Autonomic Life as the Proof of Incompleteness and Lawvere's Theorem of Fixed Point

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ABSTRACT

If finite velocity of observation propagation is taken into consideration, one is faced with a paradox or infinite regression in his own logic. It is the same as Goedel's theorem of incompleteness or Wittgenstein's paradox, and in general it can be formally replaced by Lawvere's theorem of a fixed point. To describe time evolution under finite velocity of observation propagation is not to construct a new logic in which the paradox can be removed or improved, but to accept the paradox itself. Given a system in which a paradox is deduced by taking the finite velocity of observation propagation, we can regard the whole formal description, which consists of the system and a proof sequence, toward the paradox, as an evolutionary system. A formal evolutionary system defined here is one demonstration of autonomic life.

1. INTRODUCTION

Many authors have discussed the relationship between biological systems and self-referential forms, and it has been pointed out that the metabolic system and/or self-organization shows positive feedback loops [7, 12–14]. Varela is one of the researchers who has concentrated on the self-referential property in living systems. However, in spite of his emphasis on autonomy in biological systems, his concrete models that explain some biological systems are based on a Newtonian paradigm, or on a programmable system (i.e., for a

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given input, we can determine the output under some restriction or probability) [14]. The self-referential process cannot be separated from paradox in principle; however, Varela and other researchers who refer to the self-referential form have an idea that whether it involves paradox or not depends on the descriptive manner of an observer, and that paradox is just an epistemological representation (Varela does not use the term representation). According to such an idea, paradox appears in a logical level, but if you go beyond that very level and reach the hyper-level, the paradox can be excluded. It is illustrated that a paradox is in a two-valued logic of Spencer-Brown's primary algebra embedded in Varela's extended algebra [14]. Varela referred to Lawvere's theorem, and suggested that we can construct a new logic to erase the paradox by regarding a fixed point as the new meaning value of a new logic [13].

We also pointed out the significance of self-reference in constructing formal autonomic life, but our stance is different from Varela and others [3–5]. Paradox in a self-referential form is formally described in the form of Goedel's theorem of incompleteness, and we regard it as one variation of Wittgenstein's idea [9, 15] about the existence of meaning. A skeptical ideal on the existence of meaning cannot be erased in principle, and there does not exist a logical level in which paradox can be improved [3, 4]. Therefore, we are convinced that to understand living systems is not to improve a paradox and construct a hyper-logic, but to accept the paradox itself. We cannot ignore that if one tends to improve paradox in describing a living system, he believes that the paradox must lead to death, which is a result of naive realism.

In previous papers [3–5] we discussed that the formal description of time evolution under finite velocity of observation propagation falls into infinite regression. Hence, whenever we describe objects, if we ignore the effect of finite velocity of observation propagation, we can regard any objects as machines, and otherwise we can regard any objects as autonomic systems. Therefore, the question, "Is this system autonomic or not?" is not a well-defined question.

Our stance toward an autonomic system is to accept a non-well-defined question, and it is replaced by regarding the proof of the paradox itself as a system. With respect to the feature of which paradox cannot be improved and is removed, such a system demonstrates perpetual disequilibration [5, 11]. In this paper, we adopt Lawvere's theorem of a fixed point as the proof of a fixed point (which is a general form proving logical paradox) and Goedel's theorem of incompleteness. We first estimate the relationship between infinite regression in the description of a system proceeding under finite velocity of observation propagation and paradox in the self-referential form, featuring Lawvere's theorem of a fixed point [10]. Second, we find the evolutionary system in the fixed point itself; in other words, we construct an

evolutionary system, called *self-referential system*, which constitutes a fixed point. Finally, we discuss the features of this system and refer to a future prospect of this model.

2. INTERNAL MEASUREMENT AND SELF-REFERENCE

It was discussed that any descriptions of life must involve an observer himself. Strictly speaking, the distinction between life from nonlife (i.e., machine) must depend upon the knowledge of a machine and of a man who wishes to distinguish life from the machine. However, as far as the machine is defined by something programmable [2], it is clear that the question of how to distinguish life from nonlife itself is not a well-defined question. On one hand, the programmability requires the assumption of infinite velocity of observation propagation. On the other hand, it must be a finite time to measure or observe the information in any interactions, whether the interaction is realized in what we call a machine or not.

If we ignore the effect of the finite velocity of observation propagation, all interaction or information processing can be programmed, and then we can regard all interactions as machines. Otherwise, we must regard all interactions as life. The concept of life and machine sound complementary to one another in this sense. In this section, we estimate the effect of finite velocity of observation propagation and consider the cause of unprogrammability. It is formally defined in the comparison to Lawvere's theory of fixed points.

Imagine that a system consists of many components and one component has its own internal state and transforms it through interaction with other components. For the sake of simplification, the transformation of its own internal state, which is represented by a, depends only on its own internal state, and the other component's internal state is represented by b as in Figure 1. The state values are represented by $S = \{\alpha, \beta, \gamma \ldots\}$, and we

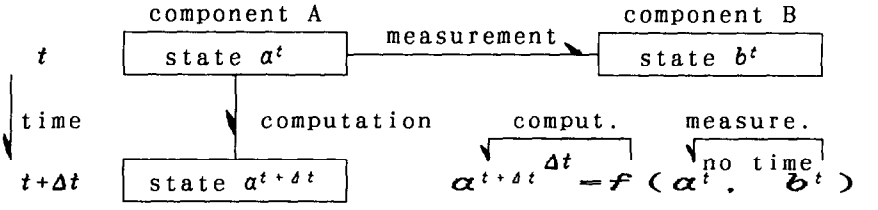


FIG. 1. Schematic diagram to express the problem of internal measurement. If we take finite velocity of observation propagation into consideration, i.e., we accept that for any measurements it takes finite time, the descriptions for such interactions must fall into infinite regression.

consider the finite time Δt to transform the internal state into consideration. Therefore, the general definition for this interaction is,

$$a^{t+\Delta t} = f_0(a^t, b^t), \tag{1}$$

where f_0 represents a specific function defining interaction. It is convenient for us to articulate the interaction process into the computation and measurement (observation) processes because we estimate the aspect of finite velocity of observation propagation. As shown in Figure 1, the output of the computation resulting from the operation of function f_0 represents computation, and the preparation of the operation of f_0 represents measurement. Note that if we accept that it takes finite time Δt to compute the transformation, the measurement defined in this sense cannot be distinguished from computation in this definition. This implies that it must take a finite time for the observation or measurement and that a process requiring the finite time must imply the transformation (change) of the state. For example, a neuron computes its own internal state by receiving input sent from other neurons, and the reception itself is realized by the diffusion-reaction process [1, 6].

If we accept (1) for the description of the interaction, we ignore the finite time to measure the other internal state because there is no time between a^t and b^t. In other words, we implicitly assume an infinite velocity of observation propagation. Therefore, if we wish to define finite velocity of observation propagation explicitly, we have to define the equations like

$$a^{t+\Delta t} = f_0(a^{t+\Delta t'}, b^t) \tag{2a}$$

$$a^{t+\Delta t'} = f_1(a^t, b^t). \tag{2b}$$

Here we define that it takes a finite time $\Delta t'$ to measure the internal state of the other component b^t. Of course the formulation like $a^{t+\Delta t} = f_0(a^t, b^{t-\Delta t'})$ cannot express a finite velocity of observation propagation because any measurement consuming finite time $\Delta t'$ implies something computing. It should be defined in the form of (2b). (2) implies that there exists a different time scale between intercomponents and intracomponents' computation. This concept is formalized in a coupled map lattice and/or Conrad's system based on enzymatic neurons.

However, (2) is just an approximation because in this definition we ignore the finite time of measurement in (2b). There is no time between a^t and b^t

again. Hence, we have to introduce another equation and define,

$$a^{t+\Delta t} = f_0(a^{t+\Delta t'}, b^t), \tag{3a}$$
$$a^{t+\Delta t'} = f_1(a^{t\Delta t''}, b^t), \tag{3b}$$
$$a^{t+\Delta t''} = f_2(a^t, b^t). \tag{3c}$$

If we do not accept the stance of approximation in which it is good enough to define a set of equations in order to express the aspect of finite velocity of observation propagation, (3) is not good enough to express the aspect, and this system of equations falls into infinite regression as

$$\begin{aligned} a^{t+\Delta t} &= f_0(a^{t+\Delta t'}, b^t), \\ a^{t+\Delta t'} &= f_1(a^{t+\Delta t''}, b^t), \\ a^{t+\Delta t''} &= f_2(a^{t+\Delta t'''}, b^t), \\ &\vdots \\ a^{t+\Delta t'''} \ldots &= f_n(a^{t+\Delta t''''} \ldots, b^t), \\ &\vdots \ . \end{aligned} \tag{4}$$

First, we define that the interaction process consists of computation and measurement, but the concept of measurement cannot be identified or distinguished from computation. In this sense, both computation and measurement can be programmable. However, if we introduce finite velocity of observation propagation in our formalization, the measurement itself is by all means embedded in infinite regression like (4). An infinite regression or an infinite series of measurements (whole system of (4)), however each element (each equation in the system (4)) is assumed to be programmable, is a problem of measurement, and this is called *internal measurement*. In other words, an infinite regression of equations like system (4) results from the assumption that the measurement process can be programmed. Now we confirm that finite velocity of observation propagation leads to the description that must involve internal measurement. To formalize internal measurement is to define a form of life.

