The Mathematical Model of Smooth and Striated Spaces and the Connectivity Problem

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Abstract

The mathematical model of smooth and striated spaces in Deleuze and Guattari's *A Thousand Plateaus* essentially rests on Riemann's manifold theory and Lautman's account of Riemannian spaces. In this paper I provide a detailed analysis of Lautman's account, arguing, however, that to discern its intricacies, and particularly what I shall call the connectivity problem, it is crucial to consider the fundamental work of Élie Cartan. This allows to address three key problems of Deleuze and Guattari's model: (1) the heterogeneity and homogeneity of spaces, and the local/global duality; (2) the connectivity of spaces and the intrinsic/extrinsic duality; and (3) the translatability of spaces and their concrete mixes. As a result, Lautman's and Cartan's philosophical-mathematical questioning proves decisive to Deleuze and Guattari's appropriation of Riemann's theory, and thus the mathematical model of smooth and striated spaces.

Keywords: Riemann, Lautman, Cartan, Riemannian spaces, Klein spaces, heterogeneity, homogeneity, local/global, connection, intrinsic/extrinsic, translation

In *A Thousand Plateaus*, Deleuze and Guattari propose a mathematical model of smooth and striated spaces. This model is fundamentally marked by the manifold theory of the German mathematician *Bernhard Riemann* (1826–66). Deleuze and Guattari's appropriation of Riemann's theory, however, rests more directly on the analysis of Riemannian

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geometry provided by the philosopher of mathematics *Albert Lautman* (1908–44). In turn, Lautman's analysis is drawn on some seminal works by one of the most important and original figures of early twentieth-century French mathematics: *Élie Cartan* (1869–1951). The aim of this paper is to show the crucial importance of this mathematical and philosophical lineage to understand Deleuze and Guattari's model.[1]

Within this lineage, Lautman takes a pivotal role, providing Deleuze and Guattari with an account of what a Riemannian space is. Lautman's account, as they report it, reads as follows:

> Riemann spaces are devoid of any kind of homogeneity. Each is characterized by the form of the expression that defines the square of the distance between two infinitely proximate (*voisins*) points. … It follows that two neighboring observers in a Riemann space can locate (*repérer*) the points in their immediate vicinity (*voisinage*) but cannot locate their spaces in relation to each other without a new convention. Each vicinity is therefore like a shred (*petit bout*) of Euclidean space, *but the linkage* (*raccordement*) *between one vicinity and the next is not defined and can be effected in an infinite number of ways. Riemann space at its most general thus presents itself as an amorphous collection of pieces* (*morceaux*) *that are juxtaposed but not attached to each other*. (Deleuze and Guattari 1987: 485)

The passage is extracted from Lautman's *Essay on the Notions of Structure and Existence in Mathematics*, first published in 1938 (Lautman 2011).

From 'Lautman's fine description', as they label it, Deleuze and Guattari derive not only the famous image of Riemannian space as 'pure patchwork', but more widely the main mathematical features defining smooth and striated spaces. These can be succinctly outlined as follows. A *smooth space* is a non-metric multiplicity, an 'amorphous', non-formal, heterogeneous space in 'continuous variation', constructed by local operations and 'accumulation of proximities' (*voisinages*) that can be multiply linked in all directions; its primary mathematical model is Riemannian space. A *striated space* is a metric multiplicity, a formal, everywhere fixed and homogeneous space of constant directions; its primary mathematical model is Euclidean space (Deleuze and Guattari 1987: 371, 383, 476–8, 483–5, 488).

Lautman's account, however, raises several questions. First, what does it mean that Riemannian spaces are absolutely devoid of homogeneity? Second, why and in which sense from their characterisation by 'the square of the distance' should it 'follow' that without a 'new convention' two 'neighboring observers' cannot locate each other? Moreover, why,

from that, should it 'therefore' derive that a 'vicinity' on a Riemannian space can be assimilated to a 'shred' of Euclidean space? Third, what is the 'linkage' (*raccordement*) to which Lautman refers, and why is it undefined and attainable in infinitely many ways? Finally, what does he properly mean by a 'most general' Riemannian space?

These questions, as we shall see shortly, depend in part on the fact that Deleuze and Guattari have omitted part of the text. It may therefore be useful to quote first of all Lautman's original and integral version of his account. Lautman writes:

> Riemann spaces are on the contrary devoid of any kind of homogeneity. Each one is characterized by the form of the expression which defines the square of the distance between two infinitely near points. This expression is called a quadratic differential form which generalizes the Euclidean formula of the distance between two points: $ds^2 = du_1^2 + du_2^2$. The Riemannian ds^2 to two dimensions is of the following form $ds^2 = g_{11}du_1^2 + g_{12}du_1du_2 + g_{21}du_2du_1 + g_{22}du_2^2$. In an n-dimensional manifold we have the general formula:
>
> $$ds^2 = \sum_{i,j}^{n} g_{ij}du_idu_j$$
>
> The g_{ij} are absolutely arbitrary coefficients, which vary from point to point. The result, as Cartan said, is that 'two neighboring observers can locate the points in a Riemann space that are in their immediate neighborhood, but they cannot, without new convention, be located with respect to one another'. Each neighborhood is therefore like a small bit of Euclidean space, but the connection from one neighborhood to the next neighborhood is not defined and can be done in an infinity of ways. The most general Riemann space is thus presented as an amorphous collection of juxtaposed pieces that aren't attached to one another. (Lautman 2011: 97–8)

At first sight, some aspects immediately appear. First, contrary to Deleuze and Guattari's version, Lautman affirms the absolute non-homogeneity of Riemannian spaces *in contrast to* something else. As we shall see, this point is decisive to understand the precise meaning of the non-homogeneity or heterogeneity of Riemannian spaces as well as what Lautman properly means by the 'most general' Riemannian space. This will allow us to individuate what we might consider the general mathematical model of smooth and striated spaces.

Second, although the mathematical formulation of Riemannian spaces was not presumably Deleuze and Guattari's primary interest, its omission notably compromises the consequent comprehension of the passage. Lautman accounts for how n-dimensional Riemannian spaces

are defined. To begin to understand what such an account implies, let us introduce some general definition. A Riemannian space (or Riemannian manifold) is essentially composed by two mathematical objects: a *manifold* of arbitrary dimension and a *metric* structure. The proper manifold underlying any Riemannian space is what we now call a *smooth manifold* (or differentiable manifold), which is a continuous and differentiable *topological space* (or topological manifold). The proper metric structure of Riemannian spaces is instead a so-called *Riemannian metric*. The Riemannian metric, roughly speaking, is what allows, through the quadratic differential form (i.e. the ds^2 formula), to determine the infinitesimal *distance* between points, curves, angles, the curvature, and so on, and thus to do *differential geometry* (i.e. geometry by means of the calculus) and, specifically, *Riemannian geometry* (i.e. differential geometry on Riemannian spaces). A smooth manifold together with a Riemannian metric forms a Riemannian space (or Riemannian manifold). Lautman explains that the quadratic form of an *n*-dimensional Riemannian space generalises the Euclidean distance or metric, which is a form of the Pythagorean theorem. This means that a Riemannian space is essentially analogous to Euclidean space, but with a fundamental difference: while in Euclidean space we can measure the distance between *any* point, in a Riemannian space this is possible *only* between *infinitely near* points. One consequence of this is that each sufficiently small neighbourhood of each point of a Riemannian space is like a 'small bit' of Euclidean space. As we shall see, this means that a Riemannian space can be locally, but in general *not* globally, assimilated to Euclidean space, and can therefore be seen as composed by a collection of infinitesimal (i.e. local) 'pieces' of Euclidean space. This is the origin of Deleuze and Guattari's image of Riemannian space as 'pure patchwork'. On these general definitions and remarks, we can already note that Deleuze and Guattari's characterisation of Riemannian space and Euclidean space as paradigmatic instances of respectively smooth and striated spaces is quite problematic. The architecture of Riemannian spaces indeed implies rather a 'mixture' of non-metric (topological) and metric (geometrical) structures, and a complex 'interaction' between the smooth and the striated. While the presence of a metric involves that Riemannian space is in fact a striated space (or metric multiplicity), its underlying smooth manifold, however, allows but importantly does not require any *metrisation*. This has suggested that, rather than Riemannian space as such, it is the topological space underlying it that may correspond to a smooth space (or non-metric multiplicity), and therefore that 'it is topology, rather than geometry, that may be

the ultimate mathematics of smooth space in Deleuze and Guattari's sense' (Plotnitsky 2006: 202). While in general agreeing with this interpretation, I shall nevertheless argue that Lautman's account may also support a different one. This will allow us to show some important, and mainly unexplored, further mathematical aspects of smooth and striated spaces.

