfrak{q}^{i}\mathfrak{p}^{i+m})^{-1}/\epsilon$	$/\mathfrak{q}^{i+m}\mathfrak{p}^{i}/\epsilon$	junior-generation consanguines of Ego
$/(\mathfrak{q}^{i}\mathfrak{p}^{i+j})^{+1}/\epsilon$	$/\mathfrak{q}^{i}\mathfrak{p}^{i+j}/\epsilon$	own- or senior-generation consanguines of Ego
$/(\mathfrak{q}^{i}\mathfrak{p}^{i+m})^{+1}/\epsilon$	$/\mathfrak{q}^{i}\mathfrak{p}^{i+m}/\epsilon$	senior-generation consanguines of Ego
$/(\mathfrak{q}^{m}\mathfrak{p}^{m+n})^{+1}/\epsilon$	$/\mathfrak{q}^{m}\mathfrak{p}^{m+n}/\epsilon$	senior-generation collaterals of Ego
$/(\mathfrak{q}^{2+i}\mathfrak{p}^{2+i+n})^{+1}/\epsilon$	$/\mathfrak{q}^{2+i}\mathfrak{p}^{2+i+n}/\epsilon$	senior-generation, $\geqslant$ 2nd-collateral-remove consanguines of Ego
$/(\mathfrak{q}^{2+i}\mathfrak{p}^{2+i+j})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{q}^{2+i}\mathfrak{p}^{2+i+j}/\epsilon$	$\geqslant$ 2nd-collateral-remove consanguines of Ego; i.e., all 'cousins,' or '$(i+1)$th cousins,' of Ego*
$/(\mathfrak{q}^{2}\mathfrak{p}^{2+j})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{q}^{2}\mathfrak{p}^{2+j}/\epsilon$	'first cousins' of Ego (broad sense)
$/(\mathfrak{q}^{3}\mathfrak{p}^{3+j})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{q}^{3}\mathfrak{p}^{3+j}/\epsilon$	'second cousins' of Ego (broad sense)
$/(\mathfrak{q}^{2+i}\mathfrak{p}^{2+i+m})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{q}^{2+i}\mathfrak{p}^{2+i+m}/\epsilon$	'$(i+1)$th cousins, (m)ce removed' of Ego†
$/(\mathfrak{q}^{2}\mathfrak{p}^{2+1})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{q}^{2}\mathfrak{p}^{3}/\epsilon$	'first cousins, once removed' of Ego
$/(\mathfrak{q}^{2}\mathfrak{p}^{2+2})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{q}^{2}\mathfrak{p}^{4}/\epsilon$	'first cousins, twice removed' of Ego
$/(\mathfrak{q}^{3}\mathfrak{p}^{3+3})^{\phi}/\epsilon$	$^{\pm}/\mathfrak{q}^{3}\mathfrak{p}^{6}/\epsilon$	'second cousins, thrice removed' of Ego

*where '1*th*' yields *first*; '2*th*,' *second*; '3*th*,' *third*; etc.
†where '1*ce*' yields *once*; '2*ce*,' *twice*; '3*ce*,' *thrice.*

Of greatest practical importance in most applications is the equivalence shown in [49], and to a lesser extent that of [50]. The strength of the formula $(\mathfrak{q}^{\nu}\mathfrak{p}^{\nu+\delta})^{\phi}$ of [49] as a

guide to the construction of efficient designations for typical anthropologically interesting Egocentric classes of consanguines is illustrated in Table 4.

For maximum compactness, the expressions in the second column of Table 4 utilize a special convention governing exponent variables. This stipulates that i and j as before (and k when needed) represent any nonnegative integer, while the letters m and n are used to stand for any *natural* number (i.e., *positive* integer). Thus:

[56] $$\left.\begin{matrix} i \\ j \\ k \end{matrix}\right\} = 0, 1, 2, \ldots$$

[57] $$\left.\begin{matrix} m \\ n \end{matrix}\right\} = 1, 2, 3, \ldots$$

All instances of m or n may be eliminated by writing $1 + i$ or $1 + j$ or $1 + k$ in their place, but the availability of these natural-number variables is a practical convenience worth retaining. Another useful convention employed in the table is the following abbreviation:

[58] $$^{\pm}/\ldots./\epsilon \text{ for } /(\ldots.)^{\pm 1}/\epsilon$$

This permits an expression such as $/(\mathfrak{q}^3 \mathfrak{p}^{3+i})^{\pm 1}/\epsilon$ to be replaced by the shorter designation $^{\pm}/\mathfrak{q}^3 \mathfrak{p}^{3+i}/\epsilon$, with no change of meaning.