This type of infinity, internal measurement, is different from a mathematical limit because a series of $f_1, f_1, f_2 \ldots$ can be formalized neither by projective nor reductive limit. Therefore, it cannot be explicitly expressed. It can be expressed only by showing a paradox that resulted from the assumption in which finite expression is possible. The general form to show a

paradox is formalized in the form of Lawvere's theorem of a fixed point [10]. We first examine this type of formalization in order to formalize internal measurement. Lawvere's theorem of a fixed point mainly consists of a diagonal argument and the assumption of self-referential property defined as follows.

DEFINITION 1: SELF-REFERENTIAL PROPERTY. An object A and an arrow $f: a \to Y^A$ has a self-referential property, if (5)

$$\begin{array}{ccc} A \times Y^A & \xrightarrow{ev} & Y \\ {\scriptstyle 1_A \times f}\uparrow & & \uparrow{\scriptstyle {}^{\forall}g} \\ A \times A & & \\ {\scriptstyle a \times {}^{\exists}b}\uparrow & & \\ 1 & \xrightarrow[{}^{\forall}\alpha]{} & A \end{array} \tag{5}$$

is commutative, i.e., for ${}^{\forall}g: A \to Y$, it hold that, for ${}^{\forall}a$, ${}^{\exists}b$ such that

$$g(a) = f(b)(a), \tag{6}$$

where $1_A: A \to A$ is an identity arrow and ev: $A \times Y^A \to Y$ is defined by $\mathrm{ev}(a, g) = g(a)$.

Lawvere proved the following theorem.

THEOREM 1: LAWVERE'S FIXED POINT THEOREM. *In Topos E, if* $f: A \to Y^A$ *has self-referential property, Y has a fixed point.*

PROOF. In Topos E, there exists Y^A, and, therefore,

$$\begin{array}{ccc} A \times Y^A & \xrightarrow{ev} & Y \\ {\scriptstyle 1_A \times f}\uparrow & & \| \\ A \times A & \xrightarrow[f']{} & Y \end{array} \tag{7}$$

is commutative. Therefore, let $g: A \to Y$ be the composition

$$\begin{aligned} g(a) &= t \circ f' \circ \Delta(a) \\ &= t \circ \mathrm{ev} \circ (1_A \times f) \circ \Delta(a) \\ &= t(f(a)(a)). \end{aligned} \tag{8}$$

for any arrow $t: Y \to Y$. Note that $\Delta: A \to A \times A$ is a diagonal arrow, where $\Delta(a) = (a, a)$.

On the other hand, because f has a self-referential property, it holds that

$$g(a) = f(b)(a). \tag{9}$$

Because (8) and (9) hold for any a, we can choose b for a, and obtain,

$$f(b)(b) = t(f(b)(b)). \tag{10}$$

where $f(b)(b)$ is a fixed point. ■

According to Lawvere's theorem of a fixed point, Goedel's theorem of incompleteness is one variation in the proof of the paradox resulted from the assumption of the self-referential property.

THEOREM 2: THEOREM OF INCOMPLETENESS. *In Boolean Topos, if* $0 \simeq 1$ *does not hold, then* $f: A \to Y^A$ *has no self-referential property.*

PROOF. Supposing that f has self-referential property. Y must have a fixed point because of Theorem 1. However, substituting $\neg$ for t, $\neg: 2 \to 2$ defined by $\neg(0) = 1$ and $\neg(1) = 0$ yields contradiction.

On one hand, that $0 \simeq 1$ does not hold prohibits that the meaning of any propositions can be both 0 and 1. This implies that the logic is consistent. On the other hand, that f has no self-referential property means that self-definability cannot be proven logically, and, for example, it means incompleteness.

The impossibility can only be shown by the paradox. The existence of the meaning in naive realism is denied by Wittgenstein and Kripke [4, 15], and this kind of proof is the same as Lawvere's theorem of a fixed point. In naive realism, it is assumed that the meaning is certain, and this causes a paradox according to Kripke. Let $a \in A$ be a word, and $g \in Y^A$ is finding a rule (referent). Hence, a word can be projected into meaning space Y, for example, a two-valued set, and be either acceptable or not. What we assume, that a meaning or a rule found for a word is certain, is something we do not have to consider about another possibility of a rule. This implies the assumption of which $f: A \to Y^A$ is surjective. In the context of Wittgenstein and Kripke, what we can by no means deny as another possibility for a referent is expressed by infinite regression. It is formally replaced by a fixed point.

Internal measurement itself can now be formalized by the proof of a fixed point. Moreover, the proof of a fixed point implies not only the destruction of naive realism, but also an alternative form of a system since we can regard the proof of a fixed point itself as a system. The system constructed in this manner must involve the engine that leads to self-contradiction in principle. Therefore, we can formalize a system that evolves under finite velocity of observation propagation using a fixed point.

3. SELF-REFERENTIAL FORM AND THE ASPECT OF EVOLUTION UNDER FINITE VELOCITY OF OBSERVATION PROPAGATION

First, let a system consist of many components called cells, and image that cells interact with each other. If the velocity of observation propagation is infinite, the interaction is uniquely decided by a precise observation, and otherwise it cannot be decided in spite of the very best observation. We define the following virtual system; If it is under infinite velocity of observation propagation, this description is possible [4]. After that, we define the aspect in which this system is under finite velocity of observation propagation, using a fixed point.

DEFINITION 2: VIRTUAL TRANSITION RULE. The collection of the symmetric rule ϕ: $A^2 \times B \to C$, $A = B = C = \{0, 1\}$ is defined by

$$R_\phi = \{\phi | A^2 \times B \to C, \phi((a, a'), b) = \phi((a', a), b)\}, \qquad (11)$$

and is called a system of the virtual transition rule.

In order to identify the aspect of local transition, we call A a set of input states from the neighboring cells. B that of the internal states, and C that of the states next to the internal states. The transition rule ϕ: $A \times B \times A \to C$ implies that a cell transforms its own internal state by receiving input from the nearest neighboring cell and is described, in general, as $a_i^{t+1} = \phi(a_{i-1}^t, a_i^t, a_{i+1}^t)$.

R_ϕ equals the class of symmetric rules called elementary cellular automata. We refer to the rule number defined by Wolfram [16]. The rule number of ϕ, which is defined by $d_k = \phi((a, a'), b)$, $k = 4a + 2b + a'$ where a, a', b and $d_k \in \{0, 1\}$, is represented by

$$\#(\phi) = \sum_{j=0}^{7} 2^j d_j. \qquad (12)$$

Now we assume that a category is cartesian closed and there exists an exponential. The reason for this assumption is discussed later. Therefore, we can define $\hat{\phi}$, the transpose of ϕ, and use an adjoint functor. For ϕ: $A^2 \times B \to C$, there uniquely exists $\hat{\phi}$: $B \to C^{A \times A}$ such that commutes (13).

$$\begin{array}{ccc} A^2 \times C^{A\times A} & \xrightarrow{\text{ev}} & C \\ {\scriptstyle 1_{A\times A}\times\hat{\phi}}\uparrow & & \| \\ A^2 \times B & \xrightarrow{\phi} & C \end{array} \tag{13}$$

Using this diagram, we can express the aspect of *internal measurement* as follows. This is as same as the definition in our previous paper [4].

DEFINITION 3: MEASUREMENT AND COMPUTATION UNDER INFINITE VELOCITY OF OBSERVATION PROPAGATION. Owing to the existence of an exponential, virtual system ϕ: $A^2 \times B \to C$ can be articulated into ev: $A^2 \times C^{A\times A} \to C$ and $\hat{\phi}$: $B \to C^{A\times A}$ such that

$$\phi((a, a'), b) = \text{ev} \circ \left(1_{A\times A} \times \hat{\phi}\right)((a, a'), b). \tag{14}$$

Here we define $\hat{\phi}$ and ev as the *measurement* of input and *computation*, respectively. This is because, at first by receiving an input (a, a'), the internal switch f: $1 \to C^{A\times A}$, by which the transition of internal state is realized, is decided, and then the transition of internal state is computed. In other words, the existence of an exponential entails what measurement and computation are expressed in a lump, and we can ignore the internal measurement in describing local interaction.

Therefore, we define that if the velocity of observation propagation is *infinite*, then (14) is satisfied.

Strictly speaking, infinite velocity of observation propagation also results from the existence of *projective limit*. In a category theory, projective limit is a limit, and if the limit is defined in a category, it can be proven that the category has products, a pullback, an equalizer, and a terminal. The definition of limit in a category implies the assumption of infinite velocity of observation propagation because it is in the definition of limit, which assumes that for any observers (Y) who observe a series of compositions of morphisms, there uniquely exists an morphism t such that $u_i = t \circ v_i$ where u_i represents observation by an observer Y, and v_i represents observation from limit. Hence, it is assumed that there exists a universal observer who can observe whole space at a moment. It is a point of view of God with infinite velocity of observation propagation. In other words, a "true" observation is constructed by using universality.

As far as velocity of observation propagation is finite, we are faced with the aspect in which operand (internal state b) cannot be independently separated from operator (internal switch f). This aspect cannot be formally described; however, we have to manage to describe it as much as possible. If we wish to describe a system in the form of a map along a time axis (i.e., for any input states, there uniquely exists an output state), the described system must involve the assumption of infinite velocity of observation propagation with a real system (which would be described) that is under finite velocity of observation propagation. Therefore, in other words, the proof of a contradictory fixed point entails the articulation of a real and virtual system. To describe this aspect and to connect it with Lawvere's fixed point theorem, we define a weak self-referential property.