Third, the metric is technically provided by the g of the ds^2 formula, and consists of a set of functions of the variables $u_1, \ldots, u_n$ (i.e. the coordinates of space). The metric encodes the *curvature*, which measures the local deviation from flatness of curves, surfaces or spaces. In other words, the curvature determines how a Riemannian space generally differs from flat Euclidean space. Lautman states that the metric coefficients (the g_{ij}) are 'absolutely arbitrary'. This is because a Riemannian space is in general of *variable* curvature (i.e. the g_{ij} are non-constant), that is to say, everywhere ('from point to point') different, although it may be also of *constant* curvature, which in principle can be positive, zero or negative. Euclidean space is the case in which the curvature is everywhere zero (i.e. flat), while the cases of constant positive and negative curvature correspond to the spaces of classical non-Euclidean geometry (i.e. spherical and hyperbolic geometry). Deleuze and Guattari's omission of this point is fatal, because the issue of the two 'neighboring observers', as we shall see, is properly a 'result' of the problems that the variability of curvature poses. This also has important implications for the understanding of smooth and striated spaces.

Finally, the issue of the 'neighboring observers' is, surprisingly, a quotation from Cartan (Cartan 1924a: 297), which Deleuze and Guattari completely omit to indicate. Nevertheless, Cartan's reference is crucial. As we shall see, the issue of the 'observers' leads in fact to what is perhaps the main problem of Lautman's account, namely, the problem of the linkage (*raccordement*) or rather *connection* of the infinitely near 'pieces' of Euclidean space. Indeed, as Lautman says, while such 'pieces' overlap, they are nevertheless 'not attached to each other', which is to say, disconnected (in a sense that we will have to clarify). In other words, while to conceive of Riemannian space as a 'patchwork' may be correct, the problematic is *how* to connect the infinitesimal 'pieces' of space to each other. I shall call this the *connectivity problem* of Riemannian spaces, which, ultimately, is what interests Deleuze and Guattari, as the emphasis (absent in the original text) they put on the entire conclusive part of the quotation suggest.

In what follows, I provide a detailed analysis of Lautman's account, considering both Lautman's philosophical and Cartan's mathematical

discussion of Riemannian spaces and Riemannian geometry. Given the technical complexity and intricate conceptual and historical developments of the mathematical notions involved, the analysis will be mainly intuitive and informal, pointing to the problems and their importance for Deleuze and Guattari's elaboration. With respect to recent studies on Deleuze's engagements with mathematics in general and Riemann's theory in particular, the most important of which are those by Arkady Plotnitsky (Plotnitsky 2006, 2009), I will provide a largely different interpretation of both Lautman's account and Deleuze and Guattari's appropriation of it. This will allow us to address three key problems at the core of smooth and striated spaces: (1) the heterogeneity and homogeneity of spaces and the local/global duality; (2) the connectivity of spaces and the intrinsic/extrinsic duality; and (3) the translatability of spaces and their concrete mixes. As a result, Lautman's and Cartan's discussion will prove decisive to understand Deleuze and Guattari's appropriation in *A Thousand Plateaus* of Riemann's manifold theory, and thus the mathematical model of smooth and striated spaces.

I. The Absolute Heterogeneity of Riemannian Spaces and the Local/Global Duality

Lautman's account opens by stating the total non-homogeneity of Riemannian spaces. Riemannian spaces are thus characterised by *absolute heterogeneity*. What does this mean? As already noted, Lautman contrasts this feature to something else. To what does he refer? In other words, contrary to what are Riemannian spaces absolutely heterogeneous? It is with this question that we have to begin.

According to Lautman, modern geometry is marked by two principal conceptions. The first of these is that introduced by Riemann in his famous 1854 Habilitation lecture 'On the Hypotheses which lie at the Foundations of Geometry', while the second is that exposed by the German mathematician Felix Klein (1849–1925) in his celebrated 1872 'Erlangen program'. Following Cartan, Lautman argues that these two conceptions are profoundly opposed to each other, and that Riemannian spaces and Riemannian geometry are in sharp contrast to Klein spaces and Klein geometries (Lautman 2011: 97).

To understand the reason for this contrast, let us first briefly recall the main idea of Klein's conception of geometry. According to Klein's Erlangen program, the various geometries (Euclidean, non-Euclidean, affine, projective, and so on) were to be founded on a unifying principle,

able to allow a unification and classification of all geometries. This principle was provided by the algebraic notion of group and the so-called *theory of transformation groups*. On this theory, a geometry (and the corresponding space) is defined by the invariant properties preserved under the action of a symmetry group, which Cartan calls, following Klein, the fundamental group. For example, Euclidean geometry (and Euclidean space) is determined by the properties of figures (such as length and angle) that remain unchanged under the action of the so-called Euclidean group, that is, the group of translations, rotations and reflections. The fundamental group of each geometry is a subgroup of a larger one associated to another geometry (e.g. the Euclidean group is a subgroup of the group of affine transformations defining affine geometry). Klein geometries (and Klein spaces) are thus encapsulated in a crescent inclusive order according to the respective symmetry group, from Euclidean geometry to projective geometry via affine geometry (Brannan et al. 2012).

Now, why are Riemannian spaces (and Riemannian geometry) deeply opposed to Klein spaces (and Klein geometries)? According to Cartan, the main feature of Klein spaces is their *absolute homogeneity*: 'all Klein spaces are *homogeneous*, in the sense that their properties are invariant under the transformations of the corresponding fundamental group' (Cartan 1927: 201). This is because Klein spaces possess a so-called *transitive* group action. Roughly speaking, this means that the group (precisely, a Lie group) acts, as Lautman puts it, 'in the same way on all points of space' (Lautman 2011: 97). However, this is possible only if the space on which the group acts is of *constant* (positive, zero or negative) curvature. This assumption makes Klein spaces *homogeneous* (i.e. everywhere the same) and *isotropic* (i.e. the same in every direction). The problem is that, in general, this does not hold for Riemannian spaces. Since their curvature is *variable* 'from point to point' (i.e. non-constant), no group action on them can operate transitively on *all* points. Consequently, except for the trivial case of the identity transformation, Riemannian spaces are generally not invariant under any group of transformations (see also Torretti 1978: 141). Therefore, Cartan concludes, 'it is clear that Riemannian geometry, ... does not fit at all within the framework of the Erlangen program, because a Riemannian manifold does not admit in general any kind of homogeneity' (Cartan 1927: 203). In other words, Riemannian spaces are *heterogeneous* (i.e. everywhere different) and *anisotropic* (i.e. different in every direction). They are characterised by *absolute heterogeneity*.

