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Moroccan ornamental quasiperiodic patterns constructed by the multigrid method

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The similarity between the structure of Islamic decorative patterns and quasicrystals has aroused the interest of several crystallographers. Many of these patterns have been analysed by different approaches, including various kinds of ornamental quasiperiodic patterns encountered in Morocco and the Alhambra (Andalusia), as well as those in the eastern Islamic world. In the present work, the interest is in the quasiperiodic patterns found in several Moroccan historical buildings constructed in the 14th century. First, the *zellige* panels (fine mosaics) decorating the Madrasas (schools) Attarine and Bou Inania in Fez are described in terms of Penrose tiling, to confirm that both panels have a quasiperiodic structure. The multigrid method developed by De Bruijn [*Proc. K. Ned. Akad. Wet. Ser. A Math. Sci.* (1981), **43**, 39–66] and reformulated by Gratias [*Tangente* (2002), **85**, 34–36] to obtain a quasiperiodic paving is then used to construct known quasiperiodic patterns from periodic patterns extracted from the Madrasas Bou Inania and Ben Youssef (Marrakech). Finally, a method of construction of heptagonal, enneagonal, tetradecagonal and octadecagonal quasiperiodic patterns, not encountered in Moroccan ornamental art, is proposed. They are built from tilings (skeletons) generated by the multigrid method and decorated by motifs obtained by craftsmen.

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1. Introduction

'In quasicrystals, we find the fascinating mosaics of the Arabic world reproduced at the level of atoms: regular patterns that never repeat themselves.' This is what the Nobel Committee wrote (Royal Swedish Academy of Sciences, 2011) regarding the Nobel prize in Chemistry attributed to Shechtman in 2011 for his discovery, in an aluminium manganese alloy, of quasicrystals with an icosahedral ordered phase, the diffraction pattern of which exhibits fine well resolved diffraction spots like in crystals but distributed according to the icosahedral symmetry that is prohibited by both the two- and the three-dimensional periodic lattice.

The simplest way of visualizing a two-dimensional quasicrystal is that followed in another context by Penrose (1974), who invented a nonperiodic tiling of a plane by two types of tiles, generated by a deterministic construction using inflation, the diffraction pattern of which consists of δ peaks that are distributed on figures with pentagonal symmetry. This tiling, which quickly became the archetype of quasicrystals, has been explained in the multigrid method developed by De Bruijn (1981) in the plane and generalized by Duneau and Katz (Duneau & Katz, 1985; Katz & Duneau, 1986), who showed that a quasiperiodic tiling could be obtained by the cut-and-projection method from a higher-dimension space.

The similarity between the *zellige* panels (fine mosaics) found in historical monuments and mausoleums and quasicrystal diffraction patterns (Fig. 1) shows that Moroccan master craftsmen have also built, since the 14th century, decorative motifs that are encountered in quasiperiodic patterns. Several authors have analysed, using different approaches, various kinds of quasiperiodic patterns encountered in Moroccan–Andalusian decorative art and in the eastern Islamic world. Makovicky (1992, 2004, 2007, 2008,

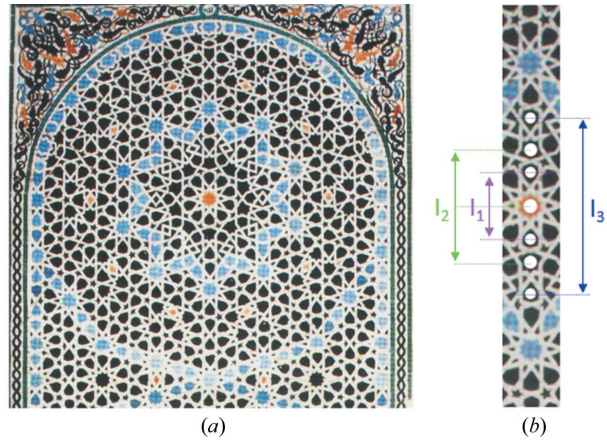


Figure 1
(*a*) *Zellige* panel of the Madrasa Attarine (Fez). (*b*) The distances between two symmetrical elements relative to the centre of the pattern are in the ratio of the golden mean.

2011), Makovicky & Fenoll Hach-Alí (1997), Makovicky *et al.* (1998), Castera & Jolis (1991) and Castera (1996, 2003) studied the octagonal and decagonal patterns, Rigby (2005), Lu & Steinhardt (2007), Saltzman (2008) and Al Ajlouni (2012) were interested in the decagonal patterns, Makovicky & Makovicky (2011) studied the dodecagonal structure, and Bonner & Pelletier (2012) constructed a sevenfold pattern. In another context, Whittaker & Whittaker (1988) proposed heptagonal and enneagonal tiling, and Franco *et al.* (1996) also proposed enneagonal quasiperiodic tiling.

In this work we shall first show, using the multigrid method, that some patterns encountered in Moroccan monuments built by the craftsmen's method called *Hasba* (Thalal *et al.*, 2011; Aboufadil *et al.*, 2013) are underlined by a quasiperiodic tiling. We then give examples of the well known decagonal patterns that adorn the Madrasas (schools) Attarine and Bou Inania (Fez), and a variant of the dodecagonal pattern of the tympanum at the entrance of the mausoleum of Moulay Idriss (Fez). These patterns have been widely analysed by Makovicky *et al.* (1998), Makovicky (2004) and Makovicky & Makovicky (2011). We shall then build new octagonal quasiperiodic patterns from periodic patterns that adorn the Madrasas Ben Youssef (Marrakech) and Bou Inania (Fez). Finally, we shall use the multigrid method to construct heptagonal, enneagonal, tetradecagonal and octadecagonal quasiperiodic patterns not encountered in Moroccan geometric art.

2. The multigrid method

Several methods are used to construct quasiperiodic tilings. One of the easiest, called the multigrid method and formulated by Gratias (2002), is discussed in this section.

We construct a set of equidistant parallel lines, characterized by a vector **V** orthogonal to the lines and of length equal to the period. We copy this set N times by performing each time a θ_n rotation in the plane; N is a generic value greater than 2, and it corresponds to the rotation order of the grid in relation to the others. We then obtain a grid, called a multigrid, constituted by N families of lines; each family is characterized by an orthogonal vector $\mathbf{V}_n$ (Fig. 2*a*).

From a central point, we place the vector $\mathbf{V}_n$ and its opposite and we trace orthogonal lines passing by their extremities. We obtain an irregular convex polygon and we trace the segments joining the centre point to the vertices of the polygon. By drawing a small circle around the central point, we obtain N sectors, each based on the polygon edge (Fig. 2*b*). In each sector, we choose a vector $\mathbf{W}_n$ of arbitrary length and direction, and we associate it with the corresponding line family.

We then construct parallelograms $P_{i,j}$ formed by the vectors $\mathbf{W}_i$ and $\mathbf{W}_j$ taken in pairs. These parallelograms will constitute the paving tiles.

We assume that the families of lines are distributed in a generic way, so that the lines intersect in pairs and there are no triple or multiple intersections. With each intersection $I_{i,j}$ between lines L_i and L_j, we associate the parallelogram $P_{i,j}$ engendered by the vectors $\mathbf{W}_i$ and $\mathbf{W}_j$ (Figs. 3*a* and 3*b*). The tiles are arranged according to the sequence of intersection of the lines (Fig. 3*c*) and the whole tiling is obtained by this process (Fig. 3*d*). The perfectly ordered tiling obtained is a quasiperiodic lattice (except for the case $N = 2$, which corresponds to a periodic lattice).

It is shown (Jaric, 1989) that these tilings are the result of cutting a periodic N-dimensional object in an irrational direction. This construction leads to remarkable results when choosing a symmetric multigrid constructed from a single family of lines copied N times by the same angle of rotation $\theta_n = 2\pi/N$ and taking $\mathbf{W}_n$ equal to the vector $\mathbf{V}_n$.