DEFINITION 4: WEAKLY SELF-REFERENTIAL PROPERTY. An object A and an arrow $\hat{\phi}: B \to C^{A\times A}$ has weak self-referential property, iff for any $((a, a'), f): 1 \to A^2 \times C^{A\times A}$, there exists $b: 1 \to B$ such that

$$f(a, a') = \hat{\phi}(b)(a, a'). \tag{15}$$

An observer's behavior manages to describe a system whether it is under finite or infinite velocity of observation propagation, and can be replaced by weak self-referential property that will be discussed later. We prepare a more detailed definition for local transition rules.

DEFINITION 5: CODE FOR INTERNAL SWITCH. For any internal switch $f: A^2 \to C$, the code for a switch $\#(f)$ is defined by,

$$\#(f) = \sum_{j=0}^{3} 2^j r_j, \tag{16}$$

where

$$r_j = f(\alpha, \beta), \qquad j = 2\alpha + \beta. \tag{17}$$

DEFINITION 6: f WHICH CONSTITUTES ϕ (f-CONSTITUTION). A rule $\phi: A^2 \times B \to C$ is defined to consist of f_p and f_q (the pair $\langle f_p, f_q \rangle$) if for any arrow $((a, a'), b): 1 \to A^2 \times B$, there exists f_k either of $k = p$ or q such that,

$$\phi((a, a'), b) = f_k \circ \pi((a, a'), b), \tag{18}$$

where π: $A^2 \times B \to A^2$ is defined by

$$\pi((a, a'), b) = (a, a'). \tag{19}$$

EXAMPLE. For a rule ϕ: $A^2 \times B \to C$ defined by

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),0) & ((0,1),0) & ((1,0),0) & ((1,1),0) \\ \phi((a,a'),b) & 0 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),1) & ((0,1),1) & ((1,0),1) & ((1,1),1) \\ \phi((a,a'),b) & 0 & 0 & 0 & 0, \end{array} \tag{20}$$

ϕ consists of the pair of internal switch, either $\langle f_0, f_6 \rangle$ or $\langle f_2, f_4 \rangle$.

Using the definition of the f-constitution, we define self-satisfaction of a transition rule. It is replaced by the description of a system under finite velocity of observation propagation assuming infinite velocity of observation propagation.

DEFINITION 7: SELF-SATISFACTION OF VIRTUAL RULE. A rule ϕ: $A^2 \times B \to C$ is self-satisfied if for any arrow $((a, a'), b)$: $1 \to A^2 \times B$, ϕ consists of f_0 and f_{15}.

We can prove the following proposition here.

PROPOSITION 1. There exists a self-satisfied rule in virtual system R_ϕ.

PROOF. A rule ϕ: $1 \to R_\phi$ can be defined in the form of

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),x_0) & ((0,1),x_1) & ((1,0),x_2) & ((1,1),x_3) \\ \phi((a,a'),b) & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),x_4) & ((0,1),x_5) & ((1,0),x_6) & ((1,1),x_7) \\ \phi((a,a'),b) & 1 & 1 & 1 & 1, \end{array} \tag{21}$$

iff ϕ consists f_0 and f_{15}.

Therefore, when it is satisfied that

$$x_j = 1 - x_{j+4}, \tag{22}$$

$x_j: 1 \to B(=\{0, 1\})$, $j = 0, \ldots, 7$, ϕ can be defined as a map, which means $\phi: 1 \to R_\phi$. ■

PROPOSITION 2. $\hat{\phi}: B \to C^{A \times A}$ has a weak self-referential property iff $\phi: A^2 \times B \to C$ is self-satisfied.

PROOF.

(i) First we prove that $\hat{\phi}$ has a weak self-referential property if ϕ is self-satisfied. From the assumption, ϕ consists of f_0 and f_{15}. Simultaneously (14) implies that

$$\hat{\phi}(0) = f_p, \qquad \hat{\phi}(1) = f_q. \tag{23}$$

In addition, from definition (21), it is clear that for any $(a, a'): 1 \to A^2$,

$$f_p(a, a') \neq f_q(a, a'). \tag{24}$$

Therefore, for any $((a, a'), f): 1 \to A^2 \times C^{A \times A}$, there exists $b: 1 \to B$ such that

$$f(a, a') = \hat{\phi}(b)(a, a'). \tag{25}$$

Then $\hat{\phi}$ has a weak self-referential property.

(ii) Second, we prove the back. From the assumption, for any $(a, a'): 1 \to A^2$ there exists $b: 1 \to B$ such that

$$f_0(a, a') = \hat{\phi}(b)(a, a'), \tag{26}$$

$$\hat{\phi}(b)(a, a') = 0. \tag{27}$$

Simultaneously, we obtain similar form with (26) for f_{15}, and we obtain

$$\hat{\phi}(b)(a, a') \neq \hat{\phi}(1 - b)(a, a') \tag{28}$$

because $f_0(a, a') \neq f_{15}(a, a')$. Therefore, we prove that for any $((a, a'), b)$: $1 \rightarrow A^2 \times B$, it holds that

$$\phi((a, a'), b) = f_k \circ \pi((a, a'), b), \tag{29}$$

where k equals to either 0 or 15. It means that ϕ consists of f_0 and f_{15}. ■

Now we can estimate the aspect of interaction that proceeds under finite velocity of observation propagation. From Definition 3, we can accept that the assumption of infinite velocity of observation propagation shows the existence of Y^A. In other words, it is the existence of adjoint functors $C((-) \times B, C)$ and $C(B, C^{(-)})$ such that

$$C(A^2 \times B, C) \simeq C(B, C^{A \times A}), \tag{30}$$

where $C(X, Y)$ represents an arrow in a category C. Here we can define the one-to-one type mapping between the virtual interaction described by the virtual transition rule and internal measurement. When the velocity of observation propagation is infinite, internal measurement can be ignored because it is describable enough to express a *real* transition rule if we only define the virtual transition rule. Here a real interaction uniquely corresponds to a virtual one. Of course, there is no real existence in mathematics or logic. All we can do is to intentionally construct or define real interaction. Real interaction under infinite velocity of observation propagation can be defined by the definition of a formal definition of interaction.

On the other hand, if the real interaction that proceeds under finite velocity of observation propagation, a virtual assumption of isomorphism (30) can by no means hold in the real system. Here a one-to-one correspondence between the virtual transition rule and the internal measurement is ideal. However, if we, observers, manage to describe a system as clearly as possible, such a description always entails the contradictory assumption of infinite velocity of observation propagation. The interaction proceeding under finite velocity of observation propagation is expressed by this formalization. Imagine that a local cell that constitutes a multicellular system transforms its own internal state by receiving input from neighboring cells. It is expressed by the form of (13) or (14) in assuming the infinite velocity of observation propagation. Now, if this interaction proceeds under the finite velocity of observation propagation, it is permitted that another input can be received in the midst of computation [3]. Note that the interaction is here articulated into internal measurement and computation. Therefore, it entails misidentification of measurement and computation. It can be replaced by the mixture between $f_p, f_q: 1 \rightarrow C^{A \times A}$, of which ϕ consists, and $c: 1 \rightarrow C$. Against this mixture,

originated from the finite velocity of observation propagation, the observers can describe the real interaction, which is nothing but finding out the one-to-one correspondence between the internal switch $S_c = \{f_p, f_q\}$ and its output C [4].

If we can find that for any $(a, a'): 1 \to A^2$, the output state c, computed by f_p or f_q, is uniquely determined by,

$$c = f_p(a, a'), \tag{31}$$

and

$$1 - c = f_q(a, a'), \tag{32}$$

we can obtain the following isomorphism:

$$S_c \simeq C. \tag{33}$$

(31)–(33) can be replaced here by finding the self-satisfied rule which consists of f_0 and f_{15}. As far as it is found, there is no contradiction in which labeling f_0 (respectively, f_{15}) as 0 (respectively, 1) is not consistent with the output computed by $f_0(\alpha, \alpha')$. Hence, the virtual state (real internal switch) in the midst of the computation can be identified with the output state. Now, we show this aspect by the following definition.

DEFINITION 8: PERTURBATION THAT RESULTED FROM THE DESCRIPTION UNDER FINITE VELOCITY OF OBSERVATION PROPAGATION. Given a virtual rule $\phi: A^2 \times B \to C$, that the operation of this rule proceeds under finite velocity of observation propagation is defined with respect to the observer's descriptive manner as follows. For any $(a, a'): 1 \to A^2$, we have to find f_p and f_q such that

$$f_p(a, a') \neq f_q(a, a'), \tag{34}$$

and define $\phi_c (= \phi \text{ or } \neq \phi)$ such that consists of f_p and f_q.

The aspect, in which only either ϕ_c or ϕ cannot be expressed to describe a real rule, is an aspect under finite velocity of observation propagation. Also, we must regard both ϕ_c and ϕ as the approximated rule for the operation under finite velocity of observation propagation. Therefore, we can no longer distinguish what ϕ consists of f_p and f_q from what ϕ_c does the same, whether $\phi_c = \phi$ or not.