The precise meaning of Lautman's statement in his account is thus as follows. The absolute heterogeneity of Riemannian spaces is in contrast to the absolute homogeneity of Klein spaces. The opposition between heterogeneity and homogeneity is understood from the point of view of group theory, within the framework of the Erlangen program of unification and classification of geometry through the theory of transformation groups. From this point of view, heterogeneity and homogeneity define the degree of symmetry of spaces and geometries: absolute heterogeneity means non-invariance under any (non-trivial) symmetry group; absolute homogeneity means maximal symmetry or invariance (i.e. homogeneity and isotropy).[2]

This interpretation also explains what Lautman (as well as Cartan) properly means by the 'most general' Riemannian space. A 'most general' Riemannian space is a Riemannian space of *variable* curvature, that is, in which the metric coefficients (the g_{ij}) are not further restricted to the special case of spaces (already considered in Riemann's lecture) of *constant* positive, zero or negative curvature. Indeed, these spaces are all included in Klein geometries, and correspond to the spherical (i.e. Riemann's non-Euclidean), parabolic (i.e. Euclidean) and hyperbolic (i.e. Lobachevskian) geometries, respectively; in short, the three classical Euclidean and non-Euclidean geometries. Thus, all spaces of constant curvature are only special, homogeneous (and isotropic) cases of Riemann's most general non-constant, heterogeneous (and anisotropic) spaces. But whereas constant curvature spaces are enclosed in Klein geometries, the 'most general' Riemannian spaces are not.

Arkady Plotnitsky has suggested that Deleuze and Guattari, in elaborating the concept of smooth space, understand the 'most general' Riemannian space as the topological space that underlies any Riemannian space (Plotnitsky 2006: 201). This interpretation is arguably the most plausible and consistent with Deleuze and Guattari's view. One reason is that by considering the underlying topological space, rather than Riemannian space as such, it may provide in fact the mathematical model of smooth spaces: while a metric structure or striation is allowed in topological spaces (every smooth manifold is *metrisable*, for instance), this is not necessarily required. Another reason is that such interpretation accounts for Deleuze and Guattari's more general tendency to privilege the topological, rather than geometrical, character of smooth spaces, in contrast to the metric character of striated spaces, and thus to consider topology, instead of geometry, the primary mathematics of the smooth (202).

However, Plotnitsky's interpretation presents a major problem, namely, that it is not consistent with Lautman's and Cartan's interpretation. As we are seeing, they always mean a Riemannian space, 'most general' or not, as endowed with a Riemannian metric, and thus defined by the ds^2 quadratic form. Although topological considerations, as we shall see, are certainly involved, their main argument remains within geometry, and in particular within the difficulties to conciliate Riemannian geometry with the modern group-theoretical approach promoted by Klein's Erlangen program. Of course, this does not invalidate Plotnitsky's interpretation, as far as it is aimed to account for Deleuze and Guattari's view. Nevertheless, it leaves open the question whether there is some trace in Deleuze and Guattari of Lautman's and Cartan's interpretation. Despite some probably inevitable strain, my answer is positive; moreover, I suggest that it may provide what we might consider the general mathematical model of smooth and striated spaces. Let us explain how.

Throughout *A Thousand Plateaus* Deleuze and Guattari distinguish a double treatment of the variables: one that puts them in 'continuous variation', and another which consists of extracting constants from the variables. In the first case the result is a heterogeneous system, while in the second we obtain a homogeneous system (Deleuze and Guattari 1987: ch. 4). The key point of this distinction is that both kinds of system do not depend on an opposition between variables and constants, but on an opposed treatment of the variables themselves (103). The same occurs with smooth and striated spaces. We might see the first case as corresponding to the heterogeneity of smooth space, which is always in a state of continuous variation. As far as this variability implies non-invariance, smooth spaces are of variable curvature. The second case, instead, may describe the homogeneity of striated space, which is defined by extracting constants and invariants from the functions of the variables (i.e. the metric), such as, for instance, 'Euclid's invariants' (109), that is, as we have seen, the Euclidean group. Striated spaces are therefore of constant curvature.

Deleuze and Guattari maintain that the continuous variation of smooth space also affects its directions and frames of reference (Deleuze and Guattari 1987: 493). 'The variability', they write, 'the polyvocality of directions, is an essential feature of smooth spaces of the rhizome type' (382). In other words, smooth space involves ceaseless 'directional changes', 'continuous variation in direction' (384, 478, 488). Smooth spaces are thus completely anisotropic. By contrast, striated space is defined by 'constant directions', and by inertial frames of reference (488,

494), that is to say, the Galilean–Newtonian and special relativistic reference systems.[3] Striated spaces are thus completely isotropic (426, 488, 489).

Taken together, these aspects prove that the opposition between heterogeneity and homogeneity in Lautman's and Cartan's sense of the degree of symmetry of spaces and geometries also characterises Deleuze and Guattari's smooth and striated spaces. This may explain why they consider Riemannian space and Euclidean space as paradigmatic examples of smooth and striated spaces, respectively. But if so, then nothing prevents us from extending these examples, including both the 'most general' Riemannian spaces of variable curvature and the entire class of spaces of constant (positive, zero or negative) curvature encompassed by Klein geometries (such as Euclidean *and* non-Euclidean spaces). As a result, we might argue that *Riemann's absolutely heterogeneous spaces define the general mathematical model of smooth spaces*, while *Klein's absolutely homogeneous spaces define the general mathematical model of striated spaces*. It is worth noting, however, that these are only general models, providing what may be a *de jure* 'abstract distinction' of the two spaces. Eventually, we will require a more complex picture, considering their *de facto* 'mixes' (Deleuze and Guattari 1987: 475).

To begin to see this, let us return to the contrast between the Riemannian and Kleinian views of geometry. Despite the indisputable success of the group-theoretical approach of the Erlangen program, Cartan suggests, the heterogeneity of the 'most general' Riemannian spaces remains irreducible to the homogeneity of Klein spaces. However, this raises a fundamental epistemological issue. If we were to assume Riemann's view of absolutely heterogeneous spaces, then we would be left only with trivial symmetries (such as the identity transformation). Consequently, Cartan argues, 'no science of the general' would be possible according to the Erlangen program, since 'every geometry would be reduced to particular facts, without link to each other' (Cartan 1927: 202). In other words, while Riemannian spaces show that in general the Kleinian approach is limited, their heterogeneity risks, however, to make impossible science itself. This is why Cartan will seek a synthesis between the points of view of Klein and Riemann, overcoming their opposition, which he would eventually accomplish by elaborating his theory of 'generalised spaces', today known as Cartan spaces or Cartan geometries (Sharpe 1997; Akivis and Rosenfeld 1993: ch. 7).

Nevertheless, the issue is not only epistemological, nor simply confined to the developments of modern geometry. According to Cartan,

the contrast between the Kleinian and Riemannian views became evident with Einstein's general theory of relativity (Cartan 1927: 204). Lautman summarises Cartan's argument as follows: 'The distinction that . . . exists between a Klein geometry and a Riemann geometry is found between the special theory of relativity and the general theory of relativity' (Lautman 2011: 98; see also Norton 1999). The point is that Special Relativity fits perfectly within the Erlangen program. Minkowski spacetime on which the special theory of relativity rests is a flat (pseudo)-Euclidean space invariant under the group of the so-called *Lorentz transformations*. The Lorentz group is what allows one to relate the spacetime coordinates of a physical event in two different inertial reference frames. This expresses Einstein's *principle of relativity*, which states that physical laws must be invariant in any inertial frame of reference. Minkowski spacetime is then a homogeneous (and isotropic) Klein space. General Relativity, by contrast, taking gravity into account, ineludibly implies a heterogeneity. The spacetime geometry, represented by a curved (pseudo)-Riemannian space, depends on the varying strength of the gravitational field, which determines the motion of matter. 'Spacetime tells matter how to move; matter tells spacetime how to curve', in John A. Wheeler's famous dictum. Gravitation is thus the manifestation of the spacetime curvature itself, which is dynamically induced by the distribution of matter and energy. Therefore, the spacetime geometry varies 'from point to point', and consequently in General Relativity there are generally no nontrivial symmetries (Janssen 2014: 187).