The multigrid method is suitable for many variations in the choice of grid spacing sequences, their relative positions and

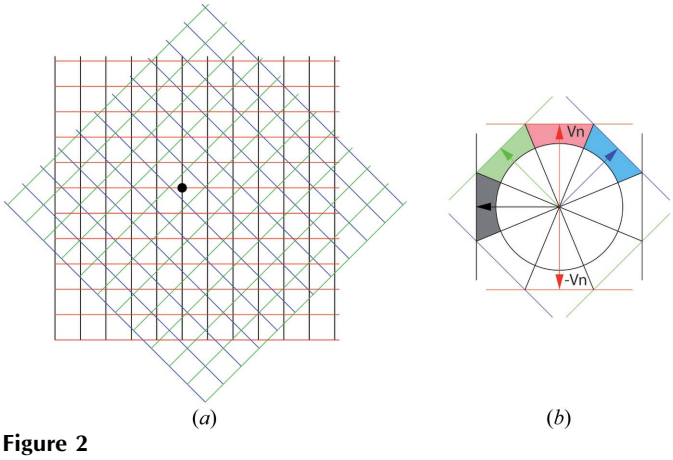


Figure 2
(*a*) A multigrid is the superposition of $N > 2$ sets of equidistant parallel lines in N distinct directions of the plane. In this case, $N = 4$. Periods in each direction are not necessarily equal. (*b*) Vectors $\mathbf{V}_n$ and their opposites are traced from the same centre to obtain a convex polygon generated by lines perpendicular to the vectors and passing by their extremities. We arbitrarily choose in each sector a vector $\mathbf{W}_n$ associated with the same family of lines.

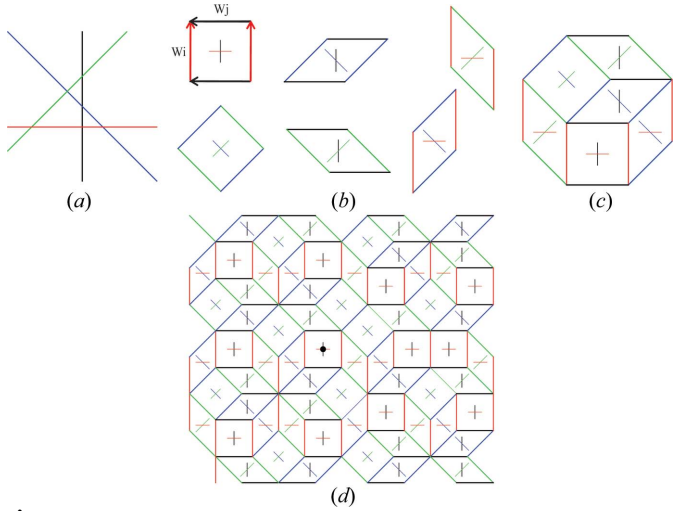


Figure 3
(*a*) Intersection of lines. (*b*) Each intersection, $I_{i,j}$, of the lines L_i and L_j of the multigrid is associated with the parallelogram $P_{i,j}$. (*c*) The paving is obtained by placing adjacent parallelograms in the exact order of intersection of the lines in the multigrid. In the case of multiple intersections, we move a family of lines by an infinitesimal amount to lift the degeneracy. (*d*) The corresponding tiling.

the vectors $\mathbf{W}_n$ that define the shape of the tiles. In all cases, we obtain uniform tilings with beautiful decorative elements. In the following, we shall use a symmetric multigrid, and the spacing sequences will be specified in each case.

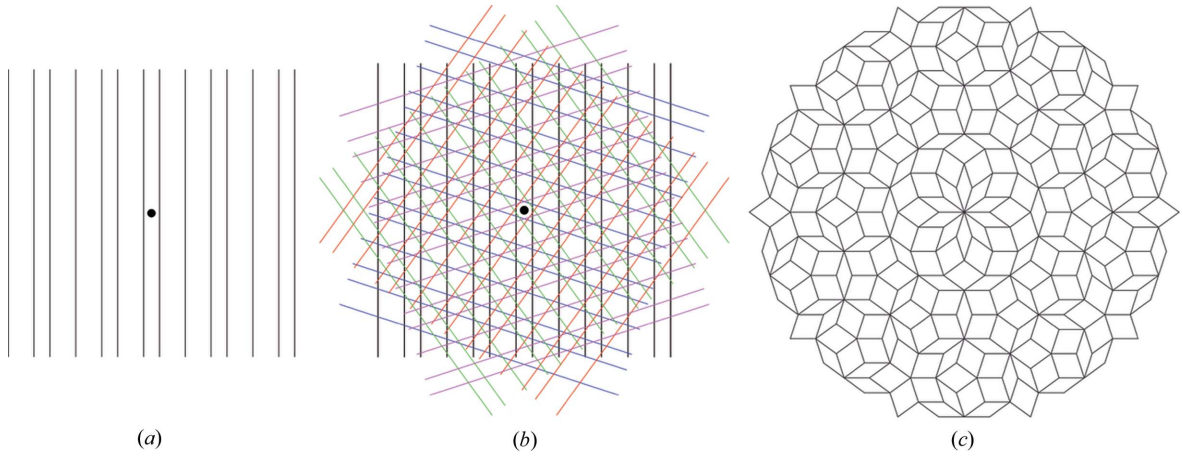


Figure 4
(*a*) Fibonacci sequence. (*b*) A 5-grid (Ammann grid). (*c*) Penrose tiling.