REMARKS. The aspect in which we manage to find ϕ_c that satisfies the form (34) is the descriptive manner in which we observers are faced with the interaction under finite velocity of observation propagation. It entails the articulation of the virtual rule ϕ_c and the real one-to-many type mapping that can be approximated into ϕ assuming the infinite velocity of observation propagation. Hence, both ϕ_c and ϕ are virtual compared to the real one-to-many type mapping. For all, we manage to describe, as highly self-consistently as possible, the description itself that is nothing but the definition of ϕ_c that paradoxically entails perturbation for description itself.

COROLLARY 1. *ϕ_c that has to be found under finite velocity of observation propagation is a self-satisfied rule.*

It is trivial. If ϕ_c consists of f_p and f_q, which satisfy the form (34), then ϕ_c also consists of f_0 and f_{15}. We here formally define how to find self-satisfied rule ϕ_c as close to the approximation of the real interaction as possible. We observers have to describe a system self-consistently. It is replaced by the description in assuming the infinite velocity of observation propagation even if our object is interacting under finite velocity of observation propagation. Hence, the contradiction of the virtual and real interactions can be replaced by that of two virtual rules, ϕ and ϕ_c. The minimamization of contradiction between ϕ and ϕ_c is the maximization of self-consistency in virtual system (i.e., our description). This is also defined later.

DEFINITION 9: CONTRADICTORY SYSTEM RESULTED FROM MEASUREMENT. Given a rule ϕ: $A^2 \times B \to C$, the contradictory system that resulted from the measurement or description itself, is defined by $\{\phi_c\}$, which is a collection of self-satisfied rules that consists of f_0 and f_{15} such that

$$\min_{p,q} \sum_{j=0}^{3} \left(|r_j(f_p) - r_j(f_0)| + |r_j(f_q) - r_j(f_{15})| \right), \tag{35}$$

$$r_j(f_p) = f_p(\alpha, \alpha'), \tag{36}$$

where ϕ consists of f_p and f_q. ϕ_c is also called a contradictory rule.

COROLLARY 2. *The aspect of the interaction (which can be described by virtual transition rule ϕ in assuming infinite velocity of observation propaga-*

tion) under finite velocity of observation propagation implies the forced assumption that $\hat{\phi}$: $B \rightarrow C^{A \times A}$ *has a weak self-referential property.*

PROOF. From Definition 8 and Corollary 1, such an aspect is replaced by finding a self-satisfied rule that satisfies Definition 10 and intentionally identifies ϕ with ϕ_c. Hence, from Proposition 2, it implies a weak self-referential property for all $\hat{\phi}$ that has no weak self-referential property. ■

A real system that is the interaction under finite velocity of observation propagation has both a weak self-referential property and the property deduced from diagonal arguments. Let the real system under finite velocity of observation propagation, which can be expressed by ϕ: $A^2 \times B \rightarrow C$ assuming infinite velocity of observation propagation, be $\langle \phi \rangle$. Now, $\langle \phi \rangle$ is a one-to-many type mapping, and then $\langle \phi \rangle$ is identified both with ϕ and ϕ_c. In other words, $\langle \phi \rangle$ has not only the properties of ϕ but also a weak self-referential property with respect to $\langle \hat{\phi} \rangle$. Now, we can replace the aspect of the interaction under finite velocity of observation propagation as follows.

DEFINITION 10: THE INTERACTION UNDER FINITE VELOCITY OF OBSERVATION PROPAGATION. A real rule under finite velocity of observation propagation, derived from a virtual rule ϕ: $A^2 \times B \rightarrow C$, is denoted by $\langle \phi \rangle$. On the one hand, a real system $\langle \phi \rangle$ has the property of ϕ (e.g., (14) is commutative). On the other hand, $\langle \hat{\phi} \rangle$ has a weak self-referential property.

REMARKS. We denote $\langle \phi \rangle$ by $\langle \phi, \phi_c \rangle$ because of the ambiguity of $\langle \phi \rangle$. Therefore, to describe the interaction under finite velocity of observation propagation, the following theorem is necessary.

THEOREM 3: FIXED POINT OF EVOLUTION UNDER FINITE VELOCITY OF OBSERVATION PROPAGATION. *In virtual system* R_ϕ, *there exists a fixed point that entails contradiction if a rule* ϕ: $A^2 \times B \rightarrow C$ *is operated under finite velocity of observation propagation.*

PROOF. Assume a rule under finite velocity of observation propagation $\langle \phi \rangle$. From Definition 10, $\langle \phi \rangle$ can be identified with ϕ, and we can use the diagonal arguments as follows.

Consider any morphism $h: C \to C$ and let $f_k: 1 \to C^{A\times A}$ be the composition that commutes (37).

$$\begin{array}{ccccccccc} & & & & A^2 \times C^{A\times A} & \xrightarrow{\text{ev}} & C & \xrightarrow{h} & C \\ & & & & {\scriptstyle 1_{A\times A}\times\langle\hat{\phi}\rangle}\uparrow & & & & \| \\ A^2 & \xrightarrow[\Delta]{} & A^2 \times A^2 & \xrightarrow[f^*]{} & A^2 \times B & \xrightarrow[\langle\phi\rangle]{} & C & \xrightarrow[h]{} & C \\ \uparrow & & & & & & & & \uparrow \\ \multicolumn{9}{c}{\text{———————————————————}} \\ & & & & & f_k & & & \end{array} \tag{37}$$

For f_k, f^* is chosen as $f^*: 1 \to C^{A\times A}$ because $B = C$. Hence, we obtain that for any (a, a'): $1 \to A^2$,

$$\begin{aligned} f_k(a, a') &= \left(h \circ \text{ev} \circ \left(1_{A\times A} \times \langle\hat{\phi}\rangle\right) \circ f^* \circ \Delta\right)(a, a') \\ &= h \circ \text{ev} \circ \left(1_{A\times A} \times \langle\hat{\phi}\rangle\right)(f^*(a, a'), (a, a')) \\ &= h\left(\langle\hat{\phi}\rangle(f^*(a, a'))(a, a'\right). \end{aligned} \tag{38}$$

Also, from Definition 10 or the forced assumption of a weak self-referential property deduced from finite velocity of observation propagation. $\langle\phi\rangle$ has weak self-referential property. We obtain

$$f_k(a, a') = \langle\hat{\phi}\rangle(b)(a, a'). \tag{39}$$

We can choose $(a*, a*')$ with $f^*(a*, a*') = b$, and then we finally obtain,

$$\hat{\phi}(b)(a*, a*') = h\left(\langle\hat{\phi}\rangle(b)(a*, a*')\right). \tag{40}$$

When we choose $\neg: C \to C$ for $h: C \to C$, $\neg(0) = 1$ and $\neg(1) = 0$, it entails contradiction. ■

THEOREM 4: CONSISTENT SYSTEM WITHOUT FIXED POINT. *If* ϕ: $A^2 \times B \to C$ *is operated under finite velocity of observation propagation, and* $\hat{\phi}$: $B \to C^{A\times A}$ *originally has a weak self-referential property, there is no fixed point. Here we call a system described by those* ϕ*s a consistent system.*

PROOF. As far as $\hat{\phi}: B \to C^{A \times A}$ has weak self-referential property, we can write $\langle \phi \rangle$ by ϕ. Consider morphism $\neg: C \to C$, and let f_k be the composition.

$$f_k(a, a') = \neg\big(\hat{\phi}(f^*(a, a'))(a, a'). \tag{41}$$

for any $(a, a'): 1 \to A^2$. Here for any $(a, a'): 1 \to A^2$, we define that $(a, a') = (s, s')$ if $f^*(a, a') = 0$, and $(a, a') = (t, t')$ otherwise. Hence,

$$f^*(s, s') = 0 \tag{42}$$

$$f^*(t, t') = 1. \tag{43}$$

Simultaneously, because ϕ is a self-satisfied rule and consists of f_p and f_q such that for any $(a, a'): 1 \to A^2, f_p(a, a') \neq f_q(a, a')$, we can define that

$$\hat{\phi}(0)(s, s') = f_p(s, s') = x \tag{44}$$

$$\hat{\phi}(1)(t, t') = f_q(t, t') = 1 - x. \tag{45}$$

Therefore, from (41) we obtain

$$f_k(s, s') = \neg\big(\hat{\phi}(f^*(s, s'))(s, s') = 1 - x \tag{46}$$

$$f_k(t, t') = \neg\big(\hat{\phi}(f^*(t, t'))(t, t') = x. \tag{47}$$

On the other hand, we also obtain for any $(\alpha, \alpha'): 1 \to A^2$, there exists b such that

$$f_k(s, s') = \hat{\phi}(b)(s, s') = 1 - x \tag{48}$$

$$f_k(t, t') = \hat{\phi}(1 - b)(t, t') = x. \tag{49}$$

Hence, on the one hand, if $b = 0$, then we obtain $f^*(s, s') = b$, but $f_k(s, s') = f_p(s, s') = x$, which contradicts (48) and (49). On the other hand, if $b = 1$, then we obtain $f^*(s, s') = 1$, which contradicts the definition of (42). Therefore, there is no $(a*, a*')$ such that $f^*(a*, a*') = b$ to connect (41) with (48). ■

As mentioned above, if ϕ is self-satisfied (i.e., $\hat{\phi}$ has weak self-referential property), contradictory morphism ϕ_c derived from ϕ leads to $\phi = \phi_c$. That is why $\langle \phi \rangle$ equals ϕ, and why there is no fixed point. Therefore, we can express $\langle \phi \rangle$ without ambiguity if ϕ is self-satisfied. It is replaced by the following definition.