However, Einstein's General Relativity, Lautman qualifies, 'does not present this complete absence of organization that characterizes the most general Riemann spaces. A physics in which the laws of the universe would vary from point to point is in effect inconceivable' (Lautman 2011: 98). As just seen, General Relativity affirms the dynamical interdependence between the spacetime geometry and the matter–energy distribution. This relation implies a heterogeneity expressed mathematically by the variable curvature of the 'most general' Riemannian spaces. As we have seen, however, this 'most general' case involves the absence of any homogeneity and isotropy, that is, any (non-trivial) symmetry or invariance. One consequence of this is that for an observer placed in such a space all positions and directions will look absolutely different 'from point to point'. The problem is that this is in conflict with the requirement of the universality and invariance of physical laws (principle of relativity), which presumes independence of any particular position and direction in space of observers. Consequently, Riemann's 'most general' spaces are, so to

speak, 'too much' heterogeneous to assure the validity of physical laws, as well as, Lautman adds, the 'determinism of physics' (99). Some 'organisation' (i.e. homogeneity) is therefore necessary.

One might think that here Lautman is simply pointing out that Einstein's theory (precisely, its cosmological models) assumes the hypothesis – known as the *cosmological principle* – for which the universe is (at very large scale) homogeneous and isotropic. Mathematically, this homogeneity (and isotropy) is translated by imposing to the (pseudo)-Riemannian space a constant curvature (which is positive in Einstein's model), therefore by limiting the 'most general' Riemannian space to a special case. Instead, Lautman's argument points to a more subtle mathematical aspect of General Relativity. The passage quoted above continues: 'Einstein's Riemannian space has what Cartan calls a Euclidean connection, that is, it is possible to locate step by step (*de proche en proche*) the different positions of an observer from each other' (Lautman 2011: 98). We shall see what this does mean. For the moment, simply note that the homogeneity required by Einstein's theory is attained by endowing the (pseudo)-Riemannian space with a new mathematical structure, namely, a *Euclidean connection*. The introduction of this additional structure, as we shall see, is the key point underlying the connectivity problem of Riemannian spaces. But before we can see this, we need to consider a further point.

According to Lautman, the contrast between Riemannian spaces and Klein spaces entails a more profound duality. The opposition between heterogeneity and homogeneity presupposes the opposition between the *local* point of view of Riemann and the *global* point of view of Klein. This duality implies two diametrically opposed methods and approaches. The local point of view starts with the determination of an infinitesimal element; then, 'following its course step by step (*de proche en proche*)', it makes progressively emerge an 'idea of the whole' (Lautman 2011: 95). By contrast, the global point of view starts from the 'structure of the whole' defining 'a totality independently of the elements that compose it' and then 'assigning elements their place before even knowing their nature' (95). Riemann's approach represents a local analytic and infinitesimal conception of geometry, for which the construction of space implies the determination of the distance between infinitely near points (i.e. the ds^2 formula). Klein's group-theoretical approach constitutes a global synthetic and structural conception, represented by group theory and further developed by set-theoretic topology and algebraic topology (46–7, 60).

The local/global duality confronts us therefore with a problematic alternative. As Lautman explains, either we start from a set with a known global structure, looking then at the conditions an element must satisfy to belong to such set; or we start from given elements endowed with some local properties and from these we try to infer the structure of the set (Lautman 2011: 101–2). In Deleuze and Guattari's view, however, this alternative is misleading, because it assimilates the relation between the local and the global to a structural and functional relation between elements and sets. This assimilation defines what they call the encompassing and encompassed characteristic of striated spaces. By starting with a (structurally) predetermined set (i.e. the encompassing), the space properties are determined through the (global) integration of the elements (i.e. the encompassed), which are subject to the conditions of set membership. According to Deleuze and Guattari, this establishes an axiomatic formalisation, which allows one to determine the invariant properties (symmetry groups, structural invariants), independently of the 'nature' of the elements. This conception defines what they call the *relative global* (Deleuze and Guattari 1987: 382, 494), in which the local point of view is entirely subjugated to the global point of view; heterogeneous smooth spaces (or Riemannian spaces) are reduced to homogeneous striated spaces (or Klein spaces). According to Deleuze and Guattari, however, the local point of view remains irreducible, because it is characterised by what they describe as a *local absolute*; that is, 'an absolute that is manifested locally, and engendered in a series of local operations of varying orientations' (382). From this point of view the space properties are not predetermined, because 'the coupling of the place and the absolute is achieved not in a centered, oriented globalization or universalization but in an infinite succession of local operations' (383). Smooth space (or Riemannian space) is thus constructed only 'step by step' by (local) integration of infinitely near 'pieces', 'piece by piece' (157, 476, 493, 574).[4]

But if so, then the local point of view is faced with a crucial problem. On the one hand, the synthetic and structural (relative global) conception of Klein spaces (or striated spaces) assimilates the local/global relation to the elements/sets relation, thus defining an axiomatic set-theoretic space. On the other hand, however, the analytic and infinitesimal (local absolute) conception of Riemannian spaces (or smooth spaces) risks not to reach any global determination. Indeed, as Lautman emphasises, 'elements composed by some process of progressive [i.e. local] development might not give rise to any entity amenable to global characteristics' (Lautman 2011: 101; translation

modified). In other words, from the strictly local point of view, the global space cannot be derived per se, because by looking at the local geometry of a given space, excluding simple cases such as Euclidean space, we can generally infer neither its global geometry nor its global topology (as, for instance, whether a space is open or closed, finite or infinite, oriented or not, with or without boundary, simply or multiply connected). Consequently, Riemann's purely local point of view prevents, or at least does not warrant, that the local integration may allow the transition from the local to the global in order to obtain an 'idea of the whole'.

What does, ultimately, involve then the contrast between the conceptions of Riemann and Klein? It is here that Deleuze rigorously points out the problem: *both* Klein space and Riemannian space involve a respective 'limit'. 'The limit of the first', Deleuze would write in the first of his *Cinema* books, 'would be empty space, but that of the second would be disconnected space' (Deleuze 1986: 187). In other words, while an absolutely homogeneous and global space would coincide, according to Deleuze, with an 'empty set', or 'emptied space' (12, 120, 217),[5] a 'most general' Riemannian space is an absolutely heterogeneous collection of infinitely near 'pieces' of space, which, however, are totally 'fragmented' and 'unlinked' to each other. Thus the question becomes: given their absolute heterogeneity and the purely local point of view, how are we to operate the linkage or connection of the Riemannian 'pieces' of space in order to construct something like a 'whole'? This question leads us to the connectivity problem of Riemannian spaces.

II. The Connectivity Problem of Riemannian Spaces and the Intrinsic/Extrinsic Duality

In his account, Lautman states that a neighbourhood (*voisinage*) on a Riemannian space is like a 'small bit' of Euclidean space. The linkage (*raccordement*) or connection between neighbourhoods, however, is undefined and not predetermined, since it can be done in an 'infinite number of ways'. What does all this mean? Note first that in Lautman's original version of his account the fact that Riemannian spaces can be described as collections of local 'pieces' of Euclidean space is introduced immediately after, and consequently to, Cartan's issue of the 'neighboring observers'. To understand Lautman's statement we have then to return first to Cartan's analyses. This, in turn, will allow us also to understand the issue of the 'observers', which, as we shall see, lies at the core of the connectivity problem of Riemannian spaces.