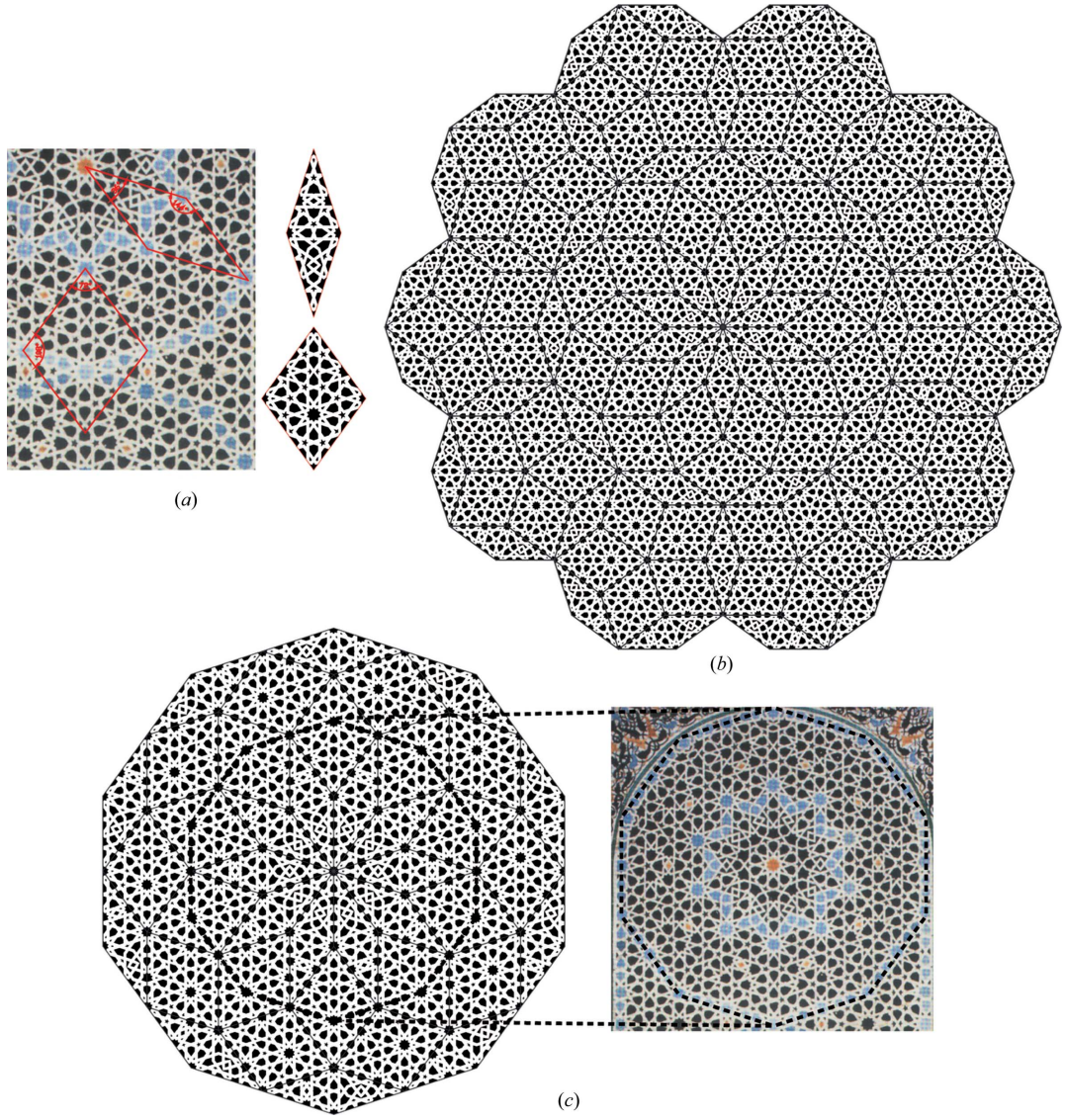


Figure 5
(*a*) A panel of the Madrasa Attarine (Sijelmassi, 1991). (*b*) The infinite quasiperiodic pattern. (*c*) A finite part of the quasiperiodic pattern.

3. Analysis of quasiperiodic patterns of some historical monuments by the multigrid method

3.1. Decagonal panels of the Madrasas Attarine and Bou Inania

Makovicky *et al.* (1998) were the first to study and explain the decagonal pattern from Madrasas Attarine and Bou Inania. The purpose of this section is to show, through these two well known examples, that the multigrid method can be adapted to study quasiperiodic patterns. We use the Penrose tiling (Fig. 4*c*) obtained, by the same process as explained in the previous section, from a 5-grid, which is a superposition of five identical grids rotated by an angle $\theta = 2\pi/5$ to one another around the fivefold axis passing through the centre (black point) (Fig. 4*b*). The parallel lines of the grids are disposed in a Fibonacci sequence (Fig. 4*a*).

3.1.1. Madrasa Attarine. The *zellige* panel (fine mosaic) of the Madrasa Attarine (MA) (Fig. 5*c*) has the symmetries 5 and 10. We define in it two rhombic tiles with vertex angles 36 and 72°, as shown in Fig. 5(*a*). If we arrange the tiles according to a Penrose tiling, we obtain an infinite quasiperiodic pattern (Fig. 5*b*). The MA panel appears as a finite part of this quasiperiodic pattern (Fig. 5*c*).

As already mentioned by several authors, we can notice the similarity of the decagonal pattern to the diffraction pattern of the quasicrystal AlMn (Shechtman *et al.*, 1984). The ratio of the distances between two symmetrical elements in the case of the panel (Fig. 1) is equal to the golden number, $\tau = (1 + 5^{1/2})/2 = 1.61803398875\ldots$.

3.1.2. Madrasa Bou Inania. The *zellige* panel of the Madrasa Bou Inania (MBI), which is a variant of the MA panel, is also extracted from an infinite decagonal quasi-