DEFINITION 11: PROGRAMMABLE SYSTEM. If ϕ is self-satisfied, we call $\langle \phi \rangle$ a programmable system.

Now we can summarize the description of time-evolution under finite velocity of observation propagation as follows. Here the behavior of the description itself entails the autonomous perturbation (i.e., the assumption of weak self-referential property) by the description itself. It is replaced by the mixture of ϕ and ϕ_c with respect to $\langle \phi \rangle$. If $\phi = \phi_c$, the system is programmable, and otherwise it entails to the existence of $(a*, a*')$ such that $f^*(a*, a*') = b$ as mentioned before. Then the fixed point is introduced.

Note that the mixture between ϕ and ϕ_c causes the condition $f^*(a*, a*') = b$. Now in order to define the mixture of ϕ and ϕ_c, we define the contradictory morphism, which represents the dynamics backwards from the future to the past. First, we prove the existence of the backward rule.

PROPOSITION 3. If $\langle \hat{\phi} \rangle \colon B \to C^{A \times A}$ has weak self-referential property, there exists a backward rule $\phi^- \colon A^2 \times C \to B$ such that, for any $((a, a'), c) \colon 1 \to A^2 \times C$,

$$\phi^-((a, a'), c) = b, \tag{50}$$

where

$$\phi_c((a, a'), b) = c. \tag{51}$$

PROOF. From that assumption, there exists $\phi' \colon A^2 \times C^{A \times A} \to B$ with respect to $\langle \phi \rangle$, such that for any $((a, a'), f) \colon 1 \to A^2 \times C^{A \times A}$,

$$\phi'((a, a'), f) = b \tag{52}$$

where

$$\phi_c((a, a'), b) = f(a, a'). \tag{53}$$

On the other hand, for ev: $A^2 \times C^{A\times A} \to C$ and π: $A^2 \times C^{A\times A} \to A^2$, there exists p: $A^2 \times C^{A\times A} \to A^2 \times C$, $p = \langle \text{ev}, \pi \rangle$ that commutes (54).

$$\begin{array}{ccccc} & & A^2 \times C^{A\times A} & & \\ & \swarrow^{\pi} & \downarrow p & \searrow^{\text{ev}} & \\ A^2 & \xleftarrow{\pi} & A^2 \times C & \xrightarrow{\pi} & C. \end{array} \tag{54}$$

Morphism p can be proved, epi, by the following. In assuming that

$$\langle s, t \rangle \circ p = \langle s, r \rangle \circ p, \tag{55}$$

where s: $A^2 \to A^2$, and t, r: $C \to C$, we obtain

$$\pi \circ \langle s, t \rangle \circ p = \pi \circ \langle s, r \rangle \circ p, \tag{56}$$

$$t \circ \pi \circ p = r \circ \pi \circ p, \tag{57}$$

$$t \circ \text{ev} = r \circ \text{ev}. \tag{58}$$

Because ev is epi, we obtain

$$t = r, \tag{59}$$

$$\langle s, t \rangle = \langle s, r \rangle. \tag{60}$$

Therefore, we can define q: $A^2 \times C \to A^2 \times C^{A\times A}$ such that for any $((a, a'), c)$: $1 \to A^2 \times C$,

$$q((a, a'), c) = ((a, a'), f), \tag{61}$$

$$f(a, a') = c. \tag{62}$$

Finally, we can obtain ϕ^-: $A^2 \times C \to B$ such that for any $((a, a'), c)$: $1 \to A^2 \times C$.

$$\begin{aligned} \phi^-((a, a'), c) &= \phi' \circ q((a, a'), c) \\ &= \phi'((a, a'), f) = b, \end{aligned} \tag{63}$$

where

$$\phi_c((a, a'), b) = f(a, a') = c. \tag{64}$$

■

From Theorems 3 and 4, if ϕ is not self-satisfied, there exist some backward rules that lead to a fixed point. Our aim is nothing but to embed a fixed point in a state space. In ordinary dynamical theory, state space consists of a point that has no area, and then we express a one-to-many type mapping with coarse graining. On the other hand, we here express a one-to-many type mapping with a fixed point. Hence, we regard a fixed point itself as an evolutionary generative system or form of life. We define the backward rule, which causes a fixed point, and call it the backward-time rule.

DEFINITION 12: BACKWARD-TIME RULE. For ϕ: $A^2 \times B \to C$, $A = B = C = \{0, 1\}$, the backward-time rule ϕ^-: $A^2 \times C \to B$ is defined by the following.

(i) If for (a_1, a'_1): $1 \to A^2$ and c_1: $1 \to C$, there uniquely exists b_1 such that

$$\phi((a_1, a'_1)\, b_1) = c_1, \tag{65}$$

we define that for (a_1, a'_1): $1 \to A^2$ and c_1: $1 \to C$,

$$\phi^-((a_1, a'_1), c_1) = b_1. \tag{66}$$

(ii) If for (a_2, a'_2): $1 \to A^2$ and c_2: $1 \to C$, there exists b_{21} and b_{22} such that

$$\phi((a_2, a'_2), b_{21}) = c_2, \tag{67a.}$$

$$\phi((a_2, a'_2), b_{22}) = c_2, \tag{67b.}$$

we arbitrarily choose b_{21} or B_{22} and define

$$\phi^-((a_2, a'_2), c_2) = b^*_2, \tag{68}$$

where b^* is either b_{21} of b_{22}.

(iii) If for (a_3, a'_3): $1 \to A^2$ and c_3: $1 \to C$, there exists no b such that

$$\phi((a_3, a'_3), b) = c_3, \tag{69}$$

we define for $(a_3, a'_3)\colon 1 \to A^2$ and $c_3\colon 1 \to C$,

$$\phi^-((a_3, a'_3), c_3) = b_2^*. \tag{70}$$

The collection of the backward-time rule, ϕ^-, which satisfies (i), (ii), and (iii), is denoted by $\{\phi^-\}$ and is called the backward system.

REMARKS. When a virtual rule ϕ is represented by $a_i^{t+1} = \phi(a_{i-1}^t, a_i^t, a_{i+1}^t)$, the backward-time rule implies $a_i^t = \phi^-(a_{i-1}^t, a_i^{t+1}, a_{i+1}^t)$. That is why ϕ^- is called the backward-time rule.

EXAMPLE. Given a virtual rule ϕ defined by

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),0) & ((0,1),0) & ((1,0),0) & ((1,1),0) \\ \phi((a,a'),b) & 0 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),1) & ((0,1),1) & ((1,0),1) & ((1,1),1) \\ \phi((a,a'),b) & 1 & 0 & 0 & 0. \end{array} \tag{71}$$

one of the contradictory systems $\{\phi_c\}$ is defined by

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),0) & ((0,1),0) & ((1,0),0) & ((1,1),0) \\ \phi_c((a,a'),b) & 0 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{cccccc} ((a,a'),b) & ((0,0),1) & ((0,1),1) & ((1,0),1) & ((1,1),1) \\ \phi_c((a,a'),b) & 1 & 0 & 0 & 1. \end{array} \tag{72}$$

From Definition 12, one of the backward-time rules can be defined by the following: for any $((a, a'), c)\colon 1 \to A^2 \times C$,

$$\phi^-((a, a'), c) = \phi((a, a'), c). \tag{73}$$

Hence, $b = \phi^{-}((a, a'), c)$ implies that

$$\begin{array}{ccccc} ((a,a'),b) & ((0,0),0) & ((0,1),0) & ((1,0),0) & ((1,1),0) \\ c & 0 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{ccccc} ((a,a'),b) & ((0,0),1) & ((0,1),1) & ((1,0),1) & ((1,1),0) \\ c & 1 & 0 & 0 & 1. \end{array} \tag{74}$$

As a result, we have constructed a one-to-many type mapping and denote it by $\langle\phi\rangle$: $A^2 \times B \to C$, which consists of ϕ, ϕ_c, and ϕ^{-}. Note that ϕ^{-} represents the mixture of ϕ and ϕ_c. On one hand, we obtain

$$\phi((1,1),0) = \phi((1,1),1), \tag{75}$$

and on the other,

$$\phi_c((1,1),0) \neq \phi_c((1,1),1). \tag{76}$$

As a result of the operation of ϕ^{-}, we obtain a constructed real one-to-many type mapping as shown by

$$\langle\phi\rangle((1,1),0) \neq \langle\phi\rangle((1,1),0). \tag{77}$$

Hence, the definition of the backward-time rule realizes the mixture of ϕ and ϕ_c. Now we show the following definition.

DEFINITION 13: THE RELATIONSHIP BETWEEN ϕ, ϕ_c, AND ϕ^{-}. The mixture between ϕ and ϕ_c that resulted from the observation of a virtual rule under finite velocity of observation propagation is defined by the form of ϕ^{-}. It entails a one-to-many type mapping $\langle\phi\rangle$ defined by the following: for any (a, a'): $1 \to A^2$ and c: $1 \to C$.

$$\langle\phi\rangle((a,a'),b) = \phi((a,a'),c), \tag{78}$$

where b is defined by

$$b = \phi^{-}((a,a'), \phi_c((a,a'),c)). \tag{79}$$

In order to prove a fixed point constructed by ϕ, ϕ_c, and ϕ^-, we here prove the following.

