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On The Articulation of Systematic-Dialectical Methodology and Mathematics

Dirk Damsma

On The Articulation Of Systematic-Dialectical Methodology And Mathematics

Dirk Damsma

ON THE ARTICULATION OF SYSTEMATIC-DIALECTICAL METHODOLOGY AND MATHEMATICS

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Universiteit van Amsterdam
op gezag van de Rector Magnificus
prof. dr. D.C. van den Boom
ten overstaan van een door het college voor promoties
ingestelde commissie,
in het openbaar te verdedigen in de Agnietenkapel
op vrijdag 9 januari 2015, te 12:00 uur

door

Dirk Folkert Damsma
geboren te De Bilt

Promotiecommissie

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Faculteit Economie en Bedrijfskunde

Acknowledgements

Geert Reuten introduced me to the methodology of systematic dialectics in his course *Politieke Economie: Staat en Globalisering* (*Political Economy: State and Globalization*). Unlike other courses, this course did not just presume the existence of markets and presume that they could be modeled using some highly unrealistic assumptions, but actually set out to prove their existence and mode of operation using systematic dialectics. For the first time in my studies, I felt I was actually learning something about our economic system and not just about the technicalities of how economists believe they can represent it. I was over the moon with enthusiasm about what I was learning, but also thought I found some mistake in Reuten & Williams (1989) book and wrote my paper on this alleged mistake. In direct defiance of Kuhn's (1970) description of how science is organized, Geert readily admitted his mistake and awarded me a very high grade for pointing out that he was wrong. As of that point onwards he developed into what Wouter Krasser once called my 'business daddy', a fatherly figure who stimulated me to pursue my academic interests further and advised me on how best to go about this. He supervised my Master Thesis, helped me get finance and land jobs and for the last ten years supervised my PhD dissertation, while John Davis enthusiastically and pro-actively facilitated us. My gratitude is immense.

I do not have the space to thank everyone that facilitated my work, career and well-being over the years. I will therefore limit myself here to the many commenters on my many drafts over the years: Arjan van den Bosch, Eric Halmans, Wouter Krasser, Marcel Boumans, Gerard Alberts, Louk Fleischhacker, Maurice Bos, Tijmen Daniels, Sietske Greeuw, David Carlson, Christopher Arthur, Paul Zarembka, Jerry Levy, Jurriaan Bendien and Henri Beaufort (referenced here as Ellsworth de Slade). Thank you all for your constructive comments. The end result has become much better because of your patient efforts.

Without my parents, this book obviously would never have been written. They supported me every step of the way, both mentally and – though I hate to admit it – financially, never complaining I was squandering my education by not capitalizing on it in a proper career. Thanks for keeping faith.

Last but not least, I owe a big thanks to my wife, Sjosjana Wetberg, who has always known that I need science to be happy and therefore stimulated me to forego career opportunities that would divert me from my purpose in life. She uncomplainingly put up with the distractedness and foregone family income that resulted from the choices she encouraged me to make. Our three boys brought structure and discipline to my days and filled the simplest things with joy and wonder. Sjosjana, Sietze, Jouke en Aiden, ik houd van jullie.

Brief Contents

Marx's critique of Hegel relates not to Hegel's dialectical method as such but mainly to Hegel's obsession with reconciling the realms of pure thought (Logic) and of nature in his theory of society. Hegel's treatment of ideas as self-contained entities rather than entities contained in man, actually inspired Marx's concept of alienation. By implication, Marx's own dialectical theory was to allow for unreconciled oppositions and materialized ideas (Chapter 1).
Since mathematics is firmly rooted in the realm of pure thought (as far as Hegel is concerned) and its application requires qualitative mediation so as to allow for measures to be formed, Hegel's obsession with reconciliation forbade him to apply mathematical techniques directly to the study of society (Chapter 2).
Marx was not hindered by such inhibitions, because part of the abstractions in society represented materialized ideas (abstractions-in-practice) and he did not believe in Hegel's imperative of reconciliation. So for him there were no fundamental philosophical objections to the articulation of mathematical models alongside, and integrated with, his systematic dialectical account. He did, however, never get round to presenting his models as part of his systematic dialectics in his stabs at mathematical modeling, such as his schemes of reproduction (Chapter 3). Hence some reconstructive work to improve these schemes is possible and called for (Chapter 4).

Abstract

In their published works, both Hegel and Marx made use of a form of a historical as well as a systematic dialectic. There is more room for quantification in Marx's system than there is in Hegel's because the imperative of exchange that is paramount to Marx determines the concepts required for Marx's dialectics as ontologically quantitative abstractions-in-practice. Hegel's system, by contrast, does not allow for the existence of such entities because his determination of the subject of mathematics as an external reflection on many distinguishable but divisible elements, forbids immediate quantitative representations of anything without first devising a suitable Measure that reunites external Quantity with internal Quality. Since Marx's schemes of reproduction, i.e. his models for the renewal and expansion of capital deal exclusively with the aforementioned abstractions-in-practice, they can be reconstructed in such a way that they are fully defendable dialectically. This requires the model's assumptions to either formally recap a dialectically arrived at result, temporarily abstract from some tendency that was already exhibited dialectically, remind the reader that some not yet exhibited entities must be considered absent at the level of abstraction the model pertains to or to anticipate the presence of not yet exhibited mechanisms crucial to the unhindered working out of a tendency that was exhibited as necessary to the working of the system as a whole. When assumptions can be presented like that, there are no fundamental obstacles to the integration of systematic dialectics and mathematics or mathematical modeling.

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Introduction

Both Hegel and Marx adopted in their work a 'historical dialectic' next to a 'systematic dialectic'. For Marx the former method is confined to his youth work (up to 1848), whereas he uses the latter method for his study of the 'system' of capitalism in his magnum opus *Capital*. This dissertation is a contemporary appraisal of Marx's work, which takes distance from Engels' interpretation of Marx's *Capital* that dominated much of the 20th century views both within Marxian circles and among their critics. This modern interpretation stems from research in the past two decades of Marx's *Capital* in the light of Hegel's systematic-dialectical works.[1] As a result the Hegelian systematic-dialectical methodology is experiencing a true revival in Marxist circles, a revival that Arthur calls "the New Dialectic" (2004: 1). On the one extreme within it, one finds materialist reevaluations of Hegel. On the other extreme, Marx is viewed as an innovator that struggled to break free from his idealist Hegelian heritage in order to create a truly new materialist dialectical methodology. What binds these new dialectical Marxists together is their acknowledgement of Hegel's profound influence on Marx's method (Arthur 2004: 2).

Although Marx studied mathematics thoroughly (Smith, Cyril 1983: 256; Smolinski 1973: 1193-1194) and utilized it in his *Capital* for two important fields at least, one does not usually find instances of its use in the writings of these Hegelian Marxists. [2] [3] The aim for this book is to critically examine whether it is methodologically possible to combine mathematical rigor with a systematic dialectical methodology and, if so, to provide an indication of how mathematics may be instrumental to a systematic dialectician and of how a systematic-dialectical perspective may help mathematical model builders.[4] The first three chapters respectively deal with differences and similarities regarding the systematic-dialectical position of Hegel and Marx, Hegel's systematic-dialectical perspective on the mathematical and finally Marx's use of mathematics (specifically with regard to his schemes of reproduction) within his systematic-

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[1] See, for example, Murray (1988), Reuten & Williams (1989), Smith (1990), Arthur (2004), which collects some of his earlier essays dating from 1988-2001, and Fraser & Burns (eds. 2000).

[2] Throughout this book the name Smith refers to Tony Smith. When referring to other Smiths (viz. Adam Smith and Cyril Smith) I cite their first names as well.

[3] Reuten (e.g. 2002a, 2004a, 2004b and 2004c) is somewhat of an exception.

[4] In this dissertation this question is only dealt with in respect to Marx's schemes of reproduction. How the indicated guidelines may be beneficial to model builders generally is beyond the scope of what can be achieved in this work.

dialectical framework. From the evaluation of the latter, it is concluded that there is room for improvement regarding the way Marx's mathematics are embedded in his overall systematic-dialectical framework. The aim for Chapter 4 is therefore to reconstruct Marx's schemes of reproduction in such a way that it makes maximum use of its place within Marx's dialectics with respect to informing its assumptions and at the same time informs how the dialectical exhibition might proceed from there.

As indicated, Chapter 1 will deal with the method of systematic dialectics in general, and it takes up the following themes. Systematic dialectics is a two-phase methodology. In the first phase, the researcher tries to get to grips with the world of the field of enquiry. To do this, one may utilize preliminary categorizations, measurement instruments, models and whatever else enhances our understanding of phenomena pertinent to the field of study (i.e., in our case, the study of capitalism). All research done in this phase endows the researcher with a partial and analytical understanding of phenomena, i.e. aspects of reality. Marx and Marxists refer to this kind of research as exploration ('Forschung') (Reuten 2000: 143). To Hegel the research methods to be used in this phase are almost exclusively confined to getting to grips with contemporary developments in the empirical sciences through desk research, but do not encompass any empirical research on the part of the dialectician himself (cf. Hegel 1830[3], 1817[1]: §259) [5].[6]

In the second phase, the dialectician systematically pieces together what he has analytically ripped apart in the first. The first question to ask in this phase is what defines the system we are interested in. To the founding father of systematic dialectics, Hegel, the answer to this question is a concept without which the whole of the field in question would be rendered unintelligible. Space, for example, is

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[5] Superscripts behind a publication year denote editions. The edition that was actually used is always cited first. Thus (1830[3], 1817[1]) means that the current text relies on the third edition of the *Encyclopädie* and that the first edition of that work was published in 1817.

[6] Note that from the vantage point of the 21st century this is still an insurmountable task for 'the empirical sciences' encompass almost all disciplines: all humanities (languages, history, economics etc), all sciences (physics, astronomy, chemistry, engineering, etc). The only two Hegel would perhaps not consider empirical are mathematics and philosophy itself. But, according to Hegel, systematic dialectics *is* philosophy and hence its practitioner should be conversant with philosophical writings; while an understanding of mathematics is required to understand everything else (even though Hegel, in contrast to Marx, did not believe *direct* applications of it were warranted anywhere, as this book will show).

Since Marx's systematic-dialectical social theory is much more limited in scope than Hegel's attempt at linking up all disciplines in one grand framework, the task posed for someone attempting to formulate a modern-day systematic-dialectical theory of capitalism is formidable but not insurmountable even if one sets himself the additional task of undertaking exploratory empirical investigations of one's own (cases in point are Reuten and Williams' 1989 *Value Form and the State* and Reuten's forthcoming *The Capitalist System* (2013 draft))

the universal principle of the natural sciences, because all material things are spatial. Hence, the study of material observables presupposes the concept Space (Hegel 1830[3], 1817[1]: §254). Marx, in adapting Hegel's methodology to the study of capitalism, found his universal principle not so much in a concept as in the most abstract expression of capitalist relations. For him, this is the commodity.[7] So, in contrast to Hegel, Marx's abstractions are not only informed by the thought about a field, but also by the reality of that field: they are abstractions-in-practice.[8]

The existence of abstractions-in-practice and the conflict-ridden take Marx had on social reality implied that he had no fundamental philosophical objections to the articulation of mathematical models alongside, and integrated with, his systematic dialectical account. As indicated, the difference of opinion between the two on this matter hinges upon the nature of the categories they appropriate for their systematic dialectics. This difference with respect to the kind of abstractions the two utilize, itself stems from their respective object of investigation. Where Hegel discusses mathematics this object is thought. His deeply philosophical question in this context is: what is it that enables us to think at all? His answer – one that, under his influence, is perhaps obvious today – is that thought requires language. If this is the case, Hegel reasons, the structure of language can inform us about the structure of the world we think about (less obvious, though relevant for the mere possibility of knowledge). Mathematical and formal thinking has a place in this structure of language, but it cannot be directly applied at more concrete levels (e.g. the level of society) without elaborate qualitative empirical considerations about these fields. So mathematical models may play a role in the empirical sciences, but not in dialectics.

Marx's subject is capitalism. Commodities in capitalist societies are of some particular use to buyers, while they represent only universal exchange value, i.e. monetary value, to sellers. This universal exchange value permeates all entities and categories in the economic domain as commodification. Consequentially, all concrete capitalist entities and categories, like commodity, price, cost, profit, value, etcetera, can also be understood abstractly, as elements in the produce of

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[7] I agree with Reuten (1989; 2013 draft), that an expression of relations is less abstract and hence less universal than the relation expressed (i.e. the exchange relation) and this, in turn, is less abstract than the institutional arrangement (i.e. the dissociation of units of production from those of consumption) upon which the relation is predicated. However, even if one were to rewrite Marx in this spirit, the abstractions involved are still 'tangibly' abstract instead of (as with Hegel) conceptually abstract only.

[8] Among other things, this claim stems from Marx's critique on Hegel (elaborated upon in Chapter 1) that he estranged thought from the thinker, and thus denied it any potential of empirical practicality. So if Marx's abstractions can indeed be legitimately called abstractions-in-practice, it appears that this was precisely what Marx was aiming for.

the system at large. In other words, where Hegel sees qualitative and quantitative reflection as reconcilable modes of *thinking* only, Marx sees this world *itself* as being both qualitatively and quantitatively constituted. So, quantities are an integral part of capitalism, rather than being externally imposed on it. It is this characteristic of capitalism that enables (mathematical) modeling methodology to be integrated with systematic dialectics all the way through, albeit with regard to the study of capitalism only (that is, amongst the systems that Marx knew of).

Once the universal principle is identified, the first thing to ask is how the principle appears in total categorial isolation (e.g. in isolation a commodity is just a product that can be of use and hence is imbued with *use* value). Secondly, one could ask how it manifests itself in the world (e.g. when a commodity is produced to be exchanged, it is primarily evaluated on the basis of its *exchange* value). Hence, the answers to those questions usually involve oppositional categories (e.g. although commodities are desired for their potential uses, they are only produced because of their exchange value). Finally, the tension between those categories needs to be resolved, in order to show why the two oppositional categories do not annihilate one another (e.g. how come useful products are produced, when the producer does not care about use values).

Resolving an opposition of this sort involves either showing how the one half of the opposition becomes the other half, or showing how the two halves can coexist (e.g. in generalized exchange in the *market*, exchange values are to some extent held in check by perceived use values). Sometimes the condition for coexistence cannot be found immediately. In that case there often are successive stages of coexistence. The first categories in this succession only partially resolve the tension between the answers to the first two questions, which is fully resolved at subsequent stages.

By asking the first two questions again about the last of the categories found, a new opposition will generally be found which can be resolved again, and so on. It is claimed that all entities and processes found in answer to the mentioned questions are *necessary* to the field under scrutiny, now represented as an interrelated system.

Chapter 2 researches Hegel's determination of the categorial foundations of mathematics. Hegel provided some tantalizing insights into the nature and essence of mathematics.[9] The aim for this chapter is to take up in particular

[9] Specifically his correct recognition of the subject of mathematics as an external reflection on many distinguishable but divisible elements, has received praise (Kol'man & Yanovskaya 1931: 5; Baer 1932: 104; Fleischhacker 1982: 194); as well as his views on the nature of the mathematical infinite (Baer 1932: 112; Ellsworth de Slade 1994: 213; Lacroix 2000: 311-315). Paterson (1997a) goes as far as to argue that the problems that the formal systems that were proposed in the 20th

Hegel's thoughts on what we would nowadays call the relations between sets and elements and his views on why these are only an intermediate step towards understanding the world in both quantitative and qualitative terms.[10] Therefore, Hegel would probably view the representation of more concrete constellations – such as society – in purely formal (i.e. set theoretical or, in this case, mathematical model) language as regressive.

Despite the amenability of capitalism to mathematical treatment, Marx presumably never got round to exhibiting the models he made alongside and integrated with his dialectics. However, as was said above, there are at least two important fields where he represents aspects of, or partial interactions in his theory in 'schematic' or algebraic form. The most famous one of these are his schemes of reproduction ('schemes' or 'models'; Tinbergen in his early work also used the term 'scheme' for what he later called 'model'). If the assumptions Marx calls upon in formulating these schemes are scrutinized from a dialectical perspective, it can be concluded that most of the assumptions indispensable for Marx's model are dialectically defendable as either foundational (formally recapping a dialectically arrived result), heuristic (temporarily abstracting from some tendency that was already exhibited dialectically), absency (merely reminding the reader that some entities have not been exhibited and thus cannot be taken into consideration at the current level of abstraction) or because they anticipate the presence of mechanisms that, though not exhibited yet, are crucial to the unhindered working out of a necessary tendency that was exhibited.

Marx did not live long enough to reintegrate the models he build with the overall structure of his work (presuming this was his intention). As a result, he hardly ever defends his assumptions dialectically and makes more of them than he needs to from a mathematical model point of view. Also, the mathematical formulations and expressions he chose are far from ideal from both a mathematical model and a systematic-dialectical point of view. So there is room for improvement on all those points. These improvements are worked out with respect to Marx's schemes in Chapter 4. Thus, these schemes and their subsequent improvement, indicate how systematic dialectical accounts may be instrumental in guiding the way to appropriate assumptions and how mathematical models may in turn guide the progress of dialectical accounts. The generalized guidelines that can be formulated as a development and generalization of this insight form the backbone of this book's conclusion.

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century as foundations for mathematics have run into, can be resolved if they are properly grounded in a Hegelian philosophy of mathematics.

[10] Although there was no set theory around in Hegel's day (its founding father Cantor, was born in 1845, while Hegel died in 1831), Hegel's Unit and Amount are clearly akin to what we would nowadays call elements and sets.

Note on the Style of Referencing and the Use of Capitalization and Emphasis in This Work.

A large part of this book is concerned with (interpretations of) the historical works of Hegel and Marx and the methodical connections between them. But since both Marx and Engels wrote in German and this text is written in English, quoting the publication date of the translation when referencing would obscure the chronological order in which the original German texts were written. This is also true regarding posthumously published texts. When referring to the latter type of text, I therefore use curled brackets - {} - placed around the years of composition of the posthumously published manuscript.

Since only the first and second edition of *Das Kapital I* were published during Marx's lifetime and under his own supervision, I deviate from this general rule when referring to the other two volumes of *Das Kapital*, that were posthumously published after thorough editing by Engels, simply because of the years of the first publication in German being so well known in Marxist circles. Engels's editorial activities have long fueled controversies regarding whether Engels had done Marx justice. The last word in these disputes might be provided by the editorial teams of the Marx-Engels Gesamtausgabe (MEGA), that from the 1970's onwards have been working on deciphering Marx's (and sometimes Engels') original manuscripts and rendering them into readable and meticulously annotated form. So as to make it easier for the reader to see the wood for the trees, I will refer to all variations by citing the year of the first publication in German, suffixing this with F for the Fowkes (translator of volume I) or Fernbach (translator of volume II and III) translation, M for Marx's manuscripts and E for Engels' editorial manuscripts (the latter two were both published in the MEGA-series). The meticulous annotations are provided by the editorial teams of the Mega in a separate volume: *Das Apparat*. Since I consider these annotations as secondary literature in their own right, I quote Mega as author and the year of publication of the volume concerned as date, when referring to the latter.

When quoting translations of German or Dutch originals, my page references are to the relevant translation. When paraphrasing, I refer to the original text. When a work comes in several editions I use superscripts behind the publication year to denote the editions (as was already indicated in note 5). The edition that was actually used is always cited first. Thus (1830^3, 1817^1) means that the current text relies on the third edition of the *Encyclopädie* and that the first edition of that work was published in 1817.

Regarding Hegel's texts, I am primarily concerned with the *Encyclopädie* (1830^3, 1817^1), but sometimes I will also refer to the *Wissenschaft*. This latter work is divided first into parts, then into books, then into segments ('Abschnitte')

and next into chapters. The chapters are subdivided into sections A, B and C which are usually, but not always, subdivided again into subsections a, b, and c. In order to enable comparisons to translations and to other editions, I will not only refer to the page number in the Suhrkamp edition of this book, but I will also specify the segment, the chapter, the section and the subsection, respectively. All references to the *Wissenschaft* are to the first book of the first part, so the part and the book in question need not be specified. Thus 1.1A means (Part 1, Book 1,) Segment 1, Chapter 1, Section A (The first chapter of Hegel 1812, 1813, 1816 has no subsections).

The *Encyclopädie* is divided into parts and subdivisions (‘Abteilungen’). Just as with the *Wissenschaft*, the subdivisions are divided into sections A, B and C which are usually, but not always, subdivided again into subsections a, b, and c. But since it is partitioned into continuously numbered and sufficiently small §§, a reference to those (notation: §#) suffices to enable comparisons to translations and to other editions than the Lasson edition usually referred to in this book. Finally, *Grundlinien der Philosophie des Rechts* (1821) (which will be referred to in examples only) is divided into parts and segments, but it too is partitioned into continuously numbered and sufficiently small §§, so a reference to those (§#) will suffice here as well.

In this book, categories that are dialectically important to Hegel (and thus function as moments – cf. Chapter 1, Section 2) will always be written with a capital letter, enabling the reader to see whether a word is used dialectically or not. In German, all nouns are written with a capital letter. So, this practice (although common among native English speaking Hegelians) has no warrant in German (Inwood 1992: 6). However, since this linguistically questionable convention usually clarifies dialectical exhibitions significantly, I will adopt it here. To avoid confusion between Hegel’s moments and Marx’s (as mainly discussed in Chapters 3 and 4), Marx’s are stressed by *italicizing* them.

1. On Marx's and Hegel's Dialectical Methods

Introduction

A central claim in the current work is that, throughout *Capital*, Marx was committed to a systematic-dialectical method inspired by Hegel. The idea that Marx's thought is somehow connected to Hegel's is acknowledged by most if not all Marxists, be it in the present or the past. Thus, it is not the Hegel–Marx connection as such that is debated, but rather the precise nature of this connection (Fraser and Burns 2000).

Most of the debate revolves around the differences between Hegel and Marx regarding their respective accounts of society and its history. And indeed there are great differences of opinion between the two thinkers regarding these topics. However, this chapter will show that these differences though ontological, are hardly epistemological in nature. Marx and Hegel both employed a form of systematic dialectics. The ontological differences are shown to imply a greater potential for the use of quantitative methods in a Marxist than a Hegelian systematic-dialectical theory of society.

To argue this, Section 1 describes the chronological order in which Hegel conceived of his systematic and historical dialectic respectively and establishes the fact that Marx criticizes Hegel's historical dialectic before starting his systematic-dialectical work in *Capital*. Next, Section 2 describes the essential elements of the systematic-dialectical method as they appear in Hegel's works and the way they shaped his historical dialectics. Thus the stage is set for an investigation in Section 3 of Marx's most important comments on Hegel and the major implications thereof for his own social theories and particularly for the method he employed in *Capital*. Finally, Section 4 discusses some contemporary authors that contend that Marx's method in the *Grundrisse* and/or *Capital* was essentially a systematic-dialectical one inspired by Hegel. The conclusion alluded to in the previous paragraph follows.

1. The chronology of Hegel's and Marx's historical and systematic dialectic

In the works of both Hegel and Marx two types of dialectical method are at play: a historical and a systematic type. The systematic type scrutinizes the architecture of a given system, while the historical type scrutinizes the mechanism by which a

system develops into another system.[11] Although both are best known for their historical dialectic – dubbed historical materialism in Marx's case and historical idealism in Hegel's – most of their work is systematic rather than historical in nature. In fact Hegel's use of the Historical dialectical method is limited to his *Vorlesungen über die Philosophie der Weltgeschichte* {1823-31}, which was based on the lectures he gave 'for the first time in the winter semester of 1822–3 in Berlin. The lectures were repeated on four occasions, in 1824–5, 1826–7, 1828–9, and 1830–1' (Brown & Hodgson 2011: 1) (note that these were not published during Hegel's lifetime which lasted until 1831, when he fell prey to a cholera epidemic). In these lectures he basically applies the same principles to history that he used to get to grips with the relation between subject and object in his *Phänomenologie des Geistes* (1807), the nature of thought in his *Wissenschaft der Logik* (1812, 1813, 1816) and society in his *Grundlinien der Philosophie des Rechts* (1821), and finally to outline the interrelations between all scientific disciplines in his *Encyclopädie der Philosophischen Wissenschaften* (1830^3, 1817^1). All of the latter four works were systematic-dialectical works. So Hegel only conceived of his historical dialectic, when his systematic-dialectical system was virtually complete.

Marx, by contrast, formulates his version of historical dialectics – together with Engels – in *Die Deutsche Ideologie* {1846} in a reaction to Hegel's before starting work on his systematic-dialectical magnum opus *Capital*.[12] As a result, as will be elaborated upon further on, historical considerations explicitly codetermine the system under scrutiny in Marx's subsequent systematic dialectics, while this is only implicitly – if at all – so for Hegel. Neither Hegel nor Marx, ever published a

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[11] Both illustrate what they think is the mechanism at work by pointing out the rise and fall of past empires or eras and their subsequent supersession by a new one. In the case of Marx, one can reasonably argue that he held that system-changes could only come about by revolutionary *Umwertunge aller Werte* (the phrase is Nietzsche's (1888) and translates as 'transvaluations of all values'), but the mechanism Hegel thinks is responsible for history's dynamism does not necessarily preclude gradual evolution of one system into another. (Footnote to the footnote: Since material redistributions are the core of Marx's historical dialectic, the multi-layered meaning Nietzsche's phrase acquires in the context of Marx, was too good to miss. However, Hegel – and by extension Marx – can be considered the last system philosophers, whereas Nietzsche was the first proponent of a new philosophical era in Germany in which 'big philosophy' got replaced by piecemeal stories accompanied by a somewhat pessimist belief that all big stories and philosophy itself is bound to fail (Schnädelbach (1984: 3) refers to these two developments as 'the 'collapse of Idealism'' and 'the age of […] 'disillusionment'' respectively). So it is unlikely Nietzsche would have approved of this admittedly anachronistic pun.)

[12] References in curled brackets refer to dates of composition of manuscripts that have been posthumously published (see the *Note on the Style of Referencing and the Use of Capitalization and Emphasis in This Work*).

comprehensive work on the method they employed as such.[13] As a result, what is involved in their dialectics and how Marx's method differs from Hegel's, must be distilled from their respective applications to realms of science.

The following two sections will attempt this distillation in chronological order. Thus, the basic principles of Hegel's systematic dialectical method are identified and described first. Next, Hegel's historical dialectical assertions are described as the outcome of his systematic dialectics (Section 2). After that, Marx's criticism on Hegel can be understood and the methodical elements of continuity and dissent described (Section 3).

2. Hegel's method.

The main tenet of systematic dialectics is that all that can be known about the world is known in language. Things that cannot be expressed in a form of language cannot actually be known at all. The upshot of this is that the basic structures of language are the basic structures of intelligibility of the world. In other words: for the world to be represented in thought, it must be representable in language. If so, the structure of language must harmonize with the structure of the world to exactly the same extent as an individual can make sense of it (a thought also expressed by Hofstädter 1979).[14] [15] It follows that the systematicity of the

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[13] Even so, in his published work Hegel is more explicit on the method than Marx who after some ten manuscript pages of 1857-58 (not prepared for publication) wrote no more than a few sentences about it in the Postface to the Second Edition of *Capital I* (1873[2], 1867[1], 1867F).

[14] The main theme in Hofstädter's work is that any sufficiently strong representational system, in mathematics, music, art, language or computer science alike, has the ability to self-reference. This ability means a Gödelian trap opens up at its core. That is, a sentence can be constructed that says of itself that it cannot be proven. If such a sentence is true, not all truths can be proven within the system. If it is not true, the system is able to prove falsehoods. So a representational system is either incomplete or inconsistent. This is similarly true for the category of self in language, leading Hofstädter to claim that 'I' is a strange loop (2007). In other words, the category 'I' is the Gödelian hole in the system of language. What Gödel did for mathematics, Hegel did for language. That is, Gödel took a formal system and turned its attention to itself. Similarly, Hegel uses language to analyze language, hoping to determine the limits of the knowledge that can be achieved within it. Completion of this project brings about what Hegel calls self-consciousness of the Absolute ('Selbsterkenntniss des Absoluten' (cf. Peperzak 1987)). In light of the above, one might say that this is the height of mankind's awareness of its own strange-loopiness.

[15] For this reason, 'learning the ropes' in any discipline, to a large extent means appropriating terminology and jargon as well. This is not only true for e.g. physicists learning about quarks or economists learning about Pareto efficiency, but also for furniture handlers that have different names for different types of trolleys, parts of elevators, etc. That is, if there is a part of the world

world's intelligibility and the fundamental interrelations between everything we can claim about it can be discovered by mapping the basic systematic relationships between categories in language.[16]

A major proviso for such a project to work is that the dialectician has sufficient and sufficiently adequate categories at his or her command to commence this mapping. Thus, systematic dialectical exhibition, or concrete determination (Reuten & Williams 1989: 18-19; cf. Smith 1990: 5), is preceded by a 'stage of appropriation' (Smith 1990: 4-5) or 'abstract determination' {Marx 1857-58: 101}; Reuten & Williams 1989: 18-19), or 'phenomenological inquiry' (Murray 2000: 36-42) in which categories are articulated. Although Hegel acknowledges the importance of 'the working out of the empirical sciences on their own account' {Hegel 1825-26: 176} and contends that '[p]hilosophy […] owes its development to the empirical sciences' (Hegel 1830[3], 1817[1]: §12), it is something he trusts can be safely delegated to its practitioners.[17] The dialectician just needs to 'become acquainted with empirical nature' {Hegel 1825-26: 175}, he does not need to engage in those studies himself.[18]

Categories are never entirely specific, unique and individual. As predicates they refer to what more particulate categories have in common and thus show what unites them (such as 'is car' or 'has value'). By doing so they obscure the differences between the particulate categories they unite. Although this renders the differences more implicit, it does not make them disappear. Thus

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that, for whatever reason, one needs to understand better than other parts, one tends to know more words pertaining to it.

[16] According to Marx, Hegel 'fell into the illusion of conceiving the real as the product of thought' {Marx 1857-58: 101}. Approvingly echoing this verdict, some Marxists (mainly those that Fraser & Burns (2000) label 'appropriationist') dismiss Hegel's thinking for Christian dogma in a philosophical guise, because they take Hegel's thinking to be on a par with biblical verses like: "In the beginning was the Word [viz. the Idea], and the Word was with God, and the Word was God. […]. All things were made through him; and without him was not anything made that hath been made."(John 1:1, 3). In post-Kuhnian terminology however, one might also interpret Hegel's thinking as a form of - and the initiation of - ontological constructivism. From that perspective, Hegel's alleged claim appears much less mystical: he simply claims that for something to exist *for us* consciousness must be able to distinguish something as existent and we can only do that if our categories are adequate. Hence, for all practical purposes at least, the world as we know it is co-created (nowadays we would say constructed) with the development of language and its subsequent systematic dialectic exhibition.

[17] When quoting translations of German or Dutch originals, my page references are to the relevant translation. When paraphrasing, I refer to the original text.

[18] As Hegel famously put it: 'The owl of Minerva spreads its wings only with the falling of the dusk' (Hegel 1821: Preface, 13), meaning that philosophy is not fortune-telling. Philosophers can only make sense of the world they find themselves in. Or, in other words: They can piece together the bits of knowledge available, but cannot create missing pieces.

categorization of the world around us acts like a two edged sword: by explicating what particulars have in common, the use of categories also implies the categorized particulars may differ in all respects to which the employed category – or the more abstract categories it is itself subsumed under – does not pertain (e.g. since a car is a vehicle, all particulars that are united by their 'carness', are also united by their 'vehicleness', but may differ in color, number of doors, horsepower, etcetera) (Smith 1990: 5-6).

Once the dialectician feels he has a sufficient grasp of the categories that s/he might need, concrete determination can begin. As alluded to above, Hegel's goal is to find the fundamental interrelations between everything we can claim. He provides an overview of this project in his *Encyclopädie* (1830^3, 1817^1). Thus the *Encyclopädie* encompasses all sciences in their systematic interconnectedness and thus also encompasses Hegel's more specific and detailed accounts of Logic and Society in his *Wissenschaft der Logik* (1812, 1813, 1816) and *Grundlinien der Philosophie des Rechts* (1821) respectively. Using the structure of the *Encyclopädie* as a template, these three works and the systematic-dialectical method of concrete determination employed therein will now be discussed.

In his *Encyclopädie der Philosophischen Wissenschaften* (1830^3, 1817^1) Hegel takes on the outline of all sciences and all knowledge in their interconnectedness. According to Hegel, this totality (and therefore the *Encyclopädie* itself) in turn consists of three parts, each of which is an object totality in its own right. An object totality is a part of the world whose intelligibility depends on one universal principle without which the totality cannot be thought (Reuten & Williams 1989: 16, 20-21).

The first part is the logic, the philosophy of our most abstract ideas of reality and the categories in which these are embodied. At this level of abstraction, it is impossible to point at something in the world and say: '*that* is what this category means'. Therefore, only the categories themselves and their categorial interrelations can be studied in the logic. Hence, Hegel calls this 'the science of the Idea in and for itself' (Hegel 1830^3, 1817^1: §18)[19]. This object totality is described at length in Hegel's *Wissenschaft der Logik* (1812, 1813, 1816). It is also the most abstract of the object totalities. Hence the *Encyclopädie* and the *Wissenschaft* start with the universal principle of the totality *of everything*, rather than of a particular *object* totality.[20] This universal principle is Being (Hegel 1830^3, 1817^1: §86; Hegel 1812, 1813, 1816: 82-83, 1.1A).[21]

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[19] All quotes are from the translation by Geraets, Suchting and Harris (1991), unless specified otherwise.

[20] Note that 'theorizing from the totality of everything' is not the same as what Stephen Hawking calls 'a theory of everything'. The latter is an audacious attempt to formulate a theory that would

The second part of the *Encyclopädie* is the philosophy of nature. Our knowledge of nature cannot alter its basic laws, although it enables us to use them to our advantage. Nature therefore is separate from our ideas about it.[22] This prompted Hegel to call this object totality 'the science of the Idea in its otherness' (Hegel 1830^3, 1817^1: §18). The category that first describes this otherness is Space, for the distinctions in Space are necessarily material in nature.[23] Taking Space as the universal principle of this object totality therefore ensures this realm stays separate from our immaterial thoughts (Hegel 1830^3, 1817^1: §254).

In the third part of the *Encyclopädie*, the philosophy of mind, Hegel sets out to describe that part of reality that is the result of human agency, viz. society (Hegel 1830^3, 1817^1: §18).[24] When we comprehend society, we can actively change it. Hegel therefore describes this object totality as the science 'of the Idea that returns into itself out of its otherness' (Hegel 1830^3, 1817^1: §18). In Hegel's view

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fit all physical observations and of which all current partial theories (specifically general relativity and the partial theories of gravity and weak, strong and electromagnetic interactions between particles) can be shown to be special cases (Hawking 1998: 213). The method for arriving at this theory is still mainly inductive, whereas Hegel's Systematic Dialectic is neither deductive nor inductive. Rather, Hegel analyses categories in language as labels for observations that at the same time produce observations. So for Hegel, the dichotomy between observations and theory does not exist.

[21] References to page numbers, would make comparison to translations and other editions harder. I have therefore opted for a different style of referencing (explicated in *Note on the Style of Referencing and the Use of Capitalization and Emphasis in This Work* that one can find immediately after the introduction to this book).

[22] This separateness is borne out in at least two ways in Hegel's introductory sections to part two of his *Encyclopädie.* First, he considers the whole object totality to be outside ('auβerlich') of the sensing individual ('sinnliches Individuum') (Hegel 1830^3, 1817^1: §245). Secondly, he consistently identifies the object of the Philosophy of Nature as a *Gegenstand* rather than an *Objekt*. The German word 'Gegenstand' also translates as resistance and hence is connotated as a concrete tangible object – a distinction lost in English that was crucial to Hegel (cf. Inwood 1992: 203-204). The word also emphasizes that Nature might resist our attempts at understanding and categorizing it. These remarks on Nature and the most adequate way of studying it add further weight to the arguments in this chapter for the claim that Marx's criticism was not generally true of Hegel's philosophy. In the context of Hegel's philosophy of right however, Marx's criticism is well taken in the sense that Hegel exhibits society as the resolution of the opposition between the object totality of the Logic and that of Nature. Thus, he effectively claims that the form society takes is only limited by people's capacity for thought and the natural limits to the realization of whatever they want it to be, which is admittedly optimistic and has quite a utopian feel to it (this argument is further elaborated upon in the next section of this chapter).

[23] More precisely, the study of nature primarily involves studying its spatial manifestations, which is not to say that its ultimate Essence may turn out to reside in a realm we would not traditionally consider spatial (that is, in multidimensional strings).

[24] A more elaborate description of this object totality is to be found in Hegel's *Grundlinien der Philosophie des Rechts* (1821).

the foundation of human agency is Spirit and since our Spirit is Free, the starting point for a comprehensive description of society is Freedom (Hegel 1830[3], 1817[1]: §382) and more specifically Free Will (1821: §4).

The idea is that an abstract principle such as Being, Space or Free Will by virtue of its two edged nature as a unifier of particulars, encompasses all concrete instances of it, albeit implicitly, not immediately. There are a lot of categories which are less abstract than their principle, but that are nevertheless far from concrete. For example, man is part of a family (meant in a rudimentary sense – man is not made in a factory but by two people), families are part of society, society is bound by law and morality and these, in Hegel's view, are the result of human agency and thus a product of Free Will (1821: §4). Hegel wants to work his way back through the elements in the last sentence. That is, he first wants to show the most abstract instances of the universal principle (and as *instances* these are already more concrete than the principle), next the most abstract instances of these, and so on until something can be said about concrete and tangible things.

Hegel and Marx refer to this process as a process of *Darstellung*. In ordinary German, *Darstellung* means representation or exhibition (cf. Inwood 1992: 257), but its root verb, *darstellen*, is composed of there (*dar*) and place or posit (*stellen*). Thus, it not only connotates the representation or exhibition of something that is already there, but also the positing (*stellung*) of new ideas, opinions and theories in a new context (*dar*). It is exactly this double meaning that is aimed for by dialectical thinkers, for a chief contention in dialectics is that the full meaning of categories can only be revealed when the way they are interconnected with other categories is comprehended. That is, when their meanings are seen to be mediated by the meanings of categories it is dialectically linked to. In this sense, a dialectical exhibition not only leads to a novel representation of existing knowledge but in so doing also creates new knowledge. Thus, the German term *Darstellung* aptly captures all the goals and claims of systematic dialectics. The double meaning of *Darstellung* is hard to capture in English, but it is this double meaning that I have in mind when I speak of 'exhibition'. When the systematic-dialectical exhibition needs to be contrasted with the pre-dialectical stage of appropriation or abstract determination (terms introduced above) I sometimes use concrete determination, for the concrete instances of a category are determined from an abstract universal principle in the unfolding of the systematic-dialectical exhibition. If this concrete determination can be completed, it is claimed this proves that the abstractions utilized in the exhibition are indeed suitable for understanding the object totality in question (Reuten & Williams 1989: 21-22).

Hegel makes this concrete determination by asking three questions about categories encountered in the exhibition. α) What does the category mean in total categorial isolation? How does it appear when viewed from the inside out?

Answering this question requires one to reflect on the *idea* behind the category in question that unites the many particulars the category subsumes. After all, the creation of categories, the decision to categorize this way rather than that, is the result of an act of thought. It is informed but not determined by sensory experience of the world. Thus it stresses what Smith calls its 'pole of unity' (Smith 1990: 5) (E.g. Free Will is a product of individual consciousness, which is universal and hence infinite when categorially isolated from all other categories (Hegel 1821: §5).) Next he asks himself: β) How does this category appear when viewed from the outside in? How does it express itself in the world?[25] Clearly, answering this question requires reflection on how the category *materializes* in the world. After all, if the category is adequate it must have been brought about by 'the working out of the empirical sciences on their own account' and hence a dialectical exhibition must eventually be able to return to material reality. The answer to this question brings out the 'pole of differences' (Smith 1990: 5) inherent in the category. (E.g. a real individual's Will is not really universal, but constrained within a person, which is only one of many and hence finite (Hegel 1821: §6).) So, from this perspective a category's intelligibility rides on its ideality and materiality being comprehended together. It is this comprehension that dialectical theory aims to bring about.

Thus, the answers to the questions α) and β) usually involve oppositional categories (e.g. Free Will exists only if it is also bounded). The tension between those categories needs to be γ) resolved in order to make sure that the category (e.g. Free Will) is not only a category, but has empirical counterparts. Resolving an opposition of this sort involves either showing how the one half of the opposition becomes the other half, or showing how the two halves can coexist (e.g. individual Free Will, when constrained by others is only potentially free, it is a Possibility, which can only be actualized to the extent determined in the remainder of *Grundlinien der Philosophie des Rechts* (Hegel 1821: §7)). Sometimes the condition for coexistence cannot be found immediately. In that case there often are successive stages of coexistence. The first categories in this succession only partially resolve the tension between α) and β), which is fully resolved at subsequent stages.

By asking the questions α) and β) again about the last of the categories found under γ), a new opposition will generally be found, which can be resolved again, and so on. The universal principle and the categories that arise from it

[25] Inside and outside correspond to the German terms Innere and Äußere. Hegel speaks of α) as inner reflection or reflection in itself ('innere Reflexion' or 'Reflexion in sich') and of β) as outer reflection or reflection in others ('äußere Reflexion' or 'Reflexion in Anderes') (Hegel 1830^3, 1817^1: §113-120).

dialectically are called moments. A moment is 'an element considered in itself, which can be conceptually [i.e. categorially] isolated, and analysed as such, but which can have no isolated existence' (Reuten & Williams 1989: 22).[26] It must be stressed that the moments found under α), β) and γ) all spring forth from reflecting on one and the same category from different points of view. Thus, there are three sides to every story: α) the inside story, β) the outside story and γ) the truth. The truth of a category is in the connection between both stories, in comprehending how the inside is responsible for the outside and the other way round, the way DNA and environmental factors make up an organism's phenotype. The 'inside' and 'outside' aspects of categories are abstractly distinguishable, but not concretely separable.

Each of the parts I, II and III of Hegel's *Encyclopädie* is further divided into subdivisions ('Abteilungen') 1, 2 and 3. These in turn are subdivided first into sections A, B and C and usually next into subsections a, b, and c. Finally, some of the subsections are subdivided into α, β and γ. The parts, subdivisions, sections and subsections relate to each other in very much the same way as α), β) and γ) do. Thus, part I, the Logic ('the science of the Idea in and for itself' – §18), relates to the most fundamental (structural relationships between) categories in language, i.e. it consists of categories without which the world would certainly be unintelligible, distinctionless white noise (such as Being, Becoming, the One and its Other) without however considering the application of these to the world itself. Next, part II, the philosophy of nature ('the science of the Idea in its otherness' – §18) considers how the categories in the Logic are altered when one applies them to nature, that is, how they are expressed in the world. Since this involves leaving the sphere of 'thinking about thinking', this transition opens up the possibility of misrepresentation (whose occurrence is amply illustrated in the history of science – cf. e.g. Bryson 2003), i.e. the possibility that the structure of language is not entirely isomorphic to the structure of the world (yet). In part III, the philosophy of mind, or, in Hegelian terms, the science 'of the idea that returns into itself out of its otherness' (§18), the inherent freedom of thought is reconciled with the material restrictions of nature by showing how self-conscious humanity can impact on nature to understand, create and change human society.

If we turn to the subdivisions of part I, we find it consists of 1) the doctrine of Being ('die Lehre vom Sein'), 2) the doctrine of Essence ('die Lehre vom Wesen')

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[26] Instead, the existence of a moment is mediated ('vermittelt') by the moments that were posited before it and dialectically follow from it. For Hegel, mediation ('Vermittlung') contrasts with immediateness ('Unmittelbarkeit'). In its immediateness Being is a distinctionless soup of everything and Free Will is utter and total chaos and anarchy, but when they are mediated by other categories, the prospects for the logic and the philosophy of society respectively become less daunting.

and 3) the doctrine of the Concept ('die Lehre vom Begriff'– §83).[27] The first of these doctrines comprises A) Quality (§86-98), B) Quantity (§99-106) and C) Measure (§107-111). All we can say at such an abstract level about the Quality of Being is that it consists of a manifold of indeterminate Ones upon which we can only externally and arbitrarily reflect, turning it into Quantity.[28] To get rid of the arbitrariness, a Qualitative Quantum is required: Measure (Damsma 2011 elaborates on this).

Exactly what type of categories one needs to get to grips with more determinate qualities is the subject of the doctrine of Essence. Not that any specific qualities can be invoked at such an abstract level yet, but the kind of categories required to allow for a reentrance of qualitative distinctions are identified and systematized at this level. In overview this doctrine is concerned with A) elusive, hidden Essence (how things are – §115-130), B) Appearance (§131-141) and C) Actuality (§142-159). Whilst at many occasions appearance may very well be all we got, it is only when it is mediated by some theory on Essence, that we understand the laws of self-development of the actual. So while Essence categories are applicable to objects, Essence is fundamentally elusive at the same time.

When objective, but elusive Essence is mediated by subjective thoughts on Being as a whole, in principle we have concretely applicable Concepts. Again, at the level of the Logic, the language refers to the type of category, not to any concrete embodiment of it. This final subdivision of the Logic consists of: A)

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[27] All of these translations are a bit tricky. Although the German 'Lehre' is always translated as doctrine (e.g. Geraets, Suchting and Harris 1991; Wallace 1873; Carlson 2003: 8; Inwood 1992: 268), the German term is much more neutral, for it does not carry the connotation of 'indoctrination' with it at all. Instead, it is derived from 'lernen', the German for 'learning' or – sometimes – 'teaching'. The German 'Wesen' (ibid.) refers to what you might call 'essential Being', the nature of something, as well as to unidentified bodies and beings, such as the building trade (das Bauwesen) or a God the speaker does not believe in (das Überwesen). Thus 'Wesen' necessarily implies some elusiveness. This is not the case with 'Essenz'. This important distinction is lost in English. Finally, 'Begriff' is derived from the German for understanding: 'begreifen' ('to grasp' literally). In its various translations as 'concept' (e.g. Arthur 1993: 64; Geraets, Suchting and Harris 1991; Inwood 1992: 58; Smith 1993: 29) or 'notion' (e.g. Wallace 1873) this connotation, if not lost, is at least severely downplayed, for noting or conceptualizing implies more of a dim *aware*ness, than an *understanding* of the matter at hand. In what follows, I will adopt today's convention in the Hegel literature and use 'category' to denote concepts in general and 'concept' when referring to Hegel's 'Begriff'.

[28] Hegel's positioning of his account of the quantitative within his overall dialectics implies he correctly recognized the subject of mathematics as an external reflection on many distinguishable but divisible elements (see footnote 9). As we will see in Chapter 2 this means that mathematical entities are never defined by referring to what they *are*, but only by how they relate to other entities that, when separated from this web of relations, escape definition as well.

Subjective Understanding (§162, §163-193), B) the Object (§162, §194-212) and C) the Idea (§162, §213-244). As such it indicates the structure by which Subjective Understanding, i.e. embodied (and thus Actualized) thought is reconciled with its Actual Object, i.e. its expressions in the Actual world. This reconciliation requires Ideas.

The general conceptual distinctions of the Logic are applicable to Hegel's philosophical system as a whole as well as to its subfields considered in themselves. Since the doctrine of Being consists of categories that are indispensable for all human understanding, the type of categories found in it best describe the Logic relative to the other two principal spheres. Considered in itself, the Logic of course displays a dialectic of 1) Being, 2) Essence and 3) Concept. Similarly, since the Philosophy of Nature relates to the way objective material things out there are represented in thought, it is best described in terms of an Essence structure, or the applied counterparts of Essence categories. When considered in itself, Nature again displays a dialectic of worldly expressions of: 1) Being (comprising Space and Time, Matter and Movement and Absolute Mechanics – §252, §253-271), 2) Essence (physics – §252, §272-336) and 3) Concept (organic physics or biology –§252, §337-376). Finally, and in the same vein, the Philosophy of Mind is supposed to resemble a structure of Concepts in that it reconciles thought with nature. As will be expected by now, its subdivisions relate to each other as: 1) Being (Subjective Spirit comprising Anthropology, Phenomenology of the Mind and Psychology – §385, §387-482), 2) Essence (society or Objective Spirit – §385, §483-552) and 3) Concept (or Absolute Spirit with philosophy as its ultimate Concept – §385, §553-577). Figure 1 summarizes and schematizes the above. As such, it is essentially a condensed version of the table of contents of the *Encyclopädie*.

I. The Logic
   1. *The Doctrine of Being*
      A. Quality
      B. Quantity
      C. Measure
   2. *The Doctrine of Essence*
      A. Essence (as ground of Existence)
      B. Appearance
      C. Actuality
   3. *The Doctrine of the Concept*
      A. Subjective Understanding
      B. The Object
      C. The Idea

II. The Philosophy of Nature
   1. *Mechanics*
   2. *Physics*
   3. *Biology*

III. The Philosophy of Mind
   1. *Subjective Spirit*
   2. *Objective Spirit*
   3. *Absolute Spirit*

Figure 1. An overview of Hegel's Encyclopädie

Hegel does not usually explicitly label the moments he discusses as α, β and γ in the works under scrutiny here. But, however implicit, the questions α, β and γ always linger in the background when Hegel determines the oppositions implicit in the exhibition of the moments and their resolution. In chapters to come Hegel's (and Marx's) ordering of categories is largely preserved, but it is consistently made explicit whether a moment emphasizes α) the idea in its conceptual isolation , β) its expression in the world or γ) the resolution of the tension between α and β. This means among other things, that some moments are brought to bear under a different heading than Hegel did or would have done.[29]

[29] This comment does not apply to Marx's works, because all of his – presumably dialectical – method is implied rather than explained (spawning the different interpretations of the nature of the Hegel-Marx connection alluded to in the introduction of e.g. Fraser and Burns 2000). It is hoped that the consistent application of the three questions mentioned, clarifies the exhibition and the method used. It certainly makes Hegel's method more transparent and at the same time serves to bring out the hidden implicit dialectics in Marx's *Capital*.

As the systematic dialectical exhibition of an object totality unfolds, the categories encountered and their interrelationships grow in number and in concreteness. In principle, the process terminates when categories are reached that are at one with themselves in the sense of being concrete enough for their abstract idea to be identical to their concrete manifestation. Another way to say this is to say that at this stage the form of the category is at one with its substance. When exhibiting society this happens when the dialectic can be shown to encompass concrete individuals; in the case of the philosophy of nature, it must be shown to encompass quarks or multi-dimensional strings (the latter, I am convinced, will turn out to be nature's true substance), while in the case of the Logic it must reach the category of concretely applicable concepts ('Begriffe') itself. That is, the Logic culminates in the conclusion that this type of concept is indispensable for thought and related to even more abstract categories than the category of concept itself.[30] Since all categories the dialectic employs are thus shown to depend on each other and on their ultimate material substance for each other's intelligibility, they are claimed to be necessary for the intelligibility (and concomitantly – at least from Hegel's perspective – the existence in thought) of the object totality. In other words, the totality of interrelated categories brings the inner nature of the object totality to light.[31] [32] This means that categories are only fully defined when

[30] We are so used to arriving at abstract categories by a process of abstracting from examples, that it is very hard to refrain from contemplating examples. This is particularly challenging when 'thinking about thinking', because categories there pertain to what can be thought and how, but have no empirical counterparts. Thus, e.g. the category of the concept is only concrete in that it indicates that concretely applicable categories exist in thought and can henceforth be applied in the culmination of other realms.

[31] Categories that are crucial for understanding nature and society are all exhibited as part of the Logic. Necessity ('Notwendigkeit') and contingency ('Zufälligkeit') are no exception. They are determinations at the level of I-2-C: Actuality ('Wirklichkeit') and are related as follows: α) Possibility, β) Contingency and γ) Necessity (Hegel 1830[3], 1817[1]: §145-149). When Actuality is considered in total categorial isolation of the world, it appears as an amalgam of α) Possibilities and at this level of abstraction it appears everything is Possible (cf. Inwood 1992: 197). This is expressed in the world as an amalgam of β) Contingencies ('Zufälligkeit'). I take this to mean that if we do not understand how the Possibilities of the Actual are limited by the empirical, we will perceive the empirical as merely Contingent or accidental (the latter is another possible translation of the root of 'Zufälligkeit', 'Zufällig') and the other way round. If we understand them together, however, and as they are interrelated in the whole, we understand their γ) Necessity. Necessity, in turn, is further determined as α) Condition ('Bedingung') and β) Case Matter ('Sache'). Thus, Condition appears as the first concretization of Possibility and Case Matter as the first concretization of Contingency.

[32] The point is that Hegel eschews any philosophical speculation about things for which no adequate categories exist yet. It is just too tempting to quote Wittgenstein in this context: 'Wovon man nicht sprechen kann, darüber muβ man schweigen' ('Whereof one cannot speak, thereof one must be silent') (1922: 90, 162 – the first edition of Wittgenstein's Tractatus was bilingual: the

the exhibition is complete. Before that happens, the categories have to remain flexible as we need to view them from different perspectives with each step that is taken towards concretization. One of the consequences of this is that rigorous definitions of categories by means of assumptions are essentially ruled out. However, evidently "a system" cannot be exhibited all at once. So the exhibition may require the positing of anticipatory assumptions, i.e. assumptions that anticipate later stages of the exhibition. This anticipation may be warranted on the basis of concrete empirical experience with the system under scrutiny. E.g. if we know that expanding production (i.e. *accumulation*) has always coincided with an expanding money base, we may assume that money will somehow expand with production until we have understood why this is the case. The argument for (temporary) adoption of this kind of assumption is twofold: 1) empirical reality seems to demand it, but more importantly 2) removal of the assumption would imply a collapse of a system whose dynamics we experience daily. However, a systematic-dialectical exhibition is never complete until all preliminary assumptions have been fully determined endogenously (cf. Reuten 2014: 9-10).