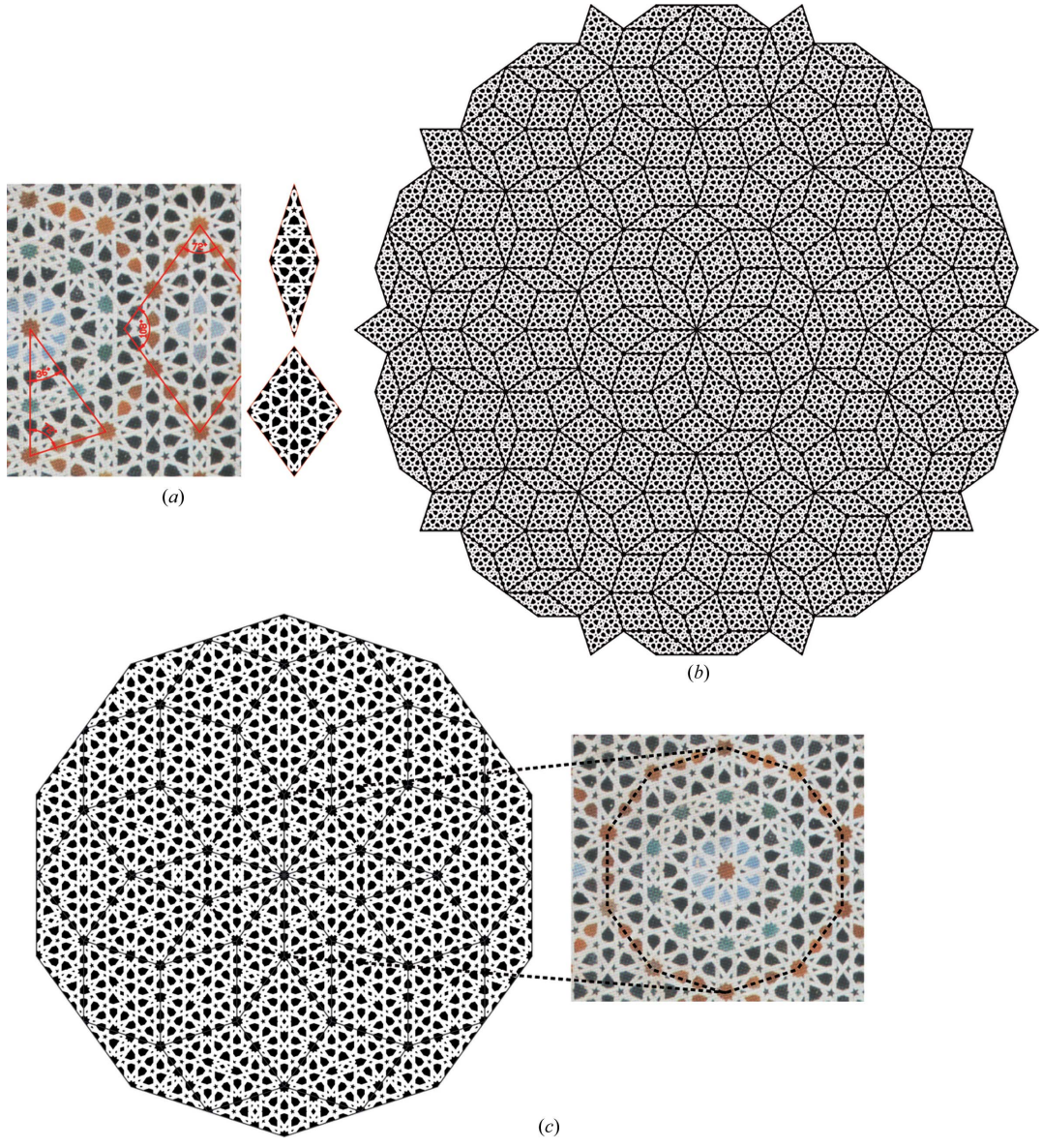


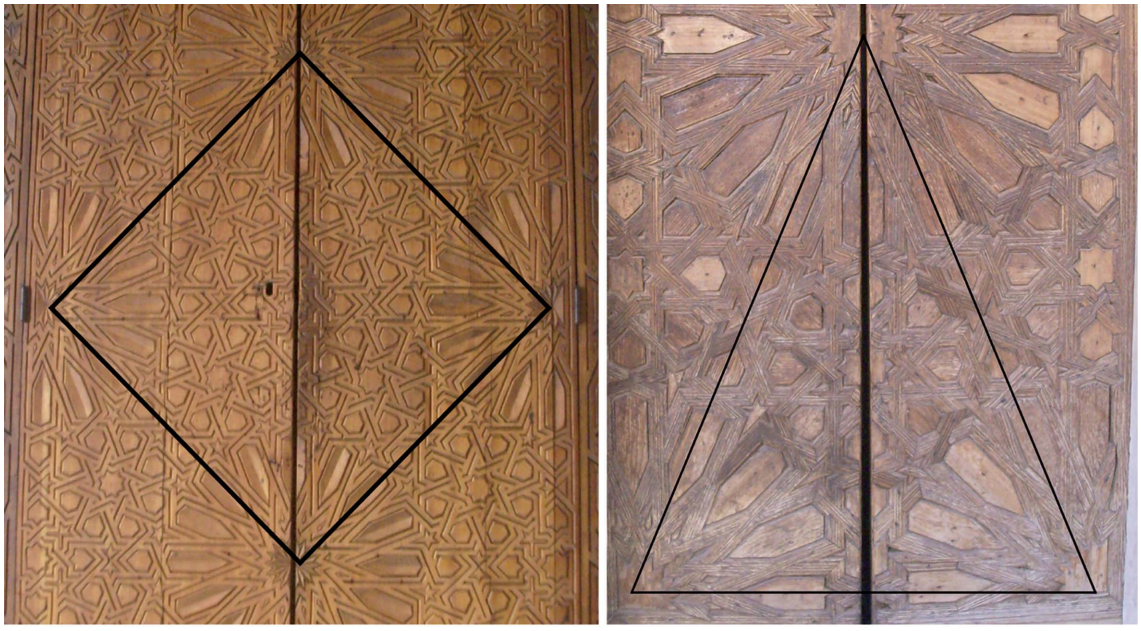
Figure 6
(*a*) A panel of the Madrasa Bou Inania. (*b*) The infinite quasiperiodic pattern. (*c*) A finite part of the quasiperiodic pattern.

periodic pattern obtained from a Penrose tiling. The two rhombus tiles have the same dimensions as in the MA panel. The difference between them is the tile decoration (Fig. 6).

Both MA and MBI panels are extracted from infinite quasiperiodic patterns. These panels, originally designed by craftsmen, can be built from decorated tiles arranged in a Penrose tiling. The difficult part of such a construction is the tile decoration. Indeed, the juxtaposition of the tiles should lead to a harmonious model. The transition from one tile to another must satisfy the artistic criteria required by the craftsmen.

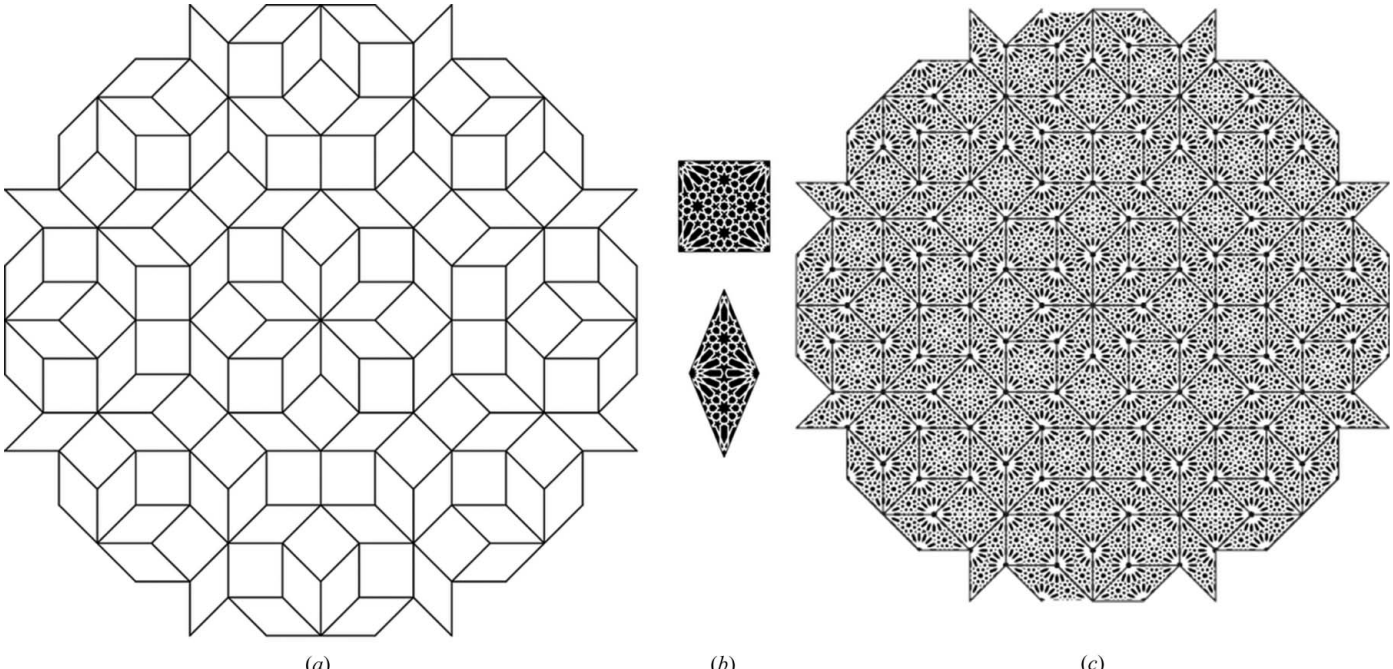
3.2. Construction of octagonal quasiperiodic patterns from known periodic patterns

In this section, we present quasiperiodic patterns built from known periodic patterns. For instance, we chose the patterns drawn on the wooden gates of the Madrasas Bou Inania (MBI) and Ben Youssef (MBY). They have the symmetry



(*a*) (*b*)

Figure 7
Wooden gates at (*a*) Madrasa Bou Inania (Fez) and (*b*) Madrasa Ben Youssef (Marrakech).



(*a*) (*b*) (*c*)

Figure 8
(*a*) Octagonal tiling. (*b*) Square and rhombic decorated tiles. (*c*) An octagonal quasiperiodic pattern.

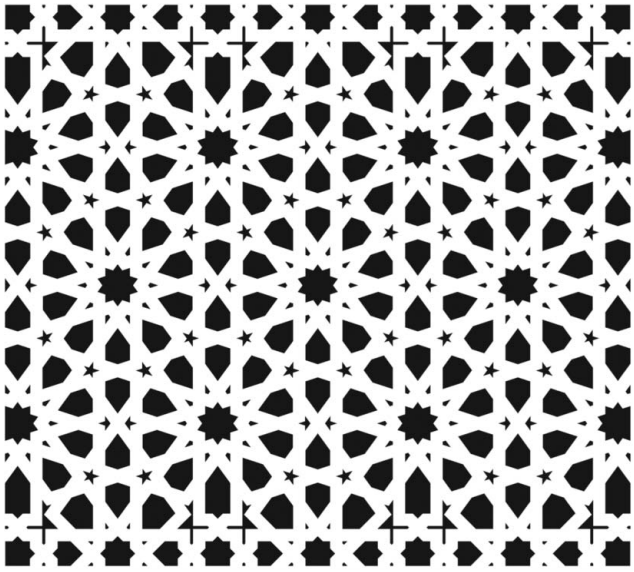


Figure 9
A tenfold pattern (Madrasa Ben Youssef, Marrakech).

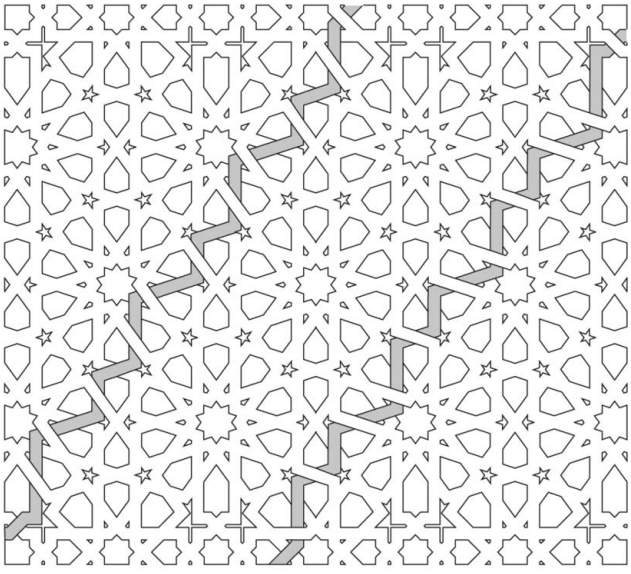


Figure 10
Continuity of ribbons in a tenfold pattern.

groups *p4mm* and *c2mm*, respectively, and the repeat units are eight- and 16-fold rosettes, respectively (Fig. 7).