After having shown the contrast between Klein spaces and Riemannian spaces, Cartan makes a decisive 'turning point', so to speak. He writes: 'if a Riemannian space does not possess an absolute homogeneity, it nevertheless possesses a kind of *infinitesimal homogeneity*; in the immediate neighbourhood (*voisinage*) of a given point it can be assimilated to a Euclidean space' (Cartan 1925b: 7; my emphasis). This point is decisive for at least two reasons. First, it allows us to see how a Riemannian space (or Riemannian manifold) is constructed. Intuitively, consider an n-dimensional topological space (or topological manifold) provided with a continuous and differentiable structure. This is what is called a *smooth manifold* or differentiable manifold (Lee 2006). On each (open) region of the manifold we introduce a coordinate system or *chart*. The (smooth) collection of all coordinate charts covering the manifold is called an *atlas*. Imagine now to attach at each point of the manifold a *tangent space*, formed by all the tangent vectors located at a point. For example, take a curved surface (i.e. a two-dimensional manifold), such as a sphere (S^2), and associate to a point a tangent plane ($\mathbf{R}^2$), composed by the tangent vectors at that point.[6] By approximating each tangent space to an infinitesimal region of the manifold, each tangent space can be seen as a 'small bit' or 'little piece' of flat, homogeneous Euclidean space. This means that the manifold possesses an 'infinitesimal homogeneity', or, equivalently, that it is *locally* Euclidean. By the quadratic differential form (i.e. the ds^2 formula), we can now introduce a *Riemannian metric*. This allows us to measure the infinitesimal (i.e. local) distance between points, curves, angles, to determine the curvature, and so on. Note that the metric properly operates on the vectors in the tangent spaces to the manifold at each point. This provides a notion of frame of reference (with arbitrary coordinates), the meaning of which, it is worth noting, is therefore only local. Reference frames can be thought of as local observers confined to the tangent spaces to the manifold at every point. An n-dimensional smooth manifold endowed with a Riemannian metric defines an n-dimensional Riemannian space (or Riemannian manifold).

Second, Euclidean space, as we have seen, is the paradigmatic example of a homogenous space. Cartan's point is therefore also decisive because, contrary to what we have seen so far, homogeneity property assumes now a primarily *local* meaning. This involves that while a Riemannian space can be locally assimilated to a homogeneous Euclidean space, in general it remains globally heterogeneous.[7] This is simply because once said that each 'piece' of Riemannian space is like a Euclidean space, we do not know whether it is also *everywhere* Euclidean, and thus not

only locally but also globally homogeneous (of course, except when a Riemannian space coincides with Euclidean space itself).[8]

Once an infinitesimal 'piece' of Riemannian space is assimilated to Euclidean space, a new problem arises, however. Cartan writes:

> The difficulty begins when one considers two neighbouring portions of space, the one surrounding a point A, the other a neighbouring point A′; they constitute two pieces (*morceaux*) of Euclidean space that are somehow isolated from each other, *until one is not able to orient them, the one with respect to the other*. (Cartan 1927: 205; my emphasis)

In a nutshell, here lies the connectivity problem of Riemannian spaces. To connect two infinitely near 'pieces' of Euclidean space, we have first of all to be able to operate their reciprocal orientation (i.e. a rotation), otherwise they would remain 'isolated' or, as Lautman says, 'not attached to each other'; in short, they would be disconnected. In other words, in order to link together the local tangent spaces (and thus the frames of reference defined on them) attached to two neighbouring points, we need to orientate reciprocally their respective vectors.

This problem arises because the vectors at a point of the manifold are in general (i.e. in the presence of a non-zero curvature) *different* from those at another point. All the tangent spaces (and reference frames), all 'pieces' of Euclidean space, at different points are *different* tangent spaces. The vectors are not fixed, they can freely rotate around their axes, and therefore their directions, in different tangent spaces, are differently oriented. Consequently, except for vectors belonging to the same tangent space, there is no way, for instance, to compare the direction of vectors belonging to different tangent spaces, or to displace them from one point to another along a path or curve. Therefore, until we have not found a means allowing a reciprocal orientation, we have merely 'a collection of little pieces (*petits morceaux*) of Euclidean space', says Cartan. And this is properly why a Riemannian space, he qualifies, is 'amorphous': because 'we have not yet connected these different pieces by defining their mutual orientation' (Cartan 1983: 89; translation modified).

However, and here is the point, it is exactly this orientation that is impossible to perform until we have not established a 'new convention':

> if two neighbouring little pieces of Riemannian space can each one be assimilated to a little piece of Euclidean space, these two little pieces are without bind to each other, they can not, *without a new convention*, be viewed as belonging to one and the same Euclidean space. (Cartan 1925b: 7)

This is the meaning of Cartan's issue of the 'neighboring observers' we read in Lautman's account. Two 'observers', and their relative reference frames, attached at two infinitely near distinct points of a Riemannian space can locate, through the metric (the ds^2), the neighbouring points belonging to the respective tangent spaces, but they cannot orientate, and thus locate, themselves reciprocally. To this end, we need to link together the 'little pieces' of space. However, this requires a 'new convention' because, except for one (unique) case we shall see shortly, the linkage is an operation *independent* of the metric, in the sense that in general it is not given by, or derived from, the metric structure. We therefore need to introduce in the manifold a *new* additional mathematical structure, and this is precisely what is known as a *connection*.[9] Moreover, since the connection is generally metric-independent, it is not predefined or predetermined, but it can be established in different manners. This is why Lautman states that the connection can be done in an 'infinite number of ways'.

The result of this discussion, then, is that to conceive of a Riemannian space as locally Euclidean is a necessary *but not* sufficient condition to connect the 'little pieces' in a 'whole' (in 'one and the same Euclidean space'). The connectivity problem is therefore closely related to the 'limit', rigorously emphasised by Deleuze, of the purely local point of view. We have a collection of disconnected 'pieces' of space, a 'pure patchwork', as Deleuze and Guattari say. However, we do not know *how* to link such 'pieces' to each other. How to deal then with this problem?

According to Cartan, it is the merit of the Italian mathematician Tullio Levi-Civita (1873–1941) to have achieved the first step, by introducing, in 1917, the notion of infinitesimal parallel transport of vectors on manifolds. This notion, Cartan explains, opens the possibility of connecting the Riemannian 'pieces' of space:

> if we attach to the points A and A′ two rectangular reference frames, we can locate, in Euclidean manner, the origin A′ of the second reference frame with respect to the first, but we can not orientate the axes of the second reference frame with respect to that of the first. The transport through Levi-Civita's parallelism precisely provides us a means to fix this orientation, since we can, thanks to it, acknowledge when two vectors of origin A and A′ must be seen as parallel. Thus, Levi-Civita's parallelism not only provides us with a relation between the vectors attached to two infinitely near points of the space, but, and this has a much greater importance, it also allows us to integrate into one and the same Euclidean space two contiguous little pieces of Riemannian space. … Riemannian space is thus seen as a collection of little pieces of

> Euclidean spaces linked together step by step (*raccordés de proche en proche*). (Cartan 1927: 205–6)

In other words, Levi-Civita's method consists of showing how, given a metric (i.e. a ds^2), two infinitely near vectors can be kept parallel to each other through their (orthogonal) projection onto two Euclidean tangent spaces, as in the previous example of the tangent plane to a sphere. The tangent spaces, and thus the frames of reference attached to them, are consequently subject to a reciprocal orientation (i.e. rotation), allowing the parallel transport of a vector (i.e. the displacement of a vector maintaining it parallel to itself) along a curve from one point to another. Levi-Civita's parallelism links thus 'step by step' the local 'pieces' of Riemannian space constructing a unique ('one and the same') Euclidean tangent space that covers, at least along a path or curve, the 'whole' manifold. It is therefore a *local* notion which allows, however, a *global* construction. According to Cartan's terminology, a Riemannian space endowed with Levi-Civita's parallelism defines a Riemannian space with a Euclidean connection (which Deleuze and Guattari, for reasons that we shall see, prefer to call Euclidean conjunction), today known as *Levi-Civita connection*.[10] This is the previously mentioned unique connection that can be derived from the metric, and thus is a metric-dependent connection (or simply, a metric connection). What does all that involve?