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In the conclusion to his *Grundlinien der Philosophie des Rechts* Hegel indicates how he thinks the methodical elements of systematic dialectics might apply to the history of societies (Hegel 1821: §341-360). But he never produced a final version of his *Philosophie der Weltgeschichte* during his lifetime - that is, beyond his lectures on the issue. Thus, as indicated in Section 1, Hegel's historical dialectics have a rather tentative and preliminary character.

In the transcripts of the lectures we have, Hegel contrasts the α) Objectivity of the State with the β) Subjectivity of the Individual and claims that originally the two were at one. This meant that laws were unnecessary, because in these small family groups and tribes people had too little self-awareness, or conversely the state too little autonomy, to meaningfully distinguish between the Will of the State and the Will of the Individual. As history progressed the two grew apart and as a result they both grew ever more self-conscious leading to a battle of the α)

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first page mentioned contains the English quote, the second one the German). In Hegel's view there is much more that can be meaningfully spoken about than in Wittgenstein's, but like Wittgenstein a century later, Hegel refuses to speak about stuff that has not yet been adequately categorized in the empirical sciences yet and thus often writes as though things that cannot be (fully) thought yet because of inadequate categorization, cannot exist either. However, it would be more accurate to say that they cannot exist in a philosophical system like Hegel's, which can only outline the theoretical Possibilities *proposed* thus far in the empirical sciences and the way language relates them to the empirical phenomena (Contingencies) *categorized* thus far for a given object totality.

'abstract Generality' of the state against the β) principle of specific Subjectivity. Out of this battle a new nexus of α) World and β) Spirit is or will eventually be born {Hegel 1823-31: 113-120}. In short, Hegel's account of world history is a tale of growing self-consciousness on the part of individuals through which they become aware of their inherent freedom. When that awareness achieves a certain height, the State has to reconcile itself with its citizens' emancipatory drive and facilitate their Freedom to the best of its ability or collapse from its internal tensions.[33] According to Hegel, the post French revolution society he lived in could *potentially* achieve that γ) Ideal State, like ancient Greece had *actually* done before (See Kedourie (1995) for a very accessible account of the historical, philosophical and other scientific influences shaping Hegel's thought and philosophical system). As such, Hegel's {1823-31} *Vorlesungen* seem to provide an *a-posteriori* justification of his decision to take Free Will as the universal principle in the realm of society.

The *a priori* justification for Hegel's starting point in this realm appears to simply be the position of the object totality vis-à-vis the other two major realms. That is, the Logic culminates in a full comprehension of our capacity for thought (and its limits), while the philosophy of nature brings about this comprehension regarding Nature. In principle then, within Hegel's system, there is no limitation to human agency creating society other than the possibilities and impossibilities inherent in Nature. Enthusiastic as he was about the architecture of his all-encompassing philosophical system it may have not dawned on Hegel when he wrote the *Grundlinien*, that history might bring about those limitations. Foreshadowing his historical dialectic in the conclusion to his philosophy of society (Hegel 1821: §341-360), it is only natural he did not completely rethink all the books he had written and thus ended up presenting historical dialectics to fit his systematic dialectics rather than the other way round, like Marx did as we will now see (cf. Reuten 2000).

[33] I use 'emancipatory' rather than 'liberal', because the political connotations that the term liberal has acquired over the years must be avoided here at all costs.

3. Marx's Comments on Hegel, their Implications and Marx's Twist on Hegel's Dialectical Method

Marx's early writings up to 1848, at age 30, can be characterized as critical reviews of important historical, economical and philosophical works. During the time Marx studied in Berlin (1836-1841), Hegelianism was the dominant philosophy there and although Marx had been a follower of Kant and Fichte throughout his studies, he grudgingly but completely converted to Hegelianism in 1841 (McLellan 1973: 28-29). So it is hardly surprising that at least three of his early manuscripts explicitly target (aspects of) Hegel's work.

First, Marx's *Zur Kritik der Hegelschen Rechtsphilosophie: Kritik des Hegelschen Staatsrechts* (written in 1843) is a scholarly work in which he meticulously fleshed out Hegel's ideas on the relation between the state, civil society and the individual and replaced them with his own.[34] His main critique is that Hegel's take on the matter is far too harmonious. After all, Hegel's starting point for his philosophy of society was Free Will. Given further that one of the fundamental premises of systematic dialectics is that tensions between α) universal categories and β) their embodiment (viz. real people) must be resolved as best as the universal principle allows, a rather harmonious depiction of society was bound to follow. Although this critical study of Hegel's Philosophy of right never got published in Marx's lifetime, its introduction got a place in the February 1844 issue of *Deutsch-Französische Jahrbücher* that Marx and Ruge edited together.[35]

Secondly, in his *Kritik der Hegelschen Dialektik und Philosophie überhaupt* (written in 1844, but never published in Marx's lifetime), Marx establishes his most comprehensive critique of Hegel's way of thinking. In it, he mainly discusses Hegel's *Phänomenologie*, but also targets the *Logik* and the *Encyclopädie*. Central to his critique is the concept of *estrangement* ('Entfremdung'). Marx claims that Hegel, by focusing on abstractions and their interrelations alone, estranges their form as thoughts from the thinking human being in which they must be embedded and thus denies them the possibility of gaining empirical content. As a result, Hegel's philosophy could never really

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[34] Surprisingly, the penguin edition of Marx's Early Writings (1975) translates the mentioned title as: *Critique of Hegel's* ***Doctrine*** *of the State*. This is wildly inaccurate, because the German word 'Recht' translates as 'right'. The use of the word Doctrine in this context would suggest that the German title was: *Kritik der Hegelschen Staats**lehre***, which clearly was not the case.

[35] The title for this introduction is adequately translated in *Early Writings* (1975) as: *A Contribution to the Critique of Hegel's Philosophy of Right: Introduction*, but since the title of the text to which it was supposed to be an introduction was not adequately translated (see footnote 34), one might easily get the impression that the two texts are more or less unrelated.

bring about the self-consciousness Hegel aimed for, but would result in self-denial instead. Another consequence was that the Hegelian system and the way it views abstractions – as products of thought only – could not allow for real alienation, i.e. for abstractions taking up a life of their own (as we will see in Chapter 3 a case can be made for the claim that the latter is going on in Capitalism).

The solution to this problem is 'to abandon abstraction and to take a look at nature, which exists *free* from abstraction' {Marx 1844: 398}, but doing so is – as we have seen in the previous section – to introduce an opposition between the 'Idea in and for itself' and the 'Idea in its otherness'. Obsessed as Hegelians are with resolving this kind of opposition, they cannot leave it at that and thus move on to resolve the tension between the two object totalities in the realm of society, or in their terms 'the Idea that returns into itself out of its otherness' and thus effectively deny the possibility of the thinking subject misrepresenting his object as well as the possible existence of real conflicts between the (interests of) several groups of people.

These critical remarks notwithstanding, Marx also devotes a couple of pages of this manuscript to 'the *positive* moments of the Hegelian dialectic within the determining limits of estrangement' {1844: 395}.[36] He puts it quite succinctly and clearly himself:

> 'Therefore, in grasping the *positive* significance of the negation, which has reference to itself, even if once again in estranged form, Hegel grasps man's self-estrangement, alienation of being, loss of objectivity and loss of reality as self-discovery, expression of being, objectification and realization. In short, he sees labour – within abstraction – as man's act of *self-creation* – and man's relation to himself as an alien being and the manifestation of himself as an alien being as the emergence of *species-consciousness* and *species-life*.' {Marx 1844: 395}[37]

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[36] The German text for 'within the determining limits of estrangement' reads 'innerhalb der Bestimmung der Entfremdung' {1844: 583} or 'within the determination of Estrangement'. That is, the word 'limit' is not to be found in the German text. 'Bestimmung' means both destination and determination and Hegel exploits this double meaning to the fullest, for in his philosophy, the determination of the concrete, means coming closer to the completion of the philosophical system and so is akin to arriving at a destination. The translators, Livingstone and Benton, have apparently tried to regain some of this double meaning by adding the word limit so as to convey the fact that the destination is already inherent in the abstraction, thus limiting the range of possible further determinations.

[37] Some points have to be made on this translation, particularly regarding the following phrase: 'Hegel grasps man's self-estrangement, alienation of being, loss of objectivity and loss of reality as self-discovery, expression of being, objectification and realization.' The German reads: 'Hegel faßt […] die Selbstentfremdung, Wesensentäußerung, Entgegenständlichung und Entwirklichung des Menschen als Selbstgewinnung, Wesensäußerung, Vergegenständlichung, Verwirklichung' {Marx 1844: 583}. The use (in compounded words) of 'Wesen', 'Gegenstand' and 'Wirklich' are

Thus, Marx's well-known concept of alienation of the workers from their product was at least partially inspired by Hegel (cf. Arthur 1986: 59-74; cf. Murray 2014). According to Marx however, Hegel failed to see how the implication of his philosophy indicated the actual existence of unresolved tension and conflict. Instead:

> '[B]ecause the conception is *formal* and *abstract*, the supersession of alienation becomes a confirmation of alienation. In other words, Hegel sees this movement of *self-creation* and *self-objectification* in the form of *self-alienation* and *self-estrangement* as the *absolute*, and hence the final *expression of human life* which has itself as its aim, is at rest in itself, and has attained its own essential nature' {Marx 1844: 396}.

Thirdly, in *Die Deutsche Ideologie* {1846} Marx – in collaboration with Engels – criticizes the fundamental premises of the German historical tradition, of which, according to the authors, Hegel was the last proponent. They write: 'The Hegelian philosophy of history is the last consequence, reduced to its "finest expression," of all this German historiography, for which it is not a question of real, nor even of political, interests, but of pure thoughts' {1846: 60}. Marx and Engels go on to reconstruct history-writing along the well-known lines of historical materialism, where the material structure of haves and have-nots as identified through their relations of production, is paralleled by a superstructure (supporting institutions, including supporting ideas about reality).[38] These material inequalities cause class struggle in which the have-nots fight for improvement of their material well-being, while the haves try to protect the status quo. If the fight is successful, a new material structure is established, accompanied by new relations of production and a new superstructure (see e.g. Shaw 1991[2], 1983[1]). Clearly, this idea of class struggle would be inconsistent with

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clearly linked to Hegel's use of these terms. In Hegel, 'Die Lehre vom Wesen' (usually translated as 'the doctrine of Essence') is the second subdivision of the Logic, while 'Die Lehre vom Sein' ('the doctrine of Being') is the first, so the use of 'being' in the translation obscures to which part of Hegel's philosophical system Marx was referring. Furthermore, the German 'Objekt' ('object') is used by Hegel in a more general sense than 'Gegenstand', which also means 'resistance', and hence is reserved by Hegel for tangible objects (cf. Inwood 1992: 203-205). Finally, 'Wirklich' (usually translated as 'actual') is reserved for the last section of Hegel's 'doctrine of Essence' and denotes reality as understood through concepts ('Begriffe') as opposed to reality as it directly confronts us (cf. Inwood 1992: 33-35, 93-95).

[38] The method of historical materialism largely got shape in the writing of *Die Deutsche Ideologie*.

a Hegelian exhibition of society starting from the universal principle of Free Will. [39]

The critical stance Marx assumes regarding most of the (German) historical, philosophical and scientific traditions generally, also implies he was less confident than Hegel that 'the working out of the empirical sciences on their own account' {Hegel 1825-26: 176} would lead to adequate categories. But if a dialectician cannot trust the categories inherited from centuries of scientific exploration and research to be adequate, simple appropriation of those categories cannot be sufficient either. It is thus understandable that Marx placed much more emphasis on pre-dialectical exploration ('Forschung') than Hegel.[40] As a result, *Capital* does not only exhibit the interrelationships between existing categories pertaining to capitalism dialectically, but also develops new categories to describe (aspects of) it from empirical material. However, as most of Marx's method in *Capital* is implied rather than explicated, the distinction is rather implicit most of the time.[41]

So it appears that Marx's critique of Hegel's supposed idealism pertained to 1) the uncritical stance Hegel displayed regarding his 'stage of appropriation' (cf. page 11), 2) his deliberate and overt estrangement of the abstract form of

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[39] Schumpeter's critique of Marx mostly pertains to his historical dialectics. He essentially claims that a historical materialist account of history oversimplifies it and is likely to lead to a sort of tunnel vision in which disconfirming facts are no longer perceived. These problems are even worse when prophecies are based on such a simplification (Schumpeter 1954[4], 1943[1]: 45-58). Although *systematic* dialectics has a lot to offer especially for disciplines studying open systems, i.e. without recourse to laboratories or controlled experiments (such as e.g. economics and sociology), Schumpeters reservations are well taken regarding *historical* dialectics. One of the problems with the latter is that conceptual meanings, dialectical categories and concomitantly their interrelationships evolve in historical time. State of the art conceptualizations and language however reflect current knowledge rather than current practice, so a systematic dialectical exhibition of the logic of language can foreshadow the logic of future systems to some degree. But to describe the systematicity of history and prophesize the future on the basis of a *historical* dialectic requires a universal principle that is somehow itself immune to history's influence on language and it is not clear how one is to ascertain this for any category, be it material inequalities and concomitant class struggle (as with Marx) self-consciousness (as with Hegel) or a different category altogether.

[40] Given the fact that most economists before, during and after Marx's day were ardent proponents of capitalism rather than neutral observers, his distrust was probably especially warranted regarding the categories that political economists up to his day had come up with. This is still true today.

[41] Marx's famous schemes of reproduction (that are elaborated upon in Chapter 3 and whose embeddedness in systematic dialectics is strengthened in Chapter 4) are a case in point. Though my analysis shows that their main structure might be conceived of through contemplating the systematic dialectical exhibition preceding it, Reuten (1998) convincingly argues that Marx conceived of his schemes as an exploratory exercise.

abstractions from their content and the person thinking them and, relatedly, 3) the categories he used to describe society with in his *Grundlinien der Philosophie des Rechts* (1821). There is little or no evidence that Marx disagreed with the epistemological premise of Hegel's systematic dialectics that all that can be known is known in language. What he did explicitly dismiss was the Hegelian premise that the tensions between the realm of pure thought (as exhibited in Hegel's Logic) and that of Nature should necessarily be resolved in a philosophy of society. It is plausible that Marx's dismissal of the universal principle Hegel chose as the starting point of his systematic-dialectical exhibition of society (and concomitantly much of its utopian result), was one of the consequences of this critique.[42] As indicated, Marx's critique of Hegel's utopian "application" of the principles Hegel 'discovered' in his *Grundlinien* to the history of societies is in turn related to his critique of its starting point.

Eleven years after Marx formulated his critical remarks on Hegel, he wrote the outline for his own systematic-dialectical account of the society he lived in: *Grundrisse der Kritik der Politischen Ökonomie* (usually referred to as the *Grundrisse*). Marx most probably did not intend to publish this manuscript and deviated significantly from it when writing *Capital*. Either way, its introduction contains the only explicit remarks Marx ever made on 'The Method of Political Economy' {Marx 1857-58: 100-108}, which, according to Marx, should clearly be a type of systematic-dialectics. Marx writes for instance:

> '[I]f I were to begin with the population, this would be a chaotic conception [*Vorstellung*] of the whole, and I would then, by means of further determination, move analytically towards ever more simple concepts [*Begriff*], from the imagined concrete towards ever thinner abstractions until I had arrived at the simplest determinations. From there the journey would have to be retraced until I had finally arrived at the population again, but this time not as the chaotic conception of a whole, but as a rich totality of many determinations and relations.' {Marx 1857-58: 100}

In this quote Marx clearly discusses both the 'stage of appropriation' and that of concrete determination using Hegelian terminology.

He goes on to point out that this method leads the concrete to appear 'in the process of thinking […] as a process of concentration, as a result, not as a point of departure, even though it is the point of departure in reality and hence also the

[42] See also Smith (2014). Another consequence might be that systematic dialecticians aspiring to carry on the dialectical work that Marx started, should accept the fact that some necessary moments and tendencies in a truly Marxist dialectic cannot be concretely grounded at the level of necessity, leading to e.g. contingent business cycles as an expression of unresolved (and unresolvable) conflicts at the heart of capitalism.

point of departure for observation and conception' {1857-58: 101}. He then criticizes Hegel for losing sight of this empirical point of departure: '[i]n this way Hegel fell into the illusion of conceiving the real as the product of thought concentrating itself' {1857-58: 101}. In effect then, Marx reemphasizes the importance of exploration ('Forschung') for systematic dialecticians here.

Next, Marx explains why a systematic dialectical exhibition of some social order is always historically specific. He does this by showing that categories like labor or capital, though they can be conceived of as transhistoric concepts, have different meanings and connotations (and concomitantly, a different place within a dialectical exhibition) in different historical societies. In the same vein he criticizes the political economists he read for seeing 'bourgeois relations in all forms of society' {Marx 1857-58: 105}. Thus, Marx (re)emphasizes the importance of understanding the historical era one wants to write about, before starting work on its systematic-dialectical exhibition. In conclusion, Marx points out that the order of historical emergence of categories may differ significantly from their systematic-dialectical ordering and that historical and systematic dialectics must therefore be clearly distinguished:

> 'It would therefore be unfeasible and wrong to let the economic categories follow one another in the same sequence as that in which they were historically decisive. Their sequence is determined, rather, by their relation to one another in modern bourgeois society, which is precisely the opposite of that which seems to be their natural order or which corresponds to historical development. The point is not the historic position of the economic relations in the succession of different forms of society. Even less is it their sequence 'in the idea' (Proudhon) (a muddy notion of historic movement). Rather, their order within modern bourgeois society.' {Marx 1857-58: 107-108}.

There is hardly any disagreement that Marx followed Hegel's Logic closely while writing the *Grundrisse*. The most elaborate study of how the two are related was conducted by Mark Meaney (2002). Others that have investigated how the Grundrisse draws upon Hegel's Logic include Arthur (2010, cf. e.g. 2003a), Postone (e.g. 2003), Bell (e.g. 2003), Fineschi (e.g. 2005) Uchida (e.g. 1988) and many others (many of whom contributed to Bellofiore, Starosta and Thomas (eds.) (2014), which is entirely devoted to critical interpretations of the Grundrisse). There is a wider variety of opinions as to whether Marx was still faithful to the systematic-dialectical method when writing Capital. At any rate, apart from the comments he made in the introduction to the *Grundrisse* (as discussed above), he was hardly explicit about his method. So in order to substantiate the pivotal claim for this work that Marx applied the systematic-dialectical method in his *Capital* as well, I will now review the work of a number of important scholars that have convincingly argued in favor of this thesis.

4. Commentators on and Studies of Marx's Dialectics

There are many contemporary Marxists that hold that Marx's theory of Capitalism was essentially conceived of by means of a critical appropriation of Hegel's systematic dialectical method by Marx (e.g. Arthur, Albritton, Carver, Fineschi (cf. e.g. 2005), Meaney, Murray, Postone, Reuten & Williams, Sekine (most notably his 1997), Smith, Uchida and many others, including myself).[43] In defense of their position a lot of them (e.g. Arthur 2004: 79; Sayers 1990; Carver 1976: 65; Murray 2014) quote the following excerpt of one of Marx's letters to Engels:

> 'What was of great use to me as regards *method* of treatment was Hegel's *Logic* at which I had taken another look by mere accident […]. If ever the time comes when such work is again possible, I should very much like to write 2 or 3 sheets making accessible to the common reader the *rational* aspect of the method which Hegel not only discovered but also mystified.' (Marx 1858a)

Since Marx never wrote these sheets or (apart from the few pages in his *Grundrisse* that were discussed above) anything else on method, we have to infer the method Marx used in *Capital* from its result, i.e. the structure of *Capital* and the reasoning in that work. This is far from straight-forward because, among other things, Marx alternates between reporting exploratory research and furthering his dialectic without clearly indicating which is which. Another complicating factor is that *Capital* is much less systematically structured using α, β, γ -like headings than Hegel's texts.

As a result, opinions among adherents of a systematic dialectical reading of Marx's *Capital* differ as to exactly how pronounced Marx's break with Hegel was and as to what was involved in Marx's variant of Hegel's method. The different commentators all hold that Marx drew on the categories in Hegel's Logic (the main structure of which was discussed in Section 2 above) while writing *Capital* (and the excerpt of Marx's letter to Engels reproduced above is often quoted as evidence). The debate focuses on what parts of this work Marx most prominently drew on and how.

Arthur (and Sekine, most elaborately in *An outline of the dialectic of capital* (1997)) establish(es) 'in detail the parallels between the categories of Hegel's Logic and the social forms exhibited in Marx's Capital' (Arthur 2004: 10). In this context, Arthur speaks of reinstating and reconstructing the 'nine-point plan'

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[43] Most or all of the publications of these authors either explicitly discuss this premise or the merits or implications of working with it. Hence, I could mention all of their works in this connection. Since this is rather impractical, I have mentioned only publication dates for the most important works of authors that I do not discuss elsewhere in this book.

'organised in Hegelian fashion according to the three moments of 'the Concept'' (2002: 47-48) that Marx provided in his *Grundrisse*. Although 'Marx himself seems to have given up his nine-point plan almost immediately […] it continued to inform his thinking. Below I reinstate it and reconstruct it' (Arthur 2002: 48). So, even though Arthur states that Marx decided against explicitly mapping the categories in Hegel's Logic onto Capital (for Marx 'seems to have given [it] up'), he holds that the mentioned parallels are nevertheless implicitly there in Capital ('it continued to inform his thinking'). In effect then, Arthur iterates between pointing out what *is* there in *Capital* and developing what *could* also have been there by reconstructing *Capital*'s text. Since my aim here is to provide a reappraisal of Marx's stance on method, I have to be careful to confine myself to Arthur's appraisal of Marx's method, rather than his own reconstruction thereof when drawing on his work.

According to Arthur the implicit parallels can be outlined as follows: 'The movement from exchange to value parallels his Doctrine of Being; the doubling of money and commodities parallels the Doctrine of Essence; and capital, positing its actualization in labor and industry, as absolute form claims all the characteristics of Hegel's Concept' (Arthur 1993: 65). Thus, Arthur views the *exchangeability* of commodities as their Quality, the ratio of exchange determined in the *bargain* as their Quantity and *value in exchange* as their Measure (Arthur 1993: 73-77, 87).

Value is an Essential condition for commodities' *exchangeability*, but since it is not an inherent property of commodities, it Appears only fleetingly in the act of *exchange* unless *money* Actualizes it (which it must in the face of generalized exchange) (Arthur 1993: 78-82, 87; Murray 1993; Murray 2014). Given *money* the (Concept of) *price* can be Subjectively determined quite independently of any individual bargain. Thus, *money* first and foremost functions as a *measure of value*. However, this value is only Objectively realized when sales actually commence, thus allowing the seller to buy the commodity s/he actually desired in the first place. This gives us the circuit of Commodity (C) – Money (M) – different Commodity (C'). Secondly then, money is a *means of circulation*. Thus, this circuit is at a constant risk of breaking down. The Idea of money is fully developed when it actualizes itself as the *end of exchange*, so that we end up with the circuit M – C – M' which determines *capital* as 'money which begets money' (Marx 1890[4], 1867[1]: Ch. 4: 170; 1867F: 256). [44] Thus, in its abstraction *capital* is posited as self-valorizing (Arthur 1993: 82-84, 87; Murray 2014).

It takes Marx only two subdivisions in a space comprising less than a fifth of *Capital I* to make the move from *exchange* to *capital* outlined above. Arthur

[44] What the letters behind the year of publication stand for and why they are used is explicated in the *Note on the Style of Referencing and the Use of Capitalization and Emphasis in This Work.*

therefore concludes that the dialectical progression in the rest of volume I and volume II and III is best represented as a dialectic of Concepts (Arthur 2002: 47). This is not to say that Marx explicitly represents it as such, but that according to Arthur it would be better to. At the same time, Arthur's comments regarding 'the nine-point plan informing Marx's thinking throughout the writing of *Capital*' imply that in Arthur's view, Marx too was writing these parts of *Capital* with this dialectic in mind. More specifically, since according to Arthur capital can already be considered as the Idea of money, most of the dialectical progression in *Capital* must be concerned with Ideas in the Hegelian sense. These, he writes, are best represented in terms of the contrast between Universality ('Algemeinheit') and Particularity ('Besonderheit') and its resolution in Individuality ('Einzelnheit') (Arthur 2002: 47). These terms supposedly represent how the three volumes of *Capital* relate to each other as well as how each volume is organized (Arthur 2002: 48-49). Thus, the distinctions between Universality, Particularity and Individuality are applicable to *Capital* as a whole as well as to each of its subfields considered in themselves in much the same way as the general conceptual distinctions of the Logic are applicable to Hegel's philosophical system as a whole as well as to its subfields.

However, Arthur is very critical of Hegel. He accuses him of thinking that 'the Idea creates Nature' (Arthur 2003b: 195). Clearly, if this is taken to mean that the world will automatically conform to whatever we think about it; there is no need whatsoever to do any empirical research adjusting our ideas to the world.[45] This type of upside-down ontology *may* have *some* relevance for 'thinking about thinking', that is at the level of the Logic, but it is unlikely to be applicable to Nature (at level II) or the Mind (at level III) (Arthur 2003b: 195-196). But Capitalism is an unlikely system that allows pure abstract thought (*value*) to gain material reality (as *money*). Hegel's Logic, then, is applicable to Capitalism, only because capitalism creates an inverted reality in which thought can indeed be said to preside over matter (Arthur 1993: 64). However, 'Marx never succeeded in making clear to himself just why Hegel's logic was so relevant to the dialectic of

[45] Note however, that it should *not* be taken to mean this. As put, the statement caricatures Arthur's position regarding Hegel's ontology in order to convey Arthur's criticism on Hegel clearly and succinctly. Arthur does not literally claim that Hegel held that 'the Idea creates Nature', as though the Idea is just God (and Hegel a very devout Christian) in a philosophical guise. The point Arthur makes is rather that the former has an ontological priority over the latter. So from Arthur's point of view, there is no real dialectic in Hegel between the world and the categories describing it. One might say that Arthur portrays Hegel as holding that the world will conform more to what we think about it (i.e. the way we have categorized it) than the other way round. The need for empirical research adjusting our ideas to the world is thus limited to the world's vast array of contingencies: its fundamental determinations are immutable. I am very grateful to both Tony Smith and Christopher Arthur for clarifying those subtleties.

capital' (Arthur 2002: 47). This argument, says Arthur, is his own innovation (2003b: 195).

As long as the inverted reality of capitalism is considered in itself, the Idea of *capital* in general can become self-subsistent relative to *many capitals.* But as soon as this realm is left and one tries to incorporate concrete people – and not just their value-expressions (such as wages and productivity) – in the system, problems arise, because people may not want to be treated as another means of production. So they may rebel (cf. Bellofiore 2008), or not: that is a wholly contingent matter. Either way, capitalism requires quantitative expressions of the value of its produce as well as the value of its inputs (like wages and productivity) to enable it to resocialize its produce and mobilize its inputs. Thus, it creates a society, whose primary focus must be on exchange values rather than use values, leading to commodity fetishism and worker alienation. These problems are similar to the problems Hegel runs into when he wants to make the transition from the Logic to the Philosophy of Nature. That is, Nature is independent of thought and hence it may 'rebel' against our classifications, just like labor may rebel against its treatment as a determinant of value only (Arthur 2003b: 196-197). Of course, if Nature 'rebels' we can only concede that our classifications were wrong, whereas a labor rebellion is more likely to result in labor being subdued again by any (contingent) means necessary. In other words, in response to a labor rebellion capitalism's basic elements (i.e. humans) are likely to be forced back into their mold rather than the mold being adjusted to the natural behavior of its basic elements (which is the only way to go when these basics elements are electrons, quarks and the like).

In short, Arthur argues that 'Marx was on the right track in borrowing logical categories from Hegel' (2002: 47), because Hegel's upside down ontology is on a par with the inverted reality of Capitalism.[46] Thus, drawing on a 'systematically muddled' (2003b: 196) philosopher enabled Marx to correctly exhibit Capitalism as an inverted reality. The upshot of this is that, by staying very close to Hegel regarding his *method* of exhibition, Marx actually dismissed the *content* of Hegel's philosophical system.

Smith's reading of *Capital* is much more favorable to Hegel. He does not grant that Hegel was unconcerned with empirical reality. On the contrary, he claims that Hegel and Hegelians as well as Marx and Marxists must appropriate their categories from elaborate empirical studies before dialectical representation can (re)commence. As we have seen, Marx and Marxists refer to this preliminary

[46] The view that the applicability of Hegel's Logic to Capitalism is due to the latter's ontological features is also apparent in Bell (e.g. 2003) and Sekine (e.g. 2003).

empirical kind of research as exploration ('Forschung') (Reuten 2000: 143).[47] So, first there is the world, second our preliminary partial categorization of it, and only when our empirical studies of a field are exhausted can we flesh out how these provisional categorizations are interrelated using systematic dialectics (Smith 1990: 3-8). But even then, the systematic dialectician, when stuck, might need to revert to exploratory types of research in order to gain a better understanding of his categories, and concomitantly their systematic dialectical interrelationships. So Hegel did not simply think the world would eventually conform to our ideas, but rather that, when we have done all we can to bring our ideas in agreement with reality, the ideas that can be shown to be systematically interrelated stand a greater chance of approximating the truth than those that resist efforts at systematization. Consequentially, Smith does not entirely reject Hegel's Philosophies of Nature and the Mind, although he is critical of a lot of its content. But, other than Arthur, he does not dismiss these philosophies for being constructed to fit a normal reality on the basis of an upside-down ontology applicable to the inverted reality of capitalism only.

At the same time, Smith agrees with Arthur that capitalist abstractions are rather peculiar in that they are both real and ideal (Smith 1990: 40-41, 93-94), but in his opinion this is not the only type of abstraction susceptible to a dialectical treatment.[48] Thus, whereas Arthur thinks that Hegel's Logic is useful only as a guide to the exhibition of capitalism, Smith argues that the whole of Hegel's system (as laid out in his *Encyclopädie*) has some merits of its own, for example with respect to properly positioning Marx's *Capital* vis-à-vis other scientific fields.[49] As a social theory, Marx's *Capital* presupposes subjective thought and

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[47] What Hegel and Hegelians call the method of the understanding ('die Methode des Verstandes') can be part of this exploration of concepts (cf. Hegel, 1817^{1}, 1830^{3}: §259), but is much narrower in scope for – as Tony Smith once pointed out – it only refers to a method in which the object of investigation is divided into separate things (or parts) in external relationships to each other, while Marxian exploration encompasses other forms of concept formation as well (such as predicate development through mathematical modeling (cf. Hausman 1992) and concept formation by pattern recognition, trying out analogies, etc.).

[48] Smith has clarified his position further by pointing out that it is the uniqueness of this peculiar type of abstraction in world history that required Marx to adjust Hegel's dialectical method accordingly. So, as far as Smith is concerned, Hegel was not necessarily wrong, he was just dealing with less peculiar content (and categories describing it) than Marx was. Since these differing contents must be and are reflected in differing logical forms, any conceptual mapping of Hegel onto Marx – such as developed by Arthur – should be ruled out in principle. Of course, as explained in the main text, if most of the content of Hegel's philosophy is dismissed, Arthur's conceptual mapping is perfectly reconcilable with the statement above concerning capitalism's peculiarity and uniqueness in world history.

[49] Readers that are familiar with Smith's (1990) book and his work generally might object that he considers capitalism a structure of Essence only and deems the logic of the Concept relevant only

malleable but essentially unchanging Nature and thus falls entirely on level III, the Philosophy of Mind.[50] So, relative to science as a whole, Concept categories are applicable here (Smith 1990: 18). Within this Philosophy however, the study of society belongs to level III-2. So, relative to other fields that study humans, be it the human mind (III-1) or human expression (III-3), Smith argues that Essence categories are most applicable. 'However within the realm of objective spirit [III-2] "civil society" is a determination on the level of ethical life (*Sittlichkeit*) [III-2-C] as opposed to abstract right [III-2-A] and morality [III-2-B]. As such it is a structure to which concept categories are applicable' (Smith 1990: 18). But at the level of ethical life itself, civil society (III-2-C-b) stands over and against the family (III-2-C-a), and it is not out to harmonize the two factions if conflicts were to arise. In Hegel's opinion, the latter is the task and the raison d'être of the State (III-2-C-c). Moreover, categories like *value*, *money* and *price*, though central to capitalism, in the last instance have very little to do with the imperatives for human survival (for – as the Cree Indian prophecy has it – 'money cannot be eaten').[51] So although capitalism Appears to be 'all about the money', this Appearance serves to hide and mystify the Essence of human sociality (i.e. safeguarding human survival). So, in the last instance, capitalism, as a form of civil society who's Appearance cannot be reconciled harmoniously with its Essence, is best described in terms of a structure of Essence (Smith 1990: 18).[52]

As far as his *mode* of representation is concerned, Smith opts for a much more general scheme than Arthur. Instead of looking for parallels between the categories in Hegel's doctrine of Essence and Marx's Capital, he exhibits the dialectic of Capital in terms of a movement from unity to difference to unity-in-difference. An abstract category unifies a multitude of particulars. Some stress what the particulars have in common (their unity), some what sets them apart (their difference) and some explicate both together (unity-in-difference). As one

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to understanding communism and since we do not live in a communist society yet the logic of the concept does not have many merits regarding the understanding of present-day society. All this is not disputed here and I do not see how saying that the whole of Hegel's system has *some* merits of its own could be considered inconsistent with Smith's opinions as to the merits of Hegel's system regarding the understanding of capitalism as they were represented above.

[50] Smith does not phrase it like this, but the point made is implicit in e.g.: 'Since our main interest is the influence of Hegel on *Capital*, the level of objective spirit is where we must focus' (1990:15)

[51] In full, the mentioned prophecy runs: 'Only after the last tree has been cut down, only after the last river has been poisoned, only after the last fish has been caught, only then you will find that money cannot be eaten.'

[52] Murray makes essentially the same point when he argues that 'domination by abstractions' as 'a theme that lies at the heart of *Capital*', implies that the realm, or object totality, of capitalism is best understood in terms of Essence logic (1993: 45; cf. Murray 2014).

moves from unity to difference and on to unity-in-difference the structure becomes more complex and the categories employed more concrete (Smith 1990: 5-6). Thus, these headings describe the general characteristics of *every* abstraction, not just of abstractions that belong to a certain Doctrine in the Logic. By implication, when a scheme like Smith's is adopted, whether there are clear parallels between Hegel's Logic and Marx's Capital or not, is immaterial to the mode of exhibition.

Although Hegel's Philosophy of society is on the same plane as Marx's (from Smith's point of view at least), their content is very different. In contrast to Marx, Hegel is hardly concerned with the material conditions of production, but rather with the articulation of α) an individual's Free Will, given that its expression is limited by β) other people's Free Will and thus is a γ) Possibility only (Hegel 1821: §4-7; cf. Hegel 1830[3], 1817[1]: §382, §487). In overview, this possibility is actualized as A) universal abstract Right (of which property right is the most prominent constituent), B) individual Morality and C) Ethical Life ('Sittlichkeit'). Hence it analyzes the political and ethical dimensions of a truly liberal society, rather than the extent to which the society we actually live in lives up to this ideal. Marx's starting point, by contrast, is his observation that capitalist specialized production, predicated on a historically given division of labor, can only work when inputs and outputs are generally exchanged in the economic domain.[53] Hence, on close inspection, the *exchange relation* appears to be the true starting point for his systematic dialectical exhibition of capitalism (Arthur 1993: 72; Smith 1990: 67-68). Though this starting point appears to be much more historically specific than free Will, both Hegel and Marx contend that 'philosophy is its own time apprehended in thoughts' (Hegel 1821: 15; cf. Smith 1990: 4; cf. Smith 2003: 187). To Marx and Marxists, it is the task of historical materialism to distinguish between 'transhistorical concepts' that belong to *all* times, and 'historically specific categories' that belong to the *theoretician's* time (in Murray's (1988) terminology these are called 'general abstractions' and 'determinate abstractions' respectively) . Systematic dialectics should appropriate the latter if it is to represent 'its own time' adequately (Reuten 2000: 141). Though Hegel and Hegelians are less outspoken about this distinction and consequentially less adamant about the proper type of categories to use, they too make use of both types of categories.

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[53] I purposely use the term 'capitalist specialized production' (or specialization for short) where most people would just speak of division of labor in an Adam Smithian (1776) sense, because theoretically there can be division of labor, predicated upon specialized laborers, without the units of production also specializing in specific products. Thus, division of labor is only a necessary, but not a sufficient condition of existence of generalized exchange.

Given specialization, one's produce is bound to differ from one's means of subsistence, so people *must* enter into exchange relations and there is no guarantee that they would have done so on their own accord anyhow. [54] This is why Marx's starting point allows for negative results, like exploitation, and Hegel's does not. Either way the philosophies at this level (III-2) investigate aspects of society that stand over and above individuals and potentially curtail their Freedom. With Hegel, individual's actions are curtailed by the need to be at least a little considerate of other people's freedoms and rights (in order to protect your own), whereas with Marx (i.e. in capitalism) individual freedom is thwarted by the imperative to engage in exchange in order to stay alive.

On close inspection, Smith and Arthur seem to concur that Capitalist societies are characterized by structures out there that individuals are dependent upon for their survival. Hence, they have very strong incentives to mechanically play by its rules as though market forces were nothing short of forces of nature.[55] But their convictions are based on a very different reading of both Hegel and Marx. For Smith, the seemingly inescapable nature of market forces is borne out by the fact that capitalism in the last instance is best represented as an Essence structure, while Arthur claims that *Capital* – and systematic dialectics generally – can only grasp the interrelations between the materialized abstractions characteristic of capitalism, but is otherwise incapable of making sense of real things and people. So capitalism can only work to the extent that it succeeds in materializing the abstractions it is predicated upon in the world. Because people are not materialized abstractions, the part they play in this process is capitalism's Achilles heel.

Smith's argument points to a strong parallel to the Philosophy of Nature (level II), because nature is the Essence structure *pur sang*. This parallel is relevant for the mere possibility of quantification. One of the reasons quantitative methods are successful in the natural sciences is that volition and subjectivity are neither present nor assumed; so that behaviors are law-like and subsuming a particular phenomenon under a law is considered satisfactory as an explanation. Since in

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[54] This is what distinguishes Marx not only from Hegel, but also from the economic mainstream. Both Hegel and mainstream economists contend that individuals enter into a bargain, because they feel that the goods they will have after the exchange will make them happier than the goods they originally possessed. The possibility that some enter the exchange relation with nothing to exchange but themselves or may only have command of inedible commodities is thus abstracted from. However, if either predicament is yours, you must exchange at any cost or die from starvation while trying. Thus, pretty much anyone who does not produce or otherwise commands food can only afford desire and happiness after the necessary exchanges have been made. This inexorable logic is missing from Hegel's and mainstream accounts alike.

[55] The very terminology of market forces, equilibrium etc. that free market apologetics are fond of using, implies a similar inescapability.

Marx's system individual humans are dependent for their survival upon capitalist relations and concomitantly on obeying its value driven imperatives, quantitative methods are potentially just as adequate for the study of this particular mode of production as they are for the study of the natural world. According to Arthur of course systematic dialectics cannot deal with real people anyhow and must *therefore* distance itself from volition and subjectivity. So although he views capitalism mostly as a concept structure, Arthur's account implies a similar potential for the use of quantitative methods in the study of capitalism as Smith's.

Such an individual dependence on structural relationships and concomitant imperatives for survival – be it real or perceived – is absent from Hegel's system for it is entirely predicated upon the very freedom of humans that Marx immediately relegates to the margins of his system. In conclusion, survival – by subsumption to the exchange relation – comes first for Marx, whereas Hegel seems to contend that when Free Will is secured (in the last instance by the State), survival is immanent.

But there is more. Although numbers and mathematical formulae can describe a lot of processes in the natural world, they are externally imposed on it: they do not *constitute* nature. I am aware that such a statement flies directly in the face of scientists that claim that 'mathematics is the grammar of the book of nature' and conclude that nature is inherently (i.e. ontologically) mathematical.[56] To me, this is just as ludicrous as to conclude that a scientific field is constituted by language from the fact that the use of language adds to the field's intelligibility. Of course it does, for language has been developed as a tool to understanding the world and the categories on which set theory is based have a qualitative basis in language

[56] From the discussion in Section 2 it may seem that it also flies in the face of Hegel. After all, Quantity is one of the most basic determinations in his Logic and hence needs to be presupposed before anything else can be understood. However, one cannot conclude from this fact that the world is ontologically mathematical. In my opinion Hegel's whole philosophy actually dismisses ontological considerations lock, stock and barrel, because it only focuses on what can be known of the world, claiming that all that can be known is known in language and hence critical scrutiny of the way categories in language are interrelated, will bring out the limits of human knowledge and understanding. Hegel often makes it sound as though what cannot be known cannot exist, but this does not logically follow from his argument. When abstractions (like *value*) take on a 'tangible' material form (viz. as *money* – which becomes increasingly less tangible nowadays through the rise of electronic means of payment: hence the inverted commas) this situation changes, which is why a systematic dialectical treatment of capitalism can have ontological implications, while this is logically impossible for systems whose categories do not have the peculiar characteristic of being both real and ideal.

(see Chapter 2 and its abridged version Damsma 2011).[57] Since set theoretical propositions form the basis of the foundational systems of mathematics, it would be very strange indeed if these structures would have no applications in the world. After all, the world informed language and language informed mathematical categories. So the fact that mathematical structures are applicable to the study of nature is a result of the way these structures came about. It has nothing to do with how nature is constituted.

In capitalism, by contrast, *value* must actualize itself as a certain Quantity of *money* for the mode of production to be viable. This universal monetary value permeates all entities and categories in the economic domain. Consequentially, all concrete capitalist entities and categories, like commodity, price, cost, profit, value, etcetera, can also be understood abstractly, as shares or elements in the produce of the system at large (Arthur 1993: 64; Arthur 2004: 79; Smith 1990: 83-94; Smith 1993: 22-23; Reuten and Williams 1989: 60-65; Murray 1993: 45; cf. Murray 2014). So, this is another reason why quantitative methods have potential in the study of capitalism. Although Hegel acknowledges the necessity of money as the quantitative measure of value, he holds that man imposes this social form on things, instead of the other way round as Marx claims.[58] So it is human volition that in the last instance determines exchange value (Arthur 1988: 27, 35) through supply and demand mechanisms. Even though these mechanisms can be mathematically formulated, entering into a bargain for Hegel is an individual choice, it is not something one must do to stay alive. Hence, capitalism's laws of motion as Hegel might formulate them are less inescapable in nature than Marx's.

In short, whereas Hegel sees qualitative and quantitative reflection as reconcilable ways of *thinking* (see Chapter 2 and its abridged version Damsma 2011 for a further explanation of this point), Marx sees the capitalist world *itself*

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[57] By saying that set theoretical categories have a qualitative basis in language, I do not mean to imply that mathematics is just another type of language. For one thing, it is much more rigorous than any other language and since its subject matter is 'external reflection on a multitude of distinguishable yet arbitrarily divisible elements' (see Chapter 2 and its abridged version Damsma 2011), it is entirely free of qualitative considerations in a way that ordinary language can never be. As a result, it can traverse universes way beyond the reach of our imagination (such as the number of elements in $P(\mathbb{R})$, i.e. the power set of $\mathbb{R}$), simply by consistently applying definitions and logical operations.

[58] As Marx wrote: 'Men make their own history, but not of their own free will; not under circumstances they themselves have chosen but under the given and inherited circumstances with which they are directly confronted' (1852^1, 1869^2, 1885^3: 146). So the individual is confronted with a material reality imposing its social form (money) on them, but historically mankind has brought this reality into existence and has the power to overturn it if a powerful enough group of people wishes to do so. As long as that does not happen, however, each individual has to yield to the value imperatives sketched out above (cf. p. 46-47 above).

as being both qualitatively and quantitatively constituted. So quantities are an integral part of the capitalist economy, rather than being externally imposed upon it (cf. Arthur 1993: 64; Smith 1990: 93-94; Smith 1993: 22-23; Reuten and Williams 1989: 65). It is this characteristic of capitalism that enables (mathematical) modeling methodology to be integrated with systematic dialectics all the way through, albeit with regard to the study of capitalism only (that is, amongst the systems that Marx knew of – or that we know of).[59]

In Chapter 3 Marx's systematic dialectical exhibition is tracked in order to substantiate the mentioned differences between Marx's conceptual abstractions and Hegel's and the associated potential for quantification and the use of quantitative methods. Despite the differences between Smith and Arthur regarding their respective modes of exhibition, the categorial *content* of both accounts is very similar, so regarding the relevant moments in *Capital,* I will draw on both accounts. However, they will be exhibited in accordance with the α)-β)-γ) format introduced in Section 2 above, which, for its general applicability, bears more resemblance to Smith's unity, difference and unity-in-difference than to Arthur's parallelization with Hegel's Logic.

Summary and Conclusions

The objective for this chapter was to show that the differences between Marx and Hegel's accounts of society though ontological, are hardly epistemological in nature and that both used a systematic- dialectical method. This method was developed by Hegel. Marx was inspired by it, but deviated from it in some important respects. To this end, Section 1 has first charted the chronological order in which Hegel and Marx developed their ideas about historical and systematic dialectics. This section showed that Hegel developed his historical dialectics to fit his systematic dialectical works, while Marx first developed his historical materialism and used the insights this gave him as an input to his systematic-dialectical outline of the capitalist system. Next, Section 2 described the fundamental premises of, essential elements in, and fundamental structure of Hegel's systematic-dialectical works so as to set the stage for understanding Marx's critique on Hegel and Marx's twist on Hegel's method. The elements in

[59] This is not to say that mathematical modeling would not be *useful* in other systems, but just that they could not be integrated with them *all the way through*, for this requires abstractions to be ontologically *as well as* epistemologically quantitative. Systems for which this is not the case can only be described mathematically to the extent that suitable Measures can be found and imposed upon it. Once again, the fact that capitalist abstractions are quantitatively constituted, implies that modeling can go much further than in other systems.

Hegel's texts that Marx was critical of as well as those he adopted and his alternative for the former were discussed in Section 3. Thus, it was established how Marx's systematic-dialectical theory was likely to differ from Hegel's .

Many (including myself) contend Marx came up with a systematic -dialectical theory of his own in the *Grundrisse* and *Capital*. Most commentators contend that in doing so, he drew on the categories in Hegel's Logic in one way or another. Especially regarding Marx's *Grundrisse* manuscript this contention is hardly disputed. Regarding *Capital* there is more diversity of opinion. To show that the latter work too can be considered a systematic-dialectical text, Section 4 discussed some scholarly work (most prominently Arthur's and Smith's) arguing this thesis.

The most fundamental premise of Hegel's work is that everything that can be known is known in language. Therefore, a deep understanding of the way that ideas about a certain realm are interrelated in language can foster new insights about the reality that those ideas pertain to, to exactly the same extent that the ideas are adequate. Both Hegel and Marx accepted this premise in my view.

However, before one can begin to investigate how ideas are interrelated in language, one must at the very least become acquainted with them. This is a process that precedes the development of the systematic-dialectical method proper. In Smith's (1990) terminology, therefore, systematic dialectics is preceded by a stage of appropriation. Reuten and Williams (1989) make a similar statement when claiming that (systematic-dialectical) 'concrete determination' is preceded by a process of 'abstract determination' in which categories (embodying ideas) are born.

Although both Hegel and Marx emphasize the importance of this stage, their opinions as to the role the dialectician or philosopher has to play in this are quite different. Where Hegel places his faith in 'the working out of the empirical sciences on their own account', Marx is decidedly more critical. For instance, he makes the sweeping claim that the whole German historical tradition has focused on pure thought and neglected actuality. In his view, Hegel was the last proponent of this erroneous school of thought. Having thus dismissed large chunks of history writing (among other disciplines), Marx could not just appropriate existing ideas to subsequently chart their interrelationships, but to a large extent felt compelled to come up with categories of his own. Thus, analytical and empirical exploratory research ('Forschung') is important to Marx alongside the systematic-dialectical investigation, whereas Hegel felt he could dispense with it. Marx repeats, re-emphasizes and clarifies this point further in his introduction to the *Grundrisse* manuscript, a few pages of which are the only ones in which Marx explicitly discusses the most appropriate method for political economy.

As to Hegel's philosophy and dialectics in general, Marx criticizes Hegel for being obsessed with overcoming conflicts and banning negativity from his

dialectics. According to Marx, Hegel correctly identified the possibility for products of labor to become alienated from the laborer but could not work out the consequences, because tensions between thought and nature always needed to be resolved in the Hegelian system, whereas a direct confrontation with nature might show how it is misrepresented in thought (so an apparent conflict is not an actual one) and/or how actually existent conflicts get disguised in more complex forms, but are never actually resolved.

It seems plausible that Marx's critique of Hegel's dialectics in general not only led him to dismiss Hegel's idea that world history is essentially a tale of an ever more pronounced actualization of individual Free Will in the world, but also led him to dismiss (most of) Hegel's *Grundlinien der Philosophie des Rechts*, including its starting point: the universal principle of Free Will. After all, Hegel's systematic-dialectical theory of society starts from this universal principle to allow this object totality to be described as 'the Idea that returns into itself out of its otherness' to resolve the tension between pure thought as 'the Idea in and for itself' and nature as 'the Idea in its otherness'. This resolution could only come to a close, if human agency was considered as unhampered and Free as possible. So to Marx, Hegel's universal principle of Free Will must have appeared as the pinnacle of his obsession with tension resolution Marx was so critical of.

As an alternative to the Hegelian conception of history, Marx asserted that changing material relations between the haves and have-nots should be considered the guiding principle to understand historical developments in society, while any historically given material constellation may be indicative of the appropriate universal principle of the societal system as it functions in a given era. The implication of Marx's critique of Hegel and his alternative conception of history is that his own systematic-dialectical starting point should 1) allow for the emergence at later stages of conflicts and negative results and 2) be embedded in Marx's historical materialist conception of history. In the introduction to his Grundrisse, Marx explicitly makes the latter point and the related remark that historical and systematic dialectical reasoning must be clearly distinguished. When exhibiting Marx's systematic-dialectical theory of capitalism in Chapter 3, it will be assessed whether the universal principle Marx starts from meets the two requirements identified above.

There is no evidence that Marx was critical of Hegel's other works or of other aspects of his method than those discussed above, albeit that the implications of his critique on Hegel's obsession with resolving oppositions (tensions between α and β) are far reaching in that any reference to the logical categories in the Doctrine of the Concept must be handled with extreme care. In fact he was often (around the time of writing the *Grundrisse*) quite enthusiastic about Hegel. Thus, Marx's criticisms mainly concern Hegel's ideas on society and its historical

development. Being critical about the received view automatically implies a larger role for empirical and analytical exploratory research ('Forschung'), for dismissing old ideas challenges one to come up with alternative ones. In post-Kuhnian terms one might say that Marx and Hegel's fundamental epistemological beliefs and method are the same, but because of their different ontological views of society and its history, their accounts of society start from a different universal principle and as a result have very different contents. The epistemological differences regarding the pre-dialectical stage of appropriation are directly related to their different views of history as well.

All this not only has implications for the universal principle Marx begins his systematic-dialectical exhibition in *Capital* with, but also for the way it draws on the categories in Hegel's Logic. This in turn has implications for whether the core capitalist categories are ontologically amenable to a quantitative treatment if seen from a systematic dialectical perspective. Quantitative methods have most potential when volition and subjectivity are neither present nor assumed so that subsumption under a law can be considered satisfactory as an explanation. In Smith's opinion this is the case for the capitalist system because in the last instance it is best described in terms of Essence categories in a Hegelian sense. This type of categories relates to the world out there only and must thus abstract from volition and subjectivity.

According to Arthur, Hegelian systematic dialectics can only deal with abstract thought. However, the pure abstract thought of *value* in capitalism has gained material reality as *money* and therefore capitalism is the only real system it can tackle, but only as long as the exhibition describes the results of the value imperatives. It cannot incorporate real people or things, but only materialized abstractions. Hence, Arthur's point of view also implies that volition and subjectivity must be abstracted from in dialectical descriptions of capitalism. So, both Smith's and Arthur's account of Marx's method, albeit on the basis of radically different readings of both Hegel and Marx, imply a huge potential for quantitative methods for the study of capitalism.

Despite these differences Smith and Arthur seem to concur that capitalist abstractions are rather peculiar in that they are both real and ideal. Again, the argument is that capitalism renders the thought of *value* tangible (as *money*) and that the concomitant value imperatives permeate all capitalist entities, relationships and processes. Hence, capitalism is largely *constituted* quantitatively. That is, not only are quantitative methods *epistemologically* useful for studying it but its core categories are also *ontologically* quantitative in ways that the categories of the natural sciences could never be.

Preview

This book is about the articulation of systematic dialectics and mathematical models that formalize or represent moments of a systematic dialectical account of capitalism. The project thus involves transitions between the qualitative and the quantitative and back again.

To understand whether such transitions are feasible and if so how they could be made, Hegel's systematic-dialectical exhibition of the mathematical is extremely useful. After all, dialectical method is a method involving categories and hence qualities. Thus, bringing it to bear on the quantitative involves a transition between the qualitative and the quantitative. Chapter 2 therefore establishes the systematic-dialectical architecture of mathematical categories and ideas by tracking Hegel's determination of these categories in his Logic and the first part of his philosophy of nature. The former establishes a systematic-dialectical foundation for what would now be called number theory, while the latter exhibits the concepts of geometry in a systematic-dialectical way.