To build an octagonal quasiperiodic pattern, we reproduce the square and rhombic tiles traced in Fig. 8. The ornament of the rhombic tiles consists of two eightfold rosettes and petals of a 16-fold rosette distributed over the four corners of the rhombus: two petals at small angles (45°) and six petals at large angles (135°). The square tile has four eightfold rosettes and four elements of a 16-fold rosette spread over the four corners of the square. The two types of tile arranged in the plane according to octagonal tiling give an octagonal quasiperiodic pattern composed of eight- and 16-fold rosettes (Fig. 8).

4. Construction of new quasiperiodic patterns

The quasiperiodic patterns are obtained from a quasiperiodic tiling, which is the skeleton of the pattern, decorated with elements extracted from ornamental patterns constructed by a method used by craftsmen, called *Hasba*.

4.1. Ornamental patterns constructed by the craftsmen's *Hasba* method

This method of construction of geometric patterns was described exhaustively by Thalal *et al.* (2011). It leads to patterns constituted of *n*-fold rosettes and *n*-star shapes, where *n* is the number of rays, for instance $n = 10$ (Fig. 9).

Patterns obtained by the *Hasba* method have interlaced ribbons of constant width, which represents the unit of measure of the pattern. Even though they are hidden in the above patterns, interlaced ribbons are naturally present. We can reveal the ribbons by extending, according to certain rules, the edges of the different shapes that constitute the patterns. By marking out the hidden ribbons, a more artistic interlaced pattern emerges from the initial one. The main condition imposed by the *Hasba* method is that the ribbons should always be continuous throughout the pattern (Fig. 10).

The *n*-fold rosettes are the elements used to decorate the skeleton constructed by the multigrid method to obtain a quasiperiodic pattern.

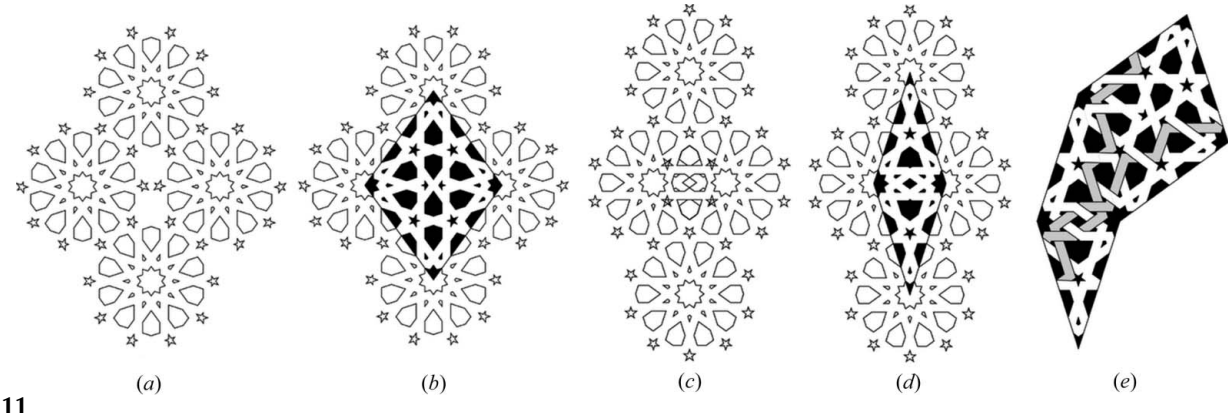
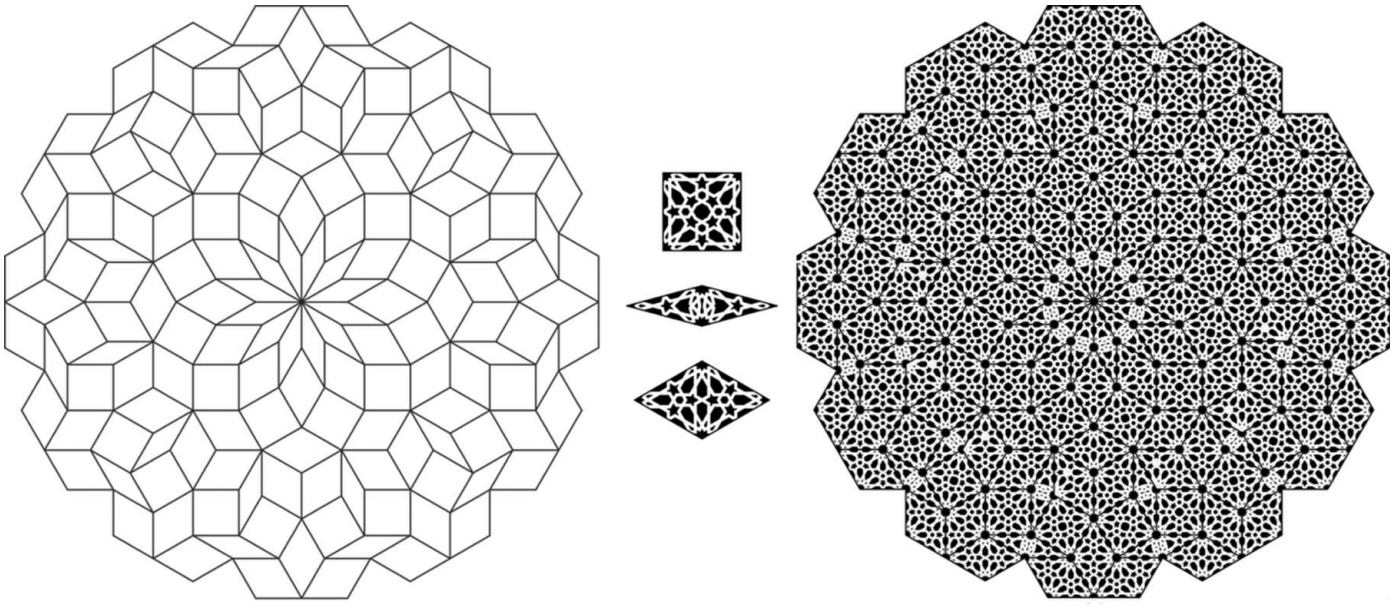


Figure 11
(*a*) Stacking of non-interpenetrating tenfold rosettes. (*b*) A tile with a vertex angle of 72°. (*c*) Stacking of interpenetrating tenfold rosettes. (*d*) A tile with a vertex angle of 36°. (*e*) Ribbon continuity across adjacent tiles.



(*a*) (*b*) (*c*)

Figure 12
(*a*) Dodecagonal tiling. (*b*) Square and rhombus tiles. (*c*) A dodecagonal quasiperiodic pattern.

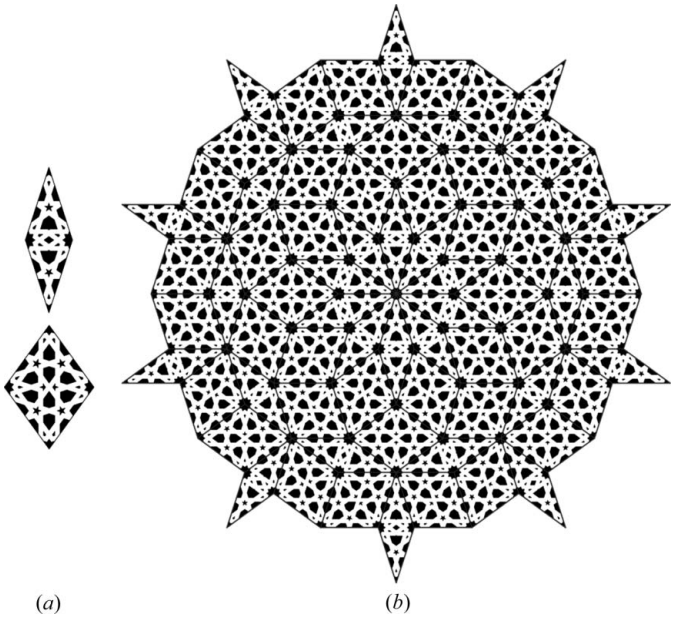
4.2. Description of the method

The combination of the *Hasba* and multigrid methods leads to quasiperiodic patterns that are not encountered in either Moroccan decorative art or diffraction patterns.