In comparison with the corresponding English glosses—which are nonideographic, and tend to be cumbersome—the reduced versions of the formal designations derived from $(\mathfrak{q}^{\nu}\mathfrak{p}^{\nu+\delta})^{\phi}$ in Table 4 are extraordinarily brief, simple, and "fundamental." Indeed, each is (1) sufficiently brief to serve as a suitable *proper name* for the geneaclass it denotes, (2) sufficiently ideographic in its construction to serve as a *structural model*[7] of the form common to all chains determining membership in the class it both denotes and names, and (3) sufficiently rigorous in its syntactical underpinnings to serve as a logically unambiguous *formal definition*[8] of the class it simultaneously denotes, names, and models. This particular combination of properties is not typical of the expressions commonly used in anthropological discourse.[9]

The full implications of these notational principles, and their practical consequences in a variety of applications, will be developed elsewhere. The applications include rigorous analyses of the kinds and quantity of information (in the technical sense of communication theory) that may be conveyed by verbal expressions typical of anthropological kinship talk. They also include the practical achievement of a goal for formal semantic analyses of kin terminologies that was stated more than twenty years ago by Greenberg (1949:63-64) but that has not yet been attained in any published componential or transformational analysis: namely, the writing of a single general or "master" structural formula for each particular system, with the attainment of immediate comparability among different systems through direct inspection of their respective general formulae.

conclusion

Reduced to barest structural essentials, Morgan's "chain of consanguinity" is a relation of the form $\mathfrak{q}^i\mathfrak{p}^j$. This is the concept at its purest, with even sex stripped away. We see in

this starkly minimal representation that consanguineal connection consists of: (1) a definite *arrangement* of (2) an indefinite but integral *number* of (3) dyadic linkages that exhibit certain *formal properties* and (4) are accepted as *parental*. With regard to the semantics of the consanguineal concept, the order in which the above ingredients are listed is also the order of (a) an increasing involvement of "content" or "substance" and (b) a decreasing objective consensus among anthropologists.

Agreement on what shall count as parenthood—the last and the least structural ingredient—is clearly hardest. Somewhat easier are the formal properties of the parental relation. (Although these properties are in themselves purely structural, concurrence on a particular set of them depends on the heterogeneity of the collection of relations that are accepted as parental.) Easier still is the second-listed ingredient, where controversy is likely to be restricted to whether certain otherwise acceptable chains are either "too long" (e.g., $\mathfrak{q}^{100}\mathrm{P}^{100}$) or else "too short" (e.g., $\mathfrak{q}^{0}\mathrm{P}^{0}$) to usefully count as consanguineal for anthropological purposes—a matter that hardly seems worth arguing about. Finally, agreement that the proper serial arrangement of linkages in a consanguineal chain is $\mathfrak{q}^{i}\mathrm{P}^{j}$ (rather than, say, $\mathrm{P}^{i}\mathfrak{q}^{j}$ or $\mathrm{P}^{i}\mathfrak{q}^{j}\mathrm{P}^{k}$) appears to be unanimous.

It is probably not fortuitous that the level of intersubjectivity achieved here is correlated with an increasing structure/substance ratio. In science generally, the pervasive historical trend away from substance in the direction of structure—toward pattern, order, number—has not been unrelated to questions of intersubjectivity. Can it be that what "really counts" about relations of the form $\mathfrak{q}^{i}\mathrm{P}^{j}$ is not what shall count as P or $\mathfrak{q}$, so much as how P's and $\mathfrak{q}$'s are to be counted? Let us hope so, for the prospect of anthropological agreement on the metaphysics of parental essences is remote. On the other hand, achieving a firm consensus on i and j and the simplest algebraic functions thereof is well within our grasp.

notes

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[2]The term 'nontransitive' is used here to mean neither transitive nor intransitive, and the term 'many-many' to mean neither left-unique nor right-unique. Since any relation exhibiting the remaining two properties, irreflexivity and antisymmetry, is by definition *asymmetric*, the list of required characteristics may be shortened to asymmetry, nontransitivity (contra Tyler 1969), and the many-many property.

[3]The precedent for including Ego among his own lineals (and hence among the consanguines of Ego) was established by Morgan, who wrote that "His position is necessarily in the lineal line" (1871:17).

[4]For mathematical reasons, the measure κ appears as the "cap-stone" at the top of Figure 5.

[5]Cf. the "normal orthogonal basis" of Birkoff and Mac Lane (1953:192-195). For orthogonal versus nonorthogonal dimensional frames in the semantic analysis of kinship terminologies, see Wallace and Atkins (1960:69-73).

[6]Ashby (1964:127-134) has emphasized the pervasiveness of the relevance of the "constraint" concept to science and has discussed its relation to information theory. For an anthropological application, see Atkins and Curtis (1969).

[7]Due to the discrete character of consanguineal chains, this modeling is simultaneously analogue and digital.

[8]The sole ambiguity in such designations is of a kind I like to call "sharply focused ambiguity," which locates the equivocation precisely where it belongs: in the intentional "outer boundary" indefiniteness of most classes defined in this manner, and—especially central—in the substantive meaning of P.

[9]The kinship designations of this paper utilize a subset of the syntactical and symbolic resources of a formal kinship metalanguage called 'Grafik' (General Relation Algebra For Investigating Kinship). Full details of the Grafik language will be set forth elsewhere.

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