Chapter 3 elaborates on the thesis that capitalism is ontologically quantitative and hence can be studied using mathematical techniques, not because Measures can be externally imposed on it, but because its very nature is quantitatively constituted. As a result, systematic-dialectical moments of capitalism can be analytically *explored* with the help of mathematical models to a further extent than is the case in most other realms.

The role of dialectics in such a modeling exercise is to bring to light and scrutinize the assumptions that models require. How this might work is investigated by means of a critical examination of Marx's schemes of reproduction and its assumptions from a systematic dialectical perspective. This examination is undertaken in Chapter 3. It is concluded that Marx's goal when drawing up these schemes was to analytically explore the categories of simple and expanded reproduction that he arrived at by systematic-dialectical means. So Marx never ventured beyond the stage of abstract determination when drawing up his schemes, but this does not mean that it is impossible or unfeasible to present these models and its assumptions as the outcome of the dialectical exhibition. Thus, a much stronger connection between the systematic-dialectical genesis of model assumptions and the formal (model) structure in which they operate is possible. Chapter 4 therefore offers a reconstruction of Marx's schemes, to show how this might be done.

In dialectical terms one might say that in this book the α) idea of mathematics (as elaborated upon in Chapter 2) is contrasted with the β) quantitatively constituted reality of capitalism, exemplified in this book by Marx's analytical, pre-systematic schemes of reproduction, that comprise two moments of the

capitalist system (Chapter 3) and that the tension between the two is resolved by showing that these analytical, pre-systematic schemes can be elevated to γ) mathematical models of particular *moments* of the capitalist system whose assumptions are dialectically informed or at least dialectically defendable, at least regarding capitalist simple and expanded reproduction (Chapter 4).

2. On the Dialectical Foundations of Mathematics

Introduction

Mathematics, like any other science, cannot justify its own foundations. For example, it needs Numbers to build up the tools by which it can apprehend Numbers.[60] Due to this circularity, mathematicians are forced to work with concepts whose genesis they cannot fathom. This circle can be broken by a reflection on the categories mathematicians work with, rather than by working out their implications (for this is already the core business of mathematicians anyhow). Hegel's methodology of systematic dialectics is instrumental in this reflection, for it entails an ordering of categories from abstract categories to concrete instances. On the basis of this order, I will show that the categories of Numbers and arithmetical operations stem from a failed attempt at making qualitative distinctions on the basis of quality alone. Further, the exhibition in this chapter clarifies the proper use of cardinal and ordinal Numbers and shows that our awareness of Time and hence of Motion presupposes distinctions in Space.

Since 'Hegel taught mathematics at both the University of Jena and the Nürnberg secondary school' (Burbidge 2006: 48), his knowledge of mathematics and its main categories must have been above average (and more than adequate for his systematic-dialectical purposes). Hegel discusses some important categories of number theory and algebra (viz. Numbers and arithmetical operations) at length in the first part of his *Wissenschaft der Logik* (1812, 1813, 1816) and more succinctly in the first subdivision of the first part of the *Encyclopädie der philosophischen Wissenschaften* (1830^3, 1817^1). Important categories of mathematical mechanics, which is akin to geometry, are discussed in the first subdivision of Hegel's philosophy of nature, which can be found in the second part of the *Encyclopädie*, but not in the *Wissenschaft*. So the *Encyclopädie* encompasses more mathematical categories than the *Wissenschaft*, and discusses them more succinctly. Thus, the *Encyclopädie* provides a more complete account of what Hegel has to say on mathematics. Moreover, I have little to add to Carlson's comprehensive discussion of the *Wissenschaft* (2000, 2002, 2003a).

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[60] In this book, categories that are dialectically important to Hegel (and thus function as moments – cf. Chapter 1, Section 2) will always be written with a capital letter. To avoid confusion between Hegel's moments and Marx's (as mainly discussed in Chapters 3 and 4), Marx's are stressed by *italicizing* them (see my *Note on the Style of Referencing and the Use of Capitalization and Emphasis in This Work* for further details and explanations).

Hence, I will confine myself to the *Encyclopädie* in this chapter. The differences and similarities between both works will be discussed in the appendix.

The central questions are how Hegel develops important mathematical categories systematically out of other more abstract categories, how this reflects on the meaning of these categories and how this in turn reflects on the mathematics in which the categories are utilized. In answering these questions it will be shown that mathematical categories presuppose abstract categories in common language. So, contrary to popular belief, the mathematical mindset is founded on languages like English, French, German, Dutch and the like. It therefore is not a language in its own right.

In the first section of this chapter a representative part of the literature on Hegel and mathematics is discussed. This helps position this chapter and hopefully provides an idea of the potential uses of systematic dialectics with regard to the philosophy of mathematics.

Hegel's determination of the quantitative is discussed in the second section and his determination of mathematical mechanics in the third. The accounts given follow the α-β-γ-format introduced in Section 1.2 and are neither quantitative nor mathematical. Rather, mathematical categories, like Discrete and Continuous Magnitude, Number, Spatial Dimensions, the Point and the Line, are ordered along other categories within Hegel's philosophical framework. In the concluding section the question will be answered what insights, if any, can be gained from this systematicdialectical perspective on the mathematical.

1. Previous Literature on Hegel and Mathematics

The literature on Hegel and mathematics falls roughly into two categories. First, some authors are searching for a philosophical understanding of mathematics and are looking for answers in Hegel's works. Second, there are those that try to elucidate, comment upon and expand Hegel's views on (certain aspects of) mathematics and especially infinity.

The reason to look for a philosophy of mathematics in Hegel lies in the rigor and precision of mathematics and definitions for mathematics. Once a category or subject is rigorously defined, it is set apart from all possibilities that are not captured by the definition. When worked with, these rigorous definitions therefore eventually call up their own negation. That is, while the mathematical implications of these definitions become clearer and clearer, so do their

shortcomings.[61] In other words, to truly understand some definition and its implications also implies an understanding of its limits. Thus, the rigorous definitions of mathematics call up their own negations (Paterson 1997a: 14; Tóth 1972: 36-38).

Tóth (1972) illustrates this point in relation to the development of non-Euclidian geometry. Interestingly, many authors, like Aristotle in the third century B.C. and Saccheri and Lambert in the 18th century A.D., already knew that a non-Euclidian geometry was possible in principle, but except for Aristotle they all dismissed this type of geometry as untrue (Tóth 1972: 20-23).[62] Thus, the Euclidian system clearly calls up its own negation, even though this negation was only accepted as a true possibility in the 19th century A. D.. Within both axiomatic geometrical systems, the other system can be shown to be false, so the two are truly oppositional.[63] But this is only a problem if an ontological status is ascribed

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[61] This statement might remind some readers of Lakatos' Proofs and Refutations in which a mathematical proof for the conjecture that 'for all regular polyhedra $V - E + F = 2$' (1976: 6) (V being the number of vertices , E the number of edges and F the number of faces), is subjected to numerous efforts at refutation by pupils in an imaginary classroom. Although many of their attempts are deemed successful by their teacher, they do not succeed in overthrowing the conjecture. Instead, the terms in the conjecture and the method of proof are changed in such a way that the conjecture and the proof are effectively immunized to falsification. But the terms and the nature of mathematical proof itself become a lot clearer in the process. Thus, Lakatos shows that the use of what Popper called 'conventionalist stratagems' (1959: 61-62) (which are usually dubbed 'immunizing' (cf. e.g. Boumans and Davis 2010)) does not always harm theories. Instead, they can play a very constructive role.

Because Lakatos' argument proceeds by means of counterexamples of statements and is cast as a conversation, it is often mistaken for a dialectical argument. But – in a Hegelian sense at least – it is not, because in systematic dialectics a category is negated as a whole by another category *at the same level of abstraction*, while the pupils in Lakatos' text try to overthrow the *abstract* general conjecture by offering *concrete* counterexamples. So the students essentially follow a Popperian procedure, allowing Lakatos to show the limits thereof. Furthermore, the mathematical proof debated in the imaginary classroom only serves as an example of the more general point Lakatos wants to make and arguing a general point by means of contemplating examples is itself an undialectical procedure.

[62] The BBC has recently replaced the abbreviations B.C. and A.D. with the religiously more neutral terms Before Common Era (B.C.E) and C.E. (Common Era) respectively. Although a good case can be made for abolishing references to Jesus Christ in a multicultural and multiethnic society, I prefer clarity over political correctness and hence will not adopt the new abbreviations until they are used more widely.

[63] Mathematically, this is only true with regard to (one of) the *axioms* of both systems. Euclidian geometry accepts the fifth axiom, which states: 'given a line *l* and a point A not on the line, there is only one line through A which does not cross *l*'. If this axiom is rejected there are two possibilities:
1) In hyperbolic geometry there is an infinite number of lines through A that do not cross *l*;
2) in elliptical geometry all lines cross.

to either of these systems of formal logic. If not, it is the positing of this opposition itself that might lead to a more comprehensive dialectical understanding of the nature of geometry (Tóth 1972: 36-40).

The fact that Hegelian philosophy can be used to make conceptual sense of the development of non-Euclidian geometry and the nature of geometry in general, is not to say that Hegel gave any account of non-Euclidian geometry in his writings. Rather, he 'fully accepted the essential validity of the Euclidian approach' (Paterson 2004/2005: 46), albeit that he criticized some of Euclid's proofs, especially when they involve superposition. His criticism was based on the fact that two distinct congruent triangles are conceptually the same. According to Hegel therefore, a pure mathematical triangle can only be congruent *with itself*. Hence, congruence must be proven from one triangle instead of from superposition of one triangle over another (Paterson 2004/2005: 37-39).

Paterson (1997a) discusses the problems that the formal systems have run into that were proposed in the 20th century as foundations for mathematics (1997: 3-10). Each of the proposed formal systems was inspired by different intuitions. In that sense each of them is deficient and can only be a foundational system for that part of mathematics that concurs with the foundational intuitions of that specific system (1997: 12).

As a solution to the problems that have arisen from this state of affairs, Paterson proposes to contrast the α) universality of mathematics itself (as a category) with β) formal systems as particular instances of foundational systems, and to proceed dialectically towards the γ) singularity of natural numbers, sets and functions (Paterson 1997a: 12-14). In such an exhibition, 'the implicit conceptual content of the formal approach will be made explicit' (1997a: 14) and 'the development will make conceptual sense of the insights which motivated the various foundational systems' (1997: 14-15).[64]

The first of these non-Euclidian possibilities implies an infinite (as opposed to one) number of parallels through A, while the second possibility implies that parallel lines are only parallel on a finite domain. So in terms of finite versus infinite the non-Euclidian axioms are truly opposed to Euclid's fifth axiom.

In mathematical practice, however, Euclidian geometry is a special case of elliptical geometry. Parallel lines on a globe, rather than a plane best represent the elliptical concept of parallelism. As the radius r of this globe approaches infinity, the elliptical system starts behaving like the Euclidian system. So the Euclidian system is the limit of the non-Euclidian elliptical system for $r \to \infty$. I am grateful to Louk Fleischhacker for help with these remarks.

[64] On Paterson's website (https://sites.google.com/site/apat1erson/) the reader will find another five papers on the desirability and merits of a Hegelian philosophy of mathematics. Three of these are about the philosophy of Number (1997b; n.d.; 2000), one is about the 'Hegelian Concept and set theory' (2007) and one (2002) is about the Hegelian philosophy of mathematics in general. In each of these, Hegelian philosophy is proposed as a solution to the problems 'which arise out of

Hegel distinguishes between the bad or metaphysical and the true infinite. According to Hegel the latter category is involved in the mathematical infinite (Lacroix 2000: 303). The bad infinite is the unreachable infinity of an endless progression and is best represented by a straight line (Inwood 1992: 141; Ellsworth de Slade 1994: 212-213). It is a Quantity beyond all Quantity in that it is forever beyond the finite: whatever operations you perform using finite quantities; the result will always be finite again (Hegel 1812, 1813, 1816: 282, 2.2Bc; Lacroix 2000: 314). Hence the bad infinite is only a potential infinity that cannot be reached by finite mathematicians. It is beyond our grasp by its very definition.

All things in the world are finite, but this fact itself is infinite. 'Finite entities develop, change, pass away and give rise to other entities' (Inwood 1992: 295) ad infinitum. This passage itself is the basis for Hegel's conception of the true infinity (Hegel 1812, 1813, 1816: 163, 1.2Cc; Lacroix 2000: 315). Thus, the mathematician deals with 'finite objectivities that thought posits in its infinite self-development' (Lacroix 2000: 315). To Hegel the most important example of an application of the true infinite in the realm of Quantity is the differential calculus. At their limit the $\mathrm{d}y$ and $\mathrm{d}x$ in the ratio $\mathrm{d}y/\mathrm{d}x$ are 'no Quanta anymore […] but have meaning only in their relation' (Hegel 1812, 1813, 1816: 295, 2.2Bc, my translation).[65] So it makes no sense to think of $\mathrm{d}y$ and $\mathrm{d}x$ as being extremely small but nonzero Quanta. Rather, it is the law that relates y and x that becomes apparent in the expression $\mathrm{d}y/\mathrm{d}x$. Thus, while at this limit $\mathrm{d}y$ and $\mathrm{d}x$ disappear as specified Quantities, their relation reappears as a qualitatively different ratio. If y and x are positively related (e.g. through a successor function), it is this relation that is the true locus of the true (quantitative) infinite, because through it the finite Quantum *x* is ceaselessly led beyond itself into the bad potential infinite. Thus, if γ) true infinity is conceived of as the law that leads α) the finite Quantum x into the β) bad potential infinity it resolves the opposition between the two (Lacroix 2000: 311-315).

In 1994 Ellsworth de Slade wrote a study on the counterparts of Hegel's true infinity in his conception of infinitesimal mathematics. In 1932 Baer published an article on Hegel and Mathematics in general. In their texts both hail the result of the last paragraph as one of the most important insights Hegel has to offer in the field of mathematics (Baer 1932: 112; Ellsworth de Slade 1994: 213).

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the existence in mathematics of self-referential, non-constructive concepts (such as *class*)' (Paterson 2002: 143).

[65] Quantum is Hegel's term for a Quantity with a specific Magnitude, a specified Quantity. These Quanta have nothing to do with Planck's packages of specific amounts of light. Exactly what is at stake will be discussed at length in the next section. In that section and the section after that, all the capitalized categories encountered thus far are elaborated upon.

Fleischhacker agrees with these authors that $\mathrm{d}y/\mathrm{d}x$ is qualitatively different from other ratios, but disagrees with calling it a ratio. He argues that at its limit $\mathrm{d}y/\mathrm{d}x$ is not a qualitatively different ratio, but a normal finite specified quantity, whereas before the limit it was still a ratio. "Not $\mathrm{d}y$ and $\mathrm{d}x$ are 'the ghosts of deceased quanta', but $\mathrm{d}y/\mathrm{d}x$ is the corpse of a deceased ratio" (Fleischhacker 1982: 148, my translation). However, as far as mathematical practice is concerned, all three authors are correct. That is, under some circumstances $\mathrm{d}y/\mathrm{d}x$ is conceived of as a quantity while under others it is best treated as a ratio.

Hegel's views on the infinite and infinitesimal mathematics are not intramathematical, but conceptual. However, as Wolff clearly shows in his 1986 text entitled *Hegel und Cauchy*, he was well versed in the research that mathematicians such as Lagrange and Cauchy have done on the subject. In this text Wolff traces how Cauchy influenced Hegel regarding the mathematical infinite and infinitesimal mathematics and discusses similarities and differences between the two (1986: 197-263).[66]

Hegel's views on mathematics have also been an inspiration to Marx and Marxists. Kol'man and Yanovskaya (1931) discuss the nature and extent of the influence of the Hegelian philosophy of mathematics on Marxism-Leninism. To them, as to Fleischhacker (1982), the most important merit of the Hegelian philosophy of Mathematics was his correct recognition of the subject matter of mathematics (Kol'man & Yanovskaya 1931: 5) (more about this in Section 2.8).

But according to Kol'man and Yanovskaya, Hegel should not have stopped there. His dialectical perspective may have helped Hegel to correctly analyze the nature of mathematics and some of its problems and shortcomings, 'but as a bourgeois philosopher who only intends to explain the world and not to change it, he does not at all pose himself the task of transforming mathematics dialectically' (Kol'man & Yanovskaya 1931: 15). This is best exemplified by Hegel's analysis of the differential calculus alluded to above. Instead of trying to transform mathematics in accordance with his analysis, Hegel states that 'these dialectical moments […] cannot be adopted by mathematics at all' (Kol'man & Yanovskaya 1931: 16) and leaves it at that. It goes without saying that Kol'man and Yanovskaya do not agree (1931: 14-18).

Finally, in the first three papers in a series on Hegel's *Wissenschaft der Logik* Carlson (2000, 2002, 2003a) gives a complete account of Hegel's determination of the quantitative in pictographic terms. His treatment in these papers is very similar to mine. That is, all of Hegel's dialectical transformations and all of the important concepts in the *Wissenschaft* are discussed, explained and when

[66] Since this is an important, but very specific detail of (the development of) Hegel's views on infinity, I will not elaborate on Wolff (1986) here.

appropriate, amended with modern-day insights. In the next two sections, I will do roughly the same for the *Encyclopädie*, although a different procedure for exhibition is adopted in those sections. Carlson exhibits Hegel's logic in the form of pictorial triads of overlapping concepts (2003a: 93-101), whereas I stick to the α) - β) - γ) -format explained in Section 2 of Chapter 1.

Furthermore, the conceptual development in the *Encyclopädie* differs at a few crucial points from that in the *Wissenschaft* and this of course is reflected in the exhibition in Section 2. The differences and similarities between the two works will be discussed in the appendix. Finally, the *Wissenschaft* does not encompass the philosophy of nature. As a result, Carlson does not discuss Hegel's determination of the concepts of mathematical mechanics, which I will do in Section 3.

2. Hegel's Determination of the Quantitative

In this section the method discussed in Section 1.2 will be used to further exhibit the systematic-dialectical determination of the quantitative and its concepts. The mathematically important concepts here are Numbers and arithmetical operations. The main gist of this section is taken from subdivision 1 of part 1 (logic) of the *Encyclopädie* (Hegel 1830^3, 1817^1: §§84-111). Since the logic is the most abstract of the object totalities, this section begins with the universal principle of everything, Being. It will take ten (out of a total of 14) subsections to dialectically determine Number from this universal principle.

Hegel seems to regard some oppositions and their resolution as self-evident, for example the opposition between Being and Nothing and its resolution in Becoming and Presence. So, the questions α) β) and γ) that were introduced in Section 1.2 as aids for clarifying systematic dialectical exhibitions are not mentioned in section A, subsection a (Hegel 1830^3, 1817^1: §§86-88). From Presence until the start of section B, Quantity, Hegel uses these questions explicitly as a tool to drive his exhibition onwards (Hegel 1830^3, 1817^1: §§89-98). After that, the quantitative and its moments (including Measure) are again not explicitly discussed this way (Hegel 1830^3, 1817^1: §§99-107). Since I think the consistently application of the α-β-γ-format to Hegel's text clarifies his exhibition and renders it more transparent, I have inserted them where they are absent in Hegel's text (i.e in Sections 2.1-4 and 2.8-14 below).

A. Quality

2.1. Being

α) As mentioned in Section 1, the universal principle of the totality of everything is Being, simply because everything we can perceive, think or imagine *is*. In total conceptual isolation this leaves us just about nowhere. Everything *is*, so viewed from the inside out, Being points to a total lack of further distinctions (Hegel 1812, 1813, 1816: 82, 1.1A; Hegel 1830^3, 1817^1: §86).[67] If Being were the only category available to us, we could give everything a name, but it would be the same name over and over again. So there would be no way to distinguish between things.[68] Hence, pure abstract Being is entirely imperceptible.

2.2. Nothing

β) To view Being from an outside perspective seems virtually impossible, for Being already encompasses everything. But by definition Being (or any other category) does not encompass its opposite, Nothing (Hegel 1812, 1813, 1816: 83, 1.1B; Hegel 1830^3, 1817^1: §87). So Nothing stands outside of Being and to acknowledge Being we need an outside perspective. Hegel regards Nothing as the outside standpoint that enables this outside perspective and therefore concludes that it is empty observation and thought itself (Hegel 1812, 1813, 1816: 82-83, 1.1A-B), which is itself every inch a Being.[69] The goal of his philosophy is to

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[67] The logic in the *Wissenschaft der Logik* and the *Encyclopädie* is the same for Being, Nothing, Becoming, Presence ('Dasein') and perhaps Something and Others. After that, terms like finitude and infinity take a rather central stage in *Wissenschaft der Logik* but not in the *Encyclopädie* (the appendix to this chapter elaborates on these and other differences). Because I'm primarily concerned with the *Encyclopädie* here, I will only refer to the *Wissenschaft* up to the point where Something and Others are exhibited.

[68] In dialectics, abstracting away from all other categories and the distinctions inherent in them is only a temporary matter. The scope and richness of language is only restricted temporarily for analytical purposes. The method does *not* propose or favor any kind of Newspeak or anything of that nature.

George Orwell introduced the term Newspeak in his famous book *1984* (1949). The idea behind it is to take away people's ability to think critically by first stripping away close conceptual relatives from words, so that people might say 'I feel good', but lose the ability to express 'I feel elated'. If people cannot express this, Big Brother hopes they cannot think or feel it either. The second step is to get rid of negative words, with the result that people can only think of their life, their society, the Party and above all Big Brother himself as 'good'. The idea that we cannot perceive the world clearly without appropriate categories is also central to systematic dialectics.

[69] Note that the Cartesian distinction between mind and matter is only maintained in this most abstract of all possible oppositions. In the remainder of the *Encyclopädie* this opposition gets further and further resolved, showing how mind and matter constitute each other and each other's understanding.

show how this is possible. That is, Hegel wants to develop an argument in which reality is not viewed from one of two poles, but in which the one pole can be shown to be part of the same totality as the other and in which the poles are mutually supportive so that the finished system is self-explanatory. Among other things, Hegel seeks to overcome the classic dichotomies between subjectivity and objectivity and between thought and things in this way (Inwood 1992: 16). Having the external standpoint Nothing doesn't help much, but at least one distinction can now be made: some things are and some are not.

Some have tried to formalize Hegel's dialectical logic using set theory (Baer 1932: 105; Kosok 1972; Priest 1989: 393-396). In set theoretical terminology one might say Hegel sought to resolve Russell's paradox long before it was formulated. This paradox runs: if V is the set of all sets (cf. Being), then its complement $C\backslash V$ (cf. Nothing) $\notin V$ because of the definition of $C\backslash V$. But at the same time $C\backslash V \notin V$ because of the definition of V (Russell 1903: 527-528). Hegel resolves this paradox by initially placing Nothing outside of Being, while stating that Nothing is empty observation and thought itself. So initially $C\backslash V$ is conceptualized external to and independent of Being (which means the paradox holds, because whoever owns this empty mind, must be a Being), but when we become conscious of the world, we come to realize that subjective Nothing is part of objective Being and conversely that the recognition of the objective (Being) requires the subjective (Nothing). So in truth the external independence posited does not hold. Surprisingly enough neither Kosok (1972) nor Baer (1932), nor Priest (1989) mentions this remarkable parallel.

2.3. Becoming

γ) Pure Being is as imperceptible as pure Nothing. If one tries to think of Being in all its entirety (so without any recourse to concrete examples), one might just as well think Nothing, because any real thought implies some distinction. As conceptually isolated abstractions then, both concepts are equally unthinkable and as such they are the same. Thus, the thought of pure Being immediately vanishes into pure Nothing and the other way round, in that neither of them can be a thought on its own (Hegel 1812, 1813, 1816: 83, 1.1C; Hegel 1830^{3}, 1817^{1}: §87). What we *can* think is exactly this disappearance of the one in the other.

The process whereby the thought of Being vanishes into Nothing and the other way around, is Becoming.[70] Thus, Becoming explicitly posits the non-thoughts of

[70] It is virtually impossible to clearly make this point in English. Unlike the German 'Werden' or the Dutch 'Worden', Becoming sounds like coming into being only, for these are the two verbs the word is made up of, despite of such expressions as 'nothing can become of it'. Werden and Worden, by contrast, are truly neutral as to the direction of the process. In short, the languages one

Being and Nothing as distinct and even oppositional concepts. At the current level of abstraction, Becoming is only change: Being Becomes Nothing and Nothing Becomes Being. Becoming is imperceptible because of this unceasing dynamism, but unlike Being and Nothing, Becoming can be thought (Hegel 1812, 1813, 1816: 83, 1.1C; Hegel 1830[3], 1817[1]: §88; Carlson 2000: 11-12; Carlson 2003b: 11-16).

2.4. Presence

γ) Paradoxically, the requirement of dynamism inherent in Becoming means that we must give a further *static* determination of Being, for if there is change, here and now must be different from there and then. If Being is determined as Being-here-and-now, we may term it Presence – my translation of Hegel's 'Dasein'. [71] One might also focus on Being-there-and-then, but by this very focus there-and-then gets determined as here-and-now, so the two determinations are conceptually the same at the current level of abstraction. So Presence is just Being with a determination. The nature of this determination does not yet matter. Presence then, is Becoming taken statically. Because of this stasis Presence can finally be perceived (albeit very abstractly), or more precisely, it is the whole of perception itself, but at the same time the category helps us to keep in mind that everything we perceive is continually undergoing change (Hegel 1812, 1813, 1816: 113, 1.1C; Hegel 1830[3], 1817[1]: §89).[72]

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can think in, in part determine what one can grasp well and what less well and the way one understands things. I am therefore convinced that one's intellect benefits from mastering more than one language, because every new language enriches one's conceptual apparatus.

[71] The German term 'Dasein' is actually more accurate, because it literally means 'being there' and something that is there can only be so at a certain point in space and time. This connotation is immediately clear from the German. The problem is that 'Dasein' is acceptable German, but 'Being-there' is very artificial English. Despite this, Geraets, Suchting and Harris have adopted this latter term in their translation (1992: 145). Suchting however contends they should have used 'Determinate Being' instead, because although the latter 'is certainly not a common English expression, […] it is not by any means a weird one' (Suchting 1991: xxxvii) (Suchting provides many other and more important arguments, but this is the most relevant one in the context of this footnote). But Suchting was outvoted by Geraets and Harris on the matter (1991: xxxii-xxxiii). Others either stress the connotation of a particular space and time (as I am doing here) or the connotation of Being. Those that use Existence instead of Presence stress this latter connotation. Other alternatives are: 'Being Determinate' (Wallace 1873: 133, §89), 'Determinate Being' (Carlson 2000) and 'Prevalence' (Reuten 2005: 79).

[72] Heisenberg showed that the more we know of a particle's position (its Presence), the less we know about its speed (Becoming) and the other way around. To be precise, his uncertainty principle states that the variance of the particle's position times the variance of the particle's speed times the particle's mass can never be smaller than Planck's constant (Hawking 1998: 72). So, we cannot know both at the same time. Rather, we need to alternate between both magnitudes in order to say anything conclusive at all. Mathematically, at the level of Becoming, speed (or whatever

2.5. Something and Other

α) Presence is Being with a determination. This means that Presence is necessarily limited in either space or time or both.[73] This very finiteness further determines it as Something (Hegel 1812, 1813, 1816: 125-126, 1.2Ba; Hegel 1830^3, 1817^1: §91).[74] The difference between Presence and Something is that Something's determination is *fixed* in time and space, while Presence is Becoming taken statically at *every* point in time and/or space. In other words, although Presence provides a static view of Becoming, it moves along with it (which is why its here-and-now is conceptually the same as its there-and-then).

β) If here-and-now is taken as the further determination of Presence as Something, the determination of there-and-then is abandoned. Yet Becoming requires Presence to be further determined as both, for if Something is fixed as here-and-now *or* there-and-then, there cannot be change. Hence, the thought of determined Presence as Something requires the thought of its Other. This Other

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'change') is the dependent variable, whereas at the level of Presence it is the independent variable. This happens because at the level of Becoming, the most we can say about reality mathematically, is that the development of the totality of all things (let's call it speed or velocity – v –, although other types of 'change' are also possible) is a function f of time t, $v = f(t)$. If you know the speed of something, you can find the distance it travelled by integrating the function for its speed by time over the interval t_0 till now. This yields: $vt = x_t = F(t)$, in which x_t stands for the distance travelled. If the speed v was constant in the first function this would have ran: $v = f(t) = v_0$ and its integral would be: $x_t = F(t) = v_0 t$. So the dependent variable v became the independent variable through our change of focus. Of course at this level of abstraction Becoming is an indeterminate Becoming that should encompass everything. It is the Becoming of the universe as a whole. Consequently, if Presence is seen as the integral of this process, it must be the integral from the big bang, t_0, till now. This knowledge, however, does not help us make any real calculations yet, because 'the theory of everything' (see footnote 20) doesn't exist yet and we do not know what the universe was like at, or immediately after the big bang and even if we did know, there is still a host of possible ways in which it might have developed due to Heisenberg's uncertainty principle. The analogy between mathematical integrals and this part of Hegel's dialectics was first brought to my attention by Wouter Krasser. Lengthy e-mail discussions with Eric Halmans on a precursor to this footnote that featured in my master's dissertation (i.e. footnote 8 in Damsma 2001: 11) have been very helpful in the course of developing this improved version. I am grateful to Maurice Bos for patiently explaining Heisenbergs uncertainty principle.

[73] Space and Time are only determined in the philosophy of nature (see Section 3 of this chapter), i.e. after the completion of the logic. So, all we can say at this stage is that Presence is just finite (period). For reasons of readability and accessibility I added the considerations of here and now (in space and time).

[74] Note that a linguistic link can now be seen to exist between Nothing and Something. Where Nothing denoted an emptiness of observation and thought, Something is the first category that provides the mind with the possibility of *Some* (instead of *No*) concrete content, quality. This link cannot be made in German. Thus dialectics provides different insights, depending on the language used.

then, is what Something is not. It is beyond Something's limits (Hegel 1830[3], 1817[1]: §§91-92).

However, Something differs from Presence – which is a *moving* here-and-now – in that it is *fixed* as either here-and-now or as there-and-then. If the word Something is restricted to either of the two poles – thereby calling it *this* Something – its Other is automatically determined as everything Something is not. Because the word 'this' is indifferent in this respect , Something and Other are only determinate categories vis-à-vis each other (Hegel 1812, 1813, 1816: 125-126, 1.2Ba). At this level of abstraction they are not yet determinations in their own right. As a first instance of Presence, Something is finitely confined by its very determinateness (or quality). But since Presence is necessarily Becoming and thus never stable, Something will sooner or later pass over into its Other. [75] Something then is firstly finite, and secondly alterable (Hegel 1830[3], 1817[1]: §§92-93).

2.6. One and Many Ones

γ) Something denotes no more than a one-sided static determination of Presence (as here-and-now or there-and-then). But due to Becoming, static determinations cannot last. So each Something Becomes its Other. This Other however, is itself also a one-sided determination of Presence, so it too may be taken as Something (after all, the word 'this' is indifferent with respect to Presence's determinations). Hence, in the process of Becoming, Something and Other are conceptually reunited as One (Hegel 1830[3], 1817[1]: §96).

γ) Because every Other may be taken as Something, it again determines an Other vis-à-vis itself and this process may be reiterated indefinitely. Thus, by reuniting Something with its Other the category One automatically leads us to acknowledge Many Ones. Or, in other words: taken statically, there is only one Presence. Taken dynamically, this Presence gets bifurcated into Something and its Other. Both Something and Other however, are static categories again. So through Something and Other, we may conceive of Becoming constituting a *series* of static entities: Many Ones. Although each One in this series is a self-contained unit that excludes Other Ones from itself, it is also true that they can only be

[75] Physicists have formulated this same point as the law of conservation of energy and mass. Put crudely, this law states that for everything that appears (as Something's Other) an equivalent amount of energy or mass disappears (from Something). Thus, ex nihilo creation of energy or mass is ruled out. Hence, each Something must have an Other. Still, the categories of appearance and disappearance are opposites. But the law that links both categories is not contradictory in a logical sense, for those categories, *as categories* do not work on the same concrete object. So the opposition between categories used in a systematic dialectical exhibition has nothing to do with logical contradictions (Wolff 1979: 342).

acknowledged as Ones because of the principle that generated the series (i.e. Becoming) (Hegel 1830[3], 1817[1]: §97).

Bringing the concepts One and Many Ones to bear under γ) is a major digression from the treatment those terms get in the *Encyclopädie*. In that book, Hegel first introduces Being-for-self ('Fürsichsein') under γ) as the union of Something and Others (1830[3], 1817[1]: §95). He generally uses this term to denote how reality appears to us before we comprehend it as being mediated by and itself mediating the other moments. Since this is still the beginning of the Logic, we have little else but indeterminate, accidental appearance by which to distinguish Something from its Other and since both Something and its Other may be determined as Something, they can be distinguished from each other by their Being-for-self (their appearance) only.

Next, under α) Hegel contends that any Being-for-self is One and under β) he clarifies that there must be Many Ones, which as self-contained units are Repulsive of one another (Hegel 1830[3], 1817[1]: §§96-97). Then, under γ) he continues that Ones are not only self-contained units, but also Ones. As such, each One is conceptually the same as each Other One and they have a relation not of Repulsion, but of Attraction (more about these two concepts in Section 2.7) (Hegel 1830[3], 1817[1]: §98).

The major problem with this treatment is that the concepts of Repulsion and Attraction are opposites. So instead of resolving a dialectical opposition under γ), on this occasion Hegel introduces one there. He rarely, if at all, does this in the rest of the *Encyclopädie*, so in order to keep this passage consistent with the rest of the exhibition in this section, I juggled the α), β) and γ) around a little bit. It must be stressed that this operation (although I contend it is an improvement) doesn't leave the meaning of the terms in question entirely unchanged.

2.7. Attraction and Repulsion

β) In the previous sections the necessary dynamism of Becoming led us beyond the determinateness of Presence as Something into the static categories of Other, One and Many Ones. As was said, Ones are all self-contained units that exclude the other Ones. In this sense the Ones are Repulsive towards each other. Through this Repulsion or reciprocal exclusion they can be conceived of as Many. Repulsion then, is the condition of existence of the Many (Hegel 1830[3], 1817[1]: §97).

α) While ceaseless Becoming led us to acknowledge Many Ones, it also implies a certain indeterminacy concerning the limit to the One. That is, when everything always changes, it is not clear where Something ends and its Other begins. The limit to the One then, is equally indeterminate. Still the finite

determinateness of Presence requires that the One be finite. Hence, there must be a limit to the One, but where this limit is, is entirely a matter of external reflection. This external reflection fences off an arbitrary part of reality as One. In that this is an arbitrary operation, one might conceive of many more Ones within this arbitrarily set limit. In this sense, the external reflection makes One out of the Many and posits the One as divisible. As such, it posits the Many Ones as mutually Attracting units, rather than self-contained mutually Repulsive Ones (cf. Hegel 1830[3], 1817[1]: §98). In a sense then, the immanent indeterminacy of Becoming gets further articulated in Attraction, while the determinacy of Something is further articulated in Repulsion.

As was alluded to in Section 2.6, Hegel regards the conceptual sameness of the Ones as the locus of Attraction instead of the indeterminacy of their Becoming. The difference between his and my treatment stems from my juggling around of the α), β) and γ). Also, the conceptual sameness of the Ones in Hegel's treatment seems to imply a regress towards only the one Presence, whereas I think the point should be that positing a One requires an arbitrary external reflection. This reading seems to be confirmed by the conceptual development towards subsequent moments such as Discrete and Continuous Magnitude.

B. Quantity

2.8. Quantity

γ) In the realm of Quality we established that the Many Ones are self-contained units through Repulsion, whose oneness can nevertheless only be determined through an arbitrary external reflection (i.e. Attraction). With this we have entered the realm of Quantity. Thus, Quantity is an external reflection on a multitude of elements that are distinguishable as Many through Repulsion, but arbitrarily divisible through Attraction (Hegel 1830[3], 1817[1]: §99).

Alberts, who writes on the nature of mathematization (1998: 18-30), Fleischhacker in his search for the object of mathematics (1982) and Dijkgraaf, who defines mathematics as 'the science of patterns and relations' (2001: 7, my translation) would all agree with this result. That is, most branches of mathematics (with the notable exception of topology) presuppose related elements. These relations may be studied for their own sake or they may give rise to patterns from which other relations can be discerned by external reflection. Thus, in terms of external reflection on many distinguishable but divisible elements, their opinion on the object and nature of mathematics and mathematical abstractions is very similar to Hegel's. To them, mathematical descriptions are descriptions in terms of patterns and relations rather than immanent qualities. Thus, to them as well as to Hegel, the object of mathematics is external structure rather than immanent

quality (Alberts 1998: 20, 27-28; Baer 1932: 104; Dijkgraaf 2001: 7; Fleischhacker 1982: 16-17). Hegel's position is remarkable for, in his time, Quantity was conceived of as a property of things rather than an external relation between indeterminate, abstract elements.[76]

2.9. Continuous and Discrete Magnitude

α) The indeterminacy of the limit to the One, means that the quantitative One is not only divisible – in which case Quantity would be confined to the set of rational numbers ℕ –, but entirely Continuous – encompassing all the real numbers ℝ. Hence, Quantity (in its moment of Attraction) is essentially given as a Continuous Magnitude (Hegel 1830^3, 1817^1: § 100).

β) Although the limit to the One is arbitrarily imposed upon the Continuity of Quantity, it is equally imperative that the One be limited one way or another. So it must be possible to stipulate Discrete elements (hence a Discrete Magnitude) within the Continuous Magnitude. In that a Discrete Magnitude excludes other Magnitudes from itself, it is the Quantitative determination of the Repulsion of the Many Ones vis-à-vis each other. Quantity then, is essentially Discrete and Continuous at the same time (Hegel 1830^3, 1817^1: § 100).

2.10. Quantum and Number [77]

γ) Quantum is a specified Quantity. In the first instance, it is a Discrete Magnitude in that whatever specification is given excludes the Other specifications from itself. However, the *range* of possible limitations to the One is Continuous. Depending on how the Continuous Magnitude is limited to arbitrarily fence of a One, the same Magnitude may have every imaginable Discrete size. Thus, the Discrete size of the Quantum crucially depends upon the way the Continuous Magnitude is arbitrarily chopped up into Many Ones. So Quantum unites both moments (Hegel 1830^3, 1817^1: §101).[78]

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[76] I am grateful to Marcel Boumans for making this remark.

[77] The main gist of this section was taken from an addendum ('Zusatz'), which cannot be found in the German edition of the *Encyclopädie* (1830^3, 1817^1) that I've used so far, but which can be found in Geraets, Suchting and Harris' (1991) translation. The Zusätze are based on lecture notes made by Hegel's students and have posthumously been edited into some of the editions of Hegel's Encyclopädie (most famously those by Suhrkamp). It is these editions that Geraets, Suchting and Harris translated the first part (the Logic) of.

[78] Nowadays, physicists tend to think of a quantum as the smallest possible package of light or other waves that can be radiated (cf. footnote 65). Low frequency quanta have little energy and high frequency quanta have high energy. Low energy quanta have little influence on a particle's speed, but hardly illuminate its position. For high-energy quanta it is the other way round. This so-called quantum hypothesis led Heisenberg to formulate his uncertainty principle (see footnote 72). An implication of this principle is that the laws of Newtonian physics break down for particles that

γ) The specification of Quantity as a Quantum requires a Number. Again, the Discrete size of this Number crucially depends on the limitations to the One chosen within Continuous Magnitude (Hegel 1830[3], 1817[1]: §102).

2.11. Unit and Amount

α) Unit is Quantity taken in its moment of Attraction. Hence, the Unit determines the limit to the One. But since this limit is indeterminate through Becoming, the One is Continuous in itself as well as into Other Ones and thus may be divided into as many smaller Ones as we please. So, a dozen, a pair, a hundred, a million etcetera may all serve as Unit (Hegel 1830[3], 1817[1]: §102).

β) From the standpoint of Repulsion, however, Ones are self-contained Units that exclude the Other Ones from themselves. As such they are distinguishable as Many, so their Amount may be determined (e.g. by counting). So every Quantum, when articulated in a Number is specified as a certain Amount of a certain Unit. Of course the Unit is chosen by an arbitrary external reflection and varies inversely to the Amount (e.g. the Number 1000 may be conceived of as 10 Amounts of the Unit 100 or the other way round) (Hegel 1830[3], 1817[1]: §102).[79]

The Magnitude of the Unit chosen depends on the story you want the Numbers to tell. For example, if you want to cover a floor with tiles and need to know the Amount needed, it makes sense to think of the tiles' surface area as Unit, but if you want to know the number of tiles that could fit into some Amount of containers, it is more sensible to view the number of tiles per container as Unit. Of course tiles, containers, floors and so on cannot be acknowledged at this level of abstraction yet. The point here is just that the same Numbers potentially tell very different stories depending on the way the Units and Amounts are specified quantitatively, even when they relate to yield the same outcome.

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are smaller than a certain limit. Below this limit nature can no longer be described by deterministic laws, but by chance only (Hawking 1998: 69-73). It must be stressed that this modern interpretation of the quantum is *not* what is at stake here.

[79] The German word is 'Anzahl'. 'Anzahl' is derived from 'An' ('to', as in 'give to') and 'Zahl' (Number), and hence has the connotation of numeration. Geraets, Suchting and Harris apparently wanted to preserve this connotation for instead of using Amount they rendered '*Anzahl* sometimes as "annumeration" and sometimes as "annumerator", thus suggesting to the unwary reader that there are two concepts, whereas in fact there is just one' (Suchting 1991: xxxiv-xxxv). My main aim here is to understand the main gist of Hegel's philosophy and explain it in English to the best of my ability. I contend that such a goal entails setting up a systematic dialectical exhibition that makes the most of the possibilities the English language has to offer, rather than trying to mimic the German language, provided that the main gist of Hegel's philosophy still comes across. So, by analogy to architecture, what I try to do is to set up a structure that performs the same *function* as Hegel's philosophy even though it is constructed from different building blocks. Staying as close as possible to Hegel's original text is therefore not imperative to me.

At this point, Hegel introduces what I call a 'side dialectic'.[80] That is, he sidesteps his main argument for a while in order to develop a further understanding of Numbers and arithmetical operations. In a sense he views this realm as an object totality in its own right. The principle of this object totality 'must come from the characteristic determinations implicit in the concept of Number itself.' (Hegel 1830^3, 1817^1: §102, my translation). These are: α) Unit and β) Amount. The γ) resolution of these two oppositional concepts cannot be given in just two moments. Instead, Hegel distinguishes four moments, three of which are arithmetical operations that he orders according to the degree to which the opposition between Unit and Amount is preserved in that operation.

γ) First of all, as Many Ones as we please, have to be taken together and Numerated. Numeration prepares a colligation of Many Ones for a quantitative treatment. However, further arithmetical operations can only be performed upon them, if what is taken together is already numerical (Hegel 1830^3, 1817^1: §102). For example, if you count the elements in a set, 1, 2, 3, …, n, you have made that set quantitative, but no further calculations can be performed upon it without a further Magnitude: a set of m elements to be added to it, or b colligations (i.e. sets) of n elements, etc. Without this further Magnitude all that is established is that all the numerated elements can be taken together as a Unit, while they are still numerable as Amount (which implies that the elements themselves are also Units). So, it is still mainly their Unit that is established so far.

γ) Next the colligated numerical Ones (i.e. sets of elements) generally consist of unequal amounts. So a different Number is applicable to each lot. Counting these lots together is the first arithmetical operation: Addition (Hegel 1830^3, 1817^1: §102).

γ) Secondly, numbers may be considered as equal rather than different (i.e. b sets of n elements), so that the moment of Unit is itself an Amount (that is, each of b sets, or units, itself consists of an Amount of n elements). Saying something

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[80] Within his §§, Hegel always first explains his main argument (in this case how Unit and Amount spring from Quantum and Number). In his *Encyclopädie* he usually, but not always, widens the left margin next. In the section with the wider margin he explains himself further either to all readers that need further explanation or to a specific group of readers such as mathematicians. The mentioned 'side dialectic' was taken from such a section.

There are similar sections in the *Wissenschaft der Logik*, which Hegel calls remarks ('Anmerkungen'). In the Petry (1970) translation of the second part of the *Encyclopädie*, the wider margined sections are also separated off from the main text under the heading 'Remark'. In the posthumously published fourth edition of the *Encyclopädie* (1832) the first part of which was translated by Geraets, Suchting and Harris (1992), you will also find additions ('Zusätze'), which are based on lecture notes of Hegel's students. In the Geraets, Suchting and Harris (1992) translation, like in the original, these are distinguished from the main text by the use of a smaller font. Petry (1970) also had the additions printed in a smaller font.

quantitative about these equal Numbers is Multiplication. In Multiplications, Unit may be taken for Amount and the other way around, because $b \cdot n = n \cdot b$. That is, in multiplication the two moments are still distinguished, but it does no longer matter what you regard as Unit and what as Amount (Hegel 1830[3], 1817[1]: §102).

γ) Thirdly, in Raising to even powers the distinction between Unit and Amount can no longer be made, because in Raising to even powers every Number only bears on itself. This means that Unit and Amount are completely equal under this operation. Hegel therefore concludes that the opposition between those two moments is entirely resolved in Raising to even powers, so there can be no other modes of calculation. However, Numbers can be taken together as well as taken apart. So next to the three positive arithmetical operations discussed, there are also three negative ones (viz. Subtraction, Division and Taking the n-root) (Hegel 1830[3], 1817[1]: §102).

When a Number is Raised to an odd power however, Unit and Amount are again unequal. If a number is Raised to the power of three, for example, this can be written as $b \cdot b^2$. In b^2 Amount equals Unit, thus forming a new Amount. With b^2 being the Amount, b must be the Unit in this expression. Thus, b^2 and b can no longer be the same ontologically when Raising to an odd power, so the difference between Unit and Amount resurfaces. Thus, when a number is Raised to an odd power, Unit and Amount are no longer equal (Carlson 2002: 36).

2.12. Limit

γ) Specifying Quantity as a Quantum means Limiting both its Unit and its Amount. The Unit is the arbitrarily chosen Limit to the Unit One. In that this Limit is quantitative as well, it is itself given as an Amount of some other arbitrary Unit chosen by external reflection. However, once a Unit is chosen, the Many Ones may be numerated into an Amount, which itself can of course be taken as Unit again. So all Quanta derive their meaning from the chosen Limit to the elements inside the set (i.e. the Units) as well as from the Limit to the Amount of elements that belong to the set. Or, in other words, a specification of a Quantum as Number requires a Limit to the One (i.e. a Unit) as well as a Limit to the Many Ones (given as Amount). Hence, 'the whole of the Quantum is identical with the Limit' (Hegel 1830[3], 1817[1]: §103, my translation; c.f. Hegel 1812, 1813, 1816: 250, 2.2Ba).

2.13. Intensive and Extensive Magnitude

α) Limiting Unit and Amount and expressing them together in a single Number sets this Number apart from other Numbers. As such the Quantum is expressed as an Intensive Magnitude or Degree (Hegel 1830[3], 1817[1]: §103; Hegel 1812, 1813, 1816: 250, 2.2Ba).

β) In itself however, this Intensive Magnitude is entirely meaningless. It derives its meaning from what it is not, from what lies beyond its Limits. That is, to specify a Quantum is to specify a Degree or Intensive Magnitude on an Extensive scale. Without this scale the Intensive Magnitude would not make sense. A hundred for example is what it is because it is one more than 99 and one less than 101. So a Quantum is always defined through its relations to other Quanta that are larger or smaller (Hegel 1812, 1813, 1816: 256, 2.2Bb). In this sense the quality of the Quantum is external to itself. This externality is explicitly expressed in Extensive Magnitude.

A Quantum is always Limited as an Intensive Magnitude within a series of other thinkable Quanta. This series is its Extensive Magnitude, the medium in which the Intensive Magnitude has its meaning. Hence, the truth of an Intensive Magnitude is in its relation to an Extensive Magnitude. The fact that this Extensive Magnitude is always beyond the Intensive Magnitude posited and the fact that every 'beyond' (e.g. 101 in the example above) can again be taken as an Intensive Magnitude, implies an infinite progression towards a beyond beyond every beyond. Thus, the Extensive Magnitude progresses towards a bad potential infinity (Hegel 1830^{3}, 1817^{1}: §104).

This progression, however, is the result of transfinite iterations of one and the same operation. E.g. assuming an element 1 as an Intensive Magnitude and a function that adds 1 to it leaves you with the – denumerable infinite – set of natural numbers $\mathbb{N}$ If you also allow subtractions, you get the set of whole numbers $\mathbb{Z}$. Allowing divisions gets you $\mathbb{Q}$ and allowing all other operations finally gets you the – overdenumerable infinite – set $\mathbb{R}$.[81] So to get a series one needs to assume an Intensive Magnitude of the Quantum and a function that specifies how to arrive at the other elements in the series. As was indicated in Section 1, this function is the locus of the true quantitative (mathematical) infinity (Hegel 1812, 1813, 1816: 260-264, 276-278, 2.2Ca-2.2Cc; Fleischhacker 1982: 143-147; Lacroix 2000: 311-315). In the Encyclopädie however, Hegel only mentions the infinite quantitative progression that takes place in Extensive Magnitude. He does not go into its resolution the way he does in the Wissenschaft. The differences between both works will be further elaborated upon in the appendix.

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[81] Mathematically, the set $\mathbb{N}$ requires three axioms. First, there is an element $1 \in \mathbb{N}$. Second, each element $\in \mathbb{N}$ has a successor that is exactly 1 element larger than the previous element. Third, all elements thus obtained $\in \mathbb{N}$. The set $\mathbb{Z}$ is obtained by expanding $\mathbb{N}$ with zero and the negative numbers. Next, $\mathbb{Q}$ is obtained by dividing $\mathbb{Z}$ by $\mathbb{N}$. I am grateful to Wouter Krasser for help with this example.

C. Measure

2.14. Measure

γ) Our careful examination of the realm of Quality left us with a multitude of elements distinguishable through Repulsion, but arbitrarily divisible through Attraction. Thus, the basis for the quantitative was a result of the failed attempt at making Qualitative distinctions through the examination of Quality alone. In the realm of Quantity however, we established that an Intensive Magnitude must go beyond itself into its Extensive Magnitude to end up beyond every Quantum. So in the end, all distinctions that were posited in the realm of Quantity dissolve in the bad potential infinity. So neither the qualitative nor the quantitative realm can stand on its own. Each ends up as the other and has it's meaning in that other. This inextricable relation between the two realms is expressed in Measure. As such, Measure is a qualitative Quantum (Hegel 1830^3, 1817^1: §106-107).

3. Hegel's Determination of Mathematical Mechanics

In this section the concepts of mathematical mechanics, which is akin to geometry, are dialectically determined. Mathematically, the most important ones here are mathematical Space, the Point, the Line, the Plane and the 'spatial figure' (i.e. Distinct Space). The argument largely depends upon part 2 (the philosophy of nature), subdivision 1, section A of the *Encyclopädie* (Hegel 1830^3, 1817^1: §§253-261).

With respect to the differences between Hegel's application of the α-β-γ-format and my own, the following comments are in order: Hegel determines the first opposition arising from Space explicitly through the questions α) and β) (Hegel 1830^3, 1817^1: §§255-256) (Sections 3.2-3 below), but the first conditions of existence of this opposition are also discussed under β) (Hegel 1830^3, 1817^1: §256), whereas I contend they should be discussed under γ) (Sections 3.4-7 below). In the remaining determination of the foundations of geometry the α), β) and γ) questions are left implicit (Hegel 1830^3, 1817^1: §§257-261) and have therefore been added in Sections 3.8-12 below.

A. Space and Time

3.1. Space

Space is the universal principle of all things other than thought, i.e. all material things, because all material things are spatial.[82] So the natural sciences presuppose space. Material observables ('Gegenstände' – see footnote 22) are the object of the natural sciences. As such they reside not in the realm of quality, which is internal to Something, but in the realm of external relations, i.e. Quantity, only (cf. Section 2.8). In conceptual isolation space is empty (i.e. devoid of distinctions), continuous (Hegel 1830^3, 1817^1: §254) and infinite.

3.2. Spatial Dimensions

α) The first distinctions that can be made within Space are between the three Spatial Dimensions. Height, length and breadth must be distinguished within Space, but in conceptual isolation these distinctions are still indeterminate. Each dimension is determined vis-à-vis the other two, but other than that it does not matter at all what direction you called height, length or breadth in the first place (Hegel 1830^3, 1817^1: §255).

3.3. The Point

β) The Point, although it is a point in Space, by definition is not spatial, the way Nothing (conceptualized as empty observation and thought itself) is *in* a Being but not itself a Being.[83] The Point resides in Space although it has neither area, nor body. One might also say that the point is 'given entirely as limit' (Paterson 2004/2005: 18).[84] As such it can function as a reference Point for qualitative distinctions in Space, but it cannot itself have material Presence ('Dasein') in Space.

Mathematically, the reference Point can only have the coordinates (0,0,0), since the dimensions height, length and breadth are still indeterminate. Hence the

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[82] As a universal principle that is not, like Being, itself part of an opposition, Space is in neither of the categories α), β), or γ).

[83] 'By definition' is not meant to imply that this mathematical definition of the Point is taken over critiquelessly. Rather, it means the conventional mathematical definition has been dialectically justified at this point.