A quasiperiodic pattern is a decorated quasiperiodic tiling obtained from an N-grid with a rotation angle $\theta = 2\pi/N$. The decorative elements of the tiles are extracted from a stacking of n-fold rosettes, where the tile vertices coincide with the centres of the rosettes (Figs. 11*b* and 11*d*).

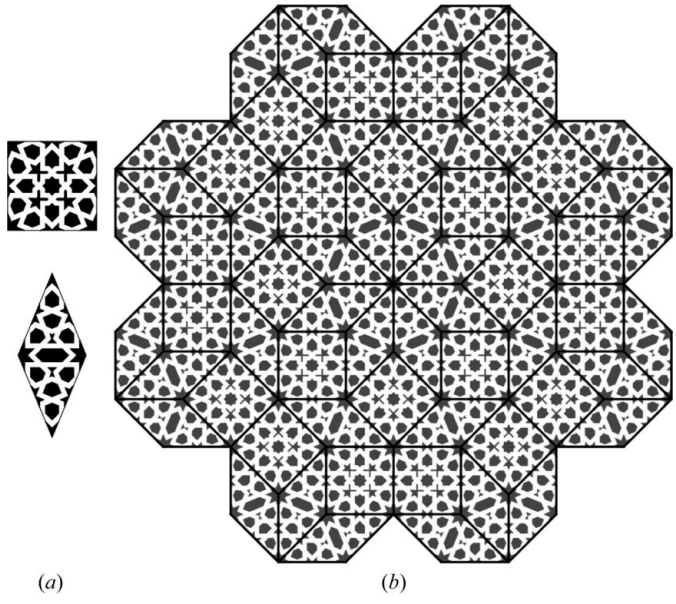
The arrangement of the tiles obeys the criterion of continuity of the ribbons required by *Hasba*, as mentioned in §4.1. Indeed, the ribbons must always be continuous across adjacent tiles over the entire pattern (Fig. 11*e*).

Furthermore, the order p of symmetry of the rosettes used to decorate the tiles must be compatible with its angles, which



(*a*) (*b*)

Figure 13
(*a*) Decorated tiles. (*b*) A decagonal quasiperiodic pattern.



(*a*) (*b*)

Figure 14
(*a*) Decorated tiles. (*b*) An octagonal quasiperiodic pattern.

are multiples of the rotation angle θ of the N-grid. It follows that the order p must satisfy the relation

$$p = k2\pi/\theta, \tag{1}$$

where k is an integer ≥ 1.

4.3. Application to known quasiperiodic patterns

The dodecagonal, decagonal and octogonal patterns have already been described by several authors who used different approaches (Makovicky & Makovicky, 2011; Makovicky, 1992, 2004, 2007, 2008, 2011; Makovicky & Fenoll Hach-Alí, 1997; Makovicky *et al.*, 1998; Al Ajlouni, 2011; Castera & Jolis, 1991; Castera, 1996, 2003). In this section, we present a variant of these three patterns obtained by the combination of the *Hasba* and multigrid methods.

4.3.1. Dodecagonal patterns. This quasiperiodic pattern is built up by a square tile and two types of rhombic tiles having respective vertex angles of 30 and 60°. The tiles are decorated, and arranged according to a dodecagonal tiling obtained from a 6-grid (Fig. 12).

4.3.2. Decagonal patterns. The decagonal tiling is obtained from a 5-grid with a rotational angle $\theta = 2\pi/5$. It consists of two rhombic tiles with vertex angles of 36 and 72°. The order of symmetry of the rosette compatible with these angles is $p = 10$. The tiles are then extracted from tenfold rosettes arranged in two configurations. In the first configuration, the rosettes do not overlap, and the tile with a vertex angle of 72° is obtained by joining the centres of four rosettes (Fig. 11*a*). The second configuration, in which the rosettes are interpenetrating, provides the tile with a vertex angle of 36° (Fig. 11*c*). The filling of the central gap in the first configuration and the overlapping arrangement of the second must satisfy the continuity rules imposed by *Hasba* (Figs. 11*b* and 11*d*).

A quasiperiodic decagonal pattern is then constructed with the two tiles according to Penrose tiling (Fig. 13). Other

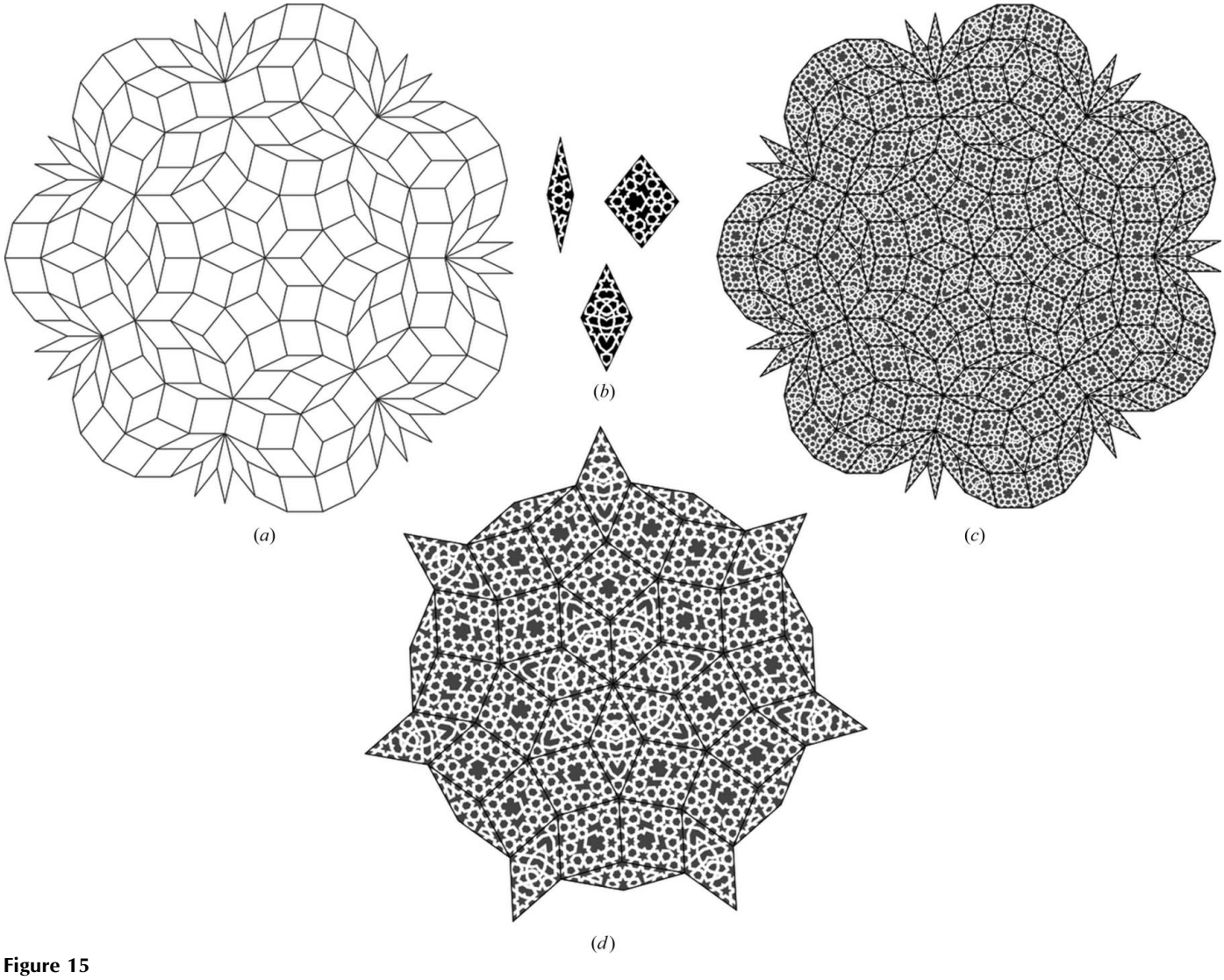


Figure 15
(*a*) Heptagonal tiling. (*b*) Decorated tiles. (*c*) A heptagonal quasiperiodic pattern. (*d*) A finite part of the quasiperiodic pattern.

configurations are possible, offering a wide variety of decagonal quasiperiodic patterns other than those described above.