According to Lautman, in addition to the local/global duality, there is a second duality. The properties of a mathematical entity can be of *intrinsic* or *extrinsic* nature. In the first case, the properties 'belong literally to the entity under consideration' (Lautman 2011: 110); in the second, on the contrary, the properties 'can only be attributed to a mathematical entity if one is referred to something other than it', namely, the 'environment that surrounds it', the 'universe within which it is embedded' (110). In the history of philosophy, Lautman argues, this duality is illustrated by the contraposition between Leibniz's intrinsic monadological conception (the internal point of view of the monads and the pre-established harmony that relates them), and the extrinsic theory of space, as a priori form of intuition, of Kant's Transcendental Aesthetic, which presupposes an 'absolute space' and the 'privileged and universal' inertial frame of reference of Newtonian physics (110–12). The intrinsic/extrinsic duality, as that between the local and the global, implies thus two divergent points of view. This divergence, Lautman argues, characterises not only the relationship between differential geometry and set-theoretic and algebraic topology, but also, and this

is what interests us here, Einstein's general theory of relativity. Let us see why.

The contrast between the intrinsic and extrinsic points of view emerged with the birth of Riemannian geometry. Riemann's differential geometry 'studies the intrinsic properties of a manifold, independently of any space in which this manifold is embedded' (Lautman 2011: 112). The notions of distance and curvature, for instance, 'are defined step by step, without exiting the manifold' (112). A Riemannian space is a space in itself, which can be neither 'thought' nor 'intuited' from an external point of view, as if it were embedded in a higher-dimensional space ($\mathbf{R}^{n+1}$). Consequently, Riemann's intrinsic geometry dismisses the presupposition of an a priori given 'absolute space' as well as any privileged reference frame (and thus, philosophically, Kantian apriorism). As we have seen, General Relativity leads this rejection to extreme consequences. With Einstein's theory, 'Geometry and physics are constituted interdependently, so that it is impossible to separate space, the Riemannian manifold and matter' (112). Riemann's intrinsic point of view seems therefore to find an exemplar confirmation in relativity theory.

And yet, according to Lautman, the extrinsic point of view is somehow reintroduced in Einstein's theory. General Relativity, as we have seen, required the introduction of an additional mathematical structure in order 'to endow the Riemann spaces that it considers with a certain homogeneity so that the laws of physics can be independent of any attachment to particular points in space' (Lautman 2011: 114). This structure is precisely the Euclidean connection defined by Levi-Civita's parallel transport (i.e. Levi-Civita connection), since it determines a homogeneity by establishing a notion of parallelism otherwise non-existent in the 'most general' Riemannian spaces. What is then the trouble envisaged by Lautman? The problem is that the notions and operations that Levi-Civita's parallelism implies and allows (the tangent space, the projection of vectors, the reciprocal rotation of the reference frames), Lautman writes, 'only make sense with respect to the space in which the manifold is embedded' (115). As Cartan puts it, 'Levi-Civita posed himself – contrary, one might say, to Riemann – at an external point of view, requiring the necessity to consider a Euclidean ambient space' (Cartan 1925a: 1). In other words, Levi-Civita's method is not only metric-dependent, but, in contrast to the intrinsic point of view of Riemannian geometry, it is also extrinsic, since it conceives of Riemannian space as if it were 'embedded in a Euclidean space to a sufficient number of dimensions' (Lautman 2011: 113).

This argument touches a crucial problem. Whenever we embed a manifold in a higher-dimensional space (which is always possible, but, importantly, not necessary), thus taking an extrinsic rather than intrinsic point of view, the consequence is that the manifold inherits the geometric (and even topological) properties of the ambient space in which it is embedded. In other words, any embedding introduces what we would call a background dependence. Mathematicians express this by saying that the manifold acquires some *induced* structure (e.g. an 'induced metric'). The same is the case with Levi-Civita's method, in which parallelism is in fact 'induced' on Riemannian space from the embedding Euclidean space (Lautman 2011: 114).[11]

Thus, although Levi-Civita's parallelism provides the linkage of the Riemannian 'pieces' of space, it reintroduces the extrinsic point of view. That implies (1) a constancy of directions (parallelism of vectors), (2) a fixed and centred global reference frame, and (3) a supplementary Euclidean ambient space. In Deleuze and Guattari's view, that means that smooth space (or Riemannian space) is entirely 'translated' in a striated space. Indeed, smooth space is characterised by (1) continuously variable directions, (2) moving and acentred reference frames, and (3) local linkages that 'do not imply an ambient space in which the multiplicity would be embedded' (Deleuze and Guattari 1987: 493; translation modified). This is why, Deleuze and Guattari argue, in contrast to Levi-Civita's extrinsic and metric-dependent Euclidean connection (or Euclidean conjunction, as they call it), the proper connection of Riemannian spaces consists of an 'accumulation' of infinitesimal neighbourhoods in an infinite local succession, 'independent of metrics' (485).[12]

According to Cartan, however, despite the limitations of Levi-Civita's approach, its importance lies neither in the parallelism nor the mere possibility of comparing vectors. Instead, 'what is essential in Levi-Civita's idea is that it provides a means to link together two infinitely near little pieces of a manifold, and it is this idea of *linkage* (*raccordement*) that is fruitful' (Cartan 1924b: 205). For Cartan, in other words, the key point is the notion of *connection* as such and the connectivity problem of Riemannian spaces. This is why he strongly stresses two important points. First, in their absolute heterogeneity, Riemannian spaces remain irreducible to the (Euclidean) tangent space on which parallel transport is defined, since it is only by introducing this 'new convention' that a Riemannian space is equipped with a Euclidean connection. Second, Levi-Civita's parallel transport provides only one type of connection among infinitely many possible. On these premises

Cartan will show that Levi-Civita connection can be generalised to other (metric-independent) types of connection (affine, projective, conformal) and that the Euclidean tangent space can be extended to whatsoever Klein space (without presuming an ambient space). In this way, Cartan will achieve his project of synthesis between Riemannian and Klein geometries. As a result, the respective 'limit' of the local (Riemann) and global (Klein) points of view, as Deleuze again acutely notes, 'are themselves re-united in the notion of the any-space-whatever' (Deleuze 1986: 187). Such a space, I would suggest, may correspond to Cartan's 'generalised spaces'. Cartan spaces (and Cartan geometries) are in fact a perfect 'mixture' of heterogeneity and homogeneity, of the local and global, intrinsic and extrinsic points of view (Sharpe 1997: 172).