[84] Paterson's 2004/2005 paper is entitled: *Hegel's Early Geometry*. In it, Paterson mainly discusses Hegel's *Geometrische Studien* (abbreviated by Paterson to GS) and his *Dissertatio Philosophica de Orbitus Planetarum*. However, 'the discussion of GS […] is fundamental for all of [Hegel's] later thought on the subject' (Paterson 2004/2005: 2) and the dialectic of Point, Plane and Distinct Space (or 'solid') is basically the same in his early and mature writings. Hence, Paterson's comments are also helpful to elucidate the *Encyclopädie's* account of geometry.

orientation of the mathematical axes cannot be determined yet , but there can be a mathematical origin (the Point 0,0,0) around which the axes may pivot freely.

3.4. The Line

γ) The opposition between Spatial Dimensions and the Point is only partially resolved in the Line (Hegel 1830^3, 1817^1: §256). A Line is defined by one direction and at least one Point it passes through.[85] It is Dimensional in that it has direction and it is positional in that it passes through a Point (and from a perspective at a right angle to the line it *is* a Point). But, just like the Point, the Line has neither area, nor body and it too is not spatial in that sense, so it still does not allow us to make any true qualitative distinctions in Space. The Line then, is the extension (unlimiting) of the Point in Space. But because it too is not spatial, it still is itself a limit, which requires further unlimiting in order to gain positive Being (Paterson 2004/2005: 29).

Mathematicians would automatically call the Line either height, length or width. In distinctionless Space this is still a matter of choice.

3.5. The Plane

γ) The opposition between Spatial Dimensions and the Point is further resolved in the Plane (Hegel 1830^3, 1817^1: §256). Two directions and at least one Point can define a Plane. The Point then defines the position of the Plane and the two directions define its orientation. The unbounded Plane can divide the unbounded Space into two, so the Plane – other than the Point and the Line – provides the first distinctions of Space. However, the Plane has no spatial existence as well, for it has an (unbounded) area, but still no body. Thus, although the Plane is the positive Being of the Line it is still a limit or negative Being at the same time (Paterson 2004/2005: 29).

Since positing the Plane involves two different directions, it must involve two dimensions as well. By definition, the third dimension is at a right angle with this Plane. Therefore, all dimensions are determined in positing the Plane. This means that the whole coordinate system is now determined.

[85] Hegel would not have accepted that a (straight) line can be defined by two points it passes through, because those points must be points in Space and understanding Space involves the Spatial Dimensions *and* the Point. Without the Spatial Dimensions therefore the two points are not necessarily points in space, so it is not clear what they define, but it cannot be a line in space (cf. Hegel 1830^3, 1817^1: §256, where he criticizes Kant's definition of a straight line).

3.6. Distinct Space

γ) At least four (flat) Planes can form an enclosing surface that separates of a Distinct (part of) Space ('einzeln Raum') (Hegel 1830[3], 1817[1]: §256). This Distinct Space finally has spatial existence, for it has area and body (content). So by using at least four Planes one can truly make distinctions in Space. Since Hegel recognizes only three Spatial Dimensions (Section 3.2.), unlimiting Distinct Space leads to its expansion or contraction without further conceptual change (Paterson 2004/2005: 33-34). Thus the opposition between the indeterminate Spatial Dimensions and the non-spatial Point is fully resolved now that Distinct Space is posited.

The dialectic of Line-Plane-Distinct Space can also be viewed as a process of integration. That is, integrating the formula for a Line, $y = ax + b$, gives us the area of the Plane beneath the graph of the line,$y = 1/2\ ax^2 + bx + c$. Integrating this again yields the content of Distinct Space, $y = 1/6\ ax^3 + ½\ bx^2 + cx + d$ (Paterson 2004/2005: 54). To do this, however, is to presuppose the existence of a coordinate system, whereas this can only be determined *after* the Plane has been posited.

3.7. Time

γ) In the subsection on Becoming (Section 2.3) we already established that everything that is, is necessarily Becoming. Becoming itself, however, can only be acknowledged if a further *static* determination of Being is given. This static determination is Presence (Section 2.4). It was said that a Presence is here and now, but the concepts here and now themselves were left implicit.

Distinct Space allows us to separate of an abstract part of Space that may be designated 'here'.[86] When we look at a Distinct part of Space, we may perceive Something. But, Something will eventually and inescapably Become its Other through Becoming. So when we confine ourselves to some designated Distinct part of Space, we may observe Becoming in action. It is this change in Space that constitutes our awareness of Time.[87] Time then, is observed Becoming ('das angeschaute Werden') (Hegel 1830[3], 1817[1]: §258; Inwood 1992: 295).

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[86] Just as every Something is actually every Other as One (Sections 2.5-2.6), in distinctionless Space every here may just as well be there as Distinct Space. Here and there are all Distinct Spaces and at this level of abstraction all Distinct Spaces are the same.

[87] Inwood writes on this subject: '[T]he measurement of time, and our perception of its passage, require movement in *space*, esp. of the heavenly bodies.' (1992: 295). True as this may be, to think of Time like this distorts the exhibition, for Motion and Matter are exhibited *after* Time. That is, Something and its Other are qualitatively different and this difference is accessible to our senses and allows us to philosophise about them, but those categories have only been posited as ideas in the realm of Logic and as ideas they are in our minds only. Their existence in the world (i.e. their

3.8. Temporal Dimensions

α) Our awareness of Time involves three Temporal Dimensions: the present, the future and the past (Hegel 1830[3], 1817[1]: §259). The present is clearly linked to Presence. Thus the present is long enough to Become aware of whatever is present (i.e. Presence), but short enough to minimize the changes in the Presence under scrutiny.

Not surprisingly this means that the present is the shortest of the three Temporal Dimensions. In the past there were a series of changes that led Presence to Become the present Something and there are a series of changes that will shape the future of the Other Ones that will Become of it. Hence there can be a lot of changes to Ones in the past and in the future, but close to none in the present. In other words, the past and future are defined by change, while the present is an almost changeless period.[88]

I am fully aware that treating Time as if it consists of three dimensions instead of only one must seem quaint. What is more, it is incongruent with the basically Euclidian determination of the Point, Line, Plane and Distinct Space in the previous sections. However, you might think of a Dimension not only as a direction, but also more generally as an unbounded medium in which something can develop. As such, a Dimension is Extensive Magnitude taken spatially or temporally.

We have seen that Presence, and hence the present, moves along with Becoming. In Hegel's defense therefore, one might say that Presence is indeed just as resilient to change as the spatial dimensions, so it can be thought of as an unbounded medium and hence as a dimension. Even if everything that is, has some definite beginning in the Big Bang the past is just as unbounded, for the present continually leaves more of it behind while it is Becoming. Finally the Becoming of the present can only be ceaseless if the future is unbounded. So in terms of unbounded mediums one may indeed think of the past, present and future as dimensions.

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existence in Space) has yet to be determined. So it is not the movement of heavenly bodies that can be observed (yet) to constitute Time, but the Presence or absence of Something or its Other in a Distinct part of Space. Alternation between Presence and absence is Becoming's condition of existence and when this can be observed in Distinct Space, there is Time.

[88] The link between Presence ('Dasein') and the present ('Gegenwart') cannot be made in German. As a result it is a lot harder to explain the Temporal Dimensions in terms of the German categories 'Werden' and 'Dasein', than in terms of Becoming and Presence with its immanent link to 'present'.

3.9. Now

β) Absolute changelessness is impossible in any period, no matter how short. So, if we want to interpret the present as a changeless period, we need to shorten it infinitesimally. An infinitesimally short and hence truly changeless period is Now (Hegel 1830[3], 1817[1]: §259). Like the Point is *in* Space, but not spatial, Now is *in* Time, but not temporal.

3.10. Place

γ) Space in its continuous emptiness and infinity can be subdivided into Distinct Spaces (Sections 3.1-3.6). Those Distinct Spaces, however, are still empty and indifferent with respect to their content. But the abstract requirement that all we can perceive, think or imagine, is Presence in a state of Becoming (Section 2.3-2.4) not only constitutes Time in Distinct Space, it also means that any Distinct Space in Time contains some kind of Presence. It is the awareness of this Presence that enables us to speak of a Distinct Space as a Place. So while Time is observed Becoming, Place is observed Presence. As such, it is the union of Distinct Space, Time, the Temporal Dimensions and Now, or the union of determinate Distinct Space, here, and Now (Hegel 1830[3], 1817[1]: §260).[89]

3.11. Motion

γ) Positing Place as the union of here and Now, still does not position that Place spatially, because the location of the Point and Distinct Space in empty, infinite and continuous Space cannot be determined for lack of a reference location which is not arbitrarily determined. Therefore every Place is just the same as every other Place, the way every Something and its Other are the same as (Many) Ones.

Because Distinct Spaces are arbitrarily fenced of parts of empty, infinite and continuous Space, there is nothing inherent in them that prohibits any One to roam freely from one such Space to another. Hence the Ones can change Place. When this happens this constitutes our awareness of Time and when it does not this constitutes our awareness of Place. As we have seen in Section 3.9, true changelessness only happens in the Now. Hence Place is the spatial Now. Since Now is infinitesimally short, it immediately passes over into Time and in Time the Ones change their Place, thus constituting Motion (Hegel 1830[3], 1817[1]: §261).[90]

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[89] This latter remark is taken from an addition ('Zusatz') to the main text. So it is not found in Hegel's own writing, but was added posthumously on the basis of transcripts of his lectures (see footnote 77).

[90] It may also be the case that Place changes its Ones. That is, there is no way to tell if things only move relative to one another in a motionless coordinate system or whether that coordinate system is itself moving, so that some things that appear to be moving are actually standing still.

3.12 Matter

γ) The passage from the spatial Now, Place, to the temporal Space, Now, i.e. Motion, still leaves every One in a notorious state of flux. Motion then is not observed Becoming (i.e. Time), but the Becoming of the natural realm as a whole. Therefore it is not the qualitative Becoming of Presence and Something and Others, but a quantitative Becoming. Quantitative Becoming is the Becoming of everything spatial, i.e. Matter. Matter therefore is the actual Presence (and not only the observed Presence) of the natural realm (Hegel 1830^3, 1817^1: §261).

Hegel does not explicitly phrase these points like this. He does call Time 'observed Becoming' ('das angeschaute Werden') (Hegel 1830^3, 1817^1: §258) and Motion 'this Becoming' (Hegel 1830^3, 1817^1: §260), but the comparisons to Presence are the result of interpretations enabled by the immanent link between Presence and Present, which does not exist between the German terms 'Dasein' and 'Gegenwart' respectively (cf. footnote 71).

Summary and Conclusions: How This Dialectic Reflects on Mathematics

In this chapter I have dealt with the question what insights, if any, can be gained from a systematic dialectical perspective on the conceptual foundations of quantitative mathematics (including set theory) and mathematical mechanics, which is akin to geometry. In systematic-dialectical methodology words are ordered according to their internal meanings and conceptual interrelations. This is done by positing and resolving opposites.

In Section 2 we established that the ultimate abstraction of the totality of everything, Being, is so devoid of distinctions that it cannot be contemplated in its abstract entirety, without thought grinding to a halt. In thinking it, one might just as well think Nothing. This however, is equally impossible. So the non-thought of Being immediately Becomes the non-thought of Nothing. The way both these thoughts Become their opposites is the first real thought in the exhibition: the thought of Becoming. The dynamism inherent in Becoming requires a further *static* determination of Being for if there is change here-and-now must be different from there-and-then. This static determination of Being is Presence. Presence is the whole of perception at any point in space and time. In this sense, it moves along with Becoming. If it did not, that is if its determination were to be fixed at some point, it is Something.

Because Something's determination is fixed, it can be left behind in the process of Becoming. When Something is left there-and-then its Other is here-and-now. But by Being here and now, this Other can itself be designated as Something. Hence Something and its Other are the same in the category One. In that the

process by which Something Becomes its Other which, taken as Something, again has an Other, is a *ceaseless* Becoming, there must be Many Ones.

As self-contained units, these Ones are distinguishable As Many through a relation of Repulsion, but because everything is in a notorious state of flux through Becoming, it is not clear where Something ends and its Other begins. Hence the limit to the One is equally indeterminate. So the One is only limited through an arbitrary external reflection. In that this limit is arbitrary, one might think of many more Ones within the unit One. So from the standpoint of Becoming the Many Ones have a relation of Attraction instead of Repulsion.

The exhibition of the realm of Quality so far only succeeded in positing that there must be Many Ones, but that the limit to the One is entirely arbitrary. Thus, all that we are left with is an external reflection on a multitude of elements that are distinguishable as Many through Repulsion, but arbitrarily divisible through Attraction. Hegel was ahead of his time in arriving at this conclusion. With it, we have entered the realm of Quantity. Quantity is a Discrete Magnitude of elements, but because the limit to these elements can only be arbitrarily determined through external reflection, it must be a Continuous Magnitude at the same time.

A Quantum is a specified Quantity expressed by a Number. In the first instance, a Number is a Discrete Magnitude that excludes other Magnitudes from itself. However, the *range* of possible Magnitudes is Continuous. That is, depending on how the One is limited, the same Magnitude may have every imaginable Discrete size. As we have seen, if the One is taken in its moment of Attraction, one may think of Many more Ones within any Unit One. Thus quantified, the One is the Unit of a Magnitude. Next, to express Magnitude in a meaningful Number, the Amount of its Units must be determined (e.g. counted).

So Quanta derive their meaning from the chosen quantitative Limit to the elements inside the set (i.e. the Units) as well as from the Limit to the Amount of elements that belong to the set. Hence, 'the whole of the Quantum is identical with the Limit' (Hegel 1830[3], 1817[1]: §103, my translation; cf. Hegel 1812, 1813, 1816: 250, 2.2Ba). A Limited Amount of a Limited Unit is expressed as an Intensive Magnitude. This Intensive Magnitude derives its meaning from what it is not (e.g. 100 is One more than 99 and One less than 101), that is, from the Extensive Magnitude it excludes. So, to specify a Quantum is to specify a Degree or Intensive Magnitude on an Extensive scale. In that an Intensive Magnitude must continually go beyond itself into its Extensive Magnitude to gain meaning, the Extensive Magnitude progresses towards a bad potential infinity, which is no longer quantitative.

So, just as our reflection on Quality led us into the realm of Quantity, our reflection on Quantity led us back into the Qualitative unreachable and unnamable bad potential infinity. The relation of both realms is made explicit in Measure. As

such, Measure is a qualitative Quantum. One of the consequences of this is that mathematical categories and techniques to Hegel are one-sided devices that need to be reconciled with the qualitative before real headway towards the actual truth can be made.

The other conclusions that will be drawn from the exhibition of Section 2 are largely based on Baer (1932). However, Baer does not present them systematically. Rather, he presupposes that the reader is quite well versed in Hegelian philosophy and proceeds to explain where Hegel stood in the mathematical debates of his time. Baer concludes that Hegel was ahead of his time on more than one occasion and that when his opinion differs from more modern views he is usually loyal to the ideas of the mathematicians of his time (1932: 109, 112-113). By contrast, the current chapter is philosophical rather than historical in nature and aims at a wider audience.

The first mathematical problem that is solved by the systematic-dialectical exhibition of Section 2 is that of number theory. To apprehend numbers with the tools of mathematics, we need to have those tools first. But to build up these tools, especially induction, we first need to have all natural numbers (the complete set N) (Baer 1932: 113). In mathematics this problem is 'solved' by assuming a 'one' and a function that increases the assumed one by one (i.e. a successor function). Although this is a way to build up the full set of natural numbers, it does not explain the origin of the mathematical mindset. Why are we able to understand the category One? What moments are presupposed in this category?

The exhibition above offers a way out of this deadlock by showing that stubbornly staying in the realm of quality automatically leads the exhibition into the realm of Quantity. This happens because the only qualitative distinction – between Something and its Other – that can be made in the realm of quality, dissolves into Many Ones, which can be likened to a set of elements. These elements are Numbers as Ones, while Becoming determines them as Many. Because Becoming is a ceaseless and infinite process the Many are further determined as the full – denumerable infinite – set of thinkable natural Numbers ($\forall x \in \mathbb{N}$: x $\in$ Many Ones). So the realm of quality must become quantitative before further headway (towards Measure) can be made.[91]

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[91] One might comment this only means Numbers are now justified linguistically rather than mathematically. So instead of solving the problem, dialectics has only shifted its locus. However, a central thesis of dialectics is that systematic-dialectical thinking presupposes categories. So it is due to the categories one has, that one can make the transition to mathematical thinking at all. It is the task of dialectics to make people conscious of this and to make them aware of the unconscious processes that must have happened in their brain in the process of acquiring language generally and specialized jargon specifically, i.e. before they started specializing on the basis of certain fixed

Thus, for Hegel the quantitative is only the necessary external reflection on the realm of Quality, without which neither realm can exist. Modern mathematics, by contrast, does not usually make true statements, but correct ones of the form: 'in a world where Euclid's axioms are true….'. Modern mathematicians do not generally care whether this world actually exists. Hence, the notion that the quantitative is the condition of existence of the Qualitative must appear alien to them. In short, mathematicians nowadays do not inquire after the things themselves but only after (hypothetical) relations between them (Baer 1932: 108). In doing so they treat the quantitative as if it were a finished actuality (Fleischhacker 1982: 125)

Now, if we accept Hegel's view that Numbers exist because of the failed attempt at making qualitative distinctions on the basis of quality alone, this means that quantitative mathematics (obviously including set theory, given the importance of concepts such as Many Ones or, equivalently, sets of elements) can never fully be this entirely free-floating subject that some modern mathematicians have made it into. Its basis (viz. the rationale behind Numbers or, more generally, sets of elements) namely is still qualitative.

On the other hand, the quantitative was determined as the realm of external reflection on a multitude of distinguishable yet arbitrarily divisible elements. This means that there is considerable scope to escape this qualitative basis by studying quantitative relations on their own account as most modern mathematicians do. But without the qualitative there would be no quantitative, hence there would be no Numbers and no mathematics. According to Kol'man and Yanovskaya this insight is one of the greatest merits of Hegelian philosophy in the field of mathematics (1931: 2, 5).

The second thing that is clarified by the exhibition in Section 2 is the proper use of ordinal and cardinal Numbers. In the 'side dialectic' of Section 2.11 it was said that Numeration prepares a qualitative colligation of Many Ones for a quantitative treatment. In the first instance Numeration involves ordinal Numbers. If one counts the Amount of elements in a finite set you have to begin somewhere, so while counting, you implicitly call One element the first, One the second etc. But if the Numeration is complete, we have arrived at an Intensive Magnitude (Baer 1932: 115).

Within that Intensive Magnitude it no longer matters which element we counted first and which second, so the arbitrary distinctions between the elements that were created by the form of the series (e.g. $1, 2, 3, \ldots, n$) of the concomitant Extensive Magnitude have now disappeared. Every element in a set of size n may

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definitions and methodologies. I am grateful to Wouter Krasser for insightful discussions on this topic.

be the n^{th} element. Then the size of the set (n) is a cardinal Number. This cardinal Number is the Intensive Magnitude of the finite set. As such the size of the set is itself a Unit: it expresses the number of elements it contains while denying them autonomy. The same goes for other sets. Next, we might want to rank the several sets according to their Intensive Magnitudes. This is only possible if underlying the Intensive Magnitudes (now taken as Units) of these sets is a more fundamental Unit. That is, the elements of both sets must be of the same size (e.g. One). If the ranking of the several sets is complete the place a particular set occupies, is again expressed in an ordinal Number (e.g. the 1st, 2nd, 3rd, …, n^{th} place in some well-ordered list) (Baer 1932: 115).

However, there is a crucial difference between the ordinal Numbers involved in numeration and those associated with a place in a ranking. The numerated Ones are Units (i.e. elements) of the same size and the ordinal Number that gets associated with each is a chance occurrence in the process of counting, so they do not denote real quantitative differences, but only superficial distinctions. The place in a well-ordered list, by contrast, is the result of real quantitative differences between the cardinal Numbers associated with sets of different sizes.

If you count the Amount of elements in the infinite set of natural numbers $\mathbb{N}$ you count not to n but to ω. ω is the infinite ordinal number associated with the last element in the set $\mathbb{N}$.[92] The cardinal Number associated with ω is $\aleph_0$, which measures the size or Intensive Magnitude of $\mathbb{N}$. By applying the power operation to $\mathbb{N}$, it is expanded with all subsets contained within it. Thus, this operation yields the nondenumerable infinite set of the real Numbers $\mathbb{R}$, of size $\aleph_1$.[93] The ordinal

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[92] If you would count first odds and then evens, you would reach the ω element twice, because both sets correspond bijectively to the set $\mathbb{N}$. So both odds and evens are of the same size as $\mathbb{N}$. This is why infinite ordinal and cardinal numbers need to be more strictly distinguished from each other than finite ones (Horsten 2004: 25).

[93] Cantor also provided an independent proof that the size of $\mathbb{R}$ is of a higher order of infinitude than $\mathbb{N}$. If you have a list of real numbers between 0 and 1:

1. 0.**0**896542 ...
2. 0.3**1**23908 ...
3. 0.90**8**9753 ...
4. ...

you can construct the diagonal number 0.018 ... If all digits $\neq 1$ in this number are replaced by the digit 1 and all digits $= 1$ are replaced by 0, the new number 0.101 ... must differ from all the numbers in the list in at least one decimal place. Hence, even the list of all elements $\in \mathbb{R}$ between 0 and 1 can always be expanded (Horsten 2004: 28). This argument does not apply to $\mathbb{N}$, because each $x \in \mathbb{N}$ has only a finite amount of positions, so the diagonal does not touch all numbers. As a consequence you can always add a number $\in \mathbb{R}$ somewhere *within* a well-ordered list, while a

Number associated with the last element of $\mathbb{R}$ is ω^{ω}. Applying the power operation to $\mathbb{R}$ yields a set of the even more infinite size $\aleph_2$ of which the last element is ω^{ω} to the power of ω^{ω} (Horsten 2004: 26). Each power set contains a tremendous Amount of sets of the previous order of infinity. The ordinal number associated with the last element of $\mathbb{R}$ for example, can also be written as $\omega^{\omega-1}\,\omega$. So you can 'count' $\omega^{\omega-1}$sets of size $\aleph_0$ within that set. Thus, in a set of size $\aleph_1$ these sets appear as actual infinite elements rather than as bad potentially infinite sets (Horsten 2004: 26-27). So just as transfinite iterations of a successor function lead a finite Intensive Magnitude into the bad potential infinity associated with its Extensive Magnitude, transfinite iterations of the power operation lead an infinite Intensive Magnitude into the bad – or infinitely worse – potential infinity associated with the size of the set of all sets V. Hence, the infinite cardinal Numbers may themselves be ranked in a well-ordered list. This implies that infinity is no longer just defined as an unreachable Extensive Magnitude beyond every finite Intensive Magnitude, but within the well ordering of the infinite cardinal Number associated with that set, can itself also be viewed as an Intensive Magnitude. As a result we can now distinguish two principles of philosophical infinity: 1) the principle of Numeration that leads a finite Intensive Magnitude into its potentially infinite Extensive Magnitude and 2) the principle of the power operation that ultimately leads an infinite Intensive Magnitude into the Extensive Magnitude associated with the set of all sets, V. Because Cantor was born after Hegel's death, Hegel cannot possibly have been aware of these points. [94]

Section four showed how our awareness of Time is the result of focusing on a Distinct (part of) Space. The awareness of Time is a prerequisite for our comprehension of Movement. Distinct Space itself presupposes Space, the Spatial Dimensions, the Point, Line and Plane. Hence, it would be misguided to think of the Line as the result of the Movement of a Point or of the Plane as resulting from a Moving Line.

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well-ordered list of the first 1,000; 1,000,000 or however many you wish elements of $\mathbb{N}$ can only be expanded *outside* the list.

[94] Although he locates the category of set at the level of Hegel's Doctrine of the concept (I-3), Paterson's (2007) Hegelian discussion of the philosophical foundations of the category of set and of Russell's paradox is strikingly similar to mine albeit that his treatment is mathematically more elaborate.

APPENDIX: COMPARISON OF THE DETERMINATION OF THE QUANTITATIVE IN THE WISSENSCHAFT AND THE ENCYCLOPÄDIE

Hegel's *Wissenschaft der Logik* is comprised of three books entitled *the logic of Being* ('die Logik des Seins'), *the logic of essence* ('die Logik des Wesens'), and *the logic of the concept* ('die Logik des Begriffs') respectively (Hegel 1812, 1813, 1816: 62, Introduction). The quantitative and its moments are entirely determined in the 457 pages long logic of Being. The whole of the Encyclopädie by contrast is only 500 pages long. Of these, only the 17 pages of Part 1, subdivision 1 correspond to the first book of the *Wissenschaft*. So, naturally the exhibition in the *Wissenschaft* is a lot more extensive, but this does not mean that the Encyclopädie only summarizes the lengthier exhibition in the *Wissenschaft*.

Because of the great differences between the two books with regard to length and detail, the *Wissenschaft* needs to be summarized before meaningful comparisons can be made. A good starting point for a meaningful summarization is the table of contents, which 'vigorously organizes the whole book' (Carlson 2000: 3). The *Wissenschaft* is divided first into parts, then into books, then into segments ('Abschnitte') and next into chapters. The chapters are subdivided into sections A, B and C which are usually, but not always, subdivided again into subsections a, b, and c. If the titles of the sections A, B and C are taken as moments in the logical progression, an exhibition with about the same level of detail as the exhibition given in the *Encyclopädie* (and reproduced in Section 2 of this chapter) is obtained.

The table of contents of the *Encyclopädie* is less detailed than the table of contents of Section 2 of this chapter. To obtain the latter I have roughly taken one emphasized moment from each § of part 1, subdivision 1 of the *Encyclopädie*, even though Hegel usually stresses a couple of words per § using d o u b l e s p a c i n g. The choices I have made, do not always correspond to the titles of sections A, B and C in the *Wissenschaft*, although they often do correspond to the titles of subsections a, b and c. Conversely, the titles of sections A, B and C in the *Wissenschaft* do often correspond to moments emphasized in Hegel's *Encyclopädie* but not in my exhibition of it. It wouldn't be fair to treat these different emphases as real differences.

In what follows, the *Wissenschaft* is therefore summarized according to the titles of sections A, B and C, but these are amended with titles of subsections when this facilitates comparisons to (my interpretation of) the exhibition in the *Encyclopädie*. Whenever the logical progression in the *Encyclopädie* can be interpreted along the lines of the *Wissenschaft* or the other way round, I will refer to both books. When a moment is unique to one of both, I will refer only to the book it is unique to.

A1. Being, Nothing, Becoming, Presence, Something and Others

The first four moments are the same in both works. That is, Being is contrasted with Nothing; Being and Nothing each Become the other, but what we perceive is not Becoming, but a stabilized Presence ('Dasein') (Hegel 1812, 1813, 1816: 82-117, 1.1A-1.2Aa; Hegel 1830^3, 1817^1: §86-89). In the *Wissenschaft,* Presence is introduced in Section A of the second chapter of the first segment. Section B is entitled Finitude and Section C Infinity. But Section B opens with a subsection on Something and Others. So the exhibition in both works is still roughly the same until this latter opposition is introduced.

However, Finitude and Infinity are clearly emphasized in the *Wissenschaft*, but not in the *Encyclopädie*. In the latter work Hegel writes: 'The categories that develop in respect of being there [i.e. Presence] only need to be indicated in a summary way' (Hegel 1830^3, 1817^1: §90) [95]. In the *Wissenschaft*, by contrast, Hegel devotes more than half a chapter (41 pages) to these categories. I will now discuss the way in which Finitude and Infinity are determined from Something and Others in the *Wissenschaft*.

A2. Qualitative Limit

γ) Something stops where its Other begins and the other way round. Hence, Something is limited by its Other which in turn is limited by Something. What they have in common is this Qualitative Limit (Hegel 1812, 1813, 1816: 135, 1.2Bb; Hegel 1830^3, 1817^1: §92).[96] The Limit must be Qualitative because the groundwork that enables a Quantitative treatment of reality is not yet complete.

A3. Finitude and Infinity

β) Something is Presence with a fixed determination, which can be left behind in the process of Becoming. As we have seen, this requires the Something to be Limited in either space or time or both. Something that is Limited is also Finite (Hegel 1812, 1813, 1816: 139-140, 1.2Bc; Hegel 1830^3, 1817^1: §92).

α) Since the determination of Something is fixed as either here-and-now or there-and-then, it will be left behind in the process of Becoming. Something

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[95] In this book all paragraph references (such as §18) are to Hegel 1830^3, 1817^1 and any citations are from the 1991 English translation by Geraets, Suchting and Harris, unless explicitly stated otherwise.

[96] Hegel does not explicitly distinguish the Qualitative Limit from the Quantitative Limit. But the category of Limit plays an important role in the first segment of the *Wissenschaft*, which is called quality, as well as in the second, which is called Quantity. There are striking differences between both types of Limit. Hence, I will make explicit distinctions between the two types of Limit in this appendix.

therefore must eventually Become its Other (e.g. when we die, our bodies dissolve and its atoms eventually build new bodies or when we grow up, we go from baby to toddler, to child to adolescent to grown-up – you can draw the Limits wherever you like). This Other is Finite again. But the very fact that each Something is doomed to perish and Become its Other, which in turn is just as Finite and thus just as doomed, is eternal (Hegel 1812, 1813, 1816: 148-149, 1.2Bc; Hegel 1830^3, 1817^1: §93-94). Finitude as a category therefore is an Infinite attribute of all Beings.

A4. True Infinite

γ) Finite Somethings incessantly Become their Others. From the point of view of this process, we perceive mainly Infinity. From the point of view of any particular Something, Finitude is perceived most clearly. The truth of Beings, however, is their Becoming as well as their Presence, that is, their Infinity as well as their Finitude. As it stands, both concepts are opposed to one another and constitute each other. Finitude is Finite because the Infinite process keeps it that way and the Infinite process constantly needs new Finite victims in order to stay an Infinite process. So both concepts return to themselves through their opposites. Neither of the two concepts can stand on its own in this self-perpetuating circle. The truth of both is therefore not in its opposite, but in the circle itself. This circle has no beginning, end or opposite. It therefore is the True Infinite (Hegel 1812, 1813, 1816: 161-163, 1.2Cc; Hegel 1830^3, 1817^1: §95).

A5. Being-for-self

γ) Hegel uses Being-for-self to denote the immediate appearance of things. So if Hegel speaks of Something Being-for-self he considers only the sensuous aspects of it and brackets out its theoretical and conceptual mediations. Being-for-self is a further determination of the True Infinite, because all Beings-for-self are individually Finite, but collectively Infinite, for collectively, Being-for-self embodies all Somethings and all Others and hence the whole of the Truly Infinite circle (Hegel 1812, 1813, 1816: 166, 1.2: 'Der Übergang'; Hegel 1830^3, 1817^1: §95).[97]

[97] 'Der Übergang' means 'the transformation'. It is a section of Chapter 1.2 that is on the same footing as the A, B and C-sections. In the whole of the first book of the *Wissenschaft*, only this chapter has such an unusual extra section.

A6. One, Many Ones, Repulsion, Attraction, Quantity, Continuous and Discrete Magnitude, Quantum, Number, Unit and Amount, Quantitative Limit and Intensive and Extensive Magnitude

Being-for-self equalizes each Something and each Other as One. As soon as the One is presented in the *Wissenschaft*, the two books are back on the same track. So, just to recap: there are Many Ones because each Other can be designated as Something, so that Something and Other are both Ones and each One must ceaselessly develop into another One through Becoming. Because the limit to the One is arbitrary, one may think of Many more Ones within a Unit One, so the Many Ones have a relation of Attraction. But at the same time the Ones are distinguishable as Many, so they must be self contained Units, that are Repulsive vis-à-vis One another. This result leads us into the realm of Quantity, i.e. that of external reflection on a multitude of elements distinguishable as Many through Repulsion, but arbitrarily divisible through Attraction.

Quantity is a Discrete Magnitude of elements, but because the limit to these elements can only be arbitrarily determined through external reflection, it must be a Continuous Magnitude at the same time. A specified Quantity is a Quantum expressed by a Number. If the elements contained in this Number, the Ones, are taken in their moment of Attraction, one may think of Many more Ones within any Unit One. Thus quantified, the One is the Unit of a Magnitude. Next, to express Magnitude in a meaningful Number, the Amount of its Units must be determined (e.g. counted) (Hegel 1812, 1813, 1816: 182-234, 1.3Ba-2.2A; Hegel 1830^{3}, 1817^{1}: §96-102).

So Quanta derive their meaning from the chosen quantitative Limit to the elements inside the set (i.e. the Units) as well as from the Limit to the Amount of elements that belong to the set (Hegel 1812, 1813, 1816: 231-234, 2.2A, 250, 2.2Ba; Hegel 1830^{3}, 1817^{1}: §103). A Limited Amount of a Limited Unit is expressed as an Intensive Magnitude. To specify a Quantum is to specify a Degree or Intensive Magnitude on an Extensive scale. In that an Intensive Magnitude must continually go beyond itself into its Extensive Magnitude to gain meaning, the Extensive Magnitude progresses towards a bad potential infinity, which is no longer quantitative (Hegel 1812, 1813, 1816: 250-253, 2.2Ba; Hegel 1830^{3}, 1817^{1}: §103; Fleischhacker 1982: 136-146).

Hegel mentions Unit and Amount in Section 2.2A of the *Wissenschaft*, but he does not devote any subsections to them (Section 2.2A does not even have subsections). These moments, then, are actually emphasized more in the *Encyclopädie* than in the *Wissenschaft*. This is rare. Furthermore, I have given the category of Quantitative Limit official status (Hegel mentions it a lot in his text, but it does not feature in the table of contents of the second segment of the

Wissenschaft). [98] The rest of the moments that were mentioned here are taken from the A, B and C-sections, so with them the comparison between the two works is particularly smooth.

A7. Quantitative Infinity

γ) As it stands, the concepts Intensive and Extensive Magnitude are opposed to each other and constitute each other through a function that specifies how to arrive at the other elements in the series. This function is the locus of the true Quantitative (mathematical) Infinity (Hegel 1812, 1813, 1816: 260-264, 276-278, 2.2Ca-2.2Cc; Fleischhacker 1982: 143-147).

A8. Direct Ratio

In the process of concrete determination it was first shown how the quantitative mindset originates in and presupposes the Qualitative moments of Being, Nothing, Becoming, Presence, Something, Others, Qualitative Limit, Finitude, Infinity, the True Infinite, Being-for-self, One, Many Ones, Repulsion and Attraction. Especially the move from Being-for-self to One and Many Ones is significant, because this transformation shows how Something and its Other are the same as One, but distinguishable as Many.

From the moment of Quantity onwards, the quantitative has been progressively freed from the qualitative moments it originated in. However, Extensive Magnitude showed that the nature of the Quantum can never be entirely determined in relation to the quantitative itself, because there is always Something beyond the Quantum that is not Limited as a Quantum (yet). This 'beyond' is the quality of the Quantum. So stubbornly staying within the realm of Quantity eventually and automatically led us back to quality, albeit the quality of the Quantum.

Since it has now been shown how the quantitative presupposes the qualitative and how the determinations of Quantity result in a return to quality, we need to

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[98] On the basis of the table of contents of the *Wissenschaft* one might conclude that the opposition between Intensive and Extensive Magnitude is determined directly from Number. In this instance Unit and Amount do not even feature in the titles of subsections. Hegel however clearly emphasizes these moments in both works. In the *Wissenschaft* for example Hegel writes: '*Amount* and *[U]nit* constitute the *moments* of number' (Hegel 1812, 1813, 1816: 232, 2.2A, Miller's 1969 translation). The systematic dialectic necessity to overcome this opposition before a new opposition can be introduced, led me to give the category of Quantitative Limit official status. In other words: I have given Unit and Amount official status in my main text about the *Encyclopädie* and this is also defendable in the case of the *Wissenschaft*. But in the latter case, this intervention entails the need for a category to bridge the gap between the opposition between Unit and Amount and the opposition between Intensive and Extensive Magnitude. This category is Quantitative Limit.

determine how the two relate. That is, to resolve this opposition we need to look for a category that explicitly relates quality to Quantity. Hegel looks for this category in mathematical relationships ('Verhältnisse'). Following the Miller translation of the *Wissenschaft*, I will call these relationships Ratios (Hegel 1812, 1813, 1816: 372, 2.3; Miller 1969: §669).[99]

γ) The Direct Ratio, $\boldsymbol{x/y = c}$ (in which $\boldsymbol{c}$ is a constant and $\boldsymbol{x}$ and $\boldsymbol{y}$ are variables), is the first of these Ratios. If it is said that a rectangle is 4 by 2 (or, equivalently, 2 by 1), its width and breadth are implicitly given as a Direct Ratio. Since the expression $\boldsymbol{x/y = c}$ may also be written as $\boldsymbol{x/c = y}$, one may either take $\boldsymbol{c}$ to be the Amount of Units $\boldsymbol{y}$ or $\boldsymbol{y}$ to be the Amount of Units $\boldsymbol{c}$. So when Unit and Amount are related through a Direct Ratio, it is still undetermined whether $\boldsymbol{x}$ or $\boldsymbol{y}$ is a Unit or an Amount. Both are what they are vis-à-vis the other. So the qualitative determinations of these Quanta are still external to this Ratio (Hegel 1812, 1813, 1816: 375, 2.3A; Fleischhacker 1982: 164; Carlson 2002: 102-103).

A9. Inverse Ratio

γ) Inverse Ratio was also discussed in the 'side dialectic' of Section 2.11 above under the heading of Multiplication. What is said about this type of relation in the *Wissenschaft* is very similar to what is said about the arithmetical operation of Multiplication in the *Encyclopädie* (and in a remark to 2.2A in the *Wissenschaft*). The most striking difference is that the determination of this Ratio in the latter work does not happen in a 'side dialectic' meant to advance a further understanding of Numbers and arithmetical operations (as was the case in the *Encyclopädie*), but forms a crucial link between Quantity and Measure. The Ratios therefore appear as manifestations of the immanent relations of concepts within the realm of Quantity. At this stage they do not appear as operations. Even though Hegel does mention the ratio ('Verhältnis') at this stage in the *Encyclopädie* as well, he does not go into any detail about the different types of ratio and their ontological import the way he does in Chapter 3 of the second segment of the *Wissenschaft* (Hegel 1830^3, 1817^1: §105-106).

In the Inverse Ratio, $\boldsymbol{x \cdot y = c}$, $\boldsymbol{x}$ and $\boldsymbol{y}$ can both be designated Unit or Amount, because $\boldsymbol{x \cdot y = y \cdot x}$. That is, in the Inverse Ratio the two moments are still distinguished, but it does no longer matter what you regard as Unit and what

[99] Verhältnis means ratio as well as relationship, but the word is usually translated into English as Ratio. As we will see, however, Hegel has a rather broad conception of a Ratio. He uses Verhältnis to denote all types of mathematical relations between undetermined Quanta x and y that have a constant outcome c, whether this is the outcome of a division ($x/y = c$), a multiplication ($x \cdot y = c$) or raising to a power ($x^2 = c$).

as Amount. If it is said that the surface of a rectangle is 8, this surface is implicitly given as an Inverse Ratio in which the length of the sides is left undetermined (i.e. $\mathbf{2 \cdot 4 = 4 \cdot 2 = 1 \cdot 8 = 8 \cdot 1 = \cdots}$ etc.). In this Ratio, the constant $\boldsymbol{c}$ limits both sides of the Ratio vis-à-vis each other as Unit versus Amount or Amount versus Unit. The constant itself is only their mutual limit. It cannot itself be considered as a Unit or an Amount (Hegel 1812, 1813, 1816: 376-380, 2.3B; Hegel 1830[3], 1817[1]: §102; Fleischhacker 1982: 164 - 165; Carlson 2002: 103-108).

A10. Ratio of Powers

γ) Like the Inverse Ratio, the Ratio of Powers was already discussed in the 'side dialectic' of Section 2.11 above. Here it fell under the heading of 'Raising to even powers'. What was said above about the differences between the Inverse Ratio in the *Wissenschaft* and Multiplication in the *Encyclopädie* also applies to the differences between the Ratio of Powers and Raising to even powers respectively.

In The Ratio of Powers the distinction between Unit and Amount can no longer be made, because in The Ratio of Powers every Number only bears on itself. So this time not only the total surface of a rectangle is given as a constant, but it is also given that $\boldsymbol{x = y}$ (i.e. the rectangle is a square). This means that Unit and Amount are completely equal under this operation. Hegel therefore concludes that the opposition between those two moments is resolved in The Ratio of Powers (Hegel 1812, 1813, 1816: 376-380, 2.3C; Hegel 1830[3], 1817[1]: §102; Carlson 2002: 108-110).

A11. Measure

γ) In the Ratio of Powers Unit quantitatively equals Amount, but not qualitatively. In the expression $\boldsymbol{x \cdot x = c}$, either $\boldsymbol{x}$ may be considered as Unit, determining the other as Amount. So the two are still qualitatively different, they are not yet at one with their concept. That is, the dialectical exhibition so far does not yet fully determine the sides of the ratio. So we need to bring the quality back into the Quantum while keeping it quantitative. 'The name of [this] partnership between quality and Quantity is Measure' (Carlson 2002: 110)

When the Intensive Magnitude of a Quantum is expanded in its Extensive Magnitude, we end up with another Quantum. What is needed in Measure is an Intensive Magnitude that can be quantitatively expanded on some large enough domain while qualitatively staying the same, so that the Quantum may change, while the nature of whatever is Measured stays intact for a while. So the quality of Measure must stay intact longer than the Quanta that are defined with respect to it (Hegel 1812, 1813, 1816: 387-394, 3).

The third segment of the first part of the *Wissenschaft* is entirely devoted to the further determinations of Measure. Within it Hegel searches for the categories and

concepts that are presupposed by a mathematical understanding of nature. So from Measure onwards, Hegel starts looking for the concepts that are presupposed by an application of the quantitative (i.e. mathematical) way of thinking to nature (Fleischhacker 1982: 171). So the purely mathematical realm of Quantity is left here. The details of this segment will therefore not be discussed any further in this appendix.

Concluding Remarks

In this appendix I tracked the differences and similarities between the determination of the quantitative in Hegel's *Encyclopädie der philosphischen Wissenschaften* and his *Wissenschaft der Logik.* Apart from differences in detail and emphases, the most important differences are situated at the end of the relevant segment(s) in the *Wissenschaft*. First of all, quantitative Infinity is absent from the *Encyclopädie*, while it is discussed extensively in the *Wissenschaft*.

Second, the *Encyclopädie* mentions the Ratio as a stepping-stone towards Measure, but leaves it at that. The *Wissenschaft*, by contrast, devotes an entire chapter to the Direct and Inverse Ratio and the Ratio of Powers. These ratios show how the Quantum is ontologically related to other Quanta.

3. Marx's Systematic Dialectics and Mathematics

Introduction

Can a systematic dialectical exhibition inform mathematics (or definitions for mathematics) or mathematical modeling and/or vice versa? The answer to such a question depends first of all on the ontological nature of the subject under investigation (the object totality) and the premises from which the investigation starts (its universal principle) (in systematic dialectics the two are intimately intertwined).

As we have seen in Chapter 1, Marx's criticism on Hegel implies that the starting point for his own dialectical theory of Capitalism should 1) allow for the emergence at later stages of conflicts and negative results and 2) be embedded in Marx's historical materialist conception of history. Just to reiterate: Marx scorned Hegel's obsession with resolving each and every opposition, arguing that misrepresentation of nature may be a source of *apparent* unresolved conflict and that the historical materialist conception of history may point to *actual* unresolved conflicts. At the same time Hegel receives praise for inspiring Marx's ideas on alienation. As a result, Marx's dialectical theory of society is likely to allow less degrees of freedom to human agency (for it is limited by nature *and* history) and allow for a more direct confrontation with empirical nature as alienated products of thought take on an empirical reality. If Marx succeeded in perceiving of such a theory, the abstractions pertaining to capitalism are likely to come out as more amenable to quantification and hence more suitable for mathematical treatment, than those pertaining to Hegel's (exhibition of) civil society.

Secondly, the answer depends on the epistemological prowess one ascribes to mathematics as a means of investigation. Chapter 2 showed that Hegel did not think highly of mathematics in this respect: mathematical categories and techniques to him are one-sided devices that need to be reconciled with the qualitative before real headway towards the actual truth can be made. The literature on Marx and mathematics discussed in Section 1, by contrast, shows not only that Marx towards the end of his life had become quite conversant with the university textbooks on mathematics of his day, but also that he endeavored to reform the basis for mathematics (especially the calculus) dialectically and toyed with a lot of ideas for the application of mathematical and formal methods to his main studies in political economy. Thus, his attitude in this respect is strikingly

different from Hegel's who contended that mathematics cannot adopt dialectical moments at all.

Having established Marx's views on the nature of capitalism and the abstractions appropriate to it (see Chapter 1), as well as his attitude towards the dialectics of mathematics and the use of mathematical techniques within a systematic dialectical exhibition of capitalism (see Section 1 below), the question becomes how these ideas could be articulated together. In order to answer this second main question for this chapter, Section 2 first of all tracks the outline of Marx's dialectics throughout the three volumes of *Capital*, so as to position Marx's 'schemes of reproduction' (which would nowadays be called models) within Marx's overall framework and illustrate the nature of Marx's abstractions. [100] It may safely be concluded from this section that Marx came a long way in formulating an alternative systematic dialectical social theory consistent with his criticisms. Next, Section 3 discusses what assumptions and formal expressions in Marx's models can be considered dialectically motivated and which cannot. Thus it is shown that the inspiration for them can be conceived of as dialectically informed and that their results also illuminate how further concretization could proceed. Section 4 concludes.

1. Marx's Acquaintance with and Ideas on Mathematics

This section discusses Marx's acquaintance with, views on and technical skill in mathematics. This will serve as a background to his use of mathematics within the systematic dialectical exhibition of *Capital* elaborated on in Section 2 and 3.

When Marx graduated from the gymnasium of Trier in 1835 'his knowledge of mathematics was considered adequate' (Struik 1948, 1997: 173; cf. Kennedy 1977: 305), but he showed no specific interest in it until after the completion of the *Grundrisse* manuscript in 1857-58, when he wrote:

> 'In elaborating the principles of economics I have been so damnably held up by errors in calculation that in despair I have applied myself to a rapid revision of algebra. I have never felt at home with arithmetic. But by making a detour via

[100] 'Model' is modern terminology. Marx used the word 'scheme' rather than 'model'. Tinbergen also used the word 'scheme' in his early writings (e.g. his 1940) for what he would later on term 'model' (e.g. in his 1957) (see Boumans 1992). Following Reuten (1999) I will use both terms interchangeably, but, like Reuten, 'I prefer the term 'schem[e]' when close to Marx's text and the term 'model' in the methodological' appraisal (in Section 3) and subsequent reconstruction of them (in Chapter 4) (the quotation can be found in Reuten 1999: 200).

> algebra, I shall quickly get back into the way of things' (Marx 1858b, cited in Struik 1948, 1997: 174; and in Kennedy 1977: 305; cf. Matthews 2002: 6-7; cf. Smolinski 1973: 1193).

From then on, Marx kept 'returning to [the study of mathematics] as a diversion during his many days of illness' (Struik 1948, 1997: 174), turning from algebra to analytical geometry and the calculus (Struik 1948, 1997: 174). Despite his original intent, 'one finds surprisingly few actual applications of mathematical methods […] to *any* practical problems' (Smolinski 1973: 1193) in Marx's notes on mathematics. Thus one may conclude that his mathematical interests increasingly shifted away from their direct practical relevance for 'the elaboration of the economic principles' and towards the study of mathematics for its own sake (Smolinski 1973: 1193). Marx, like Hegel (cf. Section 2.1), was particularly interested in (infinitesimals in) the differential calculus (Matthews 2002: 11). '[I]n 1878-83 [i.e. the last five years of his life], his main objectives became reformulating its theoretical and philosophical foundations, by showing its development from elementary algebra, to represent the operation of differentiation as a particular case of his dialectical law of "the negation of a negation"' (Smolinski 1973: 1194). While studying calculus, Marx had remarked that he found it 'a much easier branch of mathematics (so far as mere technicalities are concerned) than, say, the more advanced aspects of algebra' (Marx 1863, cited in Struik 1948, 1997: 174). Thus, it seems that Marx found 'the calculus easier than algebra' and 'algebra easier than arithmetic' (Struik 1948, 1997: 174; cf. Smolinski 1973: 1197).

Marx classified all previous methods of developing the conception of the differential that he knew about as: the mystical method of Newton-Leibnitz, the rational one of D'Alembert and the algebraic one of Lagrange. He criticized all these because they all involved the derivation of the expression for *change*, dy/dx, from *neglecting* some infinitesimally small but essentially *static* difference h (as in D'Alembert and Lagrange) or dx (as in Leibnitz) between x and $x + h$ (or dx), instead of from the dynamic *variation* of x (and concomitantly y) *itself* (Struik 1948, 1997: 179-182; Kol'man 1983: 228-231).[101] Like Hegel, he considered this procedure dialectically incorrect for it did not truly resolve Zeno's paradox of Achilles and the tortoise. It still allowed dynamic laws of motion to be derived from a reflection on static differences and thus glossed over the fact that a 'sequence of positions of [a] point at rest […] will never produce motion' (Struik

[101] Ernst Kol'man is also referred to as Kolman or Colman. Since he was a Russian mathematician, his name – which would otherwise be written in Cyrillic letters – is usually translated (or rather transcripted) along with the rest of his text, leading to the variations mentioned.

1948, 1997: 184-185; cf. Hegel 1812, 1813, 1816: 295, 2.2Bc; cf. Fleischhacker 1982: 148; cf. Section 2.2).

But, as Chapter 1 indicated, Marx had a much more positive attitude concerning the scope of mathematical formalisms than Hegel did and set out to produce his own alternative method of developing the conception of the differential that he thought lived up to his and Hegel's criticisms of infinitesimals. Hegel, by contrast, had only formulated his criticisms verbally and never bothered to rework mathematics on the basis of his dialectically derived insights, for, as Kol'man and Yanovskaya put it:[102]

> 'According to Hegel these dialectical moments, which are alien to the elementary mathematics of constant magnitudes, cannot be adopted by mathematics at all. All the attempts by mathematics to assimilate them are in vain, for since mathematics is not a science of 'concept', therefore naturally no dialectical development, no movement of its concepts and operations on its own ground is possible' (1931, 1983: 246).

Marx's method can be summed up as follows: starting from, say, $y = f(x) = x^3$, and postulating an x_1 that differs from x by some entirely arbitrary (as opposed to a small or infinitesimal) amount, we may write:

$$f(x_1) - f(x) = y_1 - y = x_1^3 - x^3 = (x_1 - x)(x_1^2 + x_1 \cdot x + x^2)$$

so that:

$$\frac{f(x_1) - f(x)}{x_1 - x} = \frac{y_1 - y}{x_1 - x} = x_1^2 + x_1 \cdot x + x^2$$

When $x_1 = x$, or $x_1 - x = 0$, we obtain:

$$0/0 = \mathrm{d}y/\mathrm{d}x = x^2 + x \cdot x + x^2 = 3x^2 \qquad \text{(Struik 1948, 1997: 183).}[103]$$

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[102] By the same token as Kol'man (see footnote 101) Sofya Yanovskaya is also referred to as Janovskaja or Ianovskaia.

[103] For some reason that eludes me, Struik first writes:
$f(x_1) - f(x) = y_1 - y = x_1^3 - x^3 = (x_1 - x)(x_1^2 + x_1 \cdot x + \boldsymbol{S}^2)$ without defining this '$\boldsymbol{S}$' or explaining where it comes from or why it is introduced. It does not resurface in the subsequent expressions, leaving the reader with the distinct impression that it was never supposed to be there in the first place.

What is dialectical about this method is, first, that 'the derivative only appears when both dy and dx are absolutely zero' (Struik 1948, 1997: 185) and second, that x is allowed to change into *any* value x_1 in its domain and not just those infinitesimally close to it (Carchedi 2008: 423). The first characteristic does away with the annihilation of infinitesimal *static* differences in order to obtain an expression for a *dynamic* relationship. Since these two are qualitatively different, the dynamic expression can only spring forth from the real disappearance or negation ('aufhebung') of the difference between x_1 and x and not from neglecting a static difference h or dx at some point. In dialectical jargon, what happens in Marx's method is that the negation of a static expression leads to a qualitatively different dynamic expression: the negation of the negation (Smith, Cyril 1983: 265). As such, the derivative is *developed* ('entwickelt') from the original expression in Marx's method and not *separated* ('losgewickelt') from some approximate expression (Kennedy 1977: 310-311). Thus, according to Carchedi, Marx shows that the potential for change is already inherent in *x*, even when no change whatsoever actually occurs (2008: 423). It is therefore 'the theorization of a temporal, real process' (2008: 423), in which the realized state of things is articulated alongside, and inseparable from, their potential for change (2008: 423-424).[104] Moreover, the second characteristic shows this change to

[104] In Carchedi's view, the concept pair of realized versus potential is crucial to Marx's dialectics, the gist of which must not be sought in Hegel. Rather, 'we should extract it from Marx's own work' (Carchedi 2008: 416). In short, Carchedi's view boils down to the articulation at each stage of the presentation of a realized phenomenon and its (sometimes contradictory) potential(s). Next, the exhibition is driven towards concreteness by introducing time, thus showing how the realized and the potential are interlinked, i.e. by what mechanism the two change into each other (Carchedi 2008: 416). Although Carchedi seems to contend otherwise, it seems to me that his position can easily be reconciled with that of most Hegelian Marxists.