4.3.3. Octagonal patterns. The same process leads to octagonal quasiperiodic patterns (Fig. 14). The decorated tiles are obtained from eightfold rosettes and arranged in an octagonal tiling.

4.4. New quasiperiodic patterns

In this section we present the heptagonal, enneagonal, tetradecagonal and octadecagonal patterns, which are not encountered in Moroccan geometric art.

Franco *et al.* (1996) proposed enneagonal tiling of the plane using geometric shapes. Seven- and ninefold quasiperiodic tilings very similar to ours were obtained by Whittaker & Whittaker (1988), who used the method of projection of N-dimensional space. In addition, another way of constructing a sevenfold tiling was presented in another context by Bonner & Pelletier (2012).

4.4.1. Heptagonal and tetradecagonal patterns. Both patterns are underlain by a heptagonal tiling obtained from a 7-grid with a rotational angle $\theta = 2\pi/7$.

Heptagonal and tetradecagonal patterns are constructed by three rhombic tiles with vertex angles of $\pi/7$, $2\pi/7$ and $3\pi/7$, and decorated by seven- and 14-fold rosettes, respectively (Figs. 15 and 16).

4.4.2. Enneagonal and octadecagonal patterns. Enneagonal and octadecagonal tilings are obtained from a 9-grid with a rotational angle $\theta = 2\pi/9$. They are constituted of four rhombic tiles with vertex angles of $\pi/9$, $2\pi/9$, $3\pi/9$ and $4\pi/9$. The

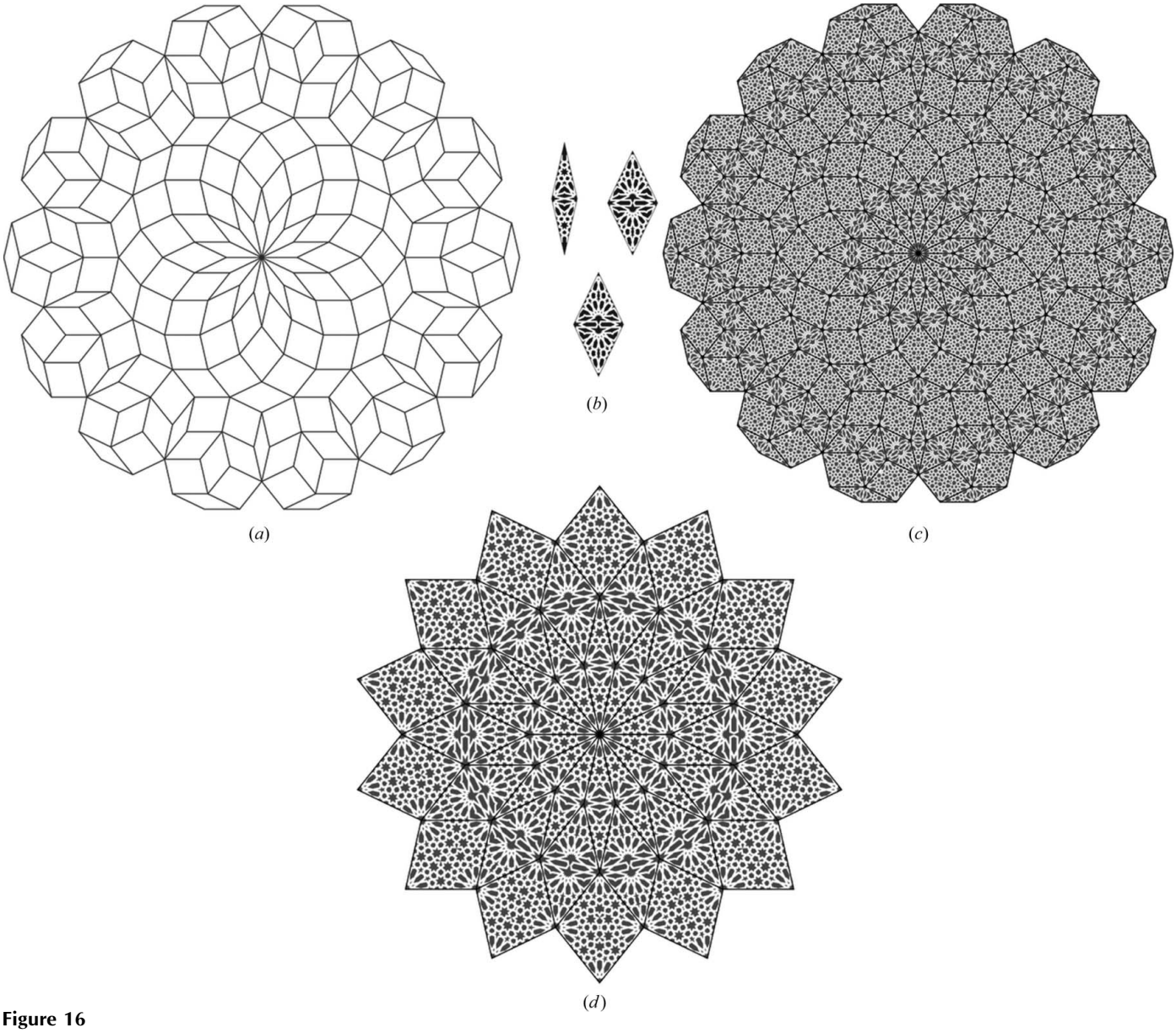


Figure 16
(*a*) Tetradecagonal tiling. (*b*) Decorated tiles. (*c*) A tetradecagonal quasiperiodic pattern. (*d*) A finite part of the quasiperiodic pattern.

decorated tiles extracted from the 18-fold rosette give an enneagonal and an octadecagonal pattern, respectively, as shown in Figs. 17 and 18.

5. Conclusion

The combination of the *Hasba* and multigrid methods confirms the previous conclusions of Makovicky *et al.* (1998) that, since the 14th century, Moroccan and Andalusian craftsmen have built quasiperiodic ornamental patterns. These uncommon patterns in Moroccan geometric art, which are different from the usual periodic patterns, were called by the craftsmen *mkhabal laâqol*, which means 'patterns that disturb the mind'. Perhaps the nonperiodic distribution of rosettes caused confusion in their minds, which were more used to working with periodic arrangements.

The method presented in this paper allows reproduction of all the traditional patterns found in historical monuments and proposes their variants. It also offers the opportunity of building new quasiperiodic patterns from periodic ones that are much more common in Moroccan and Andalusian geometric art. As the latest development in crystallography after the discovery of quasicrystals, unconventional quasiperiodic patterns such as heptagonal, tetradecagonal, enneagonal, octadecagonal and other patterns will give new impetus to Moroccan geometric art and open a new era of contemporary geometric art.

The multigrid method allows the construction of any type of plane tiling. The method is very simple and lends itself very well to computer calculation. This approach, which would be in tune with modern science, will open new horizons for a generation of artisans steeped in modern science and technology.