These results, Lautman suggests, show that both philosophically and mathematically there is a complementarity between such dualities, rather than an exclusive opposition. While the extrinsic point of view is 'philosophically inferior' – at least, Cartan says, in Levi-Civita's approach (Cartan 1927: 211) – on the other hand, the purely local point of view encounters difficulties, as the connectivity problem shows. Philosophically, the necessity of a complementarity between the intrinsic and extrinsic points of view involves overcoming the opposition between Leibniz and Kant. This seems precisely Deleuze and Guattari's attempt, at the same time neo-Leibnizian and post-Kantian, to break with the condition of closure (and of pre-established harmony) characterising Leibnizian monads, which requires a new displacement of thought from monadology to a nomadology (Deleuze and Guattari 1987: 574; Plotnitsky 2006: 205–6). This shift corresponds in 'modern mathematics', Deleuze would write in *The Fold: Leibniz and the Baroque*, to 'a fibered conception' according to which the monads directly interact with each other (Deleuze 1993: 81). Deleuze alludes here to the modern mathematical theory of fibre bundle topology, which is rooted in the developments of Cartan's 'generalised spaces', as for instance in the important works of Charles Ehresmann (1905–79).[13] These developments constitute one of the most important achievements of twentieth-century differential geometry and algebraic topology, and imply further transformations of the relationship between the local and global, intrinsic and extrinsic points of view. This leads us finally to the problem of the translatability of spaces and their concrete mixes.

III. The Translatability of Spaces and Their Concrete Mixes

Once posed the difference in kind between smooth and striated spaces, shown the general oppositions as well as the complex differences

(heterogeneity/homogeneity, local/global, intrinsic/extrinsic) defining them, the issue becomes the 'de facto mixes, and the passages from one to another' (Deleuze and Guattari 1987: 475). This is because 'the two spaces in fact exist only in mixture: smooth space is constantly being translated, transversed into a striated space; striated space is constantly being reversed, returned to a smooth space' (474). This raises the problem of the translatability of spaces; that is, the issue of the possibility and even necessity of the operations of *striation* and *smoothing* by which the spaces are reciprocally converted, assimilated, combined, transformed. Deleuze and Guattari argue that this problem is particularly complex, since the processes and operations of translation 'include as many opportunities for openings as risks of closure or stoppage' (486). The examination of Lautman's account of Riemannian spaces we have conducted so far allows us to show the importance and complexity of such problem.

Indeed, to illustrate the problem, Deleuze and Guattari report what they consider a particularly important example of translation operation, namely, 'the delicacy and complexity of the means by which Riemannian pieces of smooth space receive a Euclidean conjunction (the role of the parallelism of vectors in striating the infinitesimal)' (Deleuze and Guattari 1987: 486). This operation, they qualify, consists of overlaying 'upon each point of smooth space a Euclidean tangent space endowed with a sufficient number of dimensions, by which one reintroduces parallelism between two vectors, treating multiplicity as though it were embedded in this homogeneous and striated space' (373; translation modified). As it should be evident, Deleuze and Guattari's reference here is unequivocally to Levi-Civita's parallel transport and the Euclidean connection (according to Cartan's terminology) it establishes on Riemannian spaces. This is why they describe Levi-Civita connection as a Euclidean conjunction, precisely because 'The mode of connection proper to pieces of Riemannian space ("accumulation") is not to be confused with the Euclidean conjunction of Riemannian space ("parallelism")' (486; translation modified).

Nevertheless, we have seen that Levi-Civita's method allowed the linkage of the Riemannian 'pieces' of space, dealing in this way with the connectivity problem. This shows the necessity of the operations of translation between the smooth and the striated. While the Euclidean conjunction determines a striation of Riemannian spaces, on the other hand, some operation of translation appears to be necessary to connect the infinitely near 'pieces' of space. The Euclidean conjunction and the connection of Riemannian spaces ('accumulation') are therefore not separated, Deleuze and Guattari

stress, despite being distinct and irreducible to each other (Deleuze and Guattari 1987: 486).

The translatability of smooth and striated spaces becomes then significant for the interrelations between what Deleuze and Guattari call minor and major sciences, or also, between a 'problematic' and a 'theorematic' or 'axiomatic' conception of science. One of the principal features of major science is to imply and establish a legal or legalist model, consisting of the search of mathematical or physical universal laws. Major science constitutes and develops itself through successive operations of reduction and encompassing generalisations, extracting constants and invariants that assure the consolidation of formal (axiomatic) systems. In this sense, Klein's Erlangen program of a unification and classification of all geometries on the base of his group-theoretical approach can be seen as an example of such a legal model. Similarly, Cartan's and Lautman's epistemological issue concerning Einstein's theory (invariance and determinism of physical laws) is also exemplar. As we have seen, this required a homogenisation or striation of the 'most general' Riemannian spaces by introducing, for instance, some restriction, such as a constant curvature, or an additional structure, such as a Euclidean connection (i.e. Levi-Civita connection). With respect to these examples, Riemannian geometry appears to Deleuze and Guattari as a minor and constantly problematic conception, attested by its resistance in being encompassed in the major legal model. Thus, the minor and major duality presupposes two 'formally different' conceptions of science, although 'ontologically', Deleuze and Guattari stress, 'a single field of interaction', in which operate all forms of translation, but also of resistance (Deleuze and Guattari 1987: 367).

This is why Riemannian spaces are paradigmatic, for Deleuze and Guattari. In contrast to striated spaces, Riemannian spaces and smooth spaces seem to pertain to a 'minor geometry'. This geometry, they argue, is very limited, to the point that its 'local operations … are not even capable of general translatability' (Deleuze and Guattari 1987: 484). As we have seen, this is because of the absolute heterogeneity and purely local point of view of Riemannian spaces, which involves in general the incapability to infer the global from the local. However, it is precisely for this inability that major science introduces its normative striation, as exemplified by Levi-Civita's method. Indeed, according to such method, Deleuze and Guattari rigorously state, resuming Lautman, 'it is necessary to couple ambulant spaces [i.e. Riemannian spaces] with a space of homogeneity [i.e. the Euclidean tangent space], without which the laws of physics would depend on particular points in space' (374).

This argument implies then that while the passage to the global point of view appears to be necessary to allow the connection of Riemannian spaces, as well as a 'science of the general', to say it with Cartan, just this passage assimilates Riemann's 'minor geometry' to the legal model of major science.

Riemannian spaces and smooth spaces seem therefore strained in a twofold yoke, from which derive, as Deleuze and Guattari say, the possibilities of opening as well as the risks of closure. On the one hand, their absolute heterogeneity, locality and intrinsic features are irreducible. These characteristics, however, risk to leave them completely isolated and disconnected. On the other hand, to connect and construct a space as a 'whole', some homogenisation and striation appears to be necessary. To deal with this issue, Deleuze and Guattari's strategy seems the following. First, they acknowledge the infinitesimal (i.e. local) homogeneity of smooth spaces (Deleuze and Guattari 1987: 371). This parallels Cartan's 'turning point' in front of the contrast between the Kleinian and Riemannian views of geometry. Second, they nevertheless strongly affirm the absolute 'freedom' and 'infinity' of the ways by which the connection can be done (371, 476).[14] This parallels the two key points highlighted by Cartan, namely, the irreducibility of Riemannian spaces to the tangent space, and the 'conventionality' of the connection (of which Levi-Civita's is only a special and limited case), from which follows the possibility to elaborate infinitely many types of connection (affine, projective, conformal, and so on).