With the notable exception of those that argue that the outline of Marx's Capital is homologous to (the outline of) Hegel's Logic (such as Arthur and Sekine) (criticizing a homology, Smith (2014), provides a good and comprehensive overview of these authors and the three variants of the homology thesis), many Hegelian Marxists would readily admit that Marx's method, although inspired by Hegel, differs from Hegel's in many respects. This is why writers like Smith and I, for example, opt for general formats like unity, difference and unity-in-difference or α-β-γ respectively, to be applied to Marx's work in order to track his dialectics, rather than look for specific homologies or matching details. The end result of the application of such a format to *Capital* need not be very different from Carchedi's view of that work.

Take, for instance, the dialectics at the beginning of *Capital* as I have tracked them in Section 2 below. The account runs: α) in conceptual isolation, to be a society requires *sociation*, which encompasses social production. Hence, any *realized* society is sociate by definition. β) Yet capitalist production is privately undertaken, in *dissociation*, and capitalist societies therefore run the *potential* risk of disintegration. γ) The tension between α) and β) is resolved when private produce is allowed to move (i.e. by introducing time) from the site of production to that of consumption through the associative moment of the *exchange relation*. Thus, these approaches are

affect all of reality, whereas working with infinitesimals points to a static view of reality 'to which change is only added as an appendix' (Carchedi 2008: 17-18). Marx's, by contrast, is rooted in a dynamic ontology with respect to every element in all of reality.

So far for Marx's views on mathematics as such. As for the application of mathematical techniques to the study of political economy, Marx had at least one noteworthy intuition that did not make it into his texts for *Capital*. On May 31, 1873, Marx wrote to Engels:

> '[Y]ou know tables in which prices, calculated by percent etc. etc. are represented in their growth in the course of a year etc. showing the increases and decreases by zig-zag lines. I have repeatedly attempted, for the analysis of crises, to compute these "ups and downs" as fictional curves, and I thought (and even now I still think this possible with sufficient empirical material) to infer mathematically from this an important law of crises. Moore […] considers the problem rather impractical, and I have decided for the time being to give it up.' (Marx 1873, as cited in Kol'man 1983: 220; cf. Smolinski 1973: 1200)

Samuel Moore was Marx and Engels' advisor in mathematics and they both usually (albeit sometimes reluctantly) accepted his judgment on issues like these as the last word (Matthews 2002: 8-9). According to Kol'man, however, Moore was mistaken in this case. Had he been more conversant with 'Fourier analysis, that branch of applied mathematics which deals with the detection of latent periodicities in complex oscillatory processes', he would probably have been more supportive of Marx's attempts at finding those 'fictional curves' (1983: 220). Smolinski, by contrast, asserts that 'even though both data and analytical methods of the study of the business cycle have greatly improved since 1873, Moore's skepticism with respect to the applicability of Marx's proposal appears to be well taken even from the vantage point of the 1970s' (1973: 1200).

All in all, Marx studied at least five textbooks on calculus and two texts on algebra (Struik 1948, 1997: 176-177) and explicitly intended to use the insights he gained from these to further his 'elaboration of the economic principles'. So we can safely conclude that Marx was neither ignorant of mathematics, nor considered it inapplicable to the field of political economy generally, or socioeconomic relations specifically (Smolinski 1973: 1191 – 1193, 1201). This being said and given that he found calculus easier than algebra and algebra easier

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clearly reconcilable. That is, whether one progresses by viewing each concept from the oppositional angles α) and β) before moving on to the opposition's resolution under γ) or by looking for a concept's realization, its potential and the movement between the two, may very well amount to the same thing.

than arithmetic, it is startling that he usually, if not always, sticks to numerical examples in *Capital* even when elementary algebraic techniques, like dividing the numerator and denominator by the same symbol, could have given him a direct and, moreover, perfectly general result (Smolinski 1973: 1197).

One possible explanation could be that Marx intended *Capital* for an audience of educated laborers (among others), and assumed that algebraic operations would be slightly over their heads. But if this were the case, one would expect Marx's notes to be for the most part written down in algebraic form even when in print he reverted to numerical examples for the sake of accessibility. Moreover, had Marx algebraically determined the outcome he was after in advance of his computations, one would not expect his published works to engage in algebraic mistakes or circular reasoning because of e.g. impractically chosen numerical values, nor for him to abandon promising lines of inquiry because of computational errors. But he does all of these things both in the works that were published during his lifetime and in the draft texts first worked up for publication by Engels and later by others (Smolinski 1973: 1196-1197).[105] So a more likely explanation for Marx's predilection for numerical computations is that he 'learned the wrong methods at the wrong time', that is 'his economic system […] was already virtually completed by the time when, at the age of 40, he began studying mathematics' (Smolinski 1973: 1198-1199). By way of illustration of this fact, I have amended Reuten's 2003 table of the publication and manuscript dates of some of Marx's major works (2003: 150), with those of Marx's most important mathematical works in Table 1.

[105] In writing these lines I have greatly benefited from discussions with Geert Reuten, Harro Maas and Murat Kotan.

Table 1. Marx's mathematical texts in the context of the dates of publication of some major works

1 *First publication in German*		*2* *years 1–3*	*3* *First English translation*		*4* *Date of manuscript*	*5* *Years 1–4 Ms–German*	*6* *Years 3–4 Ms–English*
1867	*Das Kapital I*	19	1886	*Capital I*	1861, 1863, 1865-7[a]	0	19
1885	*Das Kapital II*	22	1907	*Capital II*	1865-1870, 1877-8	7	29
1894	*Das Kapital III*	15	1909	*Capital III*	1864-5	29	44
1905-10	*Theorien über den Mehrwert* (3 vols)	48-61	1952-71	*Theories of Surplus Value*[b]	1862-3	47	108
1932	*Pariser Manuskripte*	31	1963	*Economic-philosophical manuscripts*	1844	88	119
1932	*Die deutsche Ideologie*[c]	6	1938	*The German Ideology* (Parts I & III)	1845-6	86	92
1953	*Grundrisse*[d]	20	1973	*Grundrisse*[e]	1857-8	95	115
1968	*Mathematische Manuskripte*[f]	15	1983	*Marx's Mathematical Manuscripts*	1881-3	83-5	98-100
2003	*Mehrwertrate und Profitrate mathematisch behandelt*	-	-	-	1875	128	-

[a] Work on second German edition, 1867-72; and on French edition 1872-75.
[b] First Volume 1952, *A History of Economic Theories*; extracts 1951.
[c] Extracts 1902-3, 1921 and 1927.
[d] Extracts 1964 and 1971.
[e] Earlier scarcely available edition 1939-41; its Introduction was published in 1903.
[f] Parts of these manuscripts appeared in Russian translation in 1933 as *Mathematicheskie Rukopsii*.

The argument above may also explain why Marx's 1875 manuscript (finally published in 2003) entitled 'The mathematical treatment of the Rate of Surplus Value and the Rate of Profit' ('Mehrwertsrate und Profitrate Mathematisch behandelt'), which is essentially a mathematical adaptation and reformulation of Chapter 3 of volume III of Capital, never ventures beyond elementary algebra (Smolinski 1973: 1195). It is one thing to devise a theory on the basis of a model or a model and theory at the same time, but quite another to devise a model for a 17 year old theory on the basis of techniques that were alien to you at the time you conceived of the theory. In Smolinski's words: 'It would be a difficult task for Marx and, at the early stage of development of mathematical economics at the time, a pioneering venture to reformulate his economic system as a mathematical model using the tools most appropriate for that purpose, such as linear algebra, matrix algebra, and methods of finite mathematics' (Smolinski 1973: 1199), no matter how conversant Marx was with these when writing 'the mathematical treatment'. In short, mastery of mathematical techniques does not imply mastery of their applications to practical problems. Furthermore, Marx's interest in the foundation of the calculus so characteristic of dialectical thinkers was of no avail in his crisis-ridden conception of the economic system, for the calculus deals with continuous gradual changes and breaks down when leaps and discontinuities occur (Smolinski 1973: 1199).

All in all, it may be concluded that Marx wanted mathematics to reflect the systematic dialectical origins of its foundations and aspired to using such mathematical techniques as could withstand dialectical criticism to elaborate economic principles when appropriate. Seeing that his mathematical sophistication came a little late for the latter purpose, there is ample room to improve both on his technique as well as on the way mathematical formalisms are embedded in his dialectical exhibition. After Section 2 has elaborated in detail on Marx's systematic dialectical exhibition of capitalism, Section 3 will discuss his reproduction schemes as an example of Marx's attempts at the use of mathematics that - from the vantage point of the 21st century - is technically crude and only partially embedded in the overall dialectic of *Capital*.[106]

[106] Both of these caveats imply that there is room for improvement. They must not be read as criticisms levied against Marx. In Marx's day mathematical models were hardly ever used by any economist. Instead quantitative analysis was confined to presenting and reading tables exhibiting figures (see Morgan 2012 for 'a broad chronology for the historical development of modelling over the last 200 years' (2012: 7)). Naturally, Marx's models followed suit. Furthermore, Marx had barely finished his models when he deceased, so perhaps he just never got round to embedding them in his dialectics. These points are elaborated upon in the introduction to Section 3 of this chapter.

2. Marx's Exhibition of Capitalism as a System: The Systematic-Dialectical Position.

In order to assess the dialectical status of the assumptions Marx makes in his 'schemes of reproduction' (as is done in Section 3), it is necessary to locate them in Marx's overall systematic dialectical exhibition and to understand his reasoning therein. This section therefore exhibits a systematic-dialectical interpretation of some main 'moments' of Marx's *Capital.* Contentwise, it is mainly based on Smith (1990 and 1993) and Arthur's reconstruction (1993 and 2002), but the mode of exhibition is my own, that is the α)-β)-γ) format introduced in Chapter 1. Some of the terminology used emanates from Reuten and Williams (1989).

2.1. Sociation

What universal principle did Marx take as his starting point in *Capital*? Given his critique on Hegel's dialectical philosophy in general and his dismissive stance regarding Hegel's philosophy of society in particular (see Chapter 1), he was obliged to formulate an alternative for Hegel's principle of Free Will that 1) allows for 'initial contradictions and shortcomings' to be reproduced and deepened (instead of being overcome) at 'more complex and concrete theoretical levels' (Smith 2014; cf. Murray 1993: 41) and 2) is clearly embedded in a materialistically conceived of historical dialectic.

Marx looked for this alternative by analyzing the commodity. The first paragraph of *Capital* runs: 'The wealth of those societies in which the capitalist mode of production prevails, presents itself as "an immense accumulation of commodities", its unit being a single commodity. Our investigation must therefore begin with the analysis of a commodity' (Marx 1890[4], 1867[1]: 49). So it seems that the commodity is Marx's starting point. However, both Smith and Arthur argue that Marx's exhibition proper actually has a different starting point.

According to Smith, one may also read the quotation above as saying that in order to analyze *capitalism* as a specific mode of *social production* we must start with the commodity. On this reading 'the fundamental purpose of the first section of Chapter One of *Capital* is to explicate the relationships connecting a [α]) the general realm of social production; b [β]) a specific mode of social production; and c [γ]) the category that is the first determination of that specific mode' (Smith 1990: 62). This α) general realm of social production is termed '*sociation*' by Reuten and Williams (1989: 56) and Arthur adopts this terminology (1993: 71).

α) *Sociation* is the one word answer to the question what a viable society is when conceptually isolated from other categories and processes. To be viable it must socialize production somehow, ensure that women conceive and that children receive sufficient care and education (Reuten & Williams 1989: 56; cf.

Arthur 1993: 71).[107] *Sociation* refers to any and all situations in which these requirements are somehow met, but it makes just as much sense to talk about a sociate society as to talk about a Being that *is*. The very existence of a society implies that it has somehow fulfilled the mentioned requirements to a sufficient degree. So *sociation* is a universal ahistorical characteristic of any and all societies.

2.2. Dissociation

β) Having established the general requirements for any and all societies, we must move on to the specific capitalist mode of production. The paradox is that although each society must somehow socialize its produce, capitalist production is undertaken in private units that are separated from the site of consumption. So capitalist production is expressed in the world as the antithesis of social production: *dissociation.*

Arthur (1993: 71) has again adopted this terminology from Reuten and Williams (1989: 56-57). Smith seems to have a similar idea in mind when he describes the capitalist mode of production as being characterized by indirect and unrestricted sociality. Capitalism is indirectly social in that production serves private instead of social goals and unrestrictedly so in that e.g. the market may be unboundedly extended (Smith 1990: 63-65).

2.3. Association: the Exchange Relation

γ) Given that society can only survive if it socializes its produce to be used (2.1) and that capitalist production is a private affair (2.2), there must be some third mechanism that articulates dissociate production as social: the *association*. In Capitalism *association* takes the form of γ) the *exchange relation*. If private produce is exchanged it is taken from the site of the producer to that of the consumer, thus bridging the gap between the two (Marx 1867: Ch. 2: 99-108).

Just like *sociation* and *dissociation*, the term *association* was first coined in this context by Reuten and Williams (1989: 59) and was subsequently adopted by Arthur (1993: 71-72). Since the exchange relation is the first condition of existence of the capitalist dissociate mode of production, Arthur claims that 'the presentation proper […] starts with exchange' (1993: 72). Indeed, although

[107] In abstraction, whether a society is viable of course has nothing to do with whether its members are happy. Since we have abstracted away from human volition and freedom for now, what matters are the needs of the system instead of the needs of its members. From this point of view, mass raping women, taking care of them till they give birth and state raising the offspring will be just as effective as falling in love, courtship rituals and marriage, culminating in a significant probability that the couple will think it cute to 'take' children and provide loving care for them.

dissociation is capitalism's defining characteristic, this mode of production only stands a chance when mediated ('aufgehoben') by the exchange relation.

When products are produced in private units that are generally separated from where they are consumed, engaging in the *exchange relation* is an imperative and thus (Hegel's) Free Will is relegated to the margins of Marx's system right from the start. Thus the first condition (as identified in Section 1.3 and reiterated in Section 3.2.1. above) that Marx's starting point should fulfill is met, since it clearly allows for the existence (and persistence) of unresolved contradictions and antagonisms. Since Marx arrived at his starting point by way of an analysis of the *historically specific* category of the commodity, the second condition is clearly met as well.

2.4. The Commodity, Exchangeability and the Bargain

If the site of consumption is generally separated from that of production, goods are not produced for one's own use, but to be exchanged. γ) As such, they are more properly called *commodities*, because this type of good has some peculiar characteristics that goods generally do not necessarily have. Thus, *commodities* are a subspecies of goods in general.

α) First of all, *commodities* must be *exchangeable*. This requires a degree of perceived usefulness on the part of the consumer. Furthermore it must be possible to own and part with a *commodity* in delimited amounts. That is, exchange of them will only commence when the consumer assesses such and such an amount of it as potentially useful and is able to strike a β) *bargain*, exchanging this amount of x for that amount of y. The unit (e.g. weight, volume, hours, or whatever) in which these amounts are delimited is immaterial to the *bargain* struck: the ratio of exchange appears as a pure number (Arthur 1993: 74-75).

In short, γ) *commodities* are inherently α) *exchangeable* because they embody *use values* and can be owned and sold in discrete quantitatively delimited units, but when they are β) *bargained* over in the *exchange relation*, they present themselves as *exchange values* (Marx 1867: Ch. 1: 49-55). Arthur mainly emphasizes the opposition between *exchangeability* and the *bargain* in this respect (1993: 74). Marx discusses these characteristics of commodities in terms of *use value* and *exchange value* respectively and Reuten and Williams seem to have followed his lead in this respect (1989: 62-63).

2.5. Value in Exchange

In itself, a *commodity* is *exchangeable* if it is sufficiently divisible and perceived as useful. But only when it is confronted with at least one other *commodity* a *bargain* can be struck. So *bargaining* expresses two or more qualitative *use values* in a quantitative ratio of exchange. γ) In the ratio of exchange qualitatively

distinct – and as use values incomparable – commodities are commensurated as *values in exchange* (Arthur 1993: 75-76; Marx 1867: Ch. 1: 62).

2.6. The Simple, Expanded and General Commodity Form and the Money Form of Value

α) In one-off, barter exchanges *value in exchange* appears only fleetingly during the exchange itself. It is alien to the exchanged commodities and as soon as the transaction is complete and the bargain fulfilled, it is gone. This is the *simple commodity form of value* (Arthur 1993: 79-80; Smith 1990: 80; Marx 1867: Ch. 1: 63-76).

β) But since the site of consumption is institutionally structurally separated from that of production in capitalism, exchanges are an integral part of social life in capitalism. Hence, each (conceptually) isolated act of exchange can be expanded to all other commodities. Thus, some n (meters) linen does not only exchange for z (kilograms) corn and the other way round (as in the *simple commodity form*), but also mediately or immediately for k (kilograms) iron, m (sacks) potatoes, c (liters) milk, p (hours) Internet access, q (hours) escort services, etc. etc. This is the *expanded commodity form of value* (Arthur 1993: 80; Smith 1990: 80-81; Marx 1867: Ch. 1: 77-78).

γ) Since the value of any *commodity* can thus be expressed in terms of each and every other *commodity* and the other way round, each and any *commodity* can in principle be singled out to serve as a general equivalent of value. This single commodity then serves as the *general commodity form of value* (Arthur 1993: 80; Smith 1990: 81-82; Marx 1867: Ch. 1: 79-83).

γ) However, as soon as a significant part of a community singles out the same commodity to serve as general equivalent (gold being the most likely candidate in Marx's time), its character changes for then it is no longer a commodity amongst other commodities but *the* commodity: *money*. As soon as *money* is established, *value* is no longer just a relative thing to be *bargained* for. It now has a tangible pendant in the world (Marx 1867: Ch. 1: 84). Thus at this point in the exhibition, *value* is established through *money* as an abstraction-in-practice (cf. Reuten 1993; cf. Murray 1993, 2000 and 2014).

Marx illustrates all these forms of value with numerical examples. Thus the *simple commodity form* is illustrated by the formula: '20 yards of linen = 1 coat' (Marx 1890^4, 1867^1: 63; 1867F: 141), which is next *expanded* by '…or = 10 lb. tea or = 40 lb. coffee or = 1 quarter of corn or = 2 ounces of gold or = ½ ton of iron or = etc' (Marx 1890^4, 1867^1: 77; 1867F: 155). He also gives us these first two formulas in a more algebraic form: 'z commodity $A = u$ commodity B or $= \cdots$' (Marx 1890^4, 1867^1: 77; 1867F: 154). But when he illustrates the *general commodity* and the *money form*, Marx reverts to the numerical examples of

amounts of units of goods cited above, adds 'x commodity A [etc.]' (Marx 1890[4], 1867[1]F: 79, 84) and equates them to 20 yards of linen and 2 ounces of gold respectively (Marx 1890[4], 1867[1]: 79, 84; 1867F: 157, 162).[108] As said, all this is purely illustrative. Marx's numerical examples at this point play no part in driving his systematic dialectical exhibition onwards, nor do they help him to form a better understanding of his categories. So they play no part in Marx's conceptual explorations ('Forschung') either.

2.7. Money as Measure of Value, Means of Circulation and End of Exchange

α) With *money* serving as a general equivalent, *value* is no longer just a relative thing to be determined through ever so many *bargains* as there are *commodities*. Instead '[e]ach separate commodity [now] has its unique value expressed in monetary terms *prior* to coming into contact with any other commodity' (Smith 1990: 87, my emphasis). So when there is money, the seller just has to name his price and the buyer can take it, leave it or haggle over it. At any rate the bargain is between 1 unit of commodity A and the monetary expression of its exchange value, i.e. its unit price. Thus, *money* first and foremost serves as *measure of value* (Smith 1990: 87; Arthur 1993: 81-82; Marx 1867: Ch. 3: 109-118; Reuten & Williams 1989: 65).

β) But *money*, like all essential categories in capitalism, ultimately sprang forth from the necessity of *exchange* predicated upon the institutionalized separation of the sites of production and consumption given in *dissociation*. Thus, although considered in itself it is a static *measure of value*, it must mediate *exchange* between *commodities* in the capitalist world out there. So it just as much serves as *means of circulation*. The end of circulation is reestablishing dissociated production as socially useful, sociate. Thus in the first instance *money* appears to mediate between *commodities*, giving us the circuit of commodity (C) → money (M) → different commodity (C') (Marx 1867: Ch. 3: 118-143; cf. Smith 1990: 87-88; cf. Arthur 1993: 82-83).

γ) However as soon as *money* is determined as the general equivalent of all commodities, it can be immediately exchanged for any commodity, while commodities can do so only mediately via money. So from the introduction of money in the systematic-dialectical exhibition onwards, it makes more sense to have a stock of money than a stock of commodities. This means that $C \rightarrow M \rightarrow C'$, breaks down into $M \rightarrow C \rightarrow M$ and acquiring *money* instead of qualitatively

[108] Surprisingly, the abbreviation 'etc.' does feature in Marx's illustration of the *general commodity form of value*, but is absent from his illustration of the supposedly even more general *money form*.

different *commodities*, becomes the *end of exchange* (Smith 1990: 89-91; Arthur 1993: 83-84; Marx 1867: Ch. 4: 163-164).

2.8. Capital

Institutionalized *dissociation* means that privately undertaken production is only socialized when it is *exchanged*, but the aim of this *exchange* in turn is not to acquire another useful product but to acquire *money*. But of course no one would be bothered to take products to market just to end up with the same amount of *money* one started out with. People would only take this trouble if on average their *money* holdings grow in the process. Thus the aim of production is *exchange* and the aim of *exchange* is acquiring a higher sum of *money*. The first of these goals must be reached to ensure social recognition of capitalist production, whilst the attainment of the second is required to ensure continuity of production. Thus it seems that capitalist units of production are continuously throwing off value, more precisely surplus-value, ΔM, over and above the sum of *money*, M, originally laid out. So the circuit $M \rightarrow C \rightarrow M$ is now superseded ('aufgehoben') by the expanding circuit (or spiral of valorization) $M \rightarrow C \rightarrow M'$ (more *money*) (Marx 1867: Ch. 4: 165).

γ) With this, *capital* can be introduced as 'money which begets money' (Marx 1890[4], 1867[1]: Ch. 4: 170; 1867F: 256) or self-valorizing value (Marx 1890[4], 1867[1]: Ch. 4: 169; 1867F: 256; Arthur 1993: 82-84, 87; Arthur 2002: 51). *Capital* finally endows the capitalist mode of production with a structural ground for ongoing production. One can find unlimited usages for more *money*, but only limited ones for the *commodities* one happens to be able to *bargain* for at any one time, so only production for *money* is potentially continuous (Smith 1990: 98-99; Arthur 1993: 83-84). Note that, with the introduction of *capital*, our starting point, the necessity of *exchange* to socialize *dissociated* production, has all but vanished from the scope of the systematic-dialectical exhibition, whereas *value*, first introduced as a necessary facilitator of *exchange*, has now become the overriding motive of *exchange* as *money*. From this point onwards, the quality of what is *exchanged* is therefore subordinate to the quantity of *money* it *exchanges* for.[109]

2.9. Constant and Variable Capital

From the perspective of *capital* itself the realization of surplus-value is the end of exchange ($M \rightarrow C \rightarrow M'$). From the point of view of society however, *money as a means of circulation* mediates between different *commodities* ($C \rightarrow M \rightarrow C'$). So whilst *capital* requires surplus-value to survive, society merely requires

[109] The weirdness of this situation and the kind of inverted reality it can lead to, is aptly illustrated in the Cree Indian prophecy quoted in footnote 51.

resocialization of dissociated produce to take place (through $C \rightarrow M \rightarrow C'$). Hence capital can only be capital (i.e. self-valorize) if it also aims for a qualitative difference between its inputs, C (bought for M), and outputs, C' (sold for M'). With this, the focus clearly shifts towards production.

α) As self-valorizing value *capital* appears as a material process: a sum of money buying certain commodities that undergo, as worked up by labor, a qualitative transformation in production so they can be *exchanged* for a higher sum of *money* than was originally laid out. As soon as commodities enter production this way, they function as *constant capital*. Tools and raw materials thus take the (historically specific) form of *constant capital* However, the reverse does not necessarily hold: tools and raw materials not engaged in potentially self-valorizing circuits (i.e. in capitalist production with a view to exchange at a premium) are no form of *capital* but just (dormant) means of production.

β) The picture we have drawn so far is that commodities bought to enter production as *constant capital* somehow get transformed into qualitatively different commodities that can then be sold at a premium. But to understand how this transformation comes about, the self-possessed and conceptually isolated material process must once again call upon an external and intangible input: labor power. As soon as labor power is employed to further the aims of *capital*, it functions as *variable capital*. So, *variable capital* relates to labor power in the same way that *constant capital* relates to means of production: all *variable capital* consists of labor power but not all labor power usually or necessarily functions as *variable capital* (Marx 1867: Ch. 6: 214-225; cf. Murray 2014).

2.10. Accumulation

In summary the previous subsection stated that the fact that capitalist production is undertaken in private *dissociate* units together with the private ownership of *constant capital*, necessitates a form of production in which incremental growth of *money* is the driving force, but this incremental growth requires a qualitative transformation to be realized in production. This transformation comes about by applying labor power to the production process as *variable capital*. Thus, all the requirements for an ongoing spiral of valorization have now been determined, laying the basis for γ) *accumulation*, the production of commodities on an ever-expanding scale (Reuten & Williams 1989: 78; Arthur 2002: 51-52).[110]

[110] According to Arthur, *capital*'s self-valorization 'is materially grounded in exploitation' (Arthur 2002: 51). Also he says that labor 'is a difference from capital that remains in *contradictory* unity with it' (Arthur 2002: 51) and he regards constant and variable capital as 'two distinct fractions that play particular roles in the process of valorization' (Arthur 2002: 51). Supposedly these are the roles of instruments of exploitation and exploited respectively. Since exploitation can only result in valorization if the exploited can be made to part with something not originally owned by

To expand production is to expand *capital*. When the ratio between *constant* and *variable capital* remains the same, labor power and constant capital employed both grow at the rate of *accumulation*. This form of accumulation is extensive in that the increase in productive capacity is brought about solely by an increase of constant and variable capital employed. It is however much more effective to boost accumulation by means of qualitative improvements in *constant capital*, i.e. by means of technological innovations. According to Arthur these innovations can have four types of effects. First, they may 'save constant capital while eliminating laborers from the work process' (1990: 145). Second, they may save labor while keeping 'the costs of constant capital level' (1990: 145). Third, *constant capital* may be saved while the *variable capital* requirements are kept level and finally *constant capital* might be saved while the labor force required expands (as long as there are net gains this still saves costs) (1990: 145). At face value it may seem that the net effect of these changes solely determines which technological changes are implemented, but it must be noted that wages fluctuate with unemployment and that labor resistance increases with falling unemployment. So employment of labor (*variable capital*) is more risky than that of *constant capital*. As a result, the first type of innovation mentioned is preferred over the second which is preferred over the third, which in turn is preferred over the fourth. This preference not only determines which of the available options are selected for implementation but also determines which innovations get developed in the first place (Smith 1990: 144-145). As a result, intensive growth leads to a relative decline of the importance of variable capital in the production process over time (Marx 1867: Ch. 25: 762-781).[111]

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the exploiters, at least the something the exploited are made to part with must originate outside the valorization circuits. However, Arthur never makes this argument. In effect, he only mentions the chief categories involved with self-valorization, exploitation and accumulation, but never explains how these processes come about.

[111] Marx goes on to argue that these two forms of accumulation together produce an industrial reserve army (Marx 1867: Ch. 25: 781-794). This is however only true when population growth is equal to or higher than employment growth. Due to technical change, employment growth is likely to fall short of accumulation, but there is no inherent reason why population growth would always be higher. The relative decline of the role of variable capital in itself, therefore, is a necessary, but not a sufficient condition for the emergence and growth of this 'reserve army'. The debate surrounding this, is elaborated upon in the next section of this chapter.

A further consideration in this respect is the rate of depreciation. The higher this rate is, the faster old and relatively labor intensive constant capital can be replaced with new less labor intensive constant capital and the higher the rate of relative labor expulsion will be. The consequences of all these considerations are determined algebraically in Chapter 4.

2.11. The Money Capital, Production Capital and Commodity Capital Circuits

α) In conceptual isolation, *accumulation* appears as an influx of *money* into a *dissociate* unit of production, followed by an efflux and reflux of more *money* into production and so on. That is, since dissociate production is motivated first and foremost by the acquisition of ever higher sums of money, accumulation considered in itself appears as the *money capital circuit*, schematically summarized as:

$$M \longrightarrow C\{mp; lp\} \ldots P \ldots C' \longrightarrow M'$$

(Marx 1885: Ch. 1: 56, 67; adapted from Reuten 2002b: 466; cf. Arthur 1998: 100-108).

This is to be read as: a sum of money M buys means of production (constant capital) and labor power (variable capital) $\{mp;\ lp\}$. In the phase of production P, labor as variable capital transforms the means of production to create qualitatively different commodities C', which sell for a higher monetary value M' than was required for the inputs (i.e. M). This increased sum of *money* is reinvested into production only to *accumulate* an even bigger money capital. So the *money capital circuit* represents accumulation as a relation of *money capital* to itself. Thus from this point of view, labor power, means of production, production itself and commodities appear only as instruments of monetary accumulation (cf. Arthur 2002: 53-54).

β) How much *money capital* can be accumulated however, crucially depends upon the effective control of *money capital* over its instruments, in the first place its interaction with *production capital*. Thus, if we stress how capital relates to the world, the circuit looks more like this:

$$P \ldots C' \longrightarrow M. \quad M \longrightarrow C\{mp'; lp'\} \ldots P'$$

(adapted from Marx 1885: Ch. 2: 90; cf. Arthur 1998: 100-108)

Marx calls this the *production capital circuit* (Marx 1885: 90). What this stresses is that, although a bigger *money capital* (*M'*) is still the end goal (hence the full stop behind *M'*), this *money* must be reinvested if accumulation is to continue. In particular, it must engage in a new round of purchasing more *variable capital* (labor power) to transform more *constant capital* (means of production) into more qualitatively different *commodities*. Thus ongoing monetary *accumulation* requires an ongoing expansion of *production capital*. *Production capital* is a dynamic determination because in the process of production *commodities* undergo

qualitative change. Thus, as *production capital*, labor power and means of production appear mainly as *use values*. As *constant capital* confronts *variable capital* during this process, this is where *exploitation* occurs and surplus-value is created (cf. Arthur 2002: 53-54).

γ) Whether this surplus-value can be realized, however, depends on successful valorization of *commodity capital*. Thus, next to the *money* and *production capital circuits*, we also have a *commodity capital circuit*:

$$C' \longrightarrow M'. \quad M' \longrightarrow C'\{mp'; lp'\} \ldots P' \ldots C''$$

(critically adapted from Marx 1885: Ch. 3: 91, 99; cf. Arthur 1998: 100-108)

Whereas the *money capital circuit* relates *money* only to *money* as the tangible embodiment of *exchange value*, the *production capital circuit* relates it to the *use values* of labor power and means of production only. This tension is resolved in the *commodity capital circuit*, because as means of production commodities potentially associate *capital*'s produce with *capital*, but as means of subsistence they associate it with labor. So the *commodity capital circuit* unifies the overtly self-possessed *money capital circuit* with the overtly outward oriented (that is, from the point of view of capital's overriding motive) *production capital circuit* (Marx 1885: 100-103; cf. Arthur 2002: 53-54).

2.12. Fixed and Circulating Capital

Now that we know that all *capital* must in one way or another be engaged in a circuit, our attention is drawn to turnover times. On the basis of their respective turnover times *capital* can be classified as either *fixed* or *circulating capital* (Marx 1885: Ch. 8: 158-162).

α) Means of production may either be used up within a single *capital circuit* or last several circuits. While the longer lasting means of production are used in the productive process their *exchange values* diminish in proportion to their use, and when their *use value* is exhausted, so is their *exchange value*. All of their value is then transferred to the *commodities* they helped produce. But until that happens, they perform the same function over and over. As such, they are *fixed capital* in the form of *use value*. Like all *capital*, *fixed capital* is socially validated only through the circuits it enters, but it enters those circuits only mediately by means of piecemeal transfers of its value to *commodities* (Marx 1885: Ch. 8: 158-159; cf. Arthur 2002: 54).

β) Labor power, and those means of production that are used up in a single process, circulate in their entirety (or not at all). They are *circulating capital* in that all of their *use values* and all of their *exchange values* are transferred to the produced *commodity* in one go, so they need to be replaced in every new

productive cycle. This part of *capital* therefore circulates at the same pace as the *commodities* it helps produce (Marx 1885: Ch. 8: 159-160; cf. Arthur 2002: 54).

2.13. Simple Reproduction, Means of Production, Consumption Goods, Total Social Capital and Expanded Reproduction

To stay in business all capital is necessarily engaged in circuits. These circuits can only be renewed if the efflux of *money* from the *money capital circuit* M' is at least equal to the original influx *M*, so that *production capital* can be maintained and *commodities* produced at the same scale as before. To achieve this γ) *simple reproduction* of the circuits, the valorization of *commodity capital* must at least cover its cost of production (cf. Arthur 2002: 54).

Commodity capital may either be valorized by selling commodities to consumers as β) *consumption goods*, i.e. as subsistence for labor, or by selling them to producers as α) *means of production*. In the former case they disappear from the macroeconomic circuit, whereas in the latter they reenter the circuit as inputs at the same time that they leave it as outputs (Arthur 2002: 54). From the point of view of *capital*'s *reproduction*, therefore, replenishment of *means of production* is an internal affair (hence classified as α), while selling *consumption goods* requires *capital* to bridge the institutional separation of the site of production from that of consumption and thus requires *capital* to engage in external relations (hence the classification of *consumption goods* as β).

Marx schematizes (nowadays we would say models) the internal and external interactions between the production department of *means of production* and that of *consumer goods* and shows how wages, prices and quantities must be exactly proportionate to enable *simple reproduction*, i.e. ongoing production on the same scale. These proportionality requirements show that the two major departments mentioned are systematically dependent upon one another and can therefore be treated as one organic whole: γ) *total social capital* (Marx 1885: Ch. 20: 391-465). However, given the necessity of *accumulation*, *simple reproduction* is not enough for capital to stay in business. Thus we require γ) *expanded reproduction* (of *total social capital*), the proportionality requirements for which Marx's schemes (or models) show to be even more forbidding (Marx 1885: Ch. 21: 485-518; cf. Arthur 2002: 54). Both Marx's *simple* and his *expanded reproduction* schemes are elaborated upon in detail in the next section.

2.14. General Rate of Profit, Many Capitals, Competition and Minimum Prices of Production

α) *Expanded reproduction* (of *total social capital*) means that there is a *general rate of profit* (Marx 1894: Ch. 9: 164-181). This *rate of profit* is essentially an expression of *capital*'s self-valorization. Since this was the first (preliminary)

definition of *capital* given, the *rate of profit* is essentially an expression of *capital*'s relation to itself, an almost tautological definition of what it means to be *capital* (Arthur 2002: 54-55).

β) Yet the very existence of specialization, foreshadowed in *dissociation*, means that there must be *many capitals* differing in all of the respects categorially exhibited thus far. So, for example their ratios between e.g. *variable* and *constant capital* may differ, they may produce on different scales, they may sell to consumers or to other *capitals*, etc. (cf. Arthur 2002: 55).

γ) However, all these *capitals* are engaged in *competition* for the same funds (i.e. macroeconomic wages or companies' funds for replacement and accumulation of *means of production*). (They may even produce the same or similar products, but this is contingent from the present theoretical perspective). *Capital* responds to *competition* by trying to drive its cost of production down (e.g. by expulsion of labor and accumulation, i.e. production on an ever increasing scale (so that more profit can be made at a given price)) and its prices down (so as to secure a higher share of the market). γ) Thus, *minimum prices of production* tend to emerge (Marx 1894: Ch. 10: 182-209; Arthur 2002: 55).

From the categorial interactions of all categories exhibited so far, we can derive the laws of motion of capital (such as the falling rate of profit and periodic crises) (Marx 1894: Ch. 13-15: 221-277; Arthur 2002: 55). But this is a slightly different ballpark, for they deal with how the institutions and their motives interact as a whole, rather than with the systematic dialectical determination of that whole. Therefore, I will not go into them in any more detail.

3. The Role of Mathematics in Marx's Investigation and Exhibition in Capital: the Case of Marx's 'Schemes of Reproduction'

Karl Marx was both extremely ambitious and very perfectionist. 'By 1858 he planned to write six books' together containing 'a complete systematic analysis of society: economic, social, political and historic' (Reuten 2003: 149; cf. Vygodski 1965). By the end of his life in 1883, he had written enough material for the first of these books, which by then had grown to 'the three volumes of Capital that we now have', but he still thought of most of it as rough drafts unfit for publication. In his opinion only *Capital I* really came of age during his life: its first edition was published in 1867 and a second one in 1873. Luckily for us, however, Marx did recognize that his rough incomplete drafts were too valuable to disappear along with him and thus he asked his friend and publisher Friedrich Engels 'to do something with them' after his death. Engels got to work and *Capital II* was published in 1885, followed by *Capital III* in 1894. Notwithstanding Engels'

intentions to refrain from interpretation while editing Marx's drafts, he hardly left any of Marx's sentences untouched (but given Marx's perfectionism, this may have been exactly what Marx would have wanted).

It has long been impossible to flesh out exactly where Marx's text stops and Engels' editing begins. This situation started changing when work began on the Marx/Engels Gesamtausgabe (MEGA) in the 1970's. In essence the MEGA is a letter-by-letter and word-by-word transcription of all of Marx's almost illegible original manuscripts into readable form.[112] The only editing that the editorial teams of the MEGA have undertaken is the completion of certain abbreviated words. However, if a certain abbreviation allows for several possible completions, the several possibilities are meticulously acknowledged and the arguments contributing to the choice finally made spelled out in 'das Apparat' (the apparatus) that goes along with every volume in the MEGA.[113]

Engels edited Marx's schemes of reproduction into the last chapters of Part Three of *Capital II*. In these schemes Marx has endeavored to model the conditions for reproduction and accumulation pertaining to departments (or sectors) producing production and consumption goods respectively and was thus able to establish some necessary relationships between the two. In sharp contrast to contemporary economists, political economists at the time hardly made use of any mathematics at all (Morgan 2012; cf. Morgan 2003: 283-88; cf. Morgan & Knuuttila 2012). So Marx's endeavor was a pioneering venture and – considering that Marx could not build on the works of others in creating his models – an extremely creative one at that. Marx's schemes of reproduction were the first two-sector macro-economic model and as such inspired generations of (political) economists to come. Samuelson, for instance, claims that 'from the viewpoint of pure economic theory, Karl Marx can be regarded as a minor post-Ricardian ... a not uninteresting precursor (in Volume 2 of *Capital*) of Leontief's input-output analysis of circular interdependence' (1962: 12) and would later state that [o]n the basis of his schemes of reproduction 'one can claim immortal fame for Marx' (1974: 270). Furthermore, 'they have been widely acclaimed as providing the forerunner to modern growth theory, and in particular to the Harrod-Domar growth model' (Trigg 2001: 2; cf. Reuten 1999: 197-98).

Usually however, those inspired by Marx's schemes of reproduction discuss Marx's models in isolation of the structure of the systematic-dialectical exhibition

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[112] The Mega is only available in German. Hence, all quotes from it have been translated by me unless stated otherwise.

[113] As such, I consider the apparatus ('Apparat') as secondary literature in its own right and not as a variation or edition of Marx's or Engels' works. I therefore refer to pages in the apparatus to the MEGA by citing 'Mega' as author and the publication year – 2005 for Engels' editorial manuscript (i.e. 1885E) or 2008 for Marx's (1885M) manuscripts to *Capital II* – as date.

in the whole of *Capital*. This is quite possible in this case, because Marx himself used his models mainly to analytically explore some of the categories that he used in his systematic dialectical exhibition. So he probably thought of his modeling exercise as sidestepping his dialectical exhibition to do some appropriative work (Forschung) (Reuten 1998: 221-222) and by definition appropriative work is not and need not be integrated into the whole *yet* and can also be developed further without synthesizing it into the whole (although a systematic dialectician would always aim to eventually achieve the latter). This is not to say that it would have been impossible for Marx to integrate these models into his systematic-dialectical exhibition proper, just that he never got round to that stage.

In drawing up his models, Marx felt he had to call upon a number of assumptions that he usually defends by appealing to some sort of negligibility clause (i.e. along the lines: 'removal of this assumption will not change anything in the problematic or its solution'). Thus, Marx's social theory makes use of systematic dialectics to further its conceptual development from the abstract to the concrete, but also uses mathematical models for conceptual analysis at a given level of abstraction. He however, hardly calls upon his dialectical exhibition of the capitalist system as laid out in *Capital I* and *II* (and Section 2.1-2.13 above) to defend his choice of assumptions (note though that the text under consideration is a *draft*, or in fact a text based on several drafts by Marx). Thus, the exact dialectical status of the modeling choices Marx makes is still to be evaluated, as I intend to do in this section. This assessment will allow me to reconstruct the models along dialectical lines in Chapter 4 and thus provides a foundation for articulation of these models on a dialectically sound basis. On successful completion of this reconstruction, it can be claimed the models have left the stage of appropriation and can be considered mathematical (and more precisely – as it will turn out – algebraic) depictions of two necessary moments in Marx's systematic-dialectical exhibition.

The ground material for Engels' edition of Marx's 'schemes of reproduction' is to be found in his manuscript II, written between 1868 and 1870 (Mega 2008: 907), and VIII, conceived of between 1877 and 1881 (Mega 2008: 1606). Both of these were, if not unfinished contentwise, at least hardly organized when Marx bequeathed all his writings to Engels. Between the two, manuscript II was more rigorously structured and in this sense 'complete' than VIII. Towards the end of it however, Marx starts rambling off in all directions, discusses ever more other authors, and seems to lose track of his main topic (Mega 2005: 506).

Manuscript VIII (conceived of almost ten years later), by contrast, is much more focused, but according to Engels it 'too is only a provisional treatment of the subject, the main point being to set down and develop the new perspectives arrived at since manuscript II, ignoring those points on which there was nothing

new to say' (Engels 1885F: 12). Thus, VIII provides a more detailed, adequate and thought through account of the reproduction of total social capital than II, but is less structured than II and consciously omits all topics on which Marx had 'nothing new to say'.

Understandably, then, Engels, tasking himself 'to prepare the second volume of *Capital* for publication […] in such a way that it appeared not only as an integrated work, as complete as possible, but also as the exclusive work of its author, and not its editor' (Engels 1885F: 7), based most of this part of *Capital II* on manuscript VIII and only inserted passages from manuscript II when VIII was silent on that specific topic (Mega 2005: 542).

All in all, although Engels sometimes seems to have seriously misrepresented Marx's position with regard to systematic dialectics (cf. Fraser & Burns 2000: 1-23; Rockmore 2000: 95-105; McCarney 1999: 117-138), he has tried to abstain from interpretation while organizing the two mentioned manuscripts. He has however ventured to make Marx's manuscripts more accessible by replacing some idiosyncratic jargon with more everyday terminology and by (sometimes poorly) translating foreign language quotes into German (Mega 2005: 511, 519-520, 522). The terminological changes especially, often downplay Marx's frequent use of unmistakably dialectical jargon. Hence, I have based this section's appraisal of Marx's 'schemes of reproduction' on Marx's text, while adopting Engels' restructuring.

3.1. Simple Reproduction

3.1.1. The Model

Marx's systematic-dialectical exhibition leading up to the introduction of *simple reproduction* as described in Section 2.1 to 2.12 has shown, among other things, that γ) *capital* (Section 2.8) can only throw off surplus-value (as it must) if it employs labor power as β) *variable capital* to transform certain means of production, functioning as α) *constant capital* (Section 2.9), into more valuable commodities. Furthermore, Section 2.10 showed that the various forms of capital can only exist when they are engaged in *circuits* comprising both production and exchange. By articulating production and exchange together, these circuits *reproduce* the *capital* that built them.

Since *constant capital* necessarily takes the form of commodities that must be produced and both laborers and capitalists require commodities to sustain their livelihoods, the question becomes how *reproduction* of *constant capital* (more specifically means of production) is mediated by the requirement to present capitalists and laborers with an ongoing stream of commodities for their private use (more specifically means of consumption). This is the problem that Marx sets

out to investigate in his schemes (1885E: 312-317; 1885M: 340-343). Thus, the problematic depicted in his schemes is clearly inspired by his dialectics. That is, the exhibition thus far seems to be insufficient (in systematic dialectics this is always the drive to introduce a new moment) because the necessity of *capital*'s appropriation of means of production and labor power to function as *constant* and *variable capital* respectively has been established, whereas the respective origin or sustenance of these component parts, and hence their fundamental relations, are not, leading to apparent contradictions that Marx hopes to resolve through his schemes.

Since we are primarily interested here in the *function* of the commodities (i.e. their use value), we evidently have to analyze the *commodity capital circuit* (see Section 2.10 above), Marx says (1885M: 368; 1885E: 356).[114] This also was the last of the capital circuits to turn up in our exhibition and now it too appears to be lacking in concreteness. Hence, closer scrutiny is warranted. Thus, with these remarks, Marx is off to a dialectically defendable start.[115]

Since the two passages referred to above are more than 20 pages apart (and even more in Engels' editorial manuscript), I must probably explain why they nevertheless can be taken together so as to constitute a starting point for Marx's elaboration of his schemes. As indicated, the very problem Marx wants to resolve is identified in a dialectical manner in his introduction. Then follows an excursus on the topic of 'money capital as a component part of total social capital' (1885M:

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[114] More literally, Marx's sentence is the following: 'It is evidently the 3rd circulation schematic: $W'_G'_W + w_P_W'$, that we have to analyze here and for our current goal we have to do this from the point of view of the renewal of the value and substance of the individual components of W'.' (1885M: 368)

[115] The word 'start', or even 'order' for that matter, is subject to numerous qualifications in the case of these unfinished manuscripts of Marx's. Apart from the matter of organizing materials pointed out in the introduction to Section 3, there is the related matter of which materials to include in the editorial manuscript and which to dismiss. For instance, the part in manuscript II entitled *the societal circulation of constant capital, variable capital and surplus value* (1885M: 348-368) is largely missing in Engels's editorial manuscript. Most of it discusses and criticizes (or ridicules) Adam Smith's take on the matter (1885M: 350-358), just like the opening section of manuscript VIII (1885M: 698-726) and apparently this led Engels to supplant the mentioned passages in II with those in VIII.

This may be a missed chance, for Marx casually introduces no less than three of his assumptions (respectively labeled assumption **f**, **b** and **c** in this section) at the very beginning of this part of manuscript II: 'Were […] the rate of surplus-value = 100% [assumption **f**] […]. Fixed prices of the elements of production [assumption **b**] and a fixed scale of production [assumption **c**] assumed, […]' (1885M: 348). Considering the context of these assertions and the casual way they are uttered, it seems likely that, when Marx wrote these lines, it had not yet occurred to him that he would retain these assumptions throughout. At any rate, these assumptions are more formally introduced in several distinct passages later on, so perhaps Engels' judgment that this part of manuscript II could be dispensed with, is sound after all.

343-347) that, by Marx's own admission, should actually be addressed later on. Following that, Marx discusses other political economists' take on the matter (1885M: 350-368 (from II), 698-728 (from VIII); compounded by Engels into one chapter: 1885E: 322-355)[116], thus once more departing from his main line of argument.[117] He is back on track when he begins the exhibition of his model proper by pointing out the (dialectically) correct starting point for the analysis, viz. the *commodity capital circuit*. In effect then, when speaking of 'Marx's dialectically defendable start' I have focused on Marx's main argument and skipped the rest.

The first assumption that Marx introduces to delineate his model is dialectically defendable:

a. 'Furthermore it is not only assumed that products are exchanged according to their values, […]' (1885M: 369)

That is, the level of abstraction that Marx's model pertains to is below the level of *capital* in general (that at this level turns out to be departmentalized by necessity), but above the level of *many capitals*. Hence, there is no *competition* between *many capitals* yet that could induce a divergence of prices from values. However, this is not Marx's argument. Instead he defends this assumption by pointing out that a divergence between prices and values cannot influence 'the movement of social capital' (1885M: 369) in that the mass of products to be produced and exchanged is not altered by such divergence. In effect then, Marx considers it to be a negligibility assumption.[118]

The next assumption is introduced in the same sentence as the first:

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[116] Please bear in mind that what Marx labeled 'chapters' Engels christened 'parts'. Marx inserted some captions here and there in his manuscripts, but there is no indication of a structural attempt at organizing the material within his chapters. It follows that all of Engels's chapters are his own inventions, albeit that he paid close attention to Marx's casual captions in deciding where to break off one chapter and start the next one.

[117] Such departures from Marx's main line of argument can hardly be held against him. After all, Marx wrote his manuscripts in longhand in notebooks, so he could not move whole blocks of text to somewhere else as soon as he felt it appropriate, the way that modern word-processing technology allows us to. So he had to make due with little editorial comments in the text that he often placed between square brackets (like his admission that '[a]lthough the following belongs in a later part of this chapter, we already want to investigate it here' (Marx 1885M: 343)).

[118] A negligibility assumption is an assumption that assumes something away, whose influence on what the model sets out to show – such as the movement of total social capital within and between its constituent departments – (or predict) is negligible.

b. '[…], but also that no revolution in values takes place in the component parts of the productive capital' (1885M: 369).

At first sight, this assumption does not seem to be dialectically defendable. Since accumulation has been identified as the driving force for capital, we must expect the one (departmentalized) capital to take every measure available to accelerate the process. Clearly, the appropriation of new and improved means of production is among the most prominent of these measures. Hence, technological innovation geared towards accelerating accumulation is essential to capitalism. Thus, technological advances will inevitably increase the mass of products that can be produced with a given combination of constant and variable capital and their occurrence will therefore always correspond to a revolution in value.

On the other hand, the mentioned assumption was introduced in the context of *simple reproduction*. Hence, capital circulates, but all surplus-value and all wages are consumed and the total value produced stays constant (see assumption **c** below). Therefore any technical advances do not change the total value of any type of commodity produced (be it means of production or consumption goods), and as we will see below this implies that the value of every capital component is constant as well.[119]. All in all, assumption **b** is dialectically defendable as long as we are considering simple reproduction (assumption **c)**.

Considering Marx's justification for the adoption of this assumption, however, it may even be upheld in the case of expanded reproduction. He writes:

> [A]s far as revolutions in value are concerned, they change nothing in the relationships between the component parts of the annual social capital, as long as they are general and evenly distributed. In as far as they are, by contrast, partial and unevenly distributed, they represent disturbances, which *firstly* can only be understood as such in as far as they are regarded as divergences from fixed value relations; *secondly* however, given proof of the law that one part of the value of the annual product, for instance constant capital, replaces another variable capital, then a revolution in values […] would alter only the relative magnitudes of the portions of value that perform the one or the other function. (1885M: 369) [120]

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[119] This is so because assuming that the value of the means of production produced does not change and that all surplus-value and wages are consumed (assumption **c** below) means that constant capital employed cannot rise and that the sum of constant and variable capital is constant. Since offsetting a fall of constant capital with a rise in variable capital is alien to the nature of capitalism (see Section 2.10), constant capital must stay constant over time under these circumstances. As a result, there can be no redistributions among these two capital components as well (as long as we are considering simple reproduction, that is).

[120] Where I have said 'relationships', the original German speaks of 'Verhältnisse'. The German term is more adequate, for it refers both to ratio and to relationship. In this case that double

From this quote one may infer that the intended emphasis in assumption **b** is on 'in the component parts' rather than 'values'. Thus, although value revolutions due to technological innovation are part and parcel of capitalist production, these value revolutions leave the value relations *within* 'the component parts of the productive capital' unaffected, although they may very well alter the value relations *between* these parts (i.e. 'the relative magnitudes of the portions of value that perform the one or the other function'). On this interpretation there is nothing undialectical about the assumption. After all, *constant* and *variable capital* (Section 2.9) and the necessity to realize surplus-value have all been dialectically determined prior to the exhibition of the moment of *simple reproduction* (in Section 2.12). Since one of the central tenets of systematic dialectics is that what (immediately) holds for the system considered in the abstract must also mediately hold at more concrete levels, these component parts must always be present in capitalism, no matter what revolutions the system undergoes.

In his next assumption Marx defines simple reproduction:

> **c.** 'The assumption is that a social capital of a given value, like the previous year, supplies the same mass of commodity value anew and satisfies the same quantum of needs, even though the forms of the commodities may change in the reproduction process' (1885M: 728).[121]

Another variant of this assumption is to be found in a part of manuscript II which never made it into Engels' editorial manuscript (and hence was kept from the public until the 2008 publication of the MEGA). It reads: '[…] since simple *reproduction* [is] assumed, so it is assumed that the full wage of the laborers and the full surplus-value of the capitalists is consumed' (Marx 1885M: 373). This means that the sum of *constant* and *variable capital* in each department will stay the same each year. If one combines this with the better known phrasing of the assumption given in the block quote, one has to conclude that surplus-value is constant as well, since the constant value of the produce minus the constant value of the sum of the capital components, leaves surplus-value no room to maneuver.

Bearing in mind our discussion of accumulation in Section 2.10 above, assumption **c** states that extensive accumulation is abstracted from for now.

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meaning is particularly fitting, because – as the reproduction schemes will show – the *ratios* concerned are unaffected by general and unevenly distributed revolutions in value, *because* the *relations* between the component parts of capital remain unaffected.

[121] The Fernbach translation (1885F: 471) is very similar but has rendered the German *Quantum* as Quantity. Bearing Hegel's distinction between Quantity and Quantum and Hegel's influence on Marx in mind, I wanted to preserve the word Quantum here.