In addition, the relationship between mathematics and art is also a powerful tool for teachers. These complex patterns offer

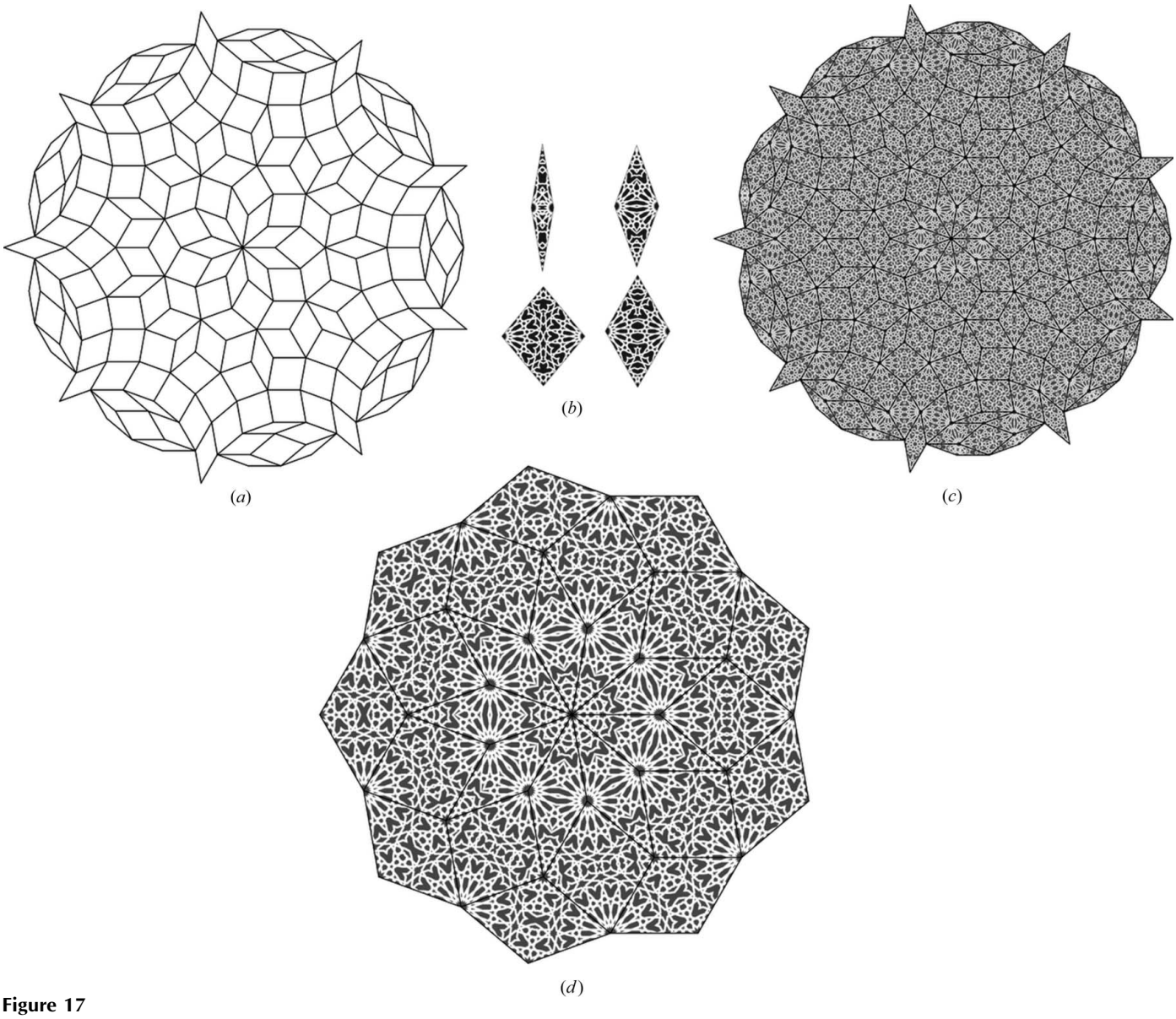


Figure 17
(*a*) Enneagonal tiling. (*b*) Decorated tiles. (*c*) An enneagonal quasiperiodic pattern. (*d*) A finite part of the quasiperiodic pattern.

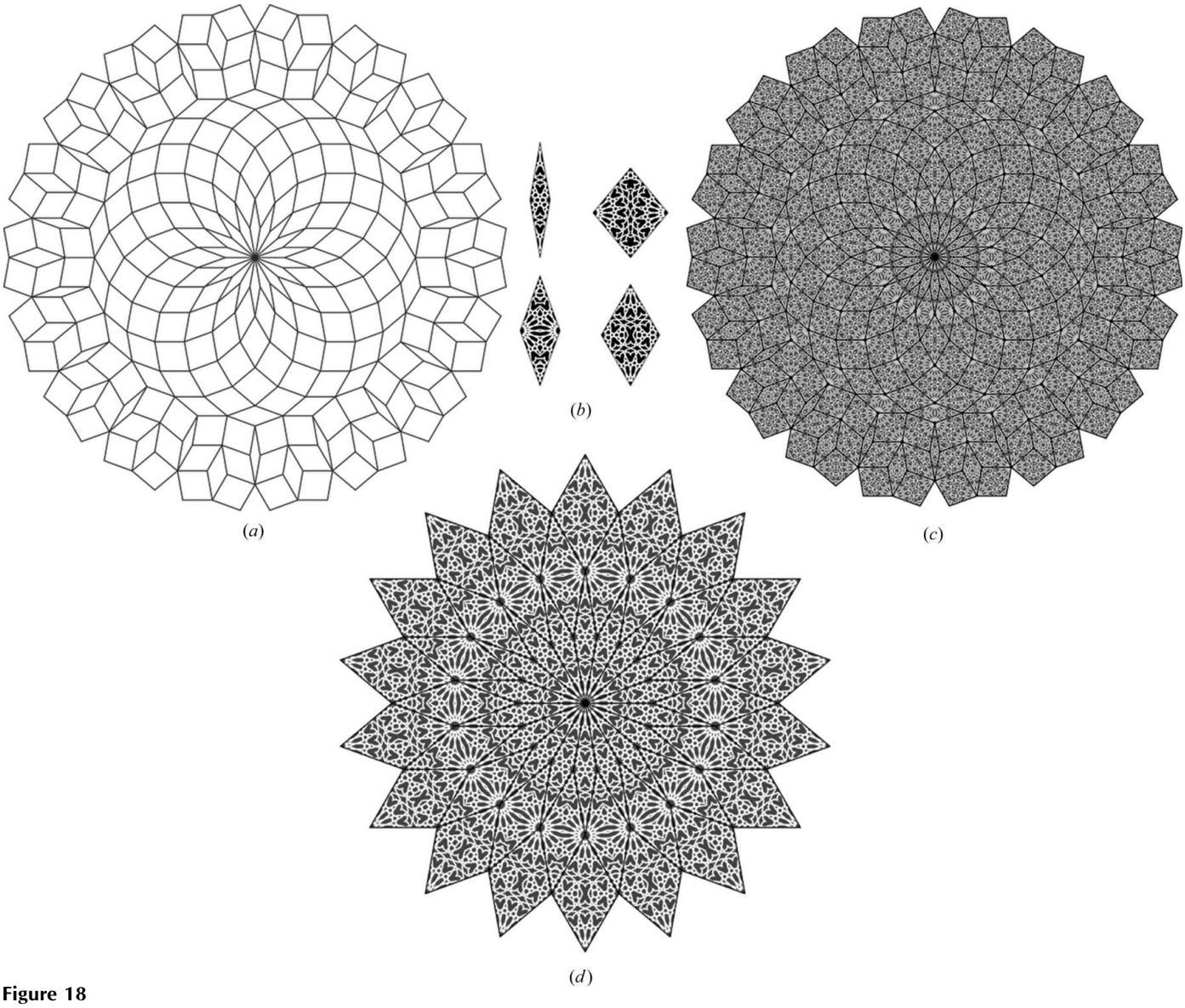


Figure 18
(*a*) Octadecagonal tiling. (*b*) Decorated tiles. (*c*) An octadecagonal quasiperiodic pattern. (*d*) A finite part of the quasiperiodic pattern.

students examples of the artistic application of symmetry and geometry. The history of geometric design contains numerous examples of collaborations between mathematicians and artists (Tennant, 2003). The construction of Islamic tiling lies on the interesting border between mathematics and art. These constructions, which have a rich history involving both mathematicians and artisans, could motivate students and increase their interest in mathematics and crystallography.

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