PROPOSITION 4. One-to-many type mapping $\langle \phi \rangle$ consists of $f_p = \langle \hat{\phi} \rangle(0)$ and $f_q = \langle \hat{\phi} \rangle(1)$ such that for any (a, a'),

$$\langle \hat{\phi} \rangle(0)(a, a') \neq \langle \hat{\phi} \rangle(1)(a, a'); \tag{80}$$

however, there exists $(a*, a*')$ such that

$$\langle \hat{\phi} \rangle(b)(a*, a*') = \neg \langle \hat{\phi} \rangle(b)(a*, a*'). \tag{81}$$

This proof is clear because of Definition 13. Now the aspect of time evolution under finite velocity of observation propagation is defined by regarding a fixed point as the evolutionary system, and the mixture (of course it is also identification) between the virtual system (ϕ) and the contradictory morphism (ϕ_c) is replaced by the definition of the backward-time rule ϕ^-. Therefore, we can prove the following proposition.

PROPOSITION 5. A virtual rule ϕ: $A^2 \times B \to C$, contradictory rule ϕ_c: $A^2 \times B \to C$, and backward-time rule ϕ^-: $A^2 \times C \to B$ constitutes a fixed point with respect to $\langle \phi \rangle$ if $\phi \neq \phi_c$.

PROOF. If for $(a*, a*')$: $1 \to A^2$ and c: $1 \to C$, there exists no b with $\phi((a*, a*')\, b) = c$, this implies that for $(a*, a*')$ and $1 - c$ there exists b^* such that

$$\phi((a*, a*'), b^*) = 1 - c, \tag{82}$$

$$\phi((a*, a*'), 1 - b^*) = 1 - c. \tag{83}$$

Hence, the backward-time rule can be defined by choosing one for $(a*, a*')$ and $1 - c$,

$$\phi^-((a*, a*'), 1 - c) = b^*. \tag{84}$$

Simultaneously, from Definition 13

$$\phi^-((a*, a*'), c) = b^* \tag{85}$$

is also defined. Therefore, $\langle\phi\rangle((a*, a*'), b^*)$ equals both c and $1 - c$. It entails the form (81). When $f_k(a*, a*')$ is defined by

$$f_k(a*, a*') = \neg\big(\langle\hat{\phi}\rangle(f^*(a*, a*'))(a*, a*'), \tag{86}$$

there exists b^* with $b^* = f^*(a*, a*')$ such that

$$f_k(a*, a*') = \langle\hat{\phi}\rangle(f^*(a*, a*'))(a*, a*') \tag{87}$$

because from (80) and (81), both $\langle\hat{\phi}\rangle(b^*)(a*, a*') = c$ and $\langle\hat{\phi}\rangle(b^*)(a*, a*') = 1 - c$ are obtained where $b^* = f^*(a*, a*')$. Finally, we obtain

$$\langle\hat{\phi}\rangle(f^*(a*, a*'))(a*, a*') = h\big(\langle\hat{\phi}\rangle(f^*(a*, a*'))(a*, a*')\big) \tag{88}$$

for any $h: C \to C$. ■

A formal description of time evolution under finite velocity of observation propagation is, in principle, unprogrammable. It is, in principle, one-to-many type mapping; however, plural possibilities are degenerated a posteriori. If we prove that it is possible to describe such an aspect formally, then it must entail a fixed point. Instead of proving the impossibility of the description, we construct a fixed point and then regard the fixed point itself as an evolutionary system that realizes one-to-many type mapping. We can summarize our construction as shown in Figure 2. The mixture between the virtual and contradictory rule is replaced by the backward-time rule, and this brings forth

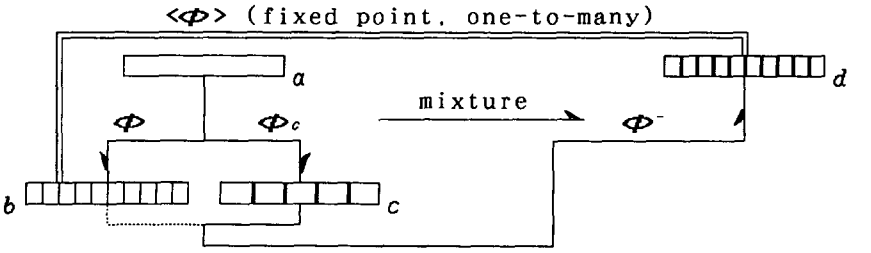


FIG. 2. The relationship between virtual morphisms ϕ, ϕ_c, ϕ^- and real time evolution $\tilde{\phi}$. The mixture of ϕ and ϕ_c that resulted from observation under finite velocity of observation propagation entails the definition of backward-time rule ϕ^-, and ϕ, ϕ_c, and ϕ^- constitute a fixed point that implies one-to-many type mapping $\langle\phi\rangle$.

a one-to-many type mapping $\langle \phi \rangle$ that is constructed by a fixed point. In the next section, the evolutionary feature in this type of one-to-many type mapping is estimated in detail.

4. SELF-REFERENTIAL SYSTEM AS AN EVOLUTIONARY GENERATIVE SYSTEM

First we formalize an evolutionary system that is nothing but a proof of incompleteness of the system to describe time evolution under finite velocity of observation propagation. A fixed point which consists of ϕ, ϕ_c, and ϕ^- is regarded as an evolutionary system and its time evolution is defined.

DEFINITION 14: SELF-REFERENTIAL SYSTEM. For ϕ: $A^2 \times B \to C$, the triplet $\langle \phi, \{\phi_c\}, \{\phi^-\} \rangle$ is called a self-referential system. Time evolution of the self-referential system is defined as follows.

$\overline{\phi}$ is the operation by the local rule ϕ over the configuration (η, ξ): $1 \to A^n \times A^2$, where η is the initial configuration and ξ is the boundary condition. Here the process of virtual computation is articulated into two phases.

(i) Phase I: The Transition from the tth to the $(t + 1)$th Step. The configuration a^{t+1} at the $t + 1$th step is defined by

$$a^{t+1} = \overline{\phi}(\eta, \xi), \tag{89}$$

and the configuration a^t at the tth step is defined by

$$a^t = \overline{\phi^-}\left(\overline{\phi_c}(\eta, \xi), \xi\right), \tag{90}$$

where ϕ_c and ϕ^- is chosen, which satisfies

$$\min_{\phi_c, \phi^-} d_H\left(\overline{\phi^-}\left(\overline{\phi_c}(\eta, \xi), \xi\right), \eta\right). \tag{91}$$

Here, d_H represents the hamming distance.

(ii) Phase II. After the operation of the phase I, the configuration η' is defined by

$$\eta' = \overline{\phi_c}\left(\overline{\phi_c}(\eta, \xi), \xi\right), \tag{92}$$

The configuration η' is regarded as the initial configuration for phase I, and then phase I and II are repeated alternately.

REMARKS. Note that a real one-to-many type mapping can be approximated into both ϕ and ϕ_c (of course the "real" one-to-many type mapping is here formally constructed). Therefore, instead of (89), we can also define $a^{t+1} = \overline{\phi_c}(\eta, \xi)$. Then we have to define $a^t = \overline{\phi^-}(\overline{\phi}(\eta, \xi), \xi)$ instead of (90). In this case, all $\overline{\phi_c}$ in the forms (91) and (92) are replaced by $\overline{\phi}$. See Figure 2 for a detailed discussion.

We have formalized an evolutionary system or expression of one-to-many type mapping as a fixed point, and call it a self-referential system. The same as the relationship between $a^t = \overline{\phi^-}(\overline{\phi_C}(\eta, \xi), \xi)$ and $a^{t+1} = \overline{\phi}(\eta, \xi)$, both $a^{t+2} = \overline{\phi^-}(\overline{\phi_c}(\overline{\phi_c}(\overline{\phi_c}(\eta, \xi), \xi), \xi), \xi)$ and $a^{t+1} = \overline{\phi}(\eta, \xi)$ bring forth a fixed point or a one-to-many type mapping. The contradictory morphism $\overline{\phi_c}$ is what we call a reversible map because for ϕ_c there exists ϕ': $A^2 \times C^{A \times A} \to B$, and then the arbitrary numbers of operation of $\overline{\phi_c}$ by no means make the configuration statistically biased. Therefore, there is no essential difference between the relation of $a^t = \overline{\phi^-}(\overline{\phi_c}(\eta, \xi), \xi)$ and $a^{t+1} = \overline{\phi}(\eta, \xi)$, and $a^{t+1} = \overline{\phi}(\eta, \xi)$ and $a^{t+2} = \overline{\phi^-}(\overline{\phi_c}(\overline{\phi_c}(\overline{\phi_c}(\eta, \xi), \xi), \xi), \xi)$.

Now we estimate the feature of time evolution by computer simulation using Definition 14. Because we consider the case $A = B = C = \{0, 1\}$, the virtual system is a symmetric elementary cellular automata (ECA), and the patterns of time evolution were estimated by the Lyapunov exponent, metric, and topological entropy, a computational theory, and an algebraic property. Their rules in ECA are classified into class 1, 2, and 3, which means a rule generates a global homogeneous pattern, generates local periodic patterns, and generates global nonhomogeneous patterns, respectively.