This is why Deleuze and Guattari emphasise the necessity of the operations of translation between the smooth and the striated, beyond any general model. Translating, they write, 'is an operation that undoubtedly consists in subjugating, overcoding, *metricizing* smooth space, in neutralizing it, but also in giving it a milieu of propagation, extension, refraction, renewal, and impulse without which it would perhaps die of its own accord' (Deleuze and Guattari 1987: 486). Against isolation and risk of self-annihilation, smooth space requires a milieu of connection, as the tangent space associated to a Riemannian space allows the linkage of the infinitely near 'pieces' of space. Therefore, beyond a *de jure* 'abstract distinction' between smooth and striated spaces, striation appears to be *de facto* necessary for the existence of the smooth. 'Nothing is ever done with: smooth space allows itself to be striated, and striated space reimparts a smooth space' (486).

For this reason, Deleuze and Guattari suggest, 'Perhaps we must say that all progress is made by and in striated space, but all becoming occurs

in smooth space' (Deleuze and Guattari 1987: 486). This, then, raises a last question: if smooth and striated spaces exist only in concrete mixes and if translation processes (striation and smoothing) are necessary, then what is the condition of possibility of both their progress *and* becoming? I would suggest, in conclusion, that the condition, as Deleuze and Guattari as well as Lautman and Cartan repeatedly argue, is that the connectivity of spaces remains in itself absolutely not predefined or predetermined. In other words, the condition of all progress and all becoming of smooth and striated spaces is that the connection may be always done in an 'infinite number of ways'.

Notes

1. In general, for Riemann see Laugwitz [1999] 2008. For Riemann and Deleuze, see Plotnitsky 2006, 2009. For Lautman, in general, see Barot 2009. For Lautman and Deleuze, see Duffy 2006, 2009, 2013. For Cartan, see Akivis and Rosenfeld 1993; Sharpe 1997. Except for Cartan 1983, all quotations from Cartan's works are my translations.
2. Note that a space may be homogeneous without being also isotropic, and vice versa. However, if it is *everywhere* isotropic, then it is necessarily also homogeneous. Spaces that are both homogeneous and isotropic are said to be maximally symmetric.
3. Deleuze and Guattari use the French terms '*repères*' and '*repères d'inertie*', corresponding to the technical notions of *frame of reference* (or reference frame) and *inertial frame of reference* (or inertial reference frame), respectively. In physics, a frame of reference is a coordinate system, assigning positions and time to physical events, together with a definite state of motion. An inertial frame of reference is one whose motion is inertial, that is to say, uniform (i.e. along a straight line) and of constant velocity (relative to another inertial reference frame). This is the case of Newtonian mechanics and Einstein's special theory of relativity (but not general theory of relativity, which extends to arbitrary non-inertial frames of reference).
4. In this regard, Deleuze often refers to the Weierstrassian local method of analytic continuation, on which see Duffy 2006; 2013: ch. 1.
5. Note that in using such expressions, it is not clear if Deleuze wants to refer to some precise mathematical or physical notion, and in the absence of further clues it is difficult to make a plausible conjecture. For this reason, in the following I will leave such point aside, without this compromising, I believe, the argument.
6. Note that this example can be misleading, because it might erroneously suggest that the manifold and the tangent spaces are embedded in a higher-dimensional ambient space ($\mathbf{R}^{n+1}$), which is possible but not necessary. I shall return to this important point later.
7. This is precisely also how Einstein's General Relativity is conceived. While spacetime is globally a curved heterogeneous (pseudo)-Riemannian space, locally it reduces to Minkowski spacetime of Special Relativity, which, as we have seen, is a flat homogeneous (pseudo)-Euclidean space.
8. Cartan's point is also decisive for his attempt of a synthesis between Klein's and Riemann's conceptions. Indeed, by considering a Riemannian space as locally Euclidean, he is able to apply the global point of view of Klein to the local point

of view of Riemann, that is to say, to transfer Klein's group-theoretical approach to Riemann's analytic-infinitesimal one. In this way, a Riemannian space can be understood as locally (but in general *not* globally) invariant under the Euclidean group.

9. Roughly speaking, a connection is what allows one to describe how certain mathematical objects such as vectors change by their infinitesimal displacement throughout the manifold in some direction. Its primary importance is to provide the modern definition of curvature. For a historical account of the notion of connection, see Bourguignon 1992.
10. Note that in the literature the Levi-Civita connection is sometimes also called Riemann connection (or Riemann–Christoffel connection). I shall maintain the first expression, in Cartan's sense of Euclidean connection, to better follow Lautman's further discussion and finally see the opposition between the Levi-Civita connection and what Deleuze and Guattari identify as the Riemann connection, which they call 'accumulation'.
11. Concerning General Relativity, Lautman's argument seems to be that it involves the same problem. While General Relativity is background-independent in that it does not rely on a *fixed* (non-dynamical) spacetime geometry, it does nevertheless depend on some background structure induced by Levi-Civita's parallelism. It is worth noting, however, that this issue was soon recognised by many mathematicians. This led Hermann Weyl (1885–1955), in 1918, to provide the first intrinsic and metric-independent formulation of Levi-Civita's parallel transport by introducing the notion of *affine connection*, which Weyl, and then Cartan, also applied to Einstein's theory; see Coleman and Korté 2001: 204.
12. This point remains, in my view, not entirely clear. Of course, Deleuze and Guattari's general strategy seems to be that of contrasting non-metric and metric spaces considering Riemannian spaces (and thus smooth spaces) as belonging to the first typology. Indeed, if we concede, following Plotnitsky's interpretation, that Deleuze and Guattari assimilate Riemannian space to its underlying (non-metric) topological space, the point is understandable. What is unclear, however, is Deleuze and Guattari's use of the notion of accumulation to describe the connection of Riemannian spaces, in opposition to Levi-Civita connection. The notion of accumulation (or accumulation point) is a topological property, related to the so-called *compactness* of topological spaces. Following Lautman, Deleuze and Guattari are right in suggesting that a topological space can be defined independently of the metric (i.e. the ds^2), as Lautman, discussing some specific aspects (such as the notion of accumulation) of Cartan's treatment of closed groups, clearly explains (Lautman 2011: 105). What is problematic, however, is Deleuze and Guattari's understanding of the notion of accumulation as a connection, which instead has a completely different meaning, as we have seen. One possible interpretation, which seems to be Plotnitsky's (Plotnitsky 2009: 201), is that Deleuze and Guattari allude to the so-called *connectedness* of topological spaces, which is also an important topological property. The problem with this interpretation is that it is unrelated to Lautman's and Cartan's discussion of what I have called the connectivity problem of Riemannian spaces, as well as to the issue of parallelism (related to Levi-Civita connection), to which Deleuze and Guattari explicitly refer. An alternative interpretation, which I can only hint at here, might be the following. As remarked in note 11 above, Weyl gave an intrinsic and metric-independent formulation of Levi-Civita's parallel transport by elaborating the notion of affine connection, which Cartan further developed. The fact that Deleuze and Guattari were aware of this might be proven by their reference to Lautman's (and thus, indirectly, Cartan's) discussion

of Levi-Civita's parallelism, in which Lautman also exposes Weyl's formulation (Deleuze and Guattari 1987: 573, n.19; Lautman 2011: 115). Deleuze and Guattari may have 'mixed' (of course, mathematically, in a questionable way) the notions of accumulation and connection (in Weyl's or Cartan's sense of affine connection) interpreting them as definable independently of any metric structure, and attributing the resulting idea to Riemannian spaces, in opposition to Levi-Civita connection, or Euclidean conjunction, as they call it. As a result, Deleuze and Guattari contrast the extrinsic and metric-dependent Levi-Civita connection to the intrinsic and metric-independent affine connection, calling the latter (although improperly) accumulation.

13. For a detailed account of fibre bundle topology in relation to Deleuze, I allow myself to refer to Calamari: 2015.
14. Deleuze will always repeat this crucial point; see, for instance, Deleuze 1986: 109, 120, 187.

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