Intensive accumulation resulting from technical change is not explicitly assumed away here, since 'the forms of the commodities may change'. As the model gets shape however, we will see that the fact that the value of means of production produced does not change, implies that neither does the total value of *constant capital* that can be employed, since all capital employed is produced in one department (assumption **d** below). So, because both the sum of *constant* and *variable capital* as well as total *constant capital* do not change from year to year, neither can the value of *variable capital* employed.[122] As a result, the magnitude of all capital components is fixed in the model of simple reproduction.

Dialectically, this seems to be a strange assumption to (re)introduce here. We have long since identified *accumulation* (both extensive and intensive) as an essential characteristic of capitalist production, so by explicitly ignoring extensive accumulation (and intensive accumulation by implication), Marx's model separates itself from the capitalist basis. However, *accumulation* is of course only possible if simple reproduction is secured (Marx 1885M: 728). Hence, it makes sense to study the maintenance of capital (i.e. its reproduction on the same scale) before moving on to full-fledged accumulation. One might say that Marx first investigates a stationary system so as to make sure his analysis of capitalism's essentially dynamic reality starts from a correct vantage point. In essence then assumption **c** secures that we focus on the relationships inherent to capitalist production before we set them in motion. This does not mean that dynamics are entirely absent from the *concept* of *simple reproduction*, just that the concept does not emphasize this aspect. In mathematical models this lack of emphasis is achieved by rigorously assuming the unemphasized aspects away (for the time

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[122] Algebraically formulated the model is the following (see Chapter 4 for details):

$$\delta c_p + v_p + s_p = x_p \quad \text{(A)}$$
$$\underline{\delta c_c + v_c + s_c = x_c \ \ +/+} \quad \text{(B)}$$
$$\delta_c + v + s = x \quad \text{(C)}$$

The requirement is that both $\delta c_p + v_p$ (i.e. the sum of constant and variable capital employed in the production of means of production) and $\delta c_c + v_c$ (those capital components employed in the production of means of consumption) as well as the value of the total produce of both departments are constant over time. Furthermore, x_p should by definition be equal to $\delta c_p + \delta c_c = \delta c$. From a purely mathematical standpoint these equalities can all be satisfied simultaneously as long as redistributions in favor of constant capital in one department are offset by an equal and opposite change in the other department, but when technical change favors constant capital in both departments (as we determined it would - see Section 2.10), all variables in the model for *simple reproduction* are fixed.

being or indefinitely).[123] As a result, models can alternate between Something's Presence and its Motion but have trouble capturing them at the same time. So by formally capturing the emphasis conceptually intended for simple reproduction the *model* separates itself from the capitalist base, even though the *concept* does not.

This procedure is slightly reminiscent of Hegel's analysis of static Presence after his introduction of dynamic Becoming and is perfectly reconcilable with the general systematic-dialectical appraisal of *Capital* developed in this chapter. That is, if one asks α) what something is in conceptual isolation, the answer points to a situation at rest that can only be given room to move through the means or medium stipulated in answering β) (how is α expressed in the world?).[124] So, as long as capital is able to renew itself, its conceptually isolated nature is unaffected (and the moment α secured) and only then can it find room to express its nature in the world through accumulation. In effect then, assumption c) is a dialectically defendable heuristic assumption.

Next, Marx formally introduces the dialectical distinction between society's two main production departments:

d. **'**Society's total yearly product breaks down into *two great departments*' – I) '*means of production*, commodities that possess a form in which they either have to enter productive consumption, or at least can enter this', and II) '*means of consumption, commodities that posses a form in which they enter the individual consumption of the capitalist and working classes*' (1885M: 370; cf. 1885E: 359).[125]

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[123] If this is done indefinitely the assumption that does so is usually referred to as a negligibility assumption. If it is done only temporarily, it is referred to as a heuristic assumption.

[124] This footnote explains why I said that this procedure is slightly reminiscent of the way Presence ('Dasein') was introduced in Chapter 2. After establishing that everything is necessarily engaged in a process of Becoming, we could only regain our footing by insisting that a static snapshot could nevertheless be taken. Only our careful analysis of this snapshot and the others that followed it eventually allowed us to give a somewhat comprehensive analysis of a system that in essence is dynamic through and through. So such a procedure is basically legitimate as long as it is only used to 'set the stage'. A static picture must be the first and not the last word in any serious account of actuality.

[125] Textually, all this is translated from Marx's Manuscript II {1868/71}. Throughout this manuscript however, Marx labeled the means of consumption department *I* and the means of production department *II*. Since he does this the other way round in his manuscript VIII {1877/81} and since Engels based his editorial activities on the latter manuscript as much as possible, it seems prudent to follow Engels' lead in adopting manuscript VIII's labeling throughout.

This departmentalization follows almost immediately from the locus of the schemes of reproduction within Marx's overall systematic dialectical exhibition. Marx presents his 'schemes of reproduction' at the end of Part Three of *Capital II*, i.e. after the several *capital circuits* (Part One) and the categories of *fixed* and *circulating capital* (Part Two) have been introduced into his general systematic-dialectical framework. Thus, it has been established that at any point in time, a part of *capital* is invested in means of production (i.e. as *constant capital*) and another part in labor power (i.e. as *variable capital*) while the rest represents potential (as P or C') or temporal (as M') surplus-value.[126] Furthermore, the fact that means of production do not cross the institutional divide between the site of production and that of consumption that we have posited as the universal principle of capitalism (i.e. through dissociation), whereas consumption goods do, makes the distinction between the production department of *means of production* and that of *consumer goods* dialectically meaningful.

In his next assumption, Marx effectively sets depreciation costs equal to the value of replacement expenditure:

e. **'**From the point of view of social capital […] – with a problem, where the question arises how the capital used up during the year can be replaced out of commodity production – only that part of the value of the employed fixed capital can initially be considered, which actually must be partially or completely replaced by new items of the same kind. Note therefore, that it is *assumed* in the following determination [*Darstellung*] of the yearly value product (in means of consumption) that the *fixed part of the value* of the constant capital transferred to the product is only equal to that *part of the value of the fixed capital* that must be replaced in kind in order to start the reproduction anew at the same scale' (1885M: 372).[127] [128]

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[126] Understood naively, the capital circuits may be taken for step-by-step descriptions of the stages capitalist production has to go through. From such a diachronous understanding of the capitalist production process, the phrase 'at any point in time' may seem incorrect. After all, if one step is taken at a time, the capitalist is either buying C, producing (P) or selling C'. But the crux of ongoing production is that no stage ever terminates: all commodities C that enter production P are replaced and when an end product C' is sold, it too is replaced by fresh produce. So from this point of view, the circuits must be understood synchronically, rather than diachronically and the aforementioned phrase is perfectly legitimate (the terminology of synchronic and diachronic was introduced by Reuten (2002c: 8-9)).

[127] Given the importance of the term Darstellung in systematic-dialectics (see page 14 above), the fact that Engels has done away with this term in extensively rewriting the quoted sentences (cf. 1885E: 360) may be a significant fact with respect to the reappraisal of Marx's methodological stance.

After Engels' editorial work, this reads much more bluntly: '[…] it is necessary to abstract at least provisionally from the portion of value transferred to the annual product during the year by the wear and tear of the fixed capital, in as much as this fixed capital is not replaced again in kind in the course of the year' (1885F: 472-473). It is this version that led Reuten to comment that 'Marx next assumes temporarily […] that there is no fixed capital' (1998: 195). But in the light of Marx's original text (unavailable in 1998), this is not the most likely interpretation. Rather, it appears that Marx assumes that 'the wear and tear of the fixed capital' is transferred in full to the value of the product and equals the fixed capital replaced 'in kind in the course of the year'.

In contrast to the assumption 'that there is no fixed capital', this assumption is very likely to hold at least on average and in the aggregate. This is because yearly depreciation allowances – by which depreciation hoards are created (see assumption **j** below) are accounted for as depreciation costs that must somehow be made up for in the revenues of the yearly produce. Thus depreciation allowances transfer to the value of the product. Although Marx does not explicitly say so, the existence of depreciation hoards implies that the value of constant capital is now composed of two components: 1) means of production used up within one capital circuit 2) depreciation allowances for means of production that last longer than one circuit (see Section 2.12 above). Even though the time it takes to complete a capital circuit depends largely on production time (cf. *Capital II*, Part Two), Marx decides to adopt the convention of opening and closing the financial accounts yearly (1885: 321) and in what follows his comments are interpreted accordingly.

The assumption under scrutiny here, says that depreciation costs incurred in any given year are equal to the value of replacement investment that year. On the current level of abstraction a capital department must be very big and hence employ a lot of very diverse means of production as *fixed capital.* Since all of these are most likely in different phases of their respective life cycles, the sum total of replacement expenditure is very likely to equal the sum total of depreciation allowances (and hence depreciation costs incurred) for any given

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[128] Even though Engels indicates in the table added to his Preface to *Das Kapital II* that this assumption was taken from manuscript II (1885F: 104), Reuten writes it was taken from manuscript VIII (1998: 195). But the table of origins ('Provenienzverzeichnis') in the apparatus ('Apparat') that accompanies Engels' editorial manuscript shows it was really taken from manuscript II (Mega 2005: 925). Indeed, the block quote above was taken from the latter manuscript.

year (i.e. $\sum \text{depreciation}_t = \sum \text{replacement expenditure}_t$).[129] Thus, macroeconomically, the depreciation costs that are contained in the value of a commodity ('the *fixed part of the value* of the constant capital transferred to the product') are indeed likely to be equal to total replacement expenditure ('that *part of the value of the fixed capital* that must be replaced in kind in order to start the reproduction anew at the same scale') in each department. So, at this level of abstraction (where our investigations is concerned with macroeconomic global aggregates only) this assumption is very likely to hold and therefore dialectically defendable.

Just before Engels moves on to the basic schematic for simple reproduction, he introduces another assumption in a rather offhand way:

f. 'For our investigation of simple reproduction, we intend to use the following schema, in which $c =$ constant capital, $v =$ variable capital, $s =$ surplus-value, and the rate of valorization s/v is taken as 100 per cent' (Marx 1885E: 360; 1885F: 473), for both departments.

In the original manuscript VIII, Marx did not even bother to introduce this assumption in a proper sentence. He simply states: 'Percentage of valorization = 100%' (1885M: 728). This absence of comments and qualifications is surprising, because the statement is by no means self-evident.

Since, as we have seen, the appropriation of surplus-value is predicated upon the existence of *variable capital*, the assumption that the ratio of s over v exhibits some degree of resilience can be granted on dialectical grounds provided that the average labor productivity is constant and given (which must be the case in simple reproduction because the sum of variable and constant capital is constant as is total constant capital – see assumption **c** above). However, we did not need assumption **f** to tell us this, since our discussion of assumption **c** already led to the conclusion that the magnitude of *all capital* components is fixed in the model of *simple reproduction*. So this assumption is redundant in the current context (but we will see below, that we do require it in the context of expanded reproduction). Either way, there is no dialectical reason for the ratio of s over v to be a 100 per cent (so I will just call it ε in my reconstruction in Chapter 4).

The equality of s/v in both departments is another matter. It may be argued that in Marx's mind this was dialectically defendable because of worker's

[129] The famous Leontief input-output table interpretation of Marx's reproduction schemes stems from this observation. (By the by, the observation is still valid when e.g. a depreciation method based on historical cost is used, although part of a replacement in kind, would then be taken to reflect accumulation rather than replacement, so that accumulation of value would not coincide with accumulation in volume.)

resistance to exploitation. That is, as we have seen, the general separation of the site of consumption and that of production (i.e. *dissociation*) forces those who do not command means of production into selling their labor power. Since no differences between different types of labor have been exhibited at this level of abstraction yet, workers will choose solely on the basis of wage level and labor intensity (to be operationalized roughly as energy expended per hour worked times the length of the working day in hours – that is, calories expended in a working day). Hourly wages and labor productivity being more or less equal, workers would then move from one department to another if a smaller part of the value of the product in the latter department fell to the capitalist and a larger part was paid out in wages. So, differences between s/v would tend to even out over time (1867[1]: 323-330; 1867F: 417-428; cf. Reuten 2004b). This reasoning is sound, *except* for the crucial assumption that labor productivity would be more or less equal. Without it, there is no reason to assume that the lowest exploitation rate would coincide with the highest wage level per calorie expended. So as long as some products by their very nature are more labor intensive than others, there is no (dialectical) reason for exploitation rates to even out (not even at this level of abstraction).

Engels introduces the basic schematic for simple reproduction from manuscript VIII in almost the same sentence as assumption **f**. It is best summarized as follows:

‘	c	v	s	x	
I.	4000	+ 1000	+ 1000	= 6000	(means of production)
II.	2000	+ 500	+ 500	= 3000	(means of consumption)
	6000	+ 1500	+ 1500	= 9000	(social gross product)

where:
I = department I, producing means of production (6000);
II= department II, producing means of consumption (3000);
c = constant capital, the value of the means of production applied;
v = variable capital, the value of the social labour power applied;
s = surplus-value, the value that is added by labour minus the replacement of the variable capital advanced’ (Reuten 1998: 196; cf. Marx 1885M: 728-731; cf. 1885E: 360-361).[130]

[130] Marx’s original notation is a mess (but since it was all taken from draft scribblings, one cannot necessarily hold this against him). The figures are the same, but in a space comprising only three lines, we find three different notations:
1. $c)$ $v)$
4000 + 1000; $K = 5000$
2. $K = 5000; = 4000c + 1000v$

When one recognizes that in general – and quite apart from the particular values of the variables chosen – the value of the constant capital employed in any one year, must be equal to the value produced during that year (given the absence of accumulation), it follows that $c_I + c_{II} = x_I = c_I + v_I + s_I$. By eliminating terms found on both sides of the equal signs, the following proportionality condition is obtained (cf. Marx 1885M: 734; cf. 1885E: 365):

$$(v + s)_I = c_{II}.$$[131]

Next, Marx assumes the organic composition of capital ($c/(c + v)$) to be equal, constant and given across departments:

g. **'**What is arbitrarily chosen here, for both departments I and II, is the ratio of *variable* to *constant capital*; arbitrary also is the *identity* of this ratio between the departments […]. This *identity* is assumed here only for the sake of simplification, and the assumption of different ratios would not

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3. […] $= 4000_c + 1000_v + 1000_m$ (1885M: 728)

Of these, only the first is partially compatible with mathematical conventions. Physicists and mathematical economists (among others) would be inclined to interpret the third as: 'in the case of c the value of this variable is 4000' and not as: '$c = 4000$'. The second seems to say: '4000 times c + 1000 times v', which of course is not the interpretation intended. Engels considerably clears up Marx's mess and renders the schemes much more insightful, but he also adopts this strange second notation (cf. 1885E: 360-361).

[131] Marx's notation is the following: $(v + s)I = c(II)$. Engels rewrote this as: $I(v + s) = IIc$. In print, the v's, s's and c's were rendered subscripts. The printed version apparently led Reuten to comment: 'Generalizing the schema, Marx uses the notation:

$$I_c + I_v + I_s = I$$
$$II_c + II_v + II_s = II$$

In what follows, we adopt the notation that has become conventional in modern Marxian economics:

$$c_1 + v_1 + s_1 = x_1$$ [etc.]' (1998: 197).

The former is not true. Looking at the notations quoted above, it may be concluded that the notation that Reuten attributes to Marx (and that Engels attributed to Marx) is actually Engels', while Marx leant towards the now conventional notation, albeit that he used Roman numerals instead of (the western notation of) Arab ones.

> change anything at all in the conditions of the problem or its solution' (1885M: 739; cf. 1885E: 370; 1885F: 483).[132]

By the same token as assumption **f** above, this assumption is mathematically and dialectically redundant in the case of simple reproduction, because the definition of simple reproduction effectively fixes all of the variables in the model for simple reproduction. When all of the variables are constant and given, it follows that all ratios between them are constant and given as well.

But in contrast to the ratio of s over v that was under consideration in assumption **f**, there is no dialectical reason to ascribe any resilience to the organic composition of capital when considering expanded reproduction. This is so, because expanded reproduction allows for both intensive and extensive accumulation. And as we have seen in Section 2.10, intensive accumulation tends to increase the amount of constant capital employed relative to variable capital and thus the organic composition of capital tends to rise each year when considering expanded reproduction. In sum, although Marx fails to integrate technological change into his subsequent schemes of *expanded reproduction*, assuming a constant and given ratio v/c is at least justifiable as long as *simple reproduction* is under scrutiny.

At first sight, the assumption that this ratio is identical across departments appears to be just a simplification indeed (as Marx stated).[133] There is, however, a possible escape route. We have long since identified the profit motive as *the* overriding motive shaping the course of capitalist development. Hence, we can safely posit at this level of abstraction that capital moves into the more profitable department until profitability evens out across departments.[134] If, following Marx, we define the profit rate as $s/(c+v)$, equalization of s/v (assumption **f** above) implies equalization of c/v.[135] The upshot of this is that acceptance of the thesis

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[132] In this rare instance, Engels took over Marx's sentence without any alterations, except for dispensing with Marx's underlinings (rendered *italic* throughout the Mega, as well as in my quotations). Hence, I could rely on Fernbach's translation for once.

[133] Moreover, Reuten (1998) calculates the κ's for the two departments that Marx implicitly uses in his schemes of *expanded reproduction* as $\kappa_I = 0{,}8$ and $\kappa_{II} = 0{,}67$. Since $0{,}8 \neq 0{,}67$, not even approximately, we may conclude that Marx implicitly dropped this part of his assumption when moving on to expanded reproduction.

[134] Marx himself postponed explicating this until he had reached a way more concrete level of abstraction in Volume III of Capital. Moreover, he posits it as the outcome of the equalities between departments he had assumed all along (i.e. as the outcome of assumption **f** and **g**) (1894: Part 1, Chapter 3). Nevertheless, all elements that justify this statement are already in place here.

[135] This is so, because the posited equality between the profit rates may be formalized as:

that profitability evens out, means that either both the rate of surplus-value and the organic composition of capital are identical in both departments or that they are both different. So if this element is dropped from assumption **f** it must also be dropped from **g**. Similarly, if it is accepted for the one ratio, it must be accepted for the other.

Immediately after the introduction of assumption **g** Marx discusses the results of his models and especially the implications of the proportionality conditions. Only after he has presented his conclusions, Marx makes it explicit that he has abstracted from foreign trade all along:

> **h.** 'Capitalist production *never* exists *without foreign trade*. […] Bringing foreign trade into an analysis of the *value of the product annually reproduced* can […] only confuse things, without providing a new moment of the problem or its solution at any point. We therefore completely abstract from it'. (1885M: 772; cf. 1885E: 433-434)

This assumption follows directly from the locus of the schemes of reproduction within Marx's overall systematic dialectics. That is, since the existence of many capitals, let alone foreign capitals, has not yet been determined, foreign trade is not even possible at this level of abstraction. Marx, however, argues for the adoption of this assumption on the grounds that the value relations between the

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$$\frac{\varepsilon_I v_I}{c_I + v_I} = \frac{\varepsilon_{II} v_{II}}{c_{II} + v_{II}}$$

from which we can derive:

$$\frac{c_I}{v_I} = \frac{\varepsilon_I}{\varepsilon_{II}} \cdot \frac{c_{II}}{v_{II}} + \frac{\varepsilon_I}{\varepsilon_{II}} - 1$$

So, c_I/v_I can only equal c_{II}/v_{II} if $\varepsilon_I = \varepsilon_{II}$ (so that $\varepsilon_I/\varepsilon_{II} = 1$). Either way, the difference between the organic composition of capital in both departments is proportional to the difference in the rate of surplus-value between the two.

The same conclusion can be drawn if variable capital is neglected (which seems empirically warranted, but should not be presumed in the abstract), but the expression is a little simpler:

$$\frac{\varepsilon_I v_I}{c_I} = \frac{\varepsilon_{II} v_{II}}{c_{II}}$$

which can be rewritten as:

$$\frac{c_I}{v_I} = \frac{\varepsilon_I}{\varepsilon_{II}} \cdot \frac{c_{II}}{v_{II}}$$

two great departments would be unaffected by foreign exchange. So, in his mind, this too is best considered a negligibility assumption.

By means of conclusion, Marx writes that although the ratios mentioned in assumption (**f** and) **g** may be arbitrarily chosen, the relationship described in the proportionality condition is a necessary one (1885M: 739), that, it might be added, clearly brings out the interdependence of the two departments.[136] With the proportionality condition we get a first glance at the fundamental relations between the two great departments ensuing from *capital*'s necessary appropriation of means of production to be employed as *constant capital* and of labor power employed as *variable capital*. Since the whole analysis was geared towards this goal, this is an important intermediate result, the implications of which Marx analyses at length in the next 30 odd pages (1885M: 740-768; cf. 1885E: 370-384, 401-430).[137]

After his lengthy elaboration of what the proportionality condition entails for capitalist productive relationships, Marx reintroduces the possibility of a mismatch between calculated depreciation costs that as such transfer to the value of the product and actual depreciation expenditure required (i.e. he drops assumption **e** above) and retains this for the remainder of his analysis of both simple and expanded reproduction. Concerning simple reproduction, the result of this is that capitalist crises would ensue from either rising or falling depreciation rates, because department I would then respectively under- or overproduce. Hence, even in the assumed absence of accumulation, crises can easily occur (Marx 1885M: 768-769; cf. 1885E: 430-432).

3.1.2. Conclusions

In my discussion of Marx's model for simple reproduction particular attention has been paid to the way and order in which he conceives of and defends his choice of assumptions. As we have seen, assumptions **a**, **c**, **d**, **e** and **h** can certainly be defended dialectically, while the dialectical defendability of **b** depends on how it is interpreted. Bearing in mind that all constant capital is produced in department I, assumptions **f** and **g** have been shown to be implicit in simple reproduction as

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[136] At this point Engels apparently decided that a few lines of clarification were in order. He writes: 'If it $[(v + s)_I]$ were smaller than II_c [i.e. c_{II}], then department II could not completely replace its constant capital; if it were larger , then an unused surplus would be left over. In both cases, the assumption of simple reproduction would be injured' (1885E: 371) (yes, 'injured', not 'destroyed' as Fernbach curiously translated the German word *verletzt*). Thus, Engels emphasizes the potential for crisis more than Marx does.

[137] The fact that Marx does not use algebraic means of generalization, but instead tries to infer general relationships from specified numerical examples, greatly contributes to the number of pages Marx requires for his analysis.

defined in assumption **c** (see the table below for a short explanation – or reminder – of what these codes stand for). This dialectical appreciation of the model assumptions and the main arguments for that appreciation in the context of simple reproduction are summarized in Table 2 below. Table 2 also shows what type of assumptions Marx thought he made. I have labeled them *negligibility* when Marx says that dropping it would leave the results unaffected; *heuristic*, where Marx intends to drop the assumption later; or *simplifying*, where simplification (of e.g. his numbers) is his only argument. When Marx did not give any classification I have filled in '-'.

		marx	appreciation	Argument
a	Values = prices	negligibility	dial +	Prior to many capitals
b	No revolution in values	negligibility	dial +	If simple ⇒ c, v & s constant
c	full surplus-value and full wage consumed	heuristic	dial +	Static existence analysed before dynamic actuality
d	Only two great departments	–	dial +	Dissociation separates the site of consumption from that of production
e	depr. cost = replacement expenditure	heuristic	dial +	macro-economic aggregates even out individual mismatches
f	s/v = 100%, constant and equal in I&II	–	redundant	Definition SR ⇒ c, v & s constant
g	Ratio v/c constant and equal in I&II	simplification	redundant	Definition SR ⇒ c, v & s constant + equality changes nothing
h	No foreign trade	negligibility	dial+	Prior to many capitals ⇒ no foreign capitals

Table 2. Marx's assumptions and their (dialectical) appraisal regarding SR.

Of all the assumptions Marx makes, only **c**, **d, e** and **h** need to be called upon to justify the proportionality condition and the potential for crisis when actual wear and tear in department II does not match the demand for replacement of *constant capital* expected by department I. This is shown more clearly in the next chapter.

All in all then, Marx has made more assumptions than were required, but if anything this means that the model results are even more general than Marx thought they were. Even though the applicability of these results is questionable because of the high level of abstraction to which they pertain, the model certainly serves its dialectical purpose of illuminating the fundamental relations between the two great departments ensuing from *capital*'s necessary appropriation of means of production to be employed as *constant capital* and of labor power

employed as *variable capital*. Moreover, since these relations could be established on the basis of dialectically defendable assumptions only, they must mediately hold on all of the more concrete levels as well. Thus, as alluded to in the introduction to this section, these relations (and the potential for disproportionate development inherent in them) are a force to be reckoned with in all models that have empirical aspirations and the praise Marx received regarding them bears testimony of this. The discussion here has shown that this praise is justified. Indeed, the relations Marx has shown are necessary and inescapable in a functioning capitalist economy.

This intermediate achievement notwithstanding, the models can only be considered successful if *accumulation* can be brought back in first, for only then is our formal analysis back on a par with all preconditions for capitalism's reproduction dialectically determined so far. To this end, Marx next draws up and analyses a model for expanded reproduction, which will be evaluated in the next sub-section.

3.2. Expanded Reproduction

3.2.1. The Model

For his analysis of expanded reproduction, Marx retains the assumptions introduced above, except the major part of the one defining simple reproduction (i.e. **c)** and the one assuming equality of depreciation costs and replacement expenditure (i.e. **e**). Regarding assumption **c** Marx only retains the idea that all wages are consumed in full. This is dialectically defendable as long as all commodities, including labor power, are produced 'at their values' (assumption **a**). In unadulterated capitalism such as is under consideration at this level of abstraction, the average value of labor power cannot be much higher than a subsistence wage, so there is not much room for labor to save any substantial sum of money and hold on to it. Of the assumptions Marx retains in full, **b** (no revolution in values), **f** ($s/v = 100\%$, constant and equal in I&II) and **g** (v/c constant and equal in I&II) were deemed dialectically justifiable (but also redundant) in the case of simple reproduction. But would this verdict still hold in the case of expanded reproduction?

Regarding assumption **b** it was already indicated that it 'may even be upheld in the case of expanded reproduction' if the assumption is interpreted as saying that 'value revolutions leave the value relations *within* 'the component parts of the productive capital' unaffected'. Regarding **f** I have said that assuming some resilience regarding the ratio of s over v is dialectically defendable, although pinning a definitive number (100%) on it is not. When suitably generalizing the schemes algebraically there is no need for such numbers either. But since Marx

presents his schemes in the form of numerical examples, this number is as good as any other. The trickiest one to maintain is **g**. If it is not relaxed or dropped when considering expanded reproduction, we are effectively focusing exclusively on extensive accumulation while ignoring the intensive aspect of that phenomenon. When modeling *expanded reproduction* in Chapter 4 I will therefore relax this assumption.[138]

Marx adds four more assumptions that he apparently considers specifically needed in the case of expanded reproduction. The first of these is:

i. there has '*already* [been] production on an expanded scale' (1885M: 791; cf. 1885E: 452)

According to Marx, this must be assumed because the means of production required by accumulating capital must somehow be on offer, before capital can decide to expand its scale of operations (Marx 1885M: 791; cf. 1885E: 452). Hence, someone must have somehow already brought them to market. Since systematic dialectics aims at understanding a given system in its entirety (i.e. as an object totality), assuming this is dialectically warranted in the sense that it formally explicates one of the characteristics of full blown capitalism. Full blown capitalism of course requires there has '*already* [been] production on an expanded scale'. So, assumption **i** is dialectically defendable as it formally explicates that systematic dialectics aims to tackle the systematicity of particular systems and not transitions between systems (the latter is the task of historical dialectics). So this assumption formally explicates the *necessary* requirement for all systematic dialectics that it must tackle the systematicity of a given or thinkable object totality (in this case capitalism) rather than its historical development.

Contemplating this assumption Reuten remarks: 'Apparently Marx does not aim to set out the transition from simple to expanded reproduction' (1998: 204). Although this is a dialectically defendable choice to make, it is a missed chance,, for later on Marx *does* describe the transition from a steady 9% growth rate to a steady 10% rate of growth. If generalized, this description might just as easily apply to a steady growth rate of 0% (as is the case for simple reproduction) accelerating to some positive number – the technique and algebra involved would

[138] Chapter 4 presents three generations of models: 1) a model for simple reproduction, 2) a model for extensive growth (i.e. with extensive accumulation only) and 3) a model for expanded reproduction (including both forms of accumulation). The first two reconstruct Marx's models in algebraical form. Thus my model for extensive growth is akin to Marx's for expanded reproduction, but since Marx's model for the latter is not on a par with his concept, I relabeled it. The third goes beyond Marx's models and as such provides a possible model for Marx's *concept* of *expanded reproduction.*

be exactly the same (see Chapter 4). Thus, it could have been shown how the dynamics of *expanded reproduction* are predicated upon static *simple reproduction* and the other way round, thus mutually validating each other. Hence, setting out the transition from simple to expanded reproduction would have brought out the model's dialectical roots and development more explicitly.[139] This, in turn, would have indicated how assumptions in mathematical models can be informed by the systematic dialectical exhibition leading up to the model, while the model results illuminate how and why the exhibition so far is still insufficiently concrete. At any rate, assumption **i** is redundant if Marx's schemes are suitably generalized algebraically, as is done in the next chapter.

After the introduction of assumption **i**, Marx goes on to explain that yearly turnover must be hoarded in gradual lumps of depreciation allowances ('one-sided sales') and discrete lumps of investment ('one-sided purchases') and assumes the two cover each other (1885M: 795; 1885E: 456; 1885F: 570; cf. Reuten 1998: 202-203). [140] . Thus, the money required for maintenance is on average available from previous hoards, so that it may be assumed that whatever part of surplus-value is invested that year is indeed used to expand a department's productive capacity. In Marx's words:

j. 'The value of one-sided purchases (from cII [i.e. c_{II}]) = one-sided sales (at cII [i.e. c_{II}])' (1885M: 795; 1885E: 456).

Since depreciation allowances and depreciation costs are mirror images, this assumption is likely to hold at this level of abstraction by the same token as assumption **e** (i.e. depreciation cost = replacement expenditure). That is, if the values of depreciation allowances and replacement expenditure are aggregated

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[139] These comments on Marx's models emanate from his systematic dialectics as I have exhibited them in the previous section. γ) *Simple reproduction* (Section 3.12) was introduced there as a moment in the systematic-dialectical exhibition. Reflection on this moment led us to distinguish between the α) *department of means of production* and the β) *department of means of consumption.* Modeling the circuits that these two departments engage in, led Marx to formulate proportionality conditions that show the interdependencies between the two departments. These interdependencies in turn led Marx to speak of the two departments together as γ) *total social capital.* So in my view the categorial transition of *simple* to *expanded reproduction* is a systematic-dialectical one, partially informed by the model of *simple reproduction*. By modeling this transition, Marx could have developed his models parallel to and in sync with his systematic-dialectical exhibition. Because *simple reproduction* to me is a dialectical moment, such a modeling exercise would result in a model of a systematic-dialectical and not a historical transition.

[140] The German terms are 'bloβer Verkauf' and 'bloβer Kauf' respectively. 'Bloβ' would literally translate as 'naked', 'bare' or 'only', but certainly not as one-sided. But I admit the translation is actually clearer.

over a whole department, the conclusion that surplus-value is only used for accumulation and capitalist consumption must hold, because *many capitals*, let alone capitals performing different functions – such as financial capital – have not been introduced yet at the level of abstraction the reproduction schemes are supposed to analyze. So for now, the system must be analyzed without recourse to credit. Marx does allow for accumulation being paid for out of previous money hoards (but explicitly assumes that away in the quote above) but given that the business cycle has not yet been dialectically introduced, there is no reason at this level of abstraction to assume any fluctuations in a department's rate of hoarding or dishoarding.[141] Consequentially, the only source of funds available to either department is yearly turnover. Thus, macroeconomically, the value of the *fixed capital* used up during a given year, equals that year's total replacement expenditure and hence the conclusion that accumulation is paid for out of surplus-value is warranted on systematic dialectical grounds.

Next, 'Marx assumes a sufficient monetary accommodation for expanded reproduction' (Reuten 1998: 203). From the vantage point of the early 21st century, this is almost a no-brainer. After all, a company that decides to expand its production will in almost all cases borrow the necessary funds from the bank. In essence, banks conjure these funds into existence ex-nihilo: they just print - or rather, credit - the money requested.[142] If the funds a company has thus acquired are invested successfully it is able to return the principal sum plus the required interest. Thus, one might say our money is essentially covered by production, so successful expansion of production goes hand-in-hand with a successful expansion of the money. But in Marx's time things were not that straightforward, since money was still covered by gold to some degree. So Marx had to make his assumption explicit:

k. **'**The only thing assumed here is that *the amount of money present in a country* is sufficient for both hoarding and accumulation.' (1885M: 800; cf. 1885E: 461)

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[141] If business cycles were considered, assumption **j** would still hold if the size of hoards were averaged out over a complete business cycle. In *Capital I*, Part Seven, Marx does not systematically introduce the business cycle, although he makes an empirical reference to the cycle (see Reuten 2004c: 274 and 294-95).

[142] Regarding the recent bail-outs of American banks, the story goes that the FED president was asked where they got all that money from. This is not a strange question to ask from the perspective of a civilian who has just lost almost everything to the failing banking system and has no way to get it back, but Bernanke clearly had trouble understanding the question as his reply was: "Why? We punched the number into a computer and credited it to their account".

From a dialectical point of view however, neither empirical reality (be it that of Marx's time or our own) needs to be fully accommodated *yet* (after all we have not introduced financial capital at this level of abstraction yet).[143] So far, it has been established that *capital* must *accumulate* to survive, is departmentalized by necessity and that neither *department* can survive without inputs from the other (indirectly so for department II). If we aim to set out to determine how these abstract requirements can be upheld at the next stage of concretization (which is still far removed from empirical reality however), the mentioned assumption is fully warranted as an anticipatory assumption. For, if it is not satisfied in the abstract (i.e., in this case, in the aggregate and on average), accumulation would sooner or later grind to a halt. So even though we might not *yet* have a dialectical basis to argue for the adoption of this assumption, we know that it must hold somehow in concrete capitalism. So by making this assumption we are anticipating later stages of concretization. Marx defends this assumption by giving four reasons why expanded reproduction requires sufficient monetary accommodation to function (1885M: 800-801; 1885F: 576). Thus, at least from a systematic-dialectical perspective, his defense is similar to mine.

Before Marx starts analyzing the effects of accumulation on the relations between capital's two great departments, he makes one final assumption:

> **l.** 'It has been extensively explained in *"Capital" (Book I)* etc., that *labor power* [is] always available on the basis of capitalist production and how, if necessary, *more labor* can be made available without expansion of the employed amount of laborers or the mass of labor power. At this moment [it is] therefore not necessary to elaborate on this further, much more to assume that the part of the *newly created money capital* that can be transformed into *variable capital* will always find the labor power on hand to transform itself into.' (1885M: 801-802; cf. 1885E: 463)

At first glance, the (possible) argument for adoption of this assumption seems to be similar to that regarding assumption **k**. That is, sufficient labor power, like sufficient monetary accommodation, must be on offer if *accumulation* is to be possible at all. Since assumption **g** (stating that the ratio of *constant* to *variable capital* is constant) implies that variable capital must tag along with the accumulation rate for constant capital, this argument seems warranted.

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[143] Of course the categories and language available to dialecticians cannot surpass the latest insights of the dialectician's time and are thus molded by empirics, but this influence is only an indirect one, until the most concrete stage of a dialectical exhibition is reached.

However, in the quote above, Marx refers back to *Capital I* to justify his assumption. But in Capital I his chief argument is that labor expelling techniques result in a rising technical composition of capital (tcc) – defined as the ratio 'between the mass of the means of production [mp] employed on the one hand, and the mass of labour necessary for their employment on the other [l]' (Reuten 2004c: 286) –, so the role labor plays relative to means of production in accumulation is an ever diminishing one (Reuten 2004c: 287-288). The problem with this argument is that it can only be adopted if assumption **g** is either dropped or modified.

If we interpret Marx's assumptions that 'products are exchanged according to their values' (assumption **a**) and that 'no revolution in values takes place in the component parts of the productive capital' (assumption **b**) to mean that values do not diverge from prices and are relatively constant in the period under consideration, a rising tcc should translate directly into a rising organic composition of capital (occ, usually defined as $occ = c/v$, but defined here as $\kappa = c/(c+v)$).[144] So with these assumptions in place the occ can be used to operationalize the tcc. Having thus operationalized the tcc, we may try and grasp the problem formally.

In formal terms, Marx assumes that the growth rate for available labor power $\Delta n/n$ is always larger than the growth rate for labor required, that at a constant wage per worker – i.e. with assumption **a** and **b** in place – is equal to the growth rate of variable capital, $\Delta v/v$. Thus it is assumed that: $\Delta n/n > \Delta v/v$. Defining the organic composition of capital as, $\kappa = c/(c+v)$ one can write for the growth rate of variable capital associated with accumulation $\Delta v_{acc}/v$:

$$\frac{\triangle v_{acc}}{v} = \frac{1/\kappa' - 1}{1/\kappa - 1} g$$

In which g is the steady state growth rate for constant capital and κ' is the organic composition of capital that goes with the latest technical composition of capital. If the latter is rising, so must the former. Hence the following condition must hold:

$$0 < \kappa < \kappa' < 1$$

Thus a rising κ will ensure that variable capital grows at a slower rate than constant capital, but given that some variable capital will always be employed, κ'

[144] The definition of occ as $\kappa = c/(c+v)$ emanates from Reuten (albeit that he labels it γ) (1998: 206) and mainly serves to keep κ between 0 and 1. Keeping it between these bounds is useful when interpreting the general algebraic formulas developed in Chapter 4.

will never reach unity and hence positive accumulation will result in more variable capital being employed (however marginally). But apart from accumulation (growth of total social capital), replacement of fixed and circulating capital will also lead *capital* to adopt new techniques. As a result of this, variable capital employed falls rather than rises with each round of depreciation. So accumulation of total social capital without an increase in variable capital employed, is perfectly possible in Marx's models, but is by no means guaranteed.[145] Therefore, without assumption(s) on the development of $\Delta n/n$, a rising technical composition of capital is insufficient to justify the conclusion that labor power is always on hand.

A possible way out of this is suggested by Zarembka (2009). In his reappraisal of Rosa Luxemburg's contribution to Marxism he first identifies the dilemma sketched above and reaches the conclusion that Marx's assumptions can only hold when there is either a continual population growth or potential growth of the proletariat (now referred to as rate of participation).[146]

A defense of assumption **l** (availability of labor power) along those lines, however, runs counter to Marx's professed goal of investigating capitalism 'in its integrity, free from all disturbing subsidiary circumstances, [so that] we must treat the whole world as one nation, and assume that capitalist production is everywhere established and has possessed itself of every branch of industry.'(Marx 1867F: 727, fn. 2). In short, he wanted to analyze an emerging societal system as if it were complete and chart out the structural interactions between the elements of this complete system. Although – as Reuten calls it – historical dialectics overarches systematic dialectics (2000: 140-152) and 'philosophy is its own time apprehended in thoughts' (Hegel 1821: 15; cf. Smith 1990: 4; cf. Smith 2003: 187), systematic and historical processes should be clearly distinguished if one is to find out whether a system *qua* system is viable.[147] So, from a systematic-dialectical perspective I cannot but agree with Marx's chosen standpoint and procedure and thus the problematic stands.

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[145] The exact determinants of the accumulation and the depreciation effects as well as their interaction are algebraically determined in the next chapter.

[146] 'Growth of the proletariat' implies that there is always someone somewhere that is not yet a wage worker or no longer a wage worker and can be coerced to become one. Examples of these people in Marx's age included housewives, children and retired people and in non-western societies also autarkic peasants and tribes. In modern times we may add stay-at-home-husbands and self-employed people (in essence they are capitalists that can be forced to become laborers again, if the market for their service or goods is bad). And indeed, these people still exist.

[147] This is not to say that one of the conclusions following from a thorough dialectical analysis of a system could not be that it is only viable as long as some of its elements resist subsumption, but such a conclusion should not be embedded in the premise.

The bottom line is that without (a) specific assumption(s) concerning the development of labor power's availability (be it by means of population growth or by forceful expansion of the proletariat), the availability of labor power must be *assumed* as a condition of existence for *accumulation*. On closer scrutiny, the argument in *Capital I* Marx presumably refers to is just not sufficient to claim that the availability of labor power has been dialectically determined. All in all then, it must be concluded that the (possible) argument for adoption of assumption **l** must indeed be similar to that regarding assumption **k**.[148] [149]

With all these assumptions in place, Marx introduces the base scheme for expanded reproduction (the scheme for year 0, one might say):

‘	c	v	s	x
I.	4000 +	1000 +	1000 =	6000
II.	1500 +	750 +	750 =	3000
	5500 +	1750 +	1750 =	9000’

(Reuten 1998: 207; cf. Marx 1885M: 810; cf. 1885E: 471).[150]

Effectively these numbers imply that constant capital worth 500 ($= x_I - c$) is available for accumulation. Next, Marx assumes that department I accumulates half of its surplus-value, i.e. 500 and spends 400 of this on constant capital and 100 on variable capital. Since accumulation in department I effectively means it has to buy more of its own produce, it is bound to succeed. Thus, the other department has to make due with whatever is left, i.e. an accumulation in constant capital worth 100. Due to assumptions **f** and **g** we end up with the following numbers after accumulation and expanded production has taken place:

‘	c	v	s	x
I.	4400 +	1100 +	1100 =	6600
II.	1600 +	800 +	800 =	3200
	6000 +	1900 +	1900 =	9800’

[148] But I am open to suggestions on how to reinterpret Marx's reference to *Capital I* in such a way that his arguments may be upheld in both places.

[149] Apart from Paul Zarembka, Jurriaan Bendien and Gerald Levy provided helpful comments on this problematic. But since Paul Zarembka actually wrote about it, his comments could be used most comprehensively. I am nevertheless very grateful to all three of these persons.

[150] Marx's notation here is almost identical to Reuten's, albeit that he did not add up his numbers for c, v and m, but only mentions the total output as: ‘{ Sum = 9000’ (1885M: 810). Engels changed this back to the questionable notation ‘$I. 4000c + 1000v + 1000m = 6000$’ (1885E: 471; cf. footnote 130 of this chapter).

(Reuten 1998: 211; cf. Marx 1885M: 812; cf. 1885E: 472).[151]

If department I continues to accumulate at the same rate next period and department II continues to pick up the pieces, then, after the next round of accumulation and production the numbers become:

‘	c	v	s	x
I.	4840	+ 1210	+ 1210	= 7260
II.	1760	+ 880	+ 880	= 3520
	6600	+ 2090	+ 2090	= 10780’

(Reuten 1998: 212; cf. Marx 1885M: 813; cf. 1885E: 473).[152]

Scrutinizing those numbers one can conclude that both departments now accumulate at the same rate again, but that this rate is higher than it could be on the basis of the first schematic given. But to achieve this, department II had to diminish its rate of accumulation in the intermediate period (the second schematic reveals this).

‘Marx calculates the schema for three more periods’ (Reuten 1998: 212) and then introduces a second example so as to show the effects of a diminishing rate of accumulation in department I. Only after that he verbally formulates the proportionality condition for expanded reproduction:

> ‘It is self-evident that – on the assumption of accumulation, $v + m(I)$ [i.e. $v_1 + m_1$] > than cII [i.e c_2] and not $= cII$, as in simple reproduction, since 1) I incorporates part of its surplus product into its own productive capital, transforms it into constant capital, but cannot simultaneously replace it with means of consumption from II. 2) I has to supply the constant capital required for accumulation within II out of its surplus product’ (Marx 1885M: 817; cf. 1885E: 475).[153]

Formalizing this, we would get:

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[151] Surprisingly, Marx starts mixing up notations again at this point (Engels’ notation is still spurious, but at least consistently so).

[152] Marx miscalculated here, but Engels comprehensively corrected his numbers.

[153] It is surprising that where Marx has ‘transforms *it*’ (1885M: 817, emphasis added), Engels has ‘transforms five sixths’ (1885E: 475). Thus, where Marx apparently draws a general conclusion from his overtly specific schemes, Engels (perhaps in pride of his superior calculations) sticks to a specific number that only holds for the values arbitrarily chosen by Marx.

$$(v + s)_I - \Delta c_I = c_{II} + \Delta c_{II}$$ (amended from Reuten 1998: 209).[154]

Algebraic methods, of course would have given Marx this result a lot quicker, a lot clearer and in an immediately general form. Moreover, such methods show that conditions for transition to either a higher or lower rate of accumulation and for steady accumulation at the same rate can be captured in just a few formulas. On top of that, careful scrutiny of these formulas shows that a few of Marx's assumptions are redundant and others can easily be relaxed. The resultant clarity makes the model's contribution to and the way it is embedded in Marx's systematic dialectics much easier to evaluate. All these insights are conveyed in the next chapter.

3.2.2. Conclusions

As with simple reproduction, I have focused on the (dialectical) defendability of Marx's assumptions here. Marx drops most of assumption **c** (defining simple reproduction) when discussing expanded reproduction. The only part of that assumption he retains is that of no savings out of wages, which is dialectically defendable to do. He has already dropped assumption **e** (depreciation cost = replacement expenditure) at the end of his discussion of simple reproduction (despite its dialectical defendability) and does not reinstate it here. Assumptions **b (**No revolution in values), **f** ($s/v = 100\%$, constant and equal in I&II) and **g** (Ratio v/c constant and equal in I&II) were deemed dialectically defendable (but also redundant) in the context of simple reproduction. Assuming revolutions in value away (assumption **b**) and assuming s/v constant (as **f** states) still are in the case of expanded reproduction (albeit for different reasons) but retaining assumption **g** is not. Retaining **g** effectively means that intensive accumulation is never brought back into the model. As a result, Marx's model of expanded reproduction did not fully reveal the conceptual grasp Marx had of the phenomenon. Marx adds four assumptions – **i** (previous reproduction on an expanded scale), **j (**accumulation out of surplus-value only), **k** (monetary accommodation) and **l** (labor abundance)– that he apparently deemed specific to expanded reproduction. All of these are dialectically defendable either because they are well founded on the systematic dialectical exhibition so far or because they anticipate what the exhibition will have to ground later on.

Regarding the latter assumptions, Marx generally seems to agree with my appraisal. So, in sharp contrast with his schemes for *simple reproduction* his own

[154] Reuten uses the now conventional notation (see footnote 131), but I wanted to stay in line with the notation Marx originally used for the proportionality condition for simple reproduction.

defense of assumptions **i**, **j**, **k** and **l** often squares with mine in that it appears to be dialectically informed. Table 3 below therefore features more labels in the column 'Marx' then Table 2 did. When an assumption is defended by referring to an earlier result in the dialectic I have labeled it 'necessary'. If Marx's reasoning seems to imply anticipation, I have written 'anticipation'. Otherwise, Table 3's set-up and the codes utilized in it are analogous to that of, and those in, Table 2.

		Marx	appreciation	argument
a	Values = prices	negligibility	dial +	Prior to many capitals
b	No revolution in values	negligibility	dial +	Changes *within* component parts do not preclude changes *between* them
c	Full surplus-value consumed dropped, but 'no savings by labor' retained	heuristic	dial +	value of labor power not generally above subsistence
d	Only two great departments	–	dial +	Dissociation separates the site of consumption from that of production
e	~~depr. cost = replacement expenditure~~	~~heuristic~~	~~dial +~~	~~macro-economic aggregates even out individual mismatches~~
f	$s/v = 100\%$, constant and equal in I&II	–	s/v constant: dial +; specific number: dial -	labor is ultimately the only source of value
g	Ratio v/c constant and equal in I&II	simplification	dial -	constancy means intensive accumulation is never brought back in
h	No foreign trade	negligibility	dial +	Prior to many capitals ⇒ no foreign capitals
i	there has already been reproduction on an expanded scale	anticipation	dial +	expansion requires prior availability + complete system analysis
j	hoards = replacement expenditure ⇒ accumulation out of surplus-value only	-	dial +	macroeconomic aggregates even out individual mismatches
k	Monetary accommodotian	anticipation	anticipation	if it were not so, capitalist accumulation would not exist
l	Labor abundance	necessity (as a result of technological development)	anticipation	if it were not so, capitalist accumulation would not exist

Table 3. Marx's assumptions and their (dialectical) appraisal regarding ER.

To justify the proportionality condition for expanded reproduction and the conditions from which disproportionalities arise, Marx needed to call on all of these assumptions. So, by contrast to simple reproduction, none of the assumptions made were redundant in the case of expanded reproduction. But most were dialectically defendable. The two exceptions were assumptions **f** and **g**.

The problem with **f** is just that a specific number was assigned to the ratio of s over v. This problem can easily be remedied by algebraic means (as is done in the next chapter) but doing so does not change Marx's conclusion at all. This is different for **g**. Modifying it to allow for technical change does make the model rather more complex, but also adds yet another source for disproportionality. So, the difficulties would only be exacerbated if technical change were factored in. Hence, Marx's conclusion that macroeconomic acceleration of growth is extremely difficult is fully warranted anyway.

Summary and Conclusions on the Role of Mathematics in Systematic Dialectical Investigation and Exhibition

At the outset of this chapter we posed the question whether a systematic dialectical exhibition can inform mathematics (or definitions for mathematics) or mathematical modeling and/or vice versa. The answer to this question depends on whether the subject of investigation is ontologically amenable to quantification and on the epistemological prowess one ascribes to mathematics as an investigative tool. Chapter 1 has already shown that capitalism and capitalist abstractions are quantitatively constituted ontologically. As to the epistemological usefulness of mathematical means of investigation for the study of capitalism, we can be brief. Section 1 showed that 1) Marx by the end of his life had become quite conversant with the university textbooks on mathematics of his day, 2) endeavored to reform the basis for mathematics (especially the differential calculus) dialectically and 3) toyed with a lot of ideas for the application of mathematical and formal methods to his main studies in political economy. So, in sharp contrast with Hegel, he clearly thought that mathematics could be improved by dialectical methods and that dialectical exhibitions could be improved by augmenting them with mathematical techniques.

His most famous attempt at the latter concerns his schemes of reproduction. Despite their merits as models, Marx did not succeed in bringing his schemes on a par with his technical prowess in algebra, nor to fully integrate them into his dialectical exhibition of capitalism, probably because he learned the wrong methods at the wrong time and mastery of mathematical techniques does not imply mastery of their applications to problems in other realms. Thus, there is ample room for improvement here and in the next chapter some promising avenues to this end will be suggested and worked out in some detail.

Now in order to evaluate the degree to which Marx succeeded in integrating his schemes of reproduction into the whole of his dialectical exhibition in *Capital* (and pinpoint possible areas for improvement), we must first come to grips with this exhibition itself and properly position these models therein. Since Marx never was very explicit concerning his method, such an exhibition always involves a substantial degree of interpretation. In this respect I especially draw on the interpretations of Smith (1990), Arthur (1993) and some of the terminology of Reuten and Williams (1989), albeit that the mode of exhibition is my own: the α-β-γ format introduced in previous chapters.

Marx's starting point is that any viable society must ensure procreation and socialization of useful products. In Reuten and Williams' terminology, a society that achieves this (i.e. *every* society by definition) is a α) *sociation*. But capitalist

production is not inherently social, for it is undertaken in private units that are separated from the site of consumption, that is in β) *dissociation*. Dissociate produce in capitalism is resocialized through exchange: γ) the *association*. Generalized exchange implies production for exchange and thus goods become γ) *commodities* that are inherently α) *exchangeable* because they embody *use values* and can be owned and sold in discrete quantitatively delimited units, but when they are β) *bargained* over in the *exchange relation*, they present themselves as *exchange values*. As soon as a bargain is struck, incomparable use values get commensurated into the same dimension: γ) *value in exchange*.

In one-off barter exchange this value appears only fleetingly during the exchange itself, i.e. it appears in its α) *simple commodity form*. But as more and more products are exchanged, the number of value relationships commodities can enter grows accordingly, so we have β) an *expanded commodity form of value*. Any commodity may next be singled out to serve as a general equivalent of value in its γ) *general commodity form*. If a significant part of society singles out the same commodity, it starts serving as a tangible embodiment of value: γ) *money*.

With the advent of *money* values become characteristics of *commodities* prior to the *bargain*. Hence, *money* first and foremost serves as α) *measure of value*. But since *money* sprang forth from the need to resocialize production through generalized *exchange*, it just as much serves as β) *means of circulation*. As such it appears to mediate between *commodities*, giving us the circuit of commodity (C) – money (M) – different commodity (C'). But since *money* can be immediately exchanged for any commodity, while commodities can do so only mediately via *money*, as of this point in the exhibition it makes more sense to have a stock of *money* than a stock of *commodities*. This means that $C - M - C'$, breaks down into $M - C - M$ and acquiring *money* instead of qualitatively different *commodities*, becomes the γ) *end of exchange.* However, $M - C - M$ only makes sense if money holdings grow in the process, giving us the circuit $M - C - M'$ (more *money*). With this, γ) *capital* can be introduced as 'money which begets money' or self-valorizing value. As such *capital* functions as a structural ground for ongoing capitalist production.

This begs the question how this self-valorization comes about. The answer lies in production: a qualitative transformation (of C into C') to be realized by means of *capital*. This is achieved by employing labor power as β) *variable capital* to transform means of production functioning as α) *constant capital*. Thus, all the requirements for an ongoing spiral of valorization have now been determined, laying the basis for γ) *accumulation*, the production of commodities on an ever-expanding scale. *Accumulation* entails growth of both *variable* and *constant capital*. This is a process of extensive growth if both capital components grow at the same rate, but to the extent that constant capital replaces variable capital, it is a

process of intensive growth. These processes can be analytically separated (as Marx does in his schemes of expanded reproduction), but in concrete capitalism the process are inseparably interrelated.