As far as we cling to Definition 9, our self-referential system is also dependent on the choice of a virtual rule ϕ for $\langle \phi, \{\phi_c\}, \{\phi^-\} \rangle$. In the case that ϕ is a class 1, the self-referential system demonstrate class 1 patterns with small perturbation, in which global homogeneous patterns are locally disturbed, but such disturbances are not propagated globally. The same as a rule of class 1, in the case that a virtual rule is class 2 and its period of local stable pattern is only 1 (patterns looks like stripes that consists of 0 and 1 bands that are parallel to the time axis), the self-referential system also demonstrates pseudo-class 2 patterns in which the 0 or 1 band is locally disturbed, but it is always attracted into the local stable band.

Of course, the feature of a self-referential system in which the virtual rule is class 1 or 2 depends on the choice of the contradictory morphisms. Definition 9 requires a minimum contradiction or perturbation that has resulted from the description itself. However, if a contradictory morphism is

chosen as a morphism that creates more contradiction, the self-referential system demonstrates a class 4 pattern even if virtual rule is class 1 or 2. A class 4 pattern is a complex pattern that consists of a global chaotic and a local periodic pattern, and in which the local periodic pattern often propagates globally. Note that class 4 patterns cannot appear in an ECA system, but can appear in rules with interaction with both the nearest and next-nearest neighbors represented by $\alpha_i^{t+1} = f(\alpha_{i-2}^t, \alpha_{i-1}^t, \alpha_i^t, \alpha_{i+1}^t, \alpha_{i+2}^t)$ [16].

An interesting feature is found in a self-referential system whose virtual rule is class 2 and whose period of local stable patterns is 2. In symmetric ECA there are only two rules, $\#(\phi) = 50$ and $\#(\phi) = 178$, which satisfy that condition. This type of self-referential system shows a characteristic pattern in which autonomic perturbation that resulted from contradictory morphism suddenly and drastically appears in a space and then it is gradually removed as if all cells were controlled by a unique virtual rule. This sequence is repeated perpetually. Figure 3 shows the time-evolution of a self-referential system whose virtual rule is $\#(\phi) = 50$, where the boundary condition is periodic. As a result of the construction of a fixed point, there always exists some sites in which the transition rule is not controlled by a virtual rule, and then it looks like perturbation in a space controlled under a unified rule. However, the perturbation gradually decreases due to the minimization of contradiction or perturbation in the form of choice of the contradictory morphism and the form (91).

In order to estimate this aspect of time evolution under finite velocity of observation propagation, we define two kind of indexes. The first index is spatial metric entropy.

DEFINITION 15: SPATIAL METRIC ENTROPY. Spatial metric entropy for a given configuration $\eta \in \{0, 1\}^n$ is defined by

$$S^{(X)} = (1/X) \sum_{j=1}^{2^X} p_j \log_2 p_j, \tag{93}$$

where X represents box size in metrics for a configuration. When η consists of n-numbers cells, the configuration is observed through a series of X-numbers cells. The probability $p_j (\Sigma p_j = 1)$ represents a frequency of appearances a box $X_j = (\alpha_1, \alpha_2, \ldots, \alpha_X) \in \{0, 1\}^X$ in η, where

$$j = \sum_{k=1}^{X} 2^k \alpha_k. \tag{94}$$

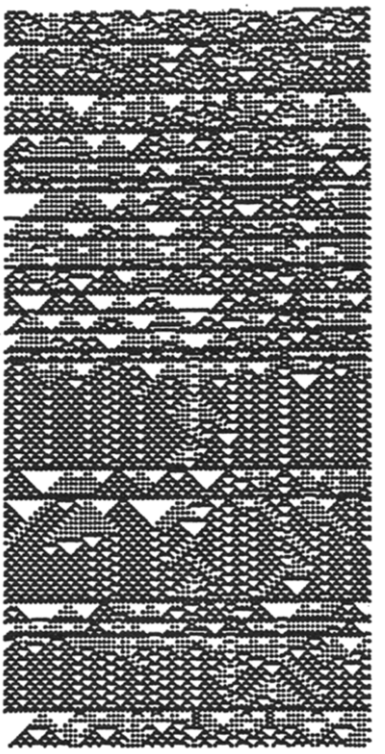


FIG. 3. Time evolution of a self-referential system whose virtual rule is $\#(\phi) = 50$. The black square represents 1 and a blank represents 0. The vertical and horizontal axes are time and space, respectively.

Because $\phi: \{0, 1\}^3 \to \{0, 1\}$, we estimate the spatial metric entropy with $X = 3$ as in Figure 4. It is clear that $S^{(3)}$ gradually increases with the progress of time and it becomes as if it were attracted into a steady state and then suddenly decreased. This sequence is repeated aperiodically.

The second index is an approximated number of rules. It shows the feature of disequilibration better.

DEFINITION 16: APPROXIMATED NUMBER OF RULES. Given a configuration $S^t = (\alpha_1^t, \alpha_2^t, \ldots, \alpha_m^t) \in \{0, 1\}^m$ at the tth step and $S^{t+1} = (\alpha_1^{t+1}, \alpha_2^{t+1}, \ldots, \alpha_m^{t+1}) \in \{0, 1\}^m$ at the $(t + 1)$th step. S^t is defined to be controlled by a unique rule ϕ, iff for $(\alpha_{j-1}^t, \alpha_j^t, \alpha_{j+1}^t) \in \{0, 1\}^3$, $j = 1, \ldots, m$, there uniquely exists $\tau \in \{0, 1\}$ with $\tau = \phi(\alpha_{j-1}^t, \alpha_j^t, \alpha_{j+1}^t)$ and $\alpha_j^{t+1} = \tau$.

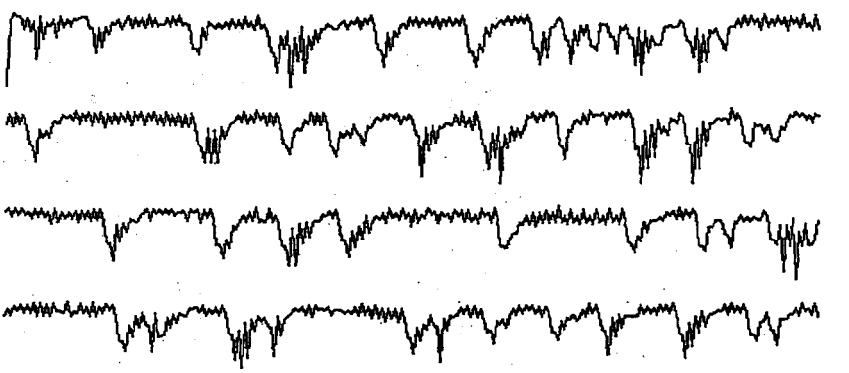


FIG. 4. Time evolution of spatial metric entropy corresponding to the time evolution of a self-referential system whose virtual rule is $\#(\phi) = 50$. system size $= 300$.

However, ϕ does not have to be defined by ε with $\varepsilon = \phi(\alpha, \beta, \gamma)$ for any $(\alpha, \beta, \gamma) \in \{0, 1\}^3$.

A configuration $\eta^t \in \{0, 1\}^N$ is defined to consist of $S_1^t = (\alpha_u^t, \alpha_{u+1}^t, \ldots, \alpha_m^t)$, $S_2^t = (\alpha_m^t, \alpha_{m+1}^t, \ldots, \alpha_l^t), \ldots, S_i^t = (\alpha_w^t, \alpha_{w+1}^t, \ldots, \alpha_N^t, \alpha_1^t, \alpha_2^t, \ldots, \alpha_r^t), \ldots,$ and $S_h^t = (\alpha_v^t, \alpha_{v+1}^t, \ldots, \alpha_{u-1}^t)$ iff for any $S_k^t = (\alpha_p^t, \alpha_{p+1}^t, \ldots, \alpha_q^t)$ that is controlled by ϕ_k and $S_{k+1}^t = (\alpha_q^t, \alpha_{q+1}^t, \ldots, \alpha_s^t)$ that is controlled by ϕ_{k+1}, there exists α_j^t, which is an element of S_k^t, such that

$$\left(\alpha_{j-1}^t, \alpha_j^t, \alpha_{j+1}^t\right) = \left(\alpha_q^t, \alpha_{q+1}^t, \alpha_{q+2}^t\right) \tag{95}$$

and

$$\alpha_j^{t+1} \neq \alpha_{q+1}^{t+1}. \tag{96}$$

This means that

$$\phi_k\left(\alpha_{j-1}^t, \alpha_j^t, \alpha_{j+1}^t\right) \neq \phi_{k+1}\left(\alpha_q^t, \alpha_{q+1}^t, \alpha_{q+2}^t\right). \tag{97}$$

Here we define the uth element of an approximated number of the rules for η^t by $\#(\eta^t, u) = h$, and define an approximated number of the rules for η^t by

$$\#(\eta^t) = (1/N) \sum_{u=1}^{N} \#(\eta^t, u). \tag{98}$$

Figure 5 shows time evolution of $\#(\eta^t)$ corresponding to the time evolution of a self-referential system whose virtual rule is $\#(\phi) = 50$. It is clear that $\#(\eta^t)$ drastically and suddenly increases, gradually decreases, and then $\#(\eta^t)$ is attracted into one or two. It looks as if $\#(\eta^t)$ were attracted into a stable fixed point; however, such a state is not final stable state, and then the diversity of the approximated number of rules is increased. Note that attracted, very small number of rules is not decided a priori, and that it is a result of the operation ϕ^- and it expresses a decision a posteriori.