If the required qualitative transformation and the forms *variable* and *constant capital* have to take (as labor power and means of production respectively) are articulated together we end up with a circuit. From the point of view of Capital's overriding motive, this is α) *the money capital circuit*:

$$M \longrightarrow C\{mp; lp\} \ldots P \ldots C' \longrightarrow M'$$

This gets expressed in the world as a continuous increase in the scale of production (P) and thus as β) *the production capital circuit*:

$$P \ldots C' \longrightarrow M. \quad M \longrightarrow C\{mp'; lp'\} \ldots P'$$

The money capital circuit commences with and culminates in exchange value, whereas the production capital circuit commences with and culminates in use value. If one takes the stock of finished products as beginning (C') and end (C'') of the capital circuit this tension is resolved since this stock represents only ideal exchange-value to the potential seller, but use-value to the buyer. Thus we get γ) *the commodity capital circuit*:

$$C' \longrightarrow M'. \quad M' \longrightarrow C'\{mp'; lp'\} \ldots P' \ldots C''$$

If means of production can enter into several capital circuits before their use-value is entirely exhausted and concomitantly their exchange-values transferred to the commodities they helped produce, they constitute α) *fixed capital*. If their use and exchange values are thus transferred in one go they constitute β) *circulating capital*.

Since *capital*'s existence depends on the circuits it is engaged in, these circuits must at least achieve γ) *simple reproduction*. That is, valorized *commodity capital* must at least cover its cost of production by selling either α) *means of production* to other capitals or β) *consumption goods* to consumers. Since the latter disappear from the macroeconomic circuit, whereas the former reenter the circuit as inputs at the same time that they leave it as outputs, it makes sense to distinguish between departments producing means of production and consumption goods respectively. Modeling the relations within and between these two departments, shows that they are systematically interrelated and can therefore be treated as one organic whole: γ) *total social capital*. This whole must *accumulate* to stay in business and hence it must engage in γ) *expanded reproduction*.

The fact that *total social capital* is *expanding*, means that there is α) a *general rate of profit*. But since *capitals* must be specialized (as was foreshadowed in *dissociation*), there must be β) *many capitals* engaged in γ) *competition* leading to γ) *minimum prices of production*.

Marx introduces *simple* and *expanded reproduction* (and their models – or schemes) in the final chapters of the last part of *Capital II*. In Section 3, I have evaluated whether the assumptions Marx makes in outlining these schemes follow from the systematic dialectical exhibition in *Capital I* and *II* up to that point. In his chapter on simple reproduction, Marx assumes (in order of appearance):

- **a.** Products are exchanged at their values
- **b.** No revolution in values takes place in the component parts of the productive capital
- **c.** The value of a department's yearly produce is constant and all surplus-value and wages are consumed (so there is no accumulation)
- **d.** Society's total yearly product breaks down into I) means of production and II) means of consumption
- **e.** Depreciation costs equal replacement expenditure
- **f.** The rate of valorization s/v (i.e. surplus-value over variable capital) is 100% for both departments
- **g.** The ratio of variable to constant capital is equal, constant and given across departments
- **h.** There is no foreign trade.

Of these, assumptions **a**, **c**, **d, e** and **h** are dialectically defendable. **a** and **h** are warranted because at the current level of abstraction we are considering relations between the two great departments of capital, but have not introduced *many capitals* yet. Hence foreign countries cannot enter the equations yet (assumption **h**) and there are no competitors yet that could induce a divergence between prices and values (assumption **a**). **c** is acceptable on dialectical grounds because it ensures that reproduction of society's productive capital is first considered in α) conceptual isolation, before it is given room to move in the world through accumulation (a step reminiscent of β). Thus, in effect it is a dialectically defendable heuristic assumption. Assumption **d** follows from the role *constant* and *variable capital* play in the *capital circuits* and the fact that *means of consumption* cross the institutional divide predicated on *dissociation*, whereas *means of production* do not. So, on the basis of the dialectical exhibition so far this seems the best way to cut the cake at this juncture. Finally, **e** is acceptable because we are considering only macroeconomic aggregates of depreciation costs

and replacement expenditure at this level of abstraction and thus any individual mismatches are likely to level out.

Whether and in what context **b** is dialectically defendable depends on how it is interpreted. If the emphasis is on 'values', it is dialectically defendable for simple reproduction only, because by virtue of assumption **c** the value of all capital components is fixed for simple reproduction. Evidently, when values cannot change at all, there cannot be a revolution in values either. Moreover, we have not introduced competition yet, so in the model all factors that could induce 'a revolution in values' are absent here and justifiably so. In the case of *expanded reproduction*, however, this argument cannot be upheld, for *accumulating capital* is bound to engage in technical innovation increasing the mass of products producible with a given combination of *constant* and *variable capital* and such innovations always correspond to revolutions in values. On the other hand, if 'component parts' is emphasized, the assumption is perfectly justified for both models. That is, the various forms *capital* takes (*variable, constant, fixed, circulating* etc.) and their roles in the *circuits* they must enter have been dialectically determined prior to drawing up the models. Thus, these component parts and the way they are interrelated have been dialectically determined before the modeling exercise starts, so it is safe to say that they will not undergo a revolution. This latter interpretation is supported by Marx's explanatory text, stating: '[A]s far as revolutions in value are concerned they change nothing in the relations between the component parts'.

Assumptions **f** and **g** are implied by assumption **c**, in a similar way as **b** is. That is, since assumption **c** basically fixes all variables in the model to a specific magnitude, the ratios between them must also be fixed. So as long as we are considering simple reproduction, assumptions **f** and **g** are redundant.

When drawing up his scheme for simple reproduction and its resultant proportionality condition, Marx only calls on the constancy of the yearly produce (assumption **c**), its departmentalization (assumption **d**), the equality of depreciation costs and replacement expenditure (assumption **e**) and the absence of foreign trade (assumption **h**). The others are actually redundant until expanded reproduction is introduced, modeled and analyzed. Marx's scheme looks like this:

	c	v	s	x	
I.	4000	+ 1000	+ 1000	= 6000	(means of production)
II.	2000	+ 500	+ 500	= 3000	(means of consumption)
	6000	+ 1500	+ 1500	= 9000	(social gross product)

where:
I = department I, producing means of production (6000);

II = department II, producing means of consumption (3000);
c = constant capital, the value of the means of production applied;
v = variable capital, the value of the social labor power applied;
s = surplus-value, the value that is added by labor minus the replacement of the variable capital advanced

Abstracting from the numbers and focusing on the relations between the two departments, Marx comes up with the following proportionality condition:

$$(v + s)_I = c_{II}$$

What this says is that department II's constant capital must be paid out of department I's wages (i.e. the value of its *variable capital*) and surplus. Failing fulfillment of this condition, at least one of the two departments goes into crisis. Hence the two form one organic whole: *total social capital*.

Since assuming accumulation away (assumption **c)** is dialectically defendable as a heuristic step only, accumulation must be brought back in for the models to be considered successful. To this end, Marx retains only one element of **c**, drops his assumption on the equality of depreciation cost and replacement expenditure (i.e. **e)** entirely and moves on to *expanded reproduction*. The element of **c** he retains throughout his schemes of expanded reproduction is the assumption that wages are consumed in full. When all commodities are exchanged at their values (assumption **a**), the value of labor power is unlikely to deviate from a subsistence wage by any significant magnitude, so retaining this element of **c** is acceptable from a dialectical perspective for the same reason that assumption **a** is.

As we have seen, the absence of revolutions in values (assumption **b**), the constancy of s/v (part of assumption **f**) and of c/v (assumption **g**) were implied by simple reproduction (assumption **c**) and were therefore more or less redundant with respect to modeling simple reproduction. With most of assumption **c** out of the way, however, this argument can no longer be upheld. For **b** this is no problem, because – as indicated – it is still dialectically defendable for expanded reproduction if one emphasizes 'component parts' rather than 'values'. As to assumption **f**, although the ratio of s/v it speaks of is a dialectically defendable element in any context, the assumption as put is way too strict. The mentioned ratio makes conceptual sense, since the appropriation of surplus-value has been shown to be predicated on the existence of *variable capital*. And given assumption **b** and **g** it can also be granted that this ratio is more or less constant and given. However the stated 100 per cent and the assumed equality across departments are entirely arbitrary choices. Assuming different ratios would complicate manipulations of the model, but as long as s/v is constant and given in

each department, the results obtained will still be similar. Assumption **g** effectively assumes technical change away. Retaining it in the context of expanded reproduction therefore effectively means that intensive accumulation is never brought back in. So this particular assumption must be modified to bring the model assumptions back on a par with the dialectical exhibition so far (see Chapter 4)

Marx introduces an extra four assumptions specifically for expanded reproduction (listed in order of appearance):

- **i.** There has already been production on an expanded scale
- **j.** The sum total of replacement expenditure on *fixed capital* equals the sum total of depreciation allowances for *fixed capital* in each department
- **k.** '[*T*]*he amount of money present in a country* is sufficient for both hoarding and accumulation.'
- **l.** There is always enough labor power on hand

All these assumptions are dialectically justifiable. Assumption **i** is acceptable because systematic dialectics should set out to analyze a complete system: full-blown capitalism, in this case. When capitalism as a system is established everywhere, 'production on an expanded scale' must already have taken place, so this can safely be assumed here. At the same time, *expanded reproduction* is predicated upon *simple reproduction*. By setting out growth acceleration from 0% to some positive number, Marx could have brought this out more clearly. That is, by thus articulating the dialectical development mathematically he could have shown how the dynamics of *expanded reproduction* are predicated upon static *simple reproduction* and the other way round, thus mutually validating each other dialectically. Chapter 4 shows how a suitable algebraic generalization of the transitions that Marx does model (i.e. growth acceleration of a steady 9% to a steady 10%) directly facilitates modeling this dialectical transition.

The upshot of **j** – and the argument for adopting it – is that only accumulation and capitalist consumption are paid for out of surplus-value and all other costs are covered elsewhere. Since at this level of abstraction there is no financial capital, *capital departments* have no other sources of funds and this conclusion is fully warranted dialectically. Assumptions **k** and **l** basically describe conditions of existence of *accumulation* and must therefore feature in any model that aims to understand the ramifications of *accumulation* for *capital*'s interrelationships.

With all these assumptions in place, Marx introduces the base scheme for *expanded reproduction*. This is similar to that for *simple reproduction*, except that the output of means of production is bigger than the constant capital used up that year, so that means of production worth $x_I - c$ are available for *accumulation*.

Marx implicitly assumes that department I always fulfills its accumulation plans and department II picks up the pieces, so that all produced means of production are sold. As long as we have only determined the existence of two *capital departments* dialectically, department I will only need to appropriate more of its own produce to get its wish in this respect and is therefore indeed bound to succeed. In the same vein as assumptions **k** and **l**, the second part of this implicit assumption is dialectically defendable as a condition of existence of balanced accumulation. That is, we know capital must *accumulate* and if we want to get a handle on the ramifications thereof, we need to assume away all obstacles to it in order to be able to eventually find out how this trick is pulled off in the real world.

By calculating the numbers in his scheme for several production periods in which accumulation and expansion take place, Marx is able to show that the result of this assumption is that growth acceleration in department I leads to growth deceleration in II. But if department I sticks to the higher growth rate it has accelerated to for more than one period and II keeps picking up the pieces this effect lasts only one period after which both departments are growing at the higher rate. Only after that, Marx verbally formulates the proportionality condition for expanded reproduction, which in more formal terms boils down to:

$$(v + s)_I - \Delta c_I = c_{II} + \Delta c_{II}$$

On the basis of the above evaluation, it may be concluded that there are at least four types of dialectically defendable assumptions. The first of these are *foundational*: they outline meaningful distinctions (such as that between the two capital departments), aggregates, categories and relations or ratios (such as s/v or v/c) pertaining to the level of abstraction being modeled. Marx's assumption concerning the departmentalization of produce (**d**) (and in their most lenient interpretations also those concerning the absence of revolutions (**b**) and constancy of s/v (**f**)) is (are) of this type. Secondly, it may be that some influence (such as foreign trade) is *absent* at the level of abstraction that the model pertains to. **a** (exchange at value), **e** (depreciation = replacement expenditure), **h** (no foreign trade), **i** (prior accumulation) and **j** (no net hoarding) are of this type. Thirdly, an assumption may formally mimic the *heuristics* of systematic dialectics, by α) conceptually isolating a – usually static – moment which is β) dynamized at the next stage of approximation. The tension between α) and β) may next be resolved in what may be called γ) a static dynamic model, e.g. describing a steady state, a predictable dynamism. Assumptions **c** (defining simple reproduction) and **g** (assuming technical change away) are of this type (albeit that Marx unjustifiably retains **g** for expanded reproduction). Finally, assumptions may *anticipate conditions of existence*. That is, if one wants to see the dynamics of a moment by

means of modeling it, obstacles to the dynamics concerned, must be assumed away (for the time being). Marx makes this type of assumption when assuming sufficient money (**k**) and labor power (**l**). The latter type of assumptions especially serve as guides to the further development of the dialectical exhibition, for the conditions of existence assumed at one level must be materially grounded at a more concrete stage (or the consequences – such as crises – endured).

The former two types of assumptions formally recap results that the systematic dialectical exhibition had previously arrived at. As such, they perform the *function* of assumptions, but from a systematic dialectical point of view they are better thought of as formally presented *conclusions* regarding the level of abstraction under consideration. Strictly speaking therefore, only the latter two types qualify as assumptions in a systematic dialectical sense. By formally mimicking heuristics one assumes some aspects away despite the fact that they had been previously exhibited. By anticipating later stages we already assume that mechanisms that have specific effects are in place before we have exhibited them. So the latter two types of assumptions are as yet ungrounded, while the former two only formalize that which has already been exhibited thus far (cf. Chapter 1, Section 2).

For Marx's schemes of reproduction, this typology implies that the systematic-dialectically informed heuristic assumption on the absence of technical change (i.e. **g**) should be dropped or modified when the transition to expanded reproduction is made. Failure to do so will result in a one-sided model for expanded reproduction that takes extensive accumulation into account only and neglects the intensive aspect of that phenomenon. So before it can be said that the model captures all essential characteristics of reproduction as it appears at the level of abstraction we are at, something must be done concerning **g**.[155] A possible way forward regarding this is suggested in Chapter 4.[156]

Alternatively, one may say that dropping **g** would not affect the proportionality conditions or the fact that growth acceleration in I leads to deceleration in II, but would only make the link between the development of *variable capital* and constant capital more stochastic. So, although it may not be the most elegant solution, one might also adopt a teleological stance regarding **g** and claim that the adoption of this assumption has not influenced the results the model set out to

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[155] In effect **g** states that technological development is kept out of the model. A neoclassical economist would be elated if this were the only partially unjustifiable assumption in any of his or her models. So the approach developed here seems very promising if I may say so myself.

[156] It is quite possible to model the development of v/c (or one of its guises as c/v or $c/(c+v)$) independently and integrate it into the reproduction schemes. At the level of abstraction of these schemes v/c would then be modeled to fall with accumulation, as less labor intensive technology is adopted with each round of accumulation. That this dialectically determined trend is not borne out empirically (Blaug 1963) is immaterial at this juncture.

attain (i.e. to chart the relationships between departments and the conditions of existence for accumulation) and that therefore there has been no harm in its adoption. But such a defense is really an admission of weakness.

In the conclusion to this book the typology just introduced is developed into a sort of ‘cookbook’ for model building. By way of illustration Chapter 4 first develops a recipe for reconstructing Marx’s reproduction schemes along dialectical lines using all four forms of dialectically justifiable assumptions in a predetermined order. The conclusion will draw on and try to generalize from the case evaluated here and reconstructed in the next chapter.

4. A Formal Dynamic Reconstruction of Marx's Schemes of Reproduction along Dialectical Lines

Introduction

In Chapter 3 it has been established that Marx had become quite conversant with the mathematical textbooks of his time when (after his 1868-70 investigations) he continued working on his schemes of reproduction in 1878. Furthermore, Section 3.3 has shown that though most of the assumptions Marx makes in these models can be defended dialectically, Marx only does so when he assumes previous accumulation (assumption **i**), sufficient monetary accommodation (**k**) and labor abundance (**l**) in the context of expanded reproduction. He defends **i** and **k** by claiming that we know from the analysis so far that the conditions assumed must somehow hold in practice, even if we do not yet know how they are brought about. The only time he did seem to try and defend an assumption by referring to earlier – and in systematic dialectics this must imply more abstract – stages (viz. **l** on labor power being always on hand), his argument required specific additional assumptions on population growth which were not (yet) borne out by the systematic dialectical exhibition up to that point and/or came across a trifle inconsistent with his professed goal of investigating capitalism as though it encompassed everyone everywhere. So his models could be much more thoroughly integrated with and embedded within Marx's dialectics than Marx did or, given his late coming-of-age in mathematical techniques, could have done during his lifetime. Moreover, Marx composed his models only in draft form, so their status is exploratory (as in 'Forschung') at best. Had he lived long enough to finish *Capital II*, he might have exhibited them as part of his dialectics. But as it stands, there is ample room for improvement in this respect. In this chapter I will try to reconstruct Marx's models along dialectical lines to see if and to what extent dialectical insights facilitate model building and if and to what extent the finished model(s) (or some equations from it) help further the systematic dialectical exhibition.

To do so, this chapter will retain only the assumptions or elements thereof that have been shown in Section 3.4 to be dialectically defendable albeit (sometimes) in their most lenient interpretation only. In effect this means that I will pretty much follow Marx in assuming constant prices (assumption **a** and **b**), two great departments (**d**) absence of growth (**c**), absence of foreign trade (**h**), accumulation out of surplus-value only (**j**), sufficient monetary accommodation (**k**) and labor abundance (**l**). As Section 3.3 has shown, setting depreciation costs equal to replacement expenditure (assumption **e**) is actually dialectically acceptable when

looking at Marx's original text. Hence assumption **e** is accepted throughout this chapter.[157] The statement made in assumption **f** that s/v = 100% in both departments is modified so as to allow for other ratios of s/v, but the postulated constancy is retained. Assumption **g** (claiming equality and constancy of the v/c across departments) is reformulated so as to allow for technical change in the context of expanded reproduction. Since assuming previous accumulation (assumption **i**) is totally redundant if an adequate algebraic technique is used, I have dropped this assumption throughout (see Table 1).

		Marx	appreciation	reconstructive action
a	Values = prices	Negligibility	absency	accept
b	No revolution in values	Negligibility	foundational	accept
c	Full surplus-value consumed, 'no savings by labor'	Heuristic	heuristic	accept for SR; drop for ER (but retain 'no savings out of wages')
d	Only two great departments	–	foundational	accept
e	depr. cost = replacement expenditure	Heuristic	absency	accept
f	$s/v = 100\%$, constant and equal in I&II	–	s/v constant: foundational; specific number: dial -	constancy accepted ; specific number dropped
g	Ratio v/c constant and equal in I&II	Simplification	redundant for SR, heuristic for EG, foundational for ER when modified[158]	drop for SR; accept for EG and modify for ER (v/c falls each year)
h	No foreign trade	Negligibility	absency	Accept
i	there has already been reproduction on an expanded scale	Anticipation	absency	drop in order to formally bring out the dialectical transition of SR tot ER
j	hoards = replacement expenditure ⇒	-	absency	Accept

[157] It is not difficult to see what happens when assumption **e** is dropped. Most importantly, taking such a measure would imply that the proportionality conditions are even harder to satisfy and thus dropping it would effectively preclude the algebraic determination of results arising from these conditions.

[158] SR = simple reproduction,
EG = extensive growth and
ER = expanded reproduction.

	accumulation out of surplus-value only			
k	Monetary accommodation	Anticipation	anticipation	Accept
l	Labor abundance	necessity (as a result of technological development)	anticipation	Accept

Table 1. Marx's assumptions, their (dialectical) appraisal and the reconstructive action taken in this chapter.

The genesis and mathematical expression of the assumptions that are retained are discussed alternately, rather than separately. Thus, it is shown how a systematic dialectical result can be formalized as a model assumption, a few of which, taken together, can be mathematically expressed and interpreted by (sometimes) calling upon yet another (set of) assumption(s). An auxiliary advantage of this presentation strategy is that it allows one to track exactly which assumptions are necessary preconditions for each intermediate result. Since systematic dialectics thus takes precedence over assumptions, the order in which the latter are presented may differ significantly from Marx's.

Furthermore, in what follows, Marx's formulas are expressed in the most general way possible. So I have replaced all of Marx's numerical values with letter-like symbols and refrain from calculating anything. Apart from preventing the pitfalls presented by (possibly) impractically chosen values, this procedure ensures that the model stays general, that is, general at its level of abstraction. This is dialectically important, for Marx's goal is to identify the processes, interactions and relationships shaping the course of *simple* (Section 1) and *expanded* (Section 3) *reproduction* of *capital*. Thus, at this level of abstraction, we are looking for qualitative and conceptual rather than quantitative results. Assigning specific values to the various variables suggests a level of concreteness we have not yet reached by any long shot.

I am not the first to reconstruct Marx's numerical schemes into a more general algebraic form: for instance, Sweezy (1942) and Koshimura (1975) cast Marx's models in terms of matrix algebra and generalize and develop it from there, while Reuten (1998: 214-217) – staying a lot closer to Marx's original numerical schemes – basically just replaces Marx's determinate figures with indeterminate symbols. Harris (1972) casts Marx's schemes in algebraical form in order to investigate the conditions that determine the possibilities for even or uneven growth and how these constraints can be mitigated when allowing capital to be reinvested in another department. The latter is a useful expansion of the model for later stages of concretization. It is not dialectically possible yet at the level of

abstraction to which Marx's modeling exercise pertains. Even though Harris's objective differs from mine, his algebraical reconstructions of the model are often similar and will therefore sometimes be referred to in the text below.

None of these accounts, however, explicitly defines variables as time-dependent. This is all the more surprising since Marx's chapters on the 'schemes of reproduction' are teeming with dynamic vocabulary. Marx speaks of transition, of development, of production periods etcetera, etcetera. Yet his numerical schemes essentially provide snapshots of intended and realized exchange arrangements each developing from the previous one. The process of development itself is described, but never modeled as an integral part of the interactions between the two departments. Thus, the systematic dialectical contention that comprehension of the static requires knowledge of its dynamics and vice versa is only present in these models in a rudimentary form. So, from a dialectical perspective there is room for improvement here as well.

Finally, as we have seen, Marx's model of expanded reproduction was one-sided in that he only reintroduced extensive accumulation into the model and generally abstracted from the intensive aspects of that phenomenon. This situation will be remedied here by providing consecutive models of *simple reproduction*, extensive growth and finally *expanded reproduction*. As Chapter 3 has shown, Marx was conceptually quite aware that the capitalist dynamics of expanded reproduction necessitate both growth of and technical change in total social capital. In his models of expanded reproduction, however, he disregards technical change. By so doing his model of expanded reproduction is one-sided in that it shows only extensive but not intensive growth. To remedy this situation, I have reintroduced technical change by reformulating Marx's assumption on it (i.e. assumption **g**) when modeling expanded reproduction. So in order to bring the *model* of expanded reproduction on a par with the *meaning* of that category as it appears in the dialectical exhibition, my model for expanded reproduction is more comprehensive than Marx's own (which is akin to what I label the model for extensive growth in this chapter). At the same time this reconstructive action clarifies the transition between simple and expanded reproduction and how it can be understood dialectically (cf. Section 3.2.13).

All in all then, the following reconstruction of Marx's models differs from his own presentation and from previous accounts in five distinct ways:

1) only the dialectically defendable (elements of) assumptions are retained,
2) dialectical reasoning rather than mathematical ease and rigor determine the order in which assumptions and equations are presented,
3) at its level of abstraction the presentation is general and therefore algebraical throughout,
4) all time-dependent variables are explicitly defined as such,

5) the intensive and extensive aspects of accumulation are integrated in a comprehensive model of expanded reproduction.

If none of these points need to be violated in the remainder of this Chapter, it has been proven that the architecture and specification of Marx's schemes can be presented as completely dialectically informed. It may well be that the reconstruction presented here also allows for new and/or more rigorous conclusions to be drawn from the model than Marx was able to do from his numerical examples. If so, that would be a nice bonus, but it is beyond the scope and ambition of this book.

1. The Model for Simple Reproduction

When building a model – any model – it makes sense to start by outlining its main concepts and distinctions. Hence, In order to reconstruct Marx's model for simple reproduction we have to start by identifying which of Marx's foundational assumption(s) are relevant to it. As Section 3.3 showed, the distinction between the production department of *means of production* and that of *consumer goods* is dialectically meaningful. So, the following *foundational* assumption in any case clearly flows from Marx's dialectics:

1. **'**Society's total yearly product breaks down into *two great departments*': a department producing means of production (i.e. current and additional constant capital) (department p) and one producing means of consumption (i.e. commodities intended for consumption out of wages paid out to variable capital, and out of capitalists' surplus-value) (department c) (cf. assumption **d** in Section 3.3).[159]

Making these departments and the major constituent parts of the value of a department's produce explicit in the model, its general outline becomes:

[159] The aim here is to present the assumptions that are indispensable for the models as clearly and succinctly as possible. Those that are interested in Marx's exact words are therefore referred to Section 3.3. In what follows I will only provide the letter that was assigned to Marx's original assumption in Section 3.3 (viz. 'cf. **d'**).

$$c_p + v_p + s_p = x_p \qquad (4.1)$$
$$c_c + v_c + s_c = x_c \quad +/+ \qquad (4.2)$$
$$c + v + s = x \qquad (4.3)$$[160]

where:

c = constant capital, the value of the means of production applied (i.e. used up that year);

v = variable capital, the value of the social labour power applied;

s = surplus-value, the value that is added by labour minus the replacement of the variable capital advanced.

x = the total value of the yearly produce in a department or the economy as a whole, that is, means of production used up plus value added.[161]

The subscripts $_p$ and $_c$ indicate whether we are considering the department producing means of production or means of consumption respectively.[162] [163]

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[160] 4.1 and 4.2 describe the decomposition of a department's total product into its main components. 4.3 gives us the decomposition of the aggregate product. Hence 4.1-3 can also be read vertically, e.g. as $s = s_p + s_c$. So, following Koshimura (1975), one might also take 4.1-3 for one matrix instead of three separate expressions. However, the aim of the reconstruction provided in this chapter is to bring Marx's models on a par with his dialectics. To this end, I want to focus exclusively on the five modifications summed up in the last paragraph of the introduction and recasting Marx's models in terms of matrix algebra was not one of them. So since Marx appeared to think of these equations as separate expressions and there is no dialectical insight to be gained from changing that, I treat Marx's equations as separate expressions as well and the labeling reflects this throughout.

[161] Note that the model is introduced as an elaboration of the commodity capital circuit (see Section 3.1.1). Thus, it should only contain flow and no stock variables. In other words, its focus is and dialectically should be on the *value* of the produce in the period considered. c therefore should not be interpreted as the *stock* of constant capital available at the start of the year, but as that part of it of which the value is transferred to the product, i.e. that period's depreciation allowance for fixed capital plus the circulating constant capital. In order to ensure that a period's production is sufficient to cover both the replacement needs for circulating capital and the expansion of fixed capital, the period t should be chosen in such a way that all fixed capital is depreciated by the end of it. If this is done, it is dialectically justifiable to assume fixed capital away. (Admittedly it is not a very elegant solution, but the models would be unnecessarily complex without adding insight without it).

[162] The definitions of c, v and s have been taken from Reuten (1998: 196) as have my definitions of the organic composition of capital (κ) (1998: 195), the rate of surplus-value (ε) and the rate of accumulation (θ) (1998: 206).

[163] This notation is not exactly Marx's. Instead Marx uses the subscripts I and II to indicate the department under consideration, where I refers to the department producing means of production and II to that producing means of consumption.

When an equation or expression is valid for each department, subscript $_i$ is used to indicate indeterminateness.

The next step in the reconstruction of the model is to identify what may safely be assumed absent at the level of abstraction the model exercise pertains to, while factoring in the relevant heuristic assumption(s). Since heuristic assumptions may hugely limit the amount of potentially complicating factors that may or may not be assumed away in virtue of the level of abstraction the model is at, they must be made explicit first (if present). Thus, in this case, the definition of *simple reproduction* must now be made explicit:

2. *Simple reproduction* means that 1) the total value of the yearly produce of both departments (x_p and x_c) is constant, 2) all surplus-value is consumed and 3) there are no savings out of wages (cf. **c**).

As we have seen, this assumption implies that all variables and the relations between them are fixed in the model (implying that fixed prices (assumption **b**) and/or ratios (**f** and **g**) need not be assumed anymore) and do not need to be explicitly defined as time dependent yet. This leaves us with only two further *assumptions on absence* to explicate:

3. There is no foreign trade (cf. **h**).

4. Aggregate depreciation costs incurred yearly equal aggregate yearly replacement expenditure (cf. **e**).

So what we see here is the decomposition of society's total product into its two main departments (assumption 1) and into its three major constituent parts *c*, *v*, and *s*. Since all means of production are produced in department p and there is no accumulation (assumption 2) or foreign trade (assumption 3) while the value of society's expenditure on constant capital c equals the value of constant capital transferred to the value of department p's and c's produce (i.e. the depreciation

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According to Reuten the Roman numerals were replaced with (the European notation of) Arab ones in 'the notation that has become conventional in modern Marxian economics' (1998: 197). Thus, where Marx has C_I and C_{II}, conventional Marxian economists have c_1 and c_2 respectively (cf. footnote 131 above). I have replaced these numerical subscripts with letters in order to facilitate dynamization of the model later on, i.e. the imputation of equations describing the development of c, v and s over time. Such dynamization of course, requires each term to be augmented with a numerical subscript for time and fitting each term with *two* numerical subscripts can only confuse things.

costs incurred by each department) in any given production period (assumption 4), we have the *implication*:

$$x_p = c \tag{4.4}$$

or, conversely:

$$x_c = v + s \tag{4.5}$$

This again implies that the flow of constant capital spent for the production of means of consumption (c_c), is equivalent to the spending on consumption by the workers (v_p) and the capitalist (s_p) in department p. Thus, in the context of *simple reproduction*, the value of the total produce of each department can only be actualized, if the following condition holds:[164]

$$v_p + s_p = c_c \tag{4.6}$$

The upshot of this condition and the most important conclusion from the model for *simple reproduction* is that the necessary capitalist institutions and processes exhibited thus far interact in ways that make smooth and harmonic reproduction of these relations apparently unlikely. In fact, nothing short of a miracle can prevent either of the two departments from getting into minor or severe imbalances. For if the year's expenditure on c_c , is larger than $v_p + s_p$ or, equally, x_p minus the expenditure on c_p during that period, production in department p falls short of the maintenance needs for department c so the latter is forced to contract its production. If it is the other way round, department p gets stuck with a stock of unsold products and thus gets into trouble.

From a systematic-dialectical perspective explicating the necessity of having two great departments (assumption 1) and modeling the way they must be interrelated has brought out that neither department can exist in conceptual isolation, so they must be thought of as an organic whole: *total social capital*. However, total social capital has been modeled thus far as being in movement but not changing. This means that dynamics must now be brought back in. As indicated in Section 2.14 the capitalist dynamics of *accumulation* take two forms: *intensive* and *extensive growth*. We will now turn to *extensive growth*, where expansion of *total social capital* is considered, but technical change is still

[164] As we have seen in Section 3.1.1, Marx calls this a proportionality condition. Since proportionality can also be taken to imply equal growth rates, Marx's term shall be avoided in the remainder of this book.

abstracted from. As such, the model that follows is an algebraic reconstruction of Marx's model for expanded reproduction.

2. Extensive Growth of Total Social Capital

Extensive growth (EG) is accumulation without technical change. Modeling it therefore requires us to modify assumption 2 (cf. **c**) so as to strip it of all elements that are not dialectically defendable. It thus becomes:

2a. There are no savings out of wages (cf. **c**).

So, now there is room for expansion of productive capacity paid for out of surplus-value (reflected in the model as increases in c. By contrast to assumption 2, assumption 2a does no longer imply fixed prices (cf. assumption **a** and **b**) and/or ratios (**f** and **g**). Even so, we have seen that the value relations *within* 'the component parts of the productive capital' form a foundation for the capitalist system being modeled here and that revolutions within these parts may therefore be assumed away (so **b** may still be upheld when there is accumulation). We have also seen that divergences of prices from values may safely be assumed away (assumption **a**) prior to the introduction of many capitals. So, we may posit as an absency assumption:

5. There are no price changes (cf. **a** and **b**) [165]

It is a bit trickier to maintain that $s/v = 100\%$ in both departments (**f**), but it has been dialectically exhibited that s/v displays some resilience. So as a foundational assumption we may posit:

[165] Assumption 5 is in fact irrelevant with respect to the relations between the model variables. However, they are relevant with respect to the interpretation of these relationships. The all-important proportionality conditions 4.6 and 4.13 for instance, would still hold if prices were not fixed, for then it is still true that the money for maintenance and accumulation of constant capital in department c must be raised from the department's sales to department p. But one cannot interpret possible mismatches in terms of material over- or underproduction and concomitant augmentation of stocks of products when prices are not fixed, just like one cannot say that a rising tcc leads to a rising occ without these assumptions in place. In short, without them a mismatch in values does not necessarily lead to mismatches in volume, although required revenue may still be adversely affected.

6. The ratio of s/v (notation: ε) is constant and given in each department (cf. **f**)

To abstract from technical change (by means of assumption **g**) is not justifiable in the context of *expanded reproduction*. But since Marx introduces it, I too will do so in order to show how my models emanate from and reconstruct Marx's. With assumption 5 in place, technical changes cannot be offset by price changes and so they will always correspond to changing value relations between the component parts of the productive capital. This means that heuristically assuming technical change away implies the following assumption may be temporarily adopted:

7. the organic composition of capital ($\kappa = c/(c+v)$) is constant and given for each department (cf. **g**).

Now, to explicate that no other funds for investment besides surplus-value have been exhibited at this level of abstraction, a further assumption has to be made as an absency assumption:

8. All expansion of the value of the yearly produce is financed out of a department's own surplus-value (cf. **j**).

Thus, a part of each department's surplus-value will now be invested in additional means of production and labor power (whatever is left is still consumed). So surplus-value is now composed of:

u = surplus-value consumed by or via capitalists ('unproductive consumption');
Δc = surplus-value accumulated in constant capital; and
Δv = surplus-value accumulated in variable capital.

These definitions imply that the following equations hold for the economy as a whole (and with appropriate subscripts, they hold for each department as well):

$c_{t+1} = c_t + \Delta c_t$
$v_{t+1} = v_t + \Delta v_t$
$x_{t+1} = x_t + \Delta x_t$

Assumption 8 implies three possible uses or destinations of surplus-value (s):

$$s_i = u_i + \Delta c_i + \Delta v_i \,. \qquad (4.7)$$

Making this explicit in the model yields:

$$c_{pt} + v_{pt} + u_{pt} + \Delta v_{pt} + \Delta c_{pt} = x_{pt} \quad (4.8)$$
$$c_{ct} + v_{ct} + u_{ct} + \Delta v_{ct} + \Delta c_{ct} = x_{ct} \quad +/+ \quad (4.9)$$
$$c_t + v_t + u_t + \Delta v_t + \Delta c_t = x_t \quad (4.10)$$

If there is room for accumulation, more means of production must have been produced than were required for maintenance purposes, so that $x_{pt} > c_t$. More precisely, production can only increase by the difference between the value of the means of production produced, the row total x_{pt}, and those used up that year, the column total c_t.

In Marx's numerical scheme for the base year (cf. 1885F: 586) this requirement is apparent from the difference of 500 between the figure reported in the upper right corner and that in the lower left (cf. 1885F: 586):

	c_0		v_0		s_0		x_0
p.	*4000*	+	*1000*	+	*1000*	=	*6000*
c.	1500	+	750	+	750	=	3000
	5500	+	1750	+	1750	=	9000

These numbers further imply that in Marx's example $\varepsilon_p = \varepsilon_c = 1$ (or a 100%) and that $\kappa_p = 0{,}8 \neq \kappa_c = 0{,}67$. So, for some reason, Marx does not maintain his assumption that the ratio of variable to constant capital is identical between departments (part of **g** in Section 3.3) in the context of extensive growth (or, in his own terminology, *expanded reproduction*).[166]

The fact that production can only increase by the difference between x_{pt} and c_t, implies that the aggregated growth rate g of overall constant capital is constrained by the amount of production in department p in the following way:

$$g_t = (x_{pt} - c_t)/c_t = \Delta c_t / c_t \quad (4.11)$$

and consequentially:

$$c_{t+1} = c_t + \Delta c_t = (1 + g_t)c_t \quad (4.12)$$

[166] He does, however, maintain this assumption in the context of simple reproduction, which is also where he originally introduced it.

By adding appropriate subscripts, the latter expression can also apply to a specific department, as long as total constant capital accumulation does not exceed production $\left(x_{pt} - c_t = \Delta c_t = \Delta c_{pt} + \Delta c_{ct}\right)$. Since for the purpose of modeling this dialectical moment we have abstracted from redistributions between c, v and s for now (assumptions 6 and 7), all value components in a department will grow at the same rate (i.e. g_p or g_c) (as we will see, divergences between the growth rates g_p and g_c pertaining to both departments can easily arise), provided that the anticipatory assumptions on monetary accommodation (**k**) and labor abundance (**l**) hold. So the following must now be assumed:

9. There is always enough money available to finance hoarding for replacement purposes and accumulation at the desired rate (cf. **k**).

10. An accumulating department will always find sufficient labor power on hand to increase the variable capital it employs by as much as its growth rate requires (cf. **l**).

Now, as we have already seen with *simple reproduction*, the only funds available for use on *constant capital* spent on the production of means of consumption in the base year come from the unproductive consumption of capitalists and laborers in department p. In the context of extensive growth this means that this influx of money to department c must serve both its replacement and its *accumulation* needs. Given however that department p also accumulates and hence will hire additional labor power spending its wages on consumption goods, this influx of money rises each year. Thus, in the context of extensive growth, the value of the total produce of each department can only be actualized, if the following condition holds:

$$v_{pt} + u_{pt} + \Delta v_{pt} = c_{ct} + \Delta c_{ct} \qquad (4.13)^{167}$$

With all ratios between *c*, *v* and *s* given through assumptions 6 and 7, we can write:

$$s_{it} = \varepsilon_i v_{it}; \quad u_{it} = (1-\theta_{it}) s_{it} = (1-\theta_{it}) \varepsilon_i v_{it}$$

[167] Products are usually bought in the same year as they are produced, so the value of x_p will only be actualized if $x_{pt} = c_{pt} + c_{ct} + \Delta c_{pt} + \Delta c_{ct} = c_{pt} + v_{pt} + u_{pt} + \Delta v_{pt} + \Delta c_{pt}$. Eliminating terms found on both sides of the equal sign yields condition 4.13.

In combination with expression 4.7 these expressions imply:

$$\Delta v_{it} + \Delta c_{it} = \theta_{it} s_{it} = \theta_{it} \varepsilon_i v_{it}$$
$$\Delta v_{it} = (1-\kappa_i)\theta_{it}\varepsilon_i v_{it}; \quad \Delta c_{it} = \kappa_i \theta_i \varepsilon_i v_{it}$$

From this it can be determined that the (departmental or macroeconomic) rate of accumulation out of surplus-value ($\theta = (\Delta v + \Delta c) / s$) is proportional to that department's (or the macroeconomic) growth rate:

$$\begin{aligned} \Delta v_{it} / v_{it} &= (1-\kappa_i)\varepsilon_i \theta_{it} \\ \Delta c_{it} / c_{it} &= \kappa_i \varepsilon_i \theta_{it} v_{it} / c_{it} = (1-\kappa_i)\varepsilon_i \theta_{it} \end{aligned} \tag{4.14}$$

The economy can grow without major disruptions or adaptations by any department (that is to say it experiences balanced growth), if $g_c = g_p = g$. Expression 4.14 tells us that in that case: $g_c = (1-\kappa_c)\varepsilon_c \theta_c = (1-\kappa_p)\varepsilon_p \theta_p = g_p$ This expression implies that balanced growth only exists if the following condition is met:

$$\frac{\theta_c}{\theta_p} = \frac{(1-\kappa_p)\varepsilon_p}{(1-\kappa_c)\varepsilon_c} \tag{4.15}$$

Since department p holds all the means of production, it can simply retain more of its own produce in order to change its rate of accumulation. So in the context of extensive growth and before exhibiting *many capitals*, we may assume:

11. Changes in department p's rate of accumulation are always actualized. Department c's actualizable rates are therefore constrained by the amount left by department p. Anticipating eventual recovery, it is assumed department c accumulates in such a way that all of department p's produce is actually sold.

So, given the assumption that $s = \varepsilon v$, condition 4.13 and the occ for both departments in the relevant production period, the maximum rate of accumulation out of surplus-value possible for department c, θ_{ct}, is a function of θ_{pt} (see the appendix for its derivation):

$$\theta_{ct} = \frac{(\varepsilon_p + 1 - \kappa_p \varepsilon_p \theta_{pt})(1/\kappa_p - 1)}{(1-\kappa_c)\varepsilon_c} \times \frac{c_{pt}}{c_{ct}} - \frac{1}{(1-\kappa_c)\varepsilon_c} \tag{4.16}$$

Alternatively, given that the relationship $\theta_i = \frac{g_i}{(1-\kappa_i)\varepsilon_i}$ holds for both departments (albeit with different parameters) the actualizable growth rate for department c (g_c) can be expressed in terms of the growth rate set by department p (g_p) (again, its derivation can be found in the appendix):

$$g_{ct} = \frac{c_{pt}}{c_{ct}}\left[\frac{(1-\kappa_p)(1+\varepsilon_p)}{\kappa_p} - g_{pt}\right] - 1 \qquad (4.17)\ ^{168}$$

This equation is similar to 14a in Harris (1972: 512). From 4.16, the ratio between the value of constant capital employed in both departments ($\frac{c_{pt}}{c_{ct}}$) required for the existence of a steady state can be found by setting $g_{ct} = g_{pt}$.

$$\begin{aligned} g_{ct} = g_{pt} &= \frac{c_{pt}}{c_{ct}}\left[\frac{(1-\kappa_p)(1+\varepsilon_p)}{\kappa_p} - g_{pt}\right] - 1 \\ \Leftrightarrow \frac{c_{pt}}{c_{ct}} &= \frac{\kappa_p(1+g_{pt})}{(1-\kappa_p)(1+\varepsilon_p) - \kappa_p g_{pt}} \end{aligned} \qquad (4.17a)$$

Since this expression features only the parameters of department p, we can say that department p determines whether the economy is at a balanced growth path or not. If it is not, but department c acts in accordance to assumption 11, the conditions for balanced growth are restored after just one period. This is so because 4.17 can be rewritten to yield:

$$(1+g_{ct})c_{ct} = c_{c,t+1} = c_{pt}\left[\frac{(1-\kappa_p)(1+\varepsilon_p)}{\kappa_p} - g_{pt}\right]$$

We further know that $c_{p,t+1} = c_{pt}(1+g_{pt})$, so

[168] When Reuten calculates the actual realized rates of growth for both departments from Marx's numerical examples, he inserts them behind the relevant row in Marx's numerical scheme for the next period (1998: 211, 212). This might leave the impression that these are the growth rates to be realized on the basis of the values of the *c* and *v* invested and the *s* realized that period, whereas the numbers he mentions in these places actually refer to the accumulation that took place in the previous period.

$$\frac{c_{p,t+1}}{c_{c,t+1}} = \frac{c_{pt}(1+g_{pt})}{c_{pt}\left[\frac{(1-\kappa_p)(1+\varepsilon_p)}{\kappa_p} - g_{pt}\right]} = \frac{\kappa_p(1+g_{pt})}{(1-\kappa_p)(1+\varepsilon_p)-\kappa_p g_{pt}}$$

This proves that if all value produced in department p is actualized (i.e. assumption 11, formalized as expression 4.16 and 4.17, holds), the steady state is restored after just one period. So, when department p sets a different growth rate for itself at time *t* and, anticipating eventual recovery, department c is assumed to buy up the leftovers, the following holds: $g_{c,t} < g_t < g_{p,t} = g_{t+1} = (x_{p,t+1} - \delta_{c,t+1})/c_{t+1} = \Delta c_{t+1}/c_{t+1} = g_{c,t+1} = g_{p,t+1}$.

In Marx's numerical example for the transition period the divergences between $g_{c,t}$ and $g_{p,t}$ and $\theta_{c,t}$ and $\theta_{p,t}$ are visible as follows:

	δc_0		v_0		u_0	Δv_0	Δc_0		x_0
p.	*4000*	+	*1000*	+	*500*	+ *100* +	*400*	=	*6000*
c.	1500	+	750	+	600	+ 50 +	100	=	3000
	5500	+	1750	+	1100	+ 150 +	500	=	9000

Scrutiny of these numbers reveals: $\theta_p = 50\% \neq \theta_c = 20\%$ and $g_p = 10\% \neq g_c = 6{,}7\%$ (cf. Marx 1885F: 587; cf. Reuten 1998: 211). The formulae presented yield exactly the same results. [169]

All in all, the model for extensive growth (which is in essence an algebraical version of Marx's model for *expanded reproduction*) shows how department c's accumulation possibilities are interconnected with changes in the rate of accumulation out of surplus-value that are initiated in department p if smooth adaptation to an altered proportional growth path is to occur. But this is not what Marx does with it. Note that, although the model describes how changes in one department should correspond to changes in the other department if crisis is to be prevented, there is no mechanism in place (nor theorized) that would compel any department to make the changes calculated in the model, at least not at this level of abstraction. In effect then, assumption 11 anticipates such a mechanism. When such an adaptation mechanism were to be concretely determined, it is unlikely to produce as smooth an adaptation as the model depicts, but we would not be living in a capitalist society anymore if the mechanism did not exist at all. So it is safe to anticipate its existence. Accumulation plans involve rather long term strategic

[169] As presented, the model connects perfectly with Marx's numerical schemes. Again, this can easily be checked by checking the results of the equations presented against Marx's numerical schemes (1885F: 586-589).

decisions. Thus, it is unlikely for intended purchases to match realized (or even realiz*able*) sales.[170]

From the valorization requirements it can be inferred that accelerating accumulation in department p, results not only in a shortage of means of production for department c, but also in deficient sales of consumption goods. Thus, department c's accumulation falls short of the mark, while it is operating at overcapacity. Or at least it appears so during transition. If department c were to respond to this situation by decreasing its demand for means of production (as seems likely), the part of assumption 10 about department c consistently picking up the pieces cannot be maintained and instead of smoothly adapting to a higher steady growth path, the economy would spiral into recession (Reuten 1998: 210). So the first question arising from those models is whether some mechanism coordinating accumulation in the two departments can be identified.

From a dialectical perspective, we may say that when capital's drive to expand (express itself in the world) through *extensive growth* is considered in isolation, we can (at least theoretically) construct a solution in which both departments can grow at the same rate. Moreover, the solution allows for changes to be made to this rate. But as noted, there is no mechanism exhibited so far that might ensure that the solution will also be found and carried out in practice.

Either way, we cannot return to the systematic-dialectical exhibition, before our model has been brought back on a par with its concept. So we must now adjust our model further to see how the two departments of total social capital would interrelate if they expand and innovate at the same time. To chart this, we now turn to the model for γ) *expanded reproduction*.

3. The Model for Expanded Reproduction

Integrating innovation into the model for *extensive growth* requires us to allow for renewal of existing *fixed constant capital* (resulting in redistributions between *c*

[170] Early 20th century Marxian economists such as Rosa Luxemburg had also drawn this conclusion. These economists however, held that this conclusion directly applied to actually existing capitalism and therefore held that this result implied the necessity for state intervention. From a dialectical point of view, by contrast, this is an intermediate result only. Coupled with the fact that the exhibition so far does not adequately capture the capitalist system in its full actuality and that at least a partial solution must be found at a more concrete level (e.g. by the introduction of *many capitals* and the concomitant divergence of prices from values). See Bellofiore (ed. 2009) for an exposition of Luxemburg's arguments, the debate surrounding them and suggested ways forward.

and *v* in favor of *c*) and growth of the total value of capital applied simultaneously. This means that the occ must now be assumed to rise every year, so we get:

7a. The organic composition of capital rises with each round of accumulation

If we redefine the organic composition of capital as $\zeta_{it} \equiv \frac{\kappa_{it}}{1-\kappa_{it}} = \frac{c_{it}}{v_{it}}$, we can formalize assumption 7a as:

$$\zeta_{i,t+1} = (1+\varphi_{it})\zeta_{it}$$

In which φ is the growth rate of the organic composition of capital. Furthermore, everything that was assumed for extensive growth is also assumed here. [171]

Because of the integration of technical change (intensive growth) into the model for extensive growth, the growth rates of the constant and variable capital start to diverge by φ so that $\frac{\Delta c_{it}}{c_{it}} - \frac{\Delta v_{it}}{v_{it}} = \varphi_{it}$. We further know that $\Delta v_{it} + \Delta c_{it} = \theta_{it} s_{it} = \theta_{it} \varepsilon_i v_{it}$, so we may write:

$$\frac{\Delta c_{it}}{c_{it}} - \frac{\Delta v_{it}}{v_{it}} = \varphi_{it} \Rightarrow \frac{\Delta c_{it}}{c_{it}} = \frac{\Delta v_{it}}{v_{it}} + \varphi_{it} \Rightarrow \Delta c_{it} = c_{it}\frac{\Delta v_{it}}{v_{it}} + \varphi_{it} c_{it} = \zeta_{it}\Delta v_{it} + \varphi_{it}\zeta_{it} v_{it}$$

$$\Rightarrow \quad \Delta v_{it} + \Delta c_{it} = \Delta v_{it} + \zeta_{it}\Delta v_{it} + \varphi_{it}\zeta_{it} v_{it} = (1+\zeta_{it})\Delta v_{it} + \varphi_{it}\zeta_{it} v_{it} = \theta_{it}\varepsilon_i v_{it}$$

$$\Rightarrow \quad \Delta v_{it} = \frac{\theta_{it}\varepsilon_i - \varphi_{it}\zeta_{it}}{1+\zeta_{it}} v_{it} \qquad \text{and} \qquad \frac{\Delta v_{it}}{v_{it}} = \frac{\theta_{it}\varepsilon_i - \varphi_{it}\zeta_{it}}{1+\zeta_{it}} \qquad (4.18)$$

$$\Rightarrow \quad \Delta c_{it} = \frac{\theta_{it}\varepsilon_i - \varphi_{it}\zeta_{it}}{1+\zeta_{it}} c_{it} + \varphi_{it} c_{it} \quad \text{and} \qquad \frac{\Delta c_{it}}{c_{it}} = \frac{\theta_{it}\varepsilon_i - \varphi_{it}\zeta_{it}}{1+\zeta_{it}} + \varphi_{it}$$

The latter two expressions describe the growth rate of variable and constant capital respectively. When integrating *intensive* and *extensive growth*, we may assume that department p takes the lead regarding both innovation and expansion, so assumption 11 becomes:

11a. Changes in department p's rate of accumulation are always actualized in

[171] For assumption 11 this means that department p takes the lead in both innovation and setting the rate of accumulation. But this can better be made explicit later on, for this assumption is not required yet for the first few equations presented.

conformity to the latest technology. Department c's actualizable rates are therefore constrained by the amount left by department p. Anticipating eventual recovery, it is assumed department c accumulates in such a way that all of department p's produce is actually sold.

As with extensive growth, this again means that the growth rate for constant capital in department c, $\frac{\Delta c_{ct}}{c_{ct}}$ can be written as a function of $\frac{\Delta c_{pt}}{c_{pt}}$ via the valorization condition for expanded reproduction (4.13 can still be used for this, as it is not affected by technological change). This function now becomes (see the appendix for its derivation):

$$\frac{\Delta c_{ct}}{c_{ct}} = \left[1+(1-\theta_{pt})\varepsilon_p - \varphi_p + \frac{\Delta c_{pt}}{c_{pt}}\right]\frac{1}{\zeta_{pt}}\frac{c_{pt}}{c_{ct}} - 1 \qquad (4.19)$$

4.19 implies that constant capital applied in department c in t+1, $c_{c,t+1}$, must be determined by the following algorithm if all produce is to be valorized:

$$[1+\frac{\Delta c_{ct}}{c_{ct}}]c_{ct} = c_{c,t+1} = \left[1+(1-\theta_p)\varepsilon_p - \varphi_{pt} + \frac{\Delta c_{pt}}{c_{pt}}\right]v_{pt}$$

Since technical change implies an ongoing shift in favor of constant capital, the relative value of variable capital will fall through time. With ε being constant, this implies a relative fall in s in each period. As a result, funds spent on means of consumption fall relative to those spent on means of production. So both departments can no longer grow at the same rate. Would this mean that department c has to make the adjustments described in 4.19 every period or would it be possible to prevent crises if both departments stick to some specific accumulation rate? To find the answer, it helps to define the profit rate as $r_{it} = \frac{s}{c+v} = \frac{\varepsilon_i}{1+\zeta_{it}}$. With this definition in place we can say that accumulation rates can only be constant through time if the following condition is met and its result constant through time (see the appendix for its derivation):

$$\frac{\Delta c_{pt}}{c_{pt}} \Big/ \frac{\Delta c_{ct}}{c_{ct}} = \frac{r_{pt}[\theta_p + \varphi_p/\varepsilon_p]}{r_{ct}[\theta_c + \varphi_c/\varepsilon_c]} = \frac{r_{pt}}{r_{ct}} \times \text{constant} = \frac{1+\zeta_{c0}(1+\varphi_c)^t}{1+\zeta_{p0}(1+\gamma_p)^t}\frac{\varepsilon_p}{\varepsilon_c} \times \text{constant} \quad (4.20)$$

This can only be true if the ratio of profit rates is constant. The expression shows further that for this to be true the organic composition of capital has to be the same in both departments in all periods, i.e.: $\varphi_c = \varphi_p = \varphi$ and $\zeta_{c0} = \zeta_{p0}$, so that $\zeta_{ct} = \zeta_{pt} \quad \forall t$. When this condition is not met, department c will have to adjust to department p by following 4.19 every period. [172] Meanwhile, it is unclear how this information could reach the other department and if it did somehow, what mechanism would entice it to act upon it so as to make sure all produce is valorized.

At any rate, the results show not only that the model for expanded reproduction is insufficiently concrete to account for all interactions taking place within total social capital, but also serves as a guide to furthering the dialectical exhibition by pointing out the exact source of the insufficiency as being the absence of a mechanism ensuring equalization of the organic composition of capital across departments.

Though Marx also concluded that a mechanism ensuring balanced growth was crucial to expanded reproduction (rechristened extensive growth in this chapter), his numerical presentation of the models that were algebraically presented in this

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[172] Understandably, any mainstream trained economist is likely to think that the workings of market supply and demand ('the invisible hand') will go a long way in ensuring that this condition is met. A first precondition for this to work is that there are *many* competing *capitals*. And indeed after the dialectical exhibition thereof and the concomitant variability of values and technical compositions of capital, smooth adaptation is perhaps less of a miracle. Firstly, the *many capitals* in department p hustling for customers are less likely to be picky as to what sort of company buys their product. So, given *many capitals* department p is likely to be less self-centered, thus limiting the need for department c to adjust to department p to prevent crises.

Harris shows mathematically that modification of the model so as to accommodate the fact that capitalists invest surplus-value wherever they expect the highest profits and not just in their own business, imparts 'greater flexibility to the solution', thus illustrating how furthering the exhibition of Marx's dialectical framework does indeed produce the conditions of existence for reproduction and accumulation (1972: 516).

Secondly, price rises in means of production may help to force department c to lower its demand for department p's produce by the required amount. Similarly, department c may be forced to lower its prices in order to get rid of its unsold products. All this diminishes the chances of crises occurring; it does not resolve the problem. After all, crises are regularly recurring, but not endemic phenomena.

It is a curious fact that Marx had written the notebooks in which he introduced *many capitals* (edited into *Capital III* by Engels), *before* he had started work on his schemes of reproduction (posthumously edited into *Capital II*, part 3). So it does not appear that the introduction of many capitals in volume *III* sprang forth from the coordination problems borne out by Marx's models at the end of volume *II*. Nevertheless the models – in their current reconstruction at least – make the transition more defendable and transparent.

chapter obscures the exact information department c would have to act upon (and how) in order to make this miracle happen. In the formulae presented here, this is directly apparent. Moreover, the formulae clarify what further data is needed as the models are brought back on a par with Marx's concept of expanded reproduction through successive model generations, the last of which was not even considered by Marx. Thus, Marx never ventured beyond the model for extensive growth (i.e. accumulation without technical change) and therefore never presented the influence of changing occ's on accumulation anyway. As a result he could not draw the conclusion that balanced growth (defined as sustained growth by equal rates in both departments) is utterly impossible when taking technical change into account. In short, my reconstruction ventures beyond Marx and its algebraic formulation prevents the inadvertent loss of information on the type of parameters and data needed to create the numerical schemes that Marx originally limited himself to.

Summary and Conclusions

If Marx's 'schemes of reproduction' are studied from a systematic dialectical perspective, i.e. as moments in the systematic dialectical exhibition of *Capital* as a whole, most of its assumptions are justifiable (as was already clear from Section 3.3). What is more, if one makes due with those that are dialectically justifiable one may still formulate the model (and even dynamically so).

So, just to reiterate: Marx introduces his models into his dialectics after his exhibition of technical change, of the accumulation of capital (and again technical change in that perspective), of the several *capital circuits of capital* and of the opposition between α) *fixed* and β) *circulating capital*. So it has been established that *capital* can only throw off surplus-value (is self-valorizing) if it continuously engages in circuits in which the labor equivalent of *variable capital* physically transforms the means of production equivalent of *constant capital* with a view to *exchange*. Maintenance and *accumulation* of *constant capital* in the shape of means of production as well as maintenance and *accumulation* of *variable capital* in the shape of labor-power together with capitalist's own consumption, requires means of production and consumption goods. Given specialization, the two types of *commodities* are produced in two discernible separate *departments* (assumption 1). At this level of abstraction *capital* is just partitioned into two *departments*, and there is no foreign trade (assumption 3).

Section 3.3 already showed that it is not warranted dialectically to assume technical change away indefinitely. At the same time it has been argued that heuristically abstracting from it for a part of the model can be defended

dialectically. These considerations led me to add a model generation to Marx's schemes of simple and expanded reproduction:

1. *Simple reproduction* in which all change is absent (after all an understanding of *dynamic* processes requires us to grasp the *static* nature of its elements first);
2. *Extensive growth*, in which capital expands but does not undergo any technical change;
3. *Expanded reproduction* which allows for both expansion and technical change and thus goes beyond Marx.

Note that Marx's model for *expanded reproduction* considers *extensive growth* only and hence bears most similarity to my model for *extensive growth*. Thus, the model for *expanded reproduction* as presented here gives a fuller account of what is involved in that concept than Marx's.

For the formal representation of Marx's models the following symbols were used:

$c =$ constant capital, the value of the means of production applied;
$v =$ variable capital, the value of the social labor power applied;
$s =$ surplus-value, the value that is added by labor minus the replacement of the variable capital advanced;
$x =$ the total value of the yearly produce in a department or the economy as a whole;
$u =$ surplus-value consumed by or via capitalists ('unproductive consumption');
$\Delta c =$ surplus-value accumulated in constant capital;
$\Delta v =$ surplus-value accumulated in variable capital;
$g =$ proportional growth rate;
g_p and $g_c =$ growth rates for department p and c respectively (during transition);
κ_p and κ_c = the organic composition of capital $= c/(c+v)$;
ζ_p and ζ_c = an alternative definition of the organic composition of capital $= c/v$
ε_p and $\varepsilon_c =$ the rate of valorization $= s/v$;
$\theta =$ the rate of accumulation out of surplus-value $= (\Delta v + \Delta c)/s$;
φ_p and φ_c = the growth rate of ζ
r_p and r_c = the profit rate $s/(c+v)$
$t =$ time, also denoted as a (numerical) subscript where the base year = *0*;

The subscripts $_p$ and $_c$ indicate whether we are considering the department producing means of production or means of consumption respectively. When an equation or expression is valid for each department, subscript $_i$ was used to indicate indeterminateness.

Given further that accumulation must be paid for out of surplus-value (assumption 10), we may write:

$$s_i = u_i + \Delta c_i + \Delta v_i \qquad \text{(4.7: definition)}$$

and for the model's main matrix:

$$\begin{aligned} c_{pt} + v_{pt} + u_{pt} + \Delta v_{pt} + \Delta c_{pt} &= x_{pt} && (4.8)\\ c_{ct} + v_{ct} + u_{ct} + \Delta v_{ct} + \Delta c_{ct} &= x_{ct} \quad +/+ && (4.9)\\ c_t + v_t + u_t + \Delta v_t + \Delta c_t &= x_t && (4.10) \end{aligned}$$

From this matrix follows the condition for valorization of all produce:

$$v_{pt} + u_{pt} + \Delta v_{pt} = c_{ct} + \Delta c_{ct} \qquad \text{(4.13: implication)}$$

This is (the ultimate abstraction of) the model for extensive growth and expanded reproduction. The model for simple reproduction, however, can be conceived of as the extreme case where all Δ's are zero, so that redistributions between the value components of total social capital and expansion of its total value are both impossible.

If there is *extensive growth* and both *departments* grow at

$$g_t = (x_{pt} - c_t)/c_t = \Delta c_t / c_t \qquad (4.11)$$

the modeled economy is on a proportional growth path and the value of *constant capital* would develop as follows:

$$c_{t+1} = c_t + \Delta c_t = (1 + g_t)c_t \qquad (4.12)$$

Since the model for *extensive growth* abstracts heuristically from technical change, all value components in a department grow at the same rate. However when department p alters its rate, it would leave less means of production on offer for department c. If the latter department buys up all that is available, its growth is related to the growth rate set by department p as follows:

$$g_{ct} = \frac{c_{pt}}{c_{ct}}\left[\frac{(1-\kappa_p)(1+\varepsilon_p)}{\kappa_p} - g_{pt}\right] - 1 \qquad (4.17)$$

If department c adjusts to changes in the accumulation rate or concomitant growth rate in department p using this formula, Marx's intuition that a new steady state, or balanced growth path will be reached in just one period can be proven to be generally true. When intensive and extensive growth are articulated together in the model (i.e. when moving on to the model for expanded reproduction), the growth rate for department c is even more adversely affected by an acceleration of growth, because growth acceleration would then go along with intensifying production so the amount of constant capital that stays in department p (and is thus never offered to department c) is higher than it would be without technical change. More precisely, the (redefined) organic composition of capital now grows in time:

$$\zeta_{i,t+1} = \frac{\kappa_{it}}{1-\kappa_{it}} = \frac{c_{it}}{v_{it}} = (1+\varphi_{it})\zeta_{it}$$

In analogy to 4.16. we may then write:

$$\frac{\Delta c_{ct}}{c_{ct}} = \left[1+(1-\theta_{pt})\varepsilon_p - \varphi_p + \frac{\Delta c_{pt}}{c_{pt}}\right]\frac{1}{\zeta_{pt}}\frac{c_{pt}}{c_{ct}} - 1 \qquad (4.19)$$

This result enables one to prove that balanced growth is impossible with ongoing technical change.

These models show how the models for the concepts of *simple reproduction*, *intensive* and *extensive growth* and *expanded reproduction* can be developed alongside and in line with the dialectics that gave rise to the concepts themselves. In so doing, the models grow ever more intricate and complex, which reflects how our understanding of the way in which the two great departments are interrelated grows more intricate and complex as well. Finally, the algebraic generalizations presented, ensure that the results found (such as the equations describing the way that the factors determining g_c interact during transition) are perfectly general and will work with all numbers. Additionally, the algebraic formalizations allow one to see how all vantage points taken until we arrived at the model for expanded reproduction can be conceived of as special cases of this most intricate and complicated model.

APPENDIX: DERIVATIONS

A1. Accumulation and growth rate for department c as a function of accumulation and growth in department p with extensive growth (expression 4.15 and 4.16)

For this derivation we need the technical and behavioral relationships which determine the movement through time of the system: the occ (κ_i), the rate of surplus value (ε_i), and the way in which the accumulation rates determine the spending on p-goods and c-goods. Subscript *i* denotes an indeterminate department and thus may be read as either *c* or *p*.

For each department, express *v* and *s* in terms of *c* (omitting the time index for the moment):

$$\kappa_i = \frac{c_i}{c_i + v_i} \Rightarrow \frac{c_i}{v_i} = \frac{\kappa_i}{1-\kappa_i} = k_i^H \quad (k_i^H \text{ as in Harris' expression 2 (1972: 508)}) \Rightarrow$$

$$v_i = \frac{1-\kappa_i}{\kappa_i} c_i \quad \text{or} \quad c_i = k_i^H v_i$$

$$\varepsilon_i = \frac{s_i}{v_i} \Rightarrow s_i = \varepsilon_i v_i \Rightarrow \qquad s_i = \varepsilon_i \frac{1-\kappa_i}{\kappa_i} c_i = \varepsilon_i c_i / k_i^H$$

Defining θ_{it} as the accumulation rate in department i at t, the division of surplus value is according to

$$u_{it} = (1-\theta_{it}) s_{it}; \quad \Delta v_{it} = (1-\kappa_i)\theta_{it} s_{it}; \qquad \Delta c_{it} = \kappa_i \theta_{it} s_{it}$$

Which keeps the ratios between c, v and s constant through time. Then apply the condition for sales actualization 4.13 and use these expressions:

$$c_{ct} + \Delta c_{ct} = v_{pt} + u_{pt} + \Delta v_{pt}$$

Because of the noted constancy of the ratios between *c*, *v* and *s*, The left hand side (LHS) of this equation can be expressed as follows:

$$c_{ct} + \Delta c_{ct} = c_{ct} + \kappa_c \theta_{ct} s_{ct} = c_{ct} + \kappa_c \theta_{ct} \varepsilon_c v_{ct} = c_{ct} + \kappa_c \theta_{ct} \varepsilon_c \frac{1-\kappa_c}{\kappa_c} c_{ct} = c_{ct} + \theta_{ct} \varepsilon_c (1-\kappa_c) c_{ct}$$

$$\Rightarrow LHS = [1 + \theta_{ct} \varepsilon_c (1-\kappa_c)] c_{ct}$$

Similarly the right hand side (RHS) can be written as:

$$v_{pt} = \frac{1-\kappa_p}{\kappa_p} c_{pt}$$

$$v_{pt} + u_{pt} + \Delta v_{pt} = v_{pt} + (1-\theta_{pt}) s_{pt} + (1-\kappa_p)\theta_{pt} s_{pt} = v_{pt} + [1-\kappa_p \theta_{pt}] \varepsilon_p v_{pt} = [1 + \varepsilon_p - \kappa_p \theta_{pt} \varepsilon_p] v_{pt}$$

$$RHS = [1 + \varepsilon_p - \kappa_p \theta_{pt} \varepsilon_p] \frac{1-\kappa_p}{\kappa_p} c_{pt}$$

Setting LHS=RHS, we get:

$$[1+\theta_{ct}\varepsilon_c(1-\kappa_c)]c_{ct} = [1+\varepsilon_p-\kappa_p\theta_{pt}\varepsilon_p]\frac{1-\kappa_p}{\kappa_p}c_{pt}$$

$$\theta_{ct}\varepsilon_c(1-\kappa_c) = \frac{c_{pt}}{c_{ct}}[1+\varepsilon_p-\kappa_p\theta_{pt}\varepsilon_p]\frac{1-\kappa_p}{\kappa_p}-1$$

So, one can write (cf. expression 4.15 above):

$$\theta_{ct} = \frac{c_{pt}}{c_{ct}}\frac{1+\varepsilon_p-\kappa_p\theta_{pt}\varepsilon_p}{\varepsilon_c(1-\kappa_c)}\frac{1-\kappa_p}{\kappa_p}-\frac{1}{\varepsilon_c(1-\kappa_c)}$$

Then derive 4.16 from 4.15. Note that the (time dependent) growth rate for department i is:

$$g_{it} = \varepsilon_i(1-\kappa_i)\theta_{it}$$

Multiplying both sides of 4.15 by $\varepsilon_c(1-\kappa_c)$ will then yield:

$$g_{ct} = \theta_{ct}\varepsilon_c(1-\kappa_c) = \frac{c_{pt}}{c_{ct}}\left(1+\varepsilon_p-\kappa_p\theta_{pt}\varepsilon_p\right)\frac{1-\kappa_p}{\kappa_p}-1$$

$$\Leftrightarrow g_{ct} = \frac{c_{pt}}{c_{ct}}\frac{1-\kappa_p}{\kappa_p}\left[(1+\varepsilon_p)-\kappa_p\theta_{pt}\varepsilon_p\right]-1$$

$$\Leftrightarrow g_{ct} = \frac{c_{pt}}{c_{ct}}\left[\frac{1-\kappa_p}{\kappa_p}(1+\varepsilon_p)-\left(1-\kappa_p\right)\theta_{pt}\varepsilon_p\right]-1$$

$$\Leftrightarrow g_{ct} = \frac{c_{pt}}{c_{ct}}\left[\frac{(1-\kappa_p)(1+\varepsilon_p)}{\kappa_p}-g_p\right]-1$$

Which is expression 4.17 (cf. Harris' 14a (1972: 512)).

A2. Constant capital's growth rate for department c for the case of expanded reproduction (expression 4.19)

Using 4.13: $v_{pt}+u_{pt}+\Delta v_{pt} = c_{ct}+\Delta c_{ct}$

And substituting from 4.18:

$$\Delta c_{ct} = \frac{\theta_{ct}\varepsilon_c-\varphi_{ct}\zeta_{ct}}{1+\zeta_{ct}}c_{ct}+\varphi_{ct}c_{ct} \quad \text{resp.} \quad \Delta v_{pt} = \frac{\theta_{pt}\varepsilon_p-\varphi_{pt}\zeta_{pt}}{1+\zeta_{pt}}v_{pt}$$

yields:

$$\Rightarrow v_{pt}+u_{pt}+\frac{\theta_{pt}\varepsilon_p-\varphi_{pt}\zeta_{pt}}{1+\zeta_{pt}}v_{pt} = c_{ct}+\frac{\theta_{ct}\varepsilon_c-\varphi_{ct}\zeta_{ct}}{1+\zeta_{ct}}c_{ct}+\varphi_{ct}c_{ct}$$

$$\Rightarrow v_{pt}+(1-\theta_{pt})\varepsilon_p v_{pt}+\frac{\theta_{pt}\varepsilon_p-\varphi_{pt}\zeta_{pt}}{1+\zeta_{pt}}v_{pt} = (1+\varphi_{ct})c_{ct}+\frac{\theta_{ct}\varepsilon_c-\varphi_{ct}\zeta_{ct}}{1+\zeta_{ct}}c_{ct}$$

so:

$$\Rightarrow \left[1+(1-\theta_{pt})\varepsilon_p+\frac{\theta_{pt}\varepsilon_p-\varphi_{pt}\zeta_{pt}}{1+\zeta_{pt}}\right]v_{pt}=\left[(1+\varphi_c)+\frac{\theta_{ct}\varepsilon_c-\varphi_{ct}\zeta_{ct}}{1+\zeta_{ct}}\right]c_{ct}$$

Or in terms of growth rates:

$$\Rightarrow \left[1+(1-\theta_{pt})\varepsilon_p-\varphi_{pt}+\varphi_{pt}+\frac{\Delta v_{pt}}{v_{pt}}\right]\frac{c_{pt}}{\zeta_{pt}}=\left[1+\frac{\Delta c_{ct}}{c_{ct}}\right]c_{ct}$$

$$\Rightarrow \frac{\Delta c_{ct}}{c_{ct}}=\left[1+(1-\theta_{pt})\varepsilon_p-\varphi_{pt}+\frac{\Delta c_{pt}}{c_{pt}}\right]\frac{1}{\zeta_{pt}}\frac{c_{pt}}{c_{ct}}-1$$

The latter is expression 4.19.

A3. The condition for constant rates of accumulation in case of expanded reproduction (expression 4.20)

$$\frac{\Delta c_{pt}}{c_{pt}}=\frac{\theta_{pt}\varepsilon_p-\varphi_{pt}\zeta_{pt}}{1+\zeta_{pt}}+\varphi_{pt}=\frac{\theta_{pt}\varepsilon_p}{1+\zeta_{pt}}+\varphi_{pt}\frac{1}{1+\zeta_{pt}}=r_{pt}[\theta_{pt}+\varphi_{pt}/\varepsilon_p]$$

$$\frac{\Delta c_{ct}}{c_{ct}}=\frac{\theta_{ct}\varepsilon_c}{1+\zeta_{ct}}+\frac{\varphi_{ct}}{1+\zeta_{ct}}=r_{ct}[\theta_{ct}+\varphi_{ct}/\varepsilon_c]$$

So, when accumulation rates and φ_i are fixed, the ratio between growth rates is proportional to the profit rates:

$$\frac{\Delta c_{pt}}{c_{pt}}\Big/\frac{\Delta c_{ct}}{c_{ct}}=\frac{r_{pt}[\theta_p+\varphi_p/\varepsilon_p]}{r_{ct}[\theta_c+\varphi_c/\varepsilon_c]}=\frac{r_{pt}}{r_{ct}}\times\text{constant}\quad \theta_{ct}=\theta_c$$

and

$$\frac{r_{pt}}{r_{ct}}=\frac{\varepsilon_p}{1+\zeta_{pt}}/\frac{\varepsilon_c}{1+\zeta_{ct}}=\frac{1+\zeta_{ct}}{1+\zeta_{pt}}\frac{\varepsilon_p}{\varepsilon_c}$$

$$\zeta_{ct}=\zeta_{c0}(1+\varphi_c)^t$$

$$\frac{r_{pt}}{r_{ct}}=\frac{1+\zeta_{c0}(1+\varphi_c)^t}{1+\zeta_{p0}(1+\varphi_p)^t}\frac{\varepsilon_p}{\varepsilon_c}$$

$$\frac{\Delta c_{pt}}{c_{pt}}\Big/\frac{\Delta c_{ct}}{c_{ct}}=\frac{r_{pt}[\theta_p+\varphi_p/\varepsilon_p]}{r_{ct}[\theta_c+\varphi_c/\varepsilon_c]}=\frac{1+\zeta_{c0}(1+\varphi_c)^t}{1+\zeta_{p0}(1+\gamma_p)^t}\frac{\varepsilon_p}{\varepsilon_c}\times\text{constant}$$

Summary and General Conclusions

The aim for this dissertation was twofold. First, it has established how a systematic dialectical perspective elucidates the nature of mathematics by clarifying the nature of the concepts that the mathematical sciences depend upon. Secondly, this dissertation has shown how a systematic dialectical perspective on capitalism may inform assumptions for mathematical models and how the results of these models can further the systematic dialectical exhibition that gave rise to the assumptions. Thus, it was shown how the two approaches mutually reinforce each other and are far from being mutually exclusive. The former was achieved by means of an investigation of Hegel's dialectical account of the mathematical (Chapter 2). The latter aim was achieved by a critical evaluation of the assumptions Marx makes when drawing up his schemes (i.e. models) of reproduction from a systematic dialectical perspective. Since these models are presented alongside Marx's systematic dialectical account of Capitalism, an exploration of the way Marx has integrated them into his overall account (Chapter 3) and the possibilities for improvement in this respect (Chapter 4), have been instrumental in indicating how a tighter fit between the systematic dialectical foundations of model assumptions, the assumptions themselves and the model may be achieved.

Before Hegel's dialectical determination of the mathematical could be presented, however, a more detailed understanding of Hegel's and Marx's historical and systematic dialectics was required (Chapter 1). Thus, the dissertation opened with a methodological chapter describing Hegel's dialectics and Marx's critique thereof as well as its implications for Marx's own dialectics. In a nutshell, the systematic dialectical method reconstructs the knowledge about a given object totality, whose intelligibility is fully dependent on one category: its universal principle. One can of course only reconstruct knowledge if one has first acquired enough of it (in the phase of appropriation).

The universal principle is a category without which no sense can be made of the totality under scrutiny. A central reference in this dissertation was Hegel's *Encyclopädie der philosophischen Wissenschaften*, in which Hegel outlines the systematic interconnections between all fields of knowledge in their totality. In his view three *object* totalities can be distinguished within this totality of all knowledge: the Logic, the realm of 'the idea in and for itself'; Nature, the realm of 'the idea in its otherness'; and society, the realm of 'the idea that returns into itself out of its otherness'. Hegel identifies Being, Space and Free Will as their respective universal principles.

In order to reconstruct the knowledge about an object totality from such a universal principle, Hegel asks himself three questions: α) how does this universal principle appear in total categorial isolation (that is if one tries to imagine it without taking recourse to any examples)?; β) how does it express itself in the world (that is, how does it appear if one tries to behold all of its instances/examples at once)?; and finally γ) how can the tension between an α) abstract thought and its β) instances be resolved? By asking the questions α) and β) again about the category found under γ) the process can start anew, until finally some γ) is found that is at one with its expression and thus would no longer yield oppositional answers to α) and β).

After his systematic dialectical accounts were complete, Hegel applied these principles to the philosophy of history, that he describes as a battle of the α) 'abstract Generality' of the state against the β) principle of specific Subjectivity that would eventually give rise to the new nexus of the γ) Ideal State. In Hegel's view this process could potentially be completed in the post French revolution society he lived in.

Marx's criticism of Hegel is two-fold. First, in his *Zur Kritik der Hegelschen Rechtsphilosophie*, he criticizes Hegel's account of society for being too harmonious. Secondly, and in my view relatedly, in his *Critique of Hegel's Philosophy and Dialectics in General* he criticizes Hegel's dialectical obsession with the resolution of conflicts, for in Marx's view this precludes Hegel from recognizing misrepresentations of Nature in thought as well as actually existent conflicts. Marx's historical materialism that largely got shape in *Die Deutsche Ideologie* (which Marx wrote in collaboration with Engels) indicates Marx's partial solution to both problems. In response to the first critique, historical materialism describes material inequalities as an ongoing cause of change and revolution, and thus identifies every society thus far, including capitalism, as a battlefield in which Free Will is anything but actualized. In response to the second critique, this conflict-ridden take on Capitalism also allows for a description of actually existent conflicts. Furthermore, Marx and Engels' account of history takes distance from the received view, thus enabling the recognition of misrepresentations in historical accounts thus far.

So, if one holds that Marx intended his social theories in the *Grundrisse* and *Capital* to be systematic dialectical (as all authors reviewed in Section 1.4 do – and I concur) it is likely to differ from Hegel's in three respects:

1. The knowledge reconstructed should be appropriated critically, so as to allow for the chance that categories developed in the empirical sciences misrepresent the matter at hand.
2. Its starting point should be materially grounded in the relations of production emanating from material inequalities.

3. Its starting point should allow for unresolved conflicts at every stage of the dialectical exhibition.

Whether Marx's attempt at formulating a systematic dialectical alternative for Hegel's social theory was successful and the ramifications this has for model building can best be discussed after Hegel's take on the mathematical is described, as was done in Chapter 2.

Hegel discusses the Quantitative and its moments as part of 1) the Doctrine of Being, which is the first of the three subdivisions in his *Logic* (the other two being 2) the Doctrine of Essence and 3) the Doctrine of the Concept). This doctrine is itself subdivided into the sections A) Quality, B) Quantity and C) Measure. The three object totalities Hegel discusses in his *Encyclopädie*, as well as the subdivisions therein and the sections of those, all relate to each other in the same way as α, β and γ do. Hence, the place Hegel reserves for his determination of the Quantitative implies that 1) Hegel conceives of the Quantitative as a fairly abstract field that is nevertheless indispensable for the understanding of everything else and 2) the Quantitative is a reflection on the hopeless multiplicity one is confronted with when trying to get to grips with all concrete instances and examples of the Qualitative at once. In overview, Hegel reasons as follows: On the basis of Quality alone we are unable to make qualitative distinctions, so we enter the realm of Quantity, which is governed by external reflections on sets of elements. The elements are arbitrarily chosen Units One and the sets are Amounts of them expressed through Number in an Intensive Magnitude that has its ultimate meaning in the bad potential infinity that develops in its Extensive Magnitude, but can only be negatively defined as being beyond the finite. So just as Quality is not sufficient to understand the absolute, so is Quantity. So we need both. That is, we need a qualitative Quantum: γ) Measure. Only though Measure there can be any hopes for practical applications of the Quantitative and its categories.

Since the Qualitative must thus dialectically precede the Quantitative, the mathematical requires the Qualitative for its existence in thought (and no one can be aware of anything that cannot be thought). The fact that mathematics helps us to comprehend reality is a result of this. It does not fit reality because 'the book of nature is written in the mathematical language' (as if nature is somehow ontologically quantitative), but because language has evolved a tight fit with reality and by implication so does mathematics.

As to clarifying the nature of the concepts that the mathematical sciences depend upon, Hegel's dialectical treatment of the mathematical, when considered in more detail, clarifies the nature of mathematical concepts like the One, the successor function and sets and elements from their systematic dialectical relations to other categories in language, rather than from their mathematical relationships only. To begin with, our understanding of Hegel's dialectical

treatment of the Quantitative and the fact that its thinkability springs from the intellect's failure to comprehend the Qualitative all at once and on its own terms, implies further that the One and the successor function have a qualitative base and need not be presupposed, as they usually are in mathematics.

Secondly, bringing Hegelian terminology to bear on set theory helps the mathematically minded to understand what Hegel was probably on about as well as helping Hegelians to understand set theory, particularly regarding the proper understanding and use of ordinal and cardinal Numbers. The ordinal number is the number you are arbitrarily assigning to each element as you are counting (that is ordering) the elements in a set. For Hegel this counting operation involves a continuous move from the elements already contained in the set we Numerated and thus considered and those for which we have not yet done so. The former as Intensive Magnitude determines what the set is and as such positively defines it, while the latter determine what it is not and thus negatively define it. Numeration then, is expanding an Intensive Magnitude into an Extensive Magnitude. So although these magnitudes change, the operation by which this is done (i.e. Numeration by using the successor function) does not. This leads Hegel to consider mathematical infinity, ∞, as the bad potential infinity that is never reached and the operation of Numeration as the true philosophical Infinity. When the size of a set's Intensive Magnitude is determined by completing the Numeration of all its elements, it does no longer matter which element was counted first and which second, for no matter where you started, the Number reached will be the same for any particular given set. From a mathematical perspective we have then reached a cardinal Number, which from a Hegelian perspective is best understood as the Intensive Magnitude or size of a finite set. As such, the size of the set is itself a Unit: it expresses the Number of elements it contains while denying them autonomy.

Hegel's point about the relationship between Intensive and Extensive Magnitudes not only elucidates the use of finite ordinal and cardinal Numbers, but also that of infinite ones. If a set is expanded with all subsets contained within it, this is equivalent to raising 2 to the power of the number of elements in that set. By analogy, since the Intensive Magnitude of the denumerable infinite set of all natural numbers N is defined as $\aleph_0$, it contains $2^{\aleph 0}$ subsets, so the size or Intensive Magnitude of the power set of $\mathbb{N}$, $P(\mathbb{N})$, that is of the continuum $\mathbb{R}$ is $2^{\aleph_0}$ and that of $P(\mathbb{R})$ is $2^{2^{\aleph_0}}$, etc. It can be proven that each set thus obtained is of a higher order of infinity than the previous set. This implies that infinite cardinal Numbers may themselves be ranked in a well ordering. So just as transfinite iterations of a successor function lead a finite Intensive Magnitude into the bad potential infinity Hegel associates with its Extensive Magnitude, transfinite iterations of the power operation lead an infinite Intensive Magnitude into the

'worst' potential infinity associated with the size of the class of all sets V. In short: even for an infinite Intensive Magnitude there exists an Extensive Magnitude through which it gains meaning.

'[S]ince the power set of $\mathbb{N}$ […], contains an enormous Amount of infinite sets as elements that therefore must be seen as complete, 'finished', limited objects', this way of thinking implies 'the existence of an enormous amount and enormously big *actually infinite sets*' (Horsten 2004: 27, my translation). This fact, together with the fact that even infinite Numbers can be ordered to fit Hegel's conceptual apparatus, at least partially dispenses with the 'badness' of Hegel's bad potential infinity in that infinity is no longer just defined as an unreachable Extensive Magnitude beyond every finite Intensive Magnitude, but within the well ordering of the infinite cardinal Number associated with that set, can itself also be viewed as an Intensive Magnitude. As a result we can now distinguish two principles of philosophical infinity: 1) the principle of Numeration that leads a finite Intensive Magnitude into its potentially infinite Extensive Magnitude and 2) the principle of the power operation that ultimately leads an infinite Intensive Magnitude into the Extensive Magnitude associated with the set of all sets, V. Because the founding father of set theory, Georg Cantor, was born after Hegel's death, Hegel cannot possibly have been aware of these points.

Chapter 3 next elaborated on Marx's dialectics. It follows Smith (1990) in his identification of Marx's universal principle as being the need for *exchange* that arises from the indirect sociality of capitalist production. This indirect sociality itself stems from the institutional separation of the site of production from that of consumption that is so characteristic of capitalism (cf. Reuten & Williams 1989: 56-57). Not only is this starting point materially grounded in Marx's historical materialist account of human history up until capitalism, it also allows for unresolved conflicts, for producers may fail to sell and consumers may be unable to buy. In either case real people have real problems caused by the other group. Therefore, in Marx's account, capitalist society is not governed by Free Will, but by the imperative for exchange and thus by 'commodification'. If products are produced to be exchanged (which determines them as *commodities*), they are produced for their *value* rather than their use. *Value in exchange* does not Measure Qualitative usefulness but rather is imposed on the product in capitalism. As such it does not predate the capitalist mode of production and would cease to exist when that mode of production would. As such, the further determination of *value in exchange* as *the money form of value* is not a Measure in a Hegelian sense at all, for rather than pinning a Quantity on a pre-existing Quality it is ontologically quantitative through and through. *Value in exchange* is an abstraction actualized through *money*. It is an abstraction-in-practice that rules our daily lives and arbitrates between life and death.

The exchange imperative not only allows for real conflicts, it also serves as an inescapable determination of human behavior and thus functions in ways that are similar to forces of nature. So, the mentioned imperative implies at least a similar potential for the application of mathematical models as is present in the natural sciences. Forces of nature and their determinants, however, must be Measured before they can be modeled. Models in the natural sciences therefore work with quantitative representations of pre-existing Qualities (such as e.g. length in physics), they are not ontologically Quantitative, whereas the entities that go into modeling capitalism are. The upshot of this is, that pure mathematical quantities can now be seen as a driving force of human behavior and modeled using purely quantitative techniques and categories. As a result, what Hegel holds for determinations on the level of abstract thinking only can be directly applied to the study of capitalism. It is this observation that led Arthur to contend that the grand structure of the three volumes of *Capital* is homologous to the structure of Hegel's *Logic*.

With indirect sociality the qualities of the product produced are immaterial to the producer. As a result, *money* as *capital* must be the *end of exchange* and each sum of *money* invested in means of production (making it into *constant capital*) must be worked up by labor (employed as *variable capital*) in order to be exchanged for a higher sum of *money* to be reinvested and *accumulated*. From a capitalist perspective, this accumulation takes the form of constant increments in money, but from the perspective of reinvestment it seems constant increases in the scale of operation are the driving force. Finally, from the perspective of society the qualitative change in the product (from raw materials to a finished commodity) seem to be the main point of production. The time a production cycle takes, can vary and so can the number of cycles for which particular means of production last. If they last for just one cycle, Marx dubs them *circulating capital* and if they last for more than one he calls them *fixed capital*. At this stage, Marx introduces his schemes of reproduction.

Though Marx's schemes of reproduction are commonly considered the first two-sector macro-economic model ever conceived of, they do not live up to their full potential as mathematical models of systematic dialectically conceived of interrelationships within capitalism. They model the interactions between *capital*'s two main departments: one producing means of production, the other consumption goods. As such, the model pertains to a rather abstract and general level in Marx's dialectical account. For the model to reflect that, its formulation should refrain from filling in any specific parameters or numbers. Instead, it should be kept as general as possible and therefore formulated purely algebraically (as Chapter 4 did).

Secondly, Marx does not present his model assumptions as though they result from his systematic dialectics, even though many of them might very well be presented like that. Since Volume II of *Capital*, in which Marx presents his schemes, was only posthumously published on the basis of the drafts and notebooks Marx bequeathed to Engels, we can only guess whether Marx would have done so had he lived long enough to write a more final draft. Either way, the schemes being presented as they are, there is room for improvement in both respects.

Specifically, a systematic dialectical account may inform four different types of assumptions:

1. foundational assumptions, that outline the important categories to model and indicate how they are related;
2. heuristic assumptions that can be used to create ever more concrete model generations, for instance by setting out a static model and allowing dynamics in later (this is what is going on when Marx first models *simple reproduction* and moves on to *expanded reproduction* later);
3. absency assumptions, that stipulate that certain influences, though not empirically absent are absent at the level of abstraction the model pertains to;
4. anticipatory assumptions that outline conditions of existence and anticipate them.

Foundational assumptions may follow directly from a systematic dialectical exhibition. After all, such an exhibition shows the categories that are important and suggests how they are interdependent, thus providing a first indication of a possible model specification. Heuristic assumptions should be sought after next. A dialectical defense of such assumptions can be that the (static) possibility of the very existence of a posited relationship should be investigated, before the (dynamic) development of such a relationship can even be contemplated. If the dialectical account indicates that a dynamic relationship is imperative, but the model cannot even ground the existence of a static one, the dialectical account so far apparently still lacks completeness and thus further mediating conditions must be dialectically determined before moving on to the dynamic model of the relationship's development. If, as was the case with Marx's schemes of reproduction, the static model does not preclude dynamics, one may of course move on to drop the heuristic assumption(s) without first returning to the systematic dialectics.

Together these two types of assumptions describe rather positively what the model is about and by implication this gives a lot of information on what is excluded from it. Yet sometimes it is helpful to reflect on the level of abstraction reached in the dialectical exhibition and explicate the influences that one has not

yet exhibited and can thus safely abstract from at the level of abstraction depicted in the model (as is the case with foreign trade at the level of abstraction Marx's schemes of reproduction pertain to). The explication of as yet unexhibited influences happens in absency assumptions. These, however, should be no more than a check on the model specification emanating from the foundational and heuristic assumptions. After all, if a certain influence was dialectically determined to be important at the level of abstraction of the model, it should have been a factor in the foundational assumptions or specifically neglected for heuristic reasons.

Finally, the assumptions that anticipate conditions of existence are in place to enable the unhindered expression of a force or tendency whose articulation has been dialectically determined to be necessary for the functioning of the object totality under scrutiny (for Marx's schemes of expanded reproduction the case in point is that of sufficient monetary accommodation and availability of labor power enabling unhindered accumulation of capital). This type of assumption differs from the absency type. The latter explicates the complete absence of something at the level of abstraction modeled (such as foreign trade). In the case of the former, by contrast, some general tendency or entity *has* been determined to exist in the abstract (such as the need for *capital* to *accumulate*), but the exact mechanism by which it might concretely come about has *not* yet been. In such cases it is dialectically OK to anticipate that possible impediments to the development that was determined as dialectically necessary, will somehow be taken away or rendered harmless at more concrete levels (e.g. monetary accommodation must somehow be offered if the abstract requirement of accumulation is to hold in the concrete). So instead of stipulating that something is *absent* at this level of abstraction, it stipulates that something we cannot yet specify must somehow be *present*. If the force or tendency in question, can be successfully expressed in a model under this type of assumption, but not when it is removed, this indicates the need to introduce institutions in the remainder of the systematic dialectical exhibition that allow the assumption to hold in practice (such as e.g. credit). (Of course, if removal of the assumption is unproblematic, it should not have been necessary to formulate it in the first place.) Thus, this fourth type of assumption is particularly useful for indicating the road ahead.

When this conceptual apparatus is applied to the assumptions Marx makes in drawing up his schemes of simple reproduction, we can identify the following as foundational (the letters indicate the order in which they were presented in *Capital II*):

b. No revolution in values takes place in the component parts of the productive capital

d. Society's total yearly product breaks down into I) means of production and II) means of consumption

f. The rate of valorization s/v (i.e. surplus-value over variable capital) is constant and given for each department (and set at 100% for both departments)

When we focus on the term 'component parts', rather than values, we can accept **b** as simply saying that productive capital can always be decomposed into *constant capital*, *variable capital* and surplus-value, a decomposition that follows directly from Marx's dialectical exhibition. Similarly, the institutional divide between the site of production and that of consumption means **d** is fully warranted dialectically. Though the terms Marx formulates it in are too strict (focusing on the 100% I placed between brackets), it is dialectically defendable to stipulate that s/v is more or less constant over time since only labor can produce value. Except then for the specific number Marx assumes, assumption **f** is defendable as a foundational assumption as well.

He also makes two heuristic assumptions:

c. The value of a department's yearly produce is constant and all surplus-value and wages are consumed (so there is no accumulation)

g. The ratio of variable to constant capital is equal, constant and given across departments

Marx himself treats **c** as a heuristic assumption. It is in place to focus on what the system is when viewed as static, before it is dynamized. When moving on to expanded reproduction he retains only one element in it: all wages are fully consumed. **g** effectively assumes technical change away. Marx retains this assumption for expanded reproduction, but since technical change is inherent in (the intensive aspect of) accumulation, this is not defendable.

Furthermore he explicates the absency of some things in 3 further assumptions:

a. Products are exchanged at their values

e. Depreciation costs equal replacement expenditure

h. There is no foreign trade.

Assumptions **a** and **h** follow from the fact that only departmentalization of capital has been dialectically determined, so distinctions within it that might induce a divergence between prices and values (such as competition or foreign trade), cannot be made yet. For the same reason, only macro-economic aggregates can be considered at this stage, so any mismatches between depreciation costs and replacement expenditure are likely to level out and **e** can be accepted as well.

Since assumption **c** implies that all values are constant, so will the ratios between them be. As a result, all more specific assumptions on values and ratios (i.e. **b**, **f** and **g**) are redundant for simple reproduction. The only assumptions Marx really needed to call upon in order to formulate his proportionality condition

were **d** (departmentalization as the foundation of his models), **c** (simple reproduction as the first case to be considered heuristically) **e** (anything depreciated gets replaced and thus represents a cost of production) and **h** (the absence of foreign trade). This condition, $(v + s)_I = c_{II}$, says that the value of consumptive expenditure in the department producing means of production is equivalent to the means of production used up in the department producing means of consumption. Thus, the condition indicates the interdependency between the two departments.

For Expanded reproduction Marx adds two more absency assumptions:

- **i.** There has already been production on an expanded scale
- **j.** The sum total of replacement expenditure on *fixed capital* equals the sum total of depreciation allowances for *fixed capital* in each department

Previous expansion (**i**) must be assumed when discussing full-fledged capitalism, for without it, it cannot be considered full-fledged. Assumption **j** effectively says that capital has no funds other than the value of their produce available from which to replace and accumulate capital and indeed the possibility of outside finance has not yet been dialectically determined.

Finally he anticipates that possible obstacles to the dialectically necessary imperative of *accumulation* will be taken away at later more concrete stages of the dialectical exhibition:

- **k.** '[*T*]*he amount of money present in a country* is sufficient for both hoarding and accumulation.'
- **l.** There is always enough labor power on hand

Here the argument is that the requirement of accumulation can only materialize concretely if somehow these conditions are met in later more concrete stages of the exhibition, even though we have not yet determined how this might be brought about.

With all these assumptions in place the proportionality condition for expanded reproduction becomes:

$$(v + s)_I - \Delta c_I = c_{II} + \Delta c_{II}$$

So, with expanded reproduction, the funds spend in the other department by the department producing means of production are lowered by the amount they accumulate, while the capital needs of the department producing means of consumption are actually higher, as it too has to accumulate.

Using the insights developed in Chapter 3, Chapter 4 proceeded to reconstruct Marx's reproduction schemes along dialectical lines. In particular it aimed to:

1) retain only the dialectically defendable (elements of) assumptions,

2) let dialectical reasoning rather than mathematical ease and rigor determine the order in which assumption and equations are presented,
3) present the whole model in its abstract generality and therefore algebraically throughout,
4) explicitly define all time-dependent variables as such.
5) integrate technical change and expansion, or the intensive and extensive aspects of accumulation respectively, in a comprehensive model of expanded reproduction.

Since Marx never brings technical change back in, a model generation had to be added to Marx's schemes to achieve the fifth aim. So Chapter 4 first discussed *simple reproduction*, next extensive growth and finally *expanded reproduction*. The first two are algebraically formulated and (where applicable) dynamized versions of Marx's own models of simple and expanded reproduction respectively. My model of expanded reproduction goes beyond Marx's. I nevertheless used Marx's term, for his *concept* encompasses technical change and, in order to achieve as tight a fit as possible between Marx's dialectics and his modeling, the model had to reflect that.

As indicated, simple reproduction requires just four assumptions, which - in keeping with my second aim - were presented in the following order:

1. Foundational: **'**Society's total yearly product breaks down into *two great departments*': a department producing means of production (i.e. current and additional constant capital) (department p) and one producing means of consumption (i.e. commodities intended for consumption out of wages paid out to variable capital, and out of capitalists' surplus-value) (department c) (cf. assumption **d** in Section 3.3).
2. Heuristic: *Simple reproduction* means that 1) the total value of the yearly produce of both departments (x_p and x_c) is constant, 2) all surplus-value is consumed and 3) there are no savings out of wages (cf. **c**).
3. Absency: There is no foreign trade (cf. **h**).
4. Absency: Aggregate depreciation costs incurred yearly equal aggregate yearly replacement expenditure (cf. **e**).

With these in place, all of Marx's results could be formulated. Since the model for simple reproduction is static, there was no insight to be gained from modeling variables as time-dependent yet. Of course this changes when modeling extensive growth.

To model extensive growth in a dialectically defendable way, the following can be assumed:

2a. Absency: There are no savings out of wages (cf. **c**).
5. Absency: There are no price changes (cf. **a** and **b**)

6. Foundational: The ratio of s/v (notation: ε) is constant and given in each department (cf. **f**)
7. Heuristic: the organic composition of capital ($\kappa = c/(c+v)$) is constant and given for each department (cf. **g**).
8. Absency: All expansion of capital is financed out of surplus-value (cf. **j**).
9. Anticipation: There is always enough money available to finance hoarding for replacement purposes and accumulation at the desired rate (cf. **k**).
10. Anticipation: An accumulating department will always find sufficient labor power on hand to increase the variable capital it employs by as much as its growth rate requires (cf. **l**).
11. Anticipation: Changes in department p's rate of accumulation are always actualized. Department c's actualizable rates are therefore constrained by the amount left by department p. Anticipating eventual recovery, it is assumed department c accumulates in such a way that all of department p's produce is actually sold.

Of these, assumptions 5 through 7 are consequences from the rewriting of assumption 2 into 2a. That is, when all the dialectically undefendable aspects of that assumption are dropped, it is no longer a heuristic, but rather an absency assumption. That is, at the level of abstraction we are now considering, wages are unlikely to exceed subsistence wages by any significant amount and so we can safely assume that savings out of wages do not occur. But now that capital is explicitly allowed to expand, we do need to rethink the way in which this might occur, leading to assumptions 5 through 7. Note that in its current formulation assumption 5 integrates the original assumptions **a** and **b** by focusing on their value aspects, changes in which and price divergences of which, they declare *absent* at this level of abstraction. So the *foundational* elements ('component parts') of assumption **b** are no longer emphasized here and its character too changes. The order in which assumptions 8 through 10 were presented and the reason they can be adopted has been sufficiently explained.

Finally, assumption 11 (which Marx never explicates at all) is only called for if we want to model the effects that a unilateral decision by department p to alter its rate of accumulation has on the other department and the economy as a whole. With the original goal being to see what happens to the conditions for valorization when introducing accumulation, we did not need the assumption immediately. The assumption can best be viewed as an anticipatory assumption. With only two departments dialectically determined, a department that wants to keep a larger part of its produce for itself (or conversely, sell a larger part of it) can do so. But since doing so regarding consumption goods does not affect accumulation, department p can take the lead. Department c is to respond according to assumption 11 so as

to limit the duration of emerging capitalist crises, and so assumption 11 is anticipatory to the extent that crises are usually overcome eventually.

As a result, the maximum rate of accumulation out of surplus-value possible for department c, θ_c, is a function of θ_p:

$$\theta_{ct} = \frac{(\varepsilon_p + 1 - \kappa_p \varepsilon_p \theta_{pt})(1/\kappa_p - 1)}{(1-\kappa_c)\varepsilon_c} \times \frac{c_{pt}}{c_{ct}} - \frac{1}{(1-\kappa_c)\varepsilon_c} \tag{4.16}$$

Alternatively, given that the relationship $\theta_i = \frac{g_i}{(1-\kappa_i)\varepsilon_i}$ holds for both departments (albeit with different parameters) the actualizable growth rate for department c (g_c) can be expressed in terms of the growth rate set by department p (g_p):

$$g_{ct} = \frac{c_{pt}}{c_{ct}} \left[\frac{(1-\kappa_p)(1+\varepsilon_p)}{\kappa_p} - g_{pt} \right] - 1 \tag{4.17}$$

This equation shows that when department p sets its g_p to g_p' during the period under scrutiny, the growth rate for both departments starts to diverge, but it has also been shown that as long as assumption 11 holds, this effect lasts only one year. Thus The following would then hold:
$g_{c0} < g_0 < g_{p0} = g_1 = (x_p1 - \delta c_1)/c_1 = \Delta c_1/c_1 = g_{c1} = g_{p1}$. But, says Marx, this is utterly unlikely since there is no dialectically determined mechanism in place yet that will entice department c somehow to consistently buy up the left-overs. But even though smooth adaptation may be unlikely, some adaptation mechanism must exist for capitalism to survive its recurring crises.

In order to go beyond Marx and model technical change as an integral part, heuristic assumption 7 and anticipatory assumption 11 from the model of extensive growth had to be modified to become:

7a. Foundational: The organic composition of capital ($\kappa = c/(c+v)$) rises with each round of accumulation (operationalized here as a calendar year).

11a. Anticipation: Changes in department p's rate of accumulation are always actualized in conformity to the latest technology. Department c's actualizable rates are therefore constrained by the amount left by department p. Anticipating eventual recovery, it is assumed department c accumulates in such a way that all of department p's produce is actually sold.

Thus formulated, assumption 7 is finally on a par with Marx's dialectics and is therefore no longer a heuristic assumption, but has become foundational. After all, *capital*'s preference for accumulating low risk *constant capital* over high risk *variable capital* that leads to a rising occ has long been shown dialectically to

ensue from the workings of *accumulation*. So 7a flows immediately from the dialectics leading up to the model. Assumption 11 is essentially retained, albeit that department p's accumulation now restricts department c in both setting the rate of accumulation and securing the latest technology for itself.

In analogy to 4.16. we may then write:

$$\frac{\Delta c_{ct}}{c_{ct}} = \left[1 + (1-\theta_{pt})\varepsilon_p - \varphi_p + \frac{\Delta c_{pt}}{c_{pt}} \right] \frac{1}{\zeta_{pt}} \frac{c_{pt}}{c_{ct}} - 1 \qquad (4.19)$$

This result enables one to prove that balanced growth is impossible with ongoing technical change.

These models show how the models for the concepts of *simple reproduction, intensive* and *extensive growth* and *expanded reproduction* can be developed alongside and in line with the dialectics that gave rise to the concepts themselves. In so doing, the models grow ever more intricate and complex, which reflects how our understanding of the way in which the two great departments are interrelated grows more intricate and complex as well. Finally, the algebraic generalizations presented, ensure that the results found (such as the equations describing the way that the factors determining g_c interact during transition) are perfectly general and will work with all numbers. Additionally, the algebraic formalizations allow one to see how all vantage points taken until we arrived at the model for expanded reproduction can be conceived of as special cases of this most intricate and complicated model.

So when *extensive growth* is integrated with *intensive growth* a solution can still be constructed (at least theoretically) in which both departments can grow at the same rate. Moreover, the solution still allows for changes to be made to this rate. But it is also clear that the relevant equations have grown into quite inelegant monsters and that finding a solution somehow requires department c to take into account its own changing occ (κ_{c1}) its previous occ (κ_{c0}) its rate of surplus-value (ε_c) and those of the other great department ($\kappa_{p1,}$ κ_{p0} and ε_p respectively). Meanwhile, it is unclear how this information could reach the other department and if it did somehow, what mechanism would entice it to act upon it so as to make sure all produce is valorized.

In overview, it can be concluded that the dialectical move from *capital*'s static renewal to its dynamic expansion can be depicted not only verbally through a systematic dialectical exhibition, but can also be represented mathematically through a series of model generations, the last of which eventually dispenses with all heuristic assumptions. All in all, careful contemplation of the dialectics of a system has been shown to give extra support to assumptions figuring in formal

and/or mathematical representations (i.e. models) of (aspects of) it, which in turn makes such mathematical representations more defensible. I expect both model builders and dialectically inclined system analysts to benefit from this result, for in the very least it has been shown here that the two ways of reasoning are actually quite compatible. I have high hopes for a fruitful debate between the two groups based on the linkages between their fields that this book has shown do exist.

List of Symbols

For the formal representation of Marx's models the following symbols were used:

$c =$ constant capital, the value of the means of production applied;
$v =$ variable capital, the value of the social labor power applied;
$s =$ surplus-value, the value that is added by labor minus the replacement of the variable capital advanced;
$x =$ the total value of the yearly produce in a department or the economy as a whole;
$u =$ surplus-value consumed by or via capitalists ('unproductive consumption');
$\Delta c =$ surplus-value accumulated in constant capital;
$\Delta v =$ surplus-value accumulated in variable capital;
$g =$ proportional growth rate;
g_p and $g_c =$ growth rates for department p and c respectively (during transition);
κ_p and κ_c = the organic composition of capital $= c/(c+v)$;
ζ_p and ζ_c = an alternative definition of the organic composition of capital $= c/v$
ε_p and $\varepsilon_c =$ the rate of valorization $= s/v$;
$\theta =$ the rate of accumulation out of surplus-value $= (\Delta v + \Delta c)/s$;
φ_p and φ_c = the growth rate of ζ
r_p and r_c = the profit rate $s/(c+v)$
$t =$ time, also denoted as a (numerical) subscript where the base year = *0*;

The subscripts $_p$ and $_c$ indicate whether we are considering the department producing means of production or means of consumption respectively. When an equation or expression is valid for each department, subscript $_i$ was used to indicate indeterminateness.

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Superscripts behind a publication year denote editions. The edition that was actually used is always cited first. Thus Hegel, G. W. F. (1830[3], 1817[1]) means that I relied on the third edition of the *Encyclopädie* throughout and that the first edition of that work was published in 1817. If a work was published in several parts, the publication years of the different parts are cited without superscripts.

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