The characteristic sequence of $\#(\eta^t)$ is explained to some extent when a virtual rule of the self-referential system is $\#(\phi) = 50$ or 178. According to Proposition 5, it is generally shown that two states as,

$$a^t = \phi^-\big((a, c), \phi_c((a, c), b)\big) \tag{99}$$

$$a^{t+1} = \phi((a, c), b) \tag{100}$$

constitute a fixed point of which for any $t: C \to C$

$$a^{t+1} = t(a^{t+1}), \tag{101}$$

partly because there exists a triplet $((\alpha, \gamma), \beta)$ and ϕ_s^- (suffix s represents an element of $\{\phi_c\}$) in some self-referential system $\langle \phi, \{\phi_c\}, \{\phi^-\}\rangle$ such that

$$\phi_s^- \circ \big((a, c), \phi_c((a, c), b)\big) = \phi_s^- \circ \big((a, c), \phi_c((a, c), \bar{b})\big) \tag{102}$$

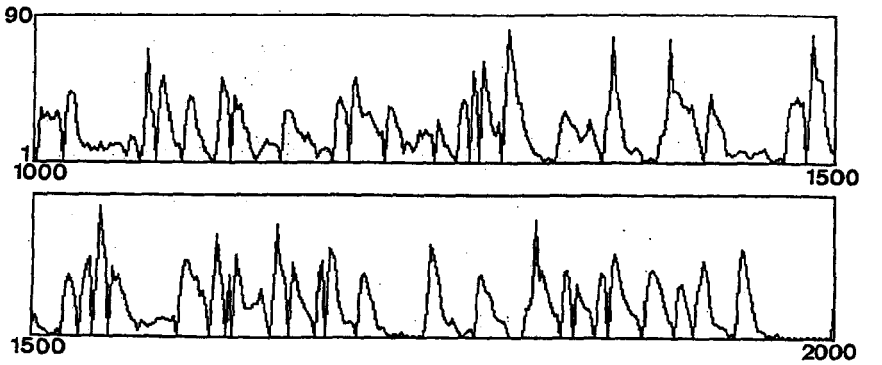


FIG. 5. Time evolution of approximated number of rules corresponding to the time evolution of a self-referential system whose virtual rule is $\#(\phi) = 50$. System size = 300.

where $\bar{b}$ represents negation of b, and

$$\phi((a,c),b)) \neq \phi((a,c),\bar{b})). \tag{103}$$

and because there exists a microscopic boundary condition (α', c') resulting from $\phi^- \circ \phi_c$, such that

$$\begin{aligned}\phi_s^- \circ ((b',b),\phi_c((b',b),a)) \\ = \phi_s^- \circ ((b'',\bar{b}),\phi_c((b'',\bar{b}),a)) = a',\end{aligned} \tag{104}$$

$$\begin{aligned}\phi_s^- \circ ((b,b^*),\phi_c((b,b^*),c)) \\ = \phi_s^- \circ ((\bar{b},b^{**}),\phi_c((\bar{b},b^{**}),c)) = c'.\end{aligned} \tag{105}$$

Therefore, we obtain a one-to-many type mapping $\langle\phi\rangle((a',c'),d) = \neg\langle\phi\rangle((a',c'),d)$.

Here we estimate the transition of a virtual rule, $\#(\phi) = 50$ and 178. Both rules show that any configurations are attracted into a local periodic pattern, and that such a periodic pattern is stabilized by $\phi((0,0),1) = 0$ and $\phi((1,1),0) = 1$. Note that

$$\phi((0,0),1) = \phi((0,0),0) \tag{106}$$

for both rules. Hence, ϕ_c is chosen either by

$$\phi_{c1}((0,0),1) = 0 \quad \text{or} \quad \phi_{c2}((0,0),0) = 0. \tag{107}$$

where both ϕ_{c1} and ϕ_{c2} satisfy

$$\phi_{ci}((0,0),1) \neq \phi_{ci}((0,0),0) \tag{108}$$

for $i = 1, 2$. Also, a one-to-many type mapping is realized either by $\langle\phi\rangle((0,0),0) = \neg\langle\phi\rangle((0,0),0)$ or $\langle\phi\rangle((0,0),1) = \neg\langle\phi\rangle((0,0),1)$, dependent on the choice of ϕ^-.

$$\phi_1^-((0,0),0) = \phi_1^-((0,0),1) = 0, \tag{109}$$

or

$$\phi_2^-((0,0),0) = \phi_2^-((0,0),1) = 1. \tag{110}$$

Note the following aspect. On the one hand, if $\langle\phi\rangle((0,0),0) = \neg\langle\phi\rangle((0,0),0)$ is defined as a result of $\phi^- \circ \phi_c$, $\langle\phi\rangle$ is not defined for $((0,0),1)$. On the other hand, if $\langle\phi\rangle((0,0),1) = \neg\langle\phi\rangle((0,0),1)$ is defined as a result of $\phi^- \circ \phi_c$, $\langle\phi\rangle$ is not defined for $(0,0),0)$. The local periodic pattern is always disturbed due to the one-to-many type mapping, and then whether the number of the triplet $((0,0),0)$ is larger than that of $((0,0),1)$ in η^t depends on the whole configuration which is before the operation of $\phi^- \circ \phi_c$. This distinguishes η' from $\eta^t = \phi^- \circ \phi_c(\eta', \xi), \xi)$. We call η' a virtual configuration. Because of phase II in the definition of a self-referential system, η' cannot appear as a real configuration. If the number of the triplet $((0,0),1)$ is larger than that of $((0,0),0)$, ϕ_2^- is chosen due to the minimization of (91), and otherwise, ϕ_1^- is chosen whether either ϕ_{c1} or ϕ_{c2} is selected.

Here as far as ϕ_1^- is chosen, virtual $\phi((0,0),1) = 0$ is transformed into real $\langle\phi\rangle((0,0),0) = 0$. Then, both $\phi((0,1),0) = 1$ and $\phi((1,0),0) = 1$ are transformed into $\langle\phi\rangle((0,0),0) = 1$. Therefore, there necessarily exists an articulation at the edge of the configuration $(\ldots, 0, 0, 0, \cdots)$ at the tth step dependent on either $\langle\phi\rangle((0,0),0) = 0$ or $\langle\phi\rangle((0,0),0) = 1$. This entails an increase of the approximated number of rules. After that, the virtual rule ϕ makes a time evolution of configuration attracted into a local periodic pattern where spatial configuration that can be attracted looks like $(\ldots 0, 1, 0, 1, 0 \ldots)$ in spite of the effect of the one-to-many type mapping. Then, the table is suddenly turned because the number of the triplet $((0,0),1)$ is larger than that of $((0,0),0)$ in the virtual configuration. Therefore, ϕ_2^- is chosen, and then the small number of a cell with $\phi((0,0),0) = 0$ are transformed into $\langle\phi\rangle((0,0),1) = 0$. This phase continues over a longer period of time. As long as it is continued, the number of cells that are contradictory to one another in the form of $\langle\phi\rangle((a,c),b) \neq \langle\phi\rangle((a,c),b)$ is gradually decreased. This implies the decrease of the approximated number of rules $\#(\eta^t)$.

However, the effect of one-to-many type mapping cannot disappear (i.e., in spite of the effort of removing the contradiction, contradiction cannot be proved). Therefore, there exists the time in which there are many cells $\langle\phi\rangle((0,0),1) = 0$ realized from virtual $\phi((0,0),0) = 0$. Also, the real time evolution, the pair $((\eta^t, \xi^t), \eta^{t+1})$, looks as if it were programmed by a unique rule, but at the next time step the number of a triplet $((0,0),0)$ suddenly increases again. As soon as the number of triplets $((0,0),0)$ is larger than that of $((0,0),1)$ in virtual configuration, ϕ_1^- is chosen. This implies the sudden increase of $\#(\eta^t)$. Hence, soon after $\#(\eta^t)$ is attracted into one or two, a very small number, it is increased drastically. This sequence is perpetually continued and cannot be terminated.

In the case in which a self-referential system whose virtual rule is class 3, proceeds, it shows a very complex pattern. It looks like a class 3 pattern, but

sometimes it shows global autonomic perturbations. We can sketch this kind of time evolution compared with that of a class 2 self-referential system. Owing to the minimization of the hamming distance defined above, a system is perpetually attracted into the system controlled by a virtual rule. However, it is also perpetually disturbed because of the existence of the contradictory rule and the backward-time rule. Therefore, however close to a virtual rule a system is, it is suddenly turned over. In other words, a self-referential system is always involved in self-contradiction, and it cannot be improved in principle. Self-contradiction itself makes a system proceed. Now we can yield the following conjecture.

CONJECTURE. The behavior of a self-referential system gradually approaches to that of a virtual rule; however, it cannot coincide in principle. Then the behavior of a self-referential system is suddenly disturbed because of the contradictory rule; thus, it is gradually close to that of a virtual rule again. The sequence is perpetually repeated nonperiodically.

Such a trend of time evolution is suggested on many features of the biological evolutionary processes whether it is ontogeny or phylogeny. If you cling to the variation and selection mechanism in evolution, you feel that the evolutionary trend here is very peculiar. If a mutant is generated at random and it is selected and stabilized by the selection process, it must be gradually propagated and prevailed in a population in general. In order to explain the sudden increasing of the mutation rate, you have to introduce a peculiar mechanism.

In biological ontogenetic processes, we find sudden increments of autonomic perturbation. For example, the neural network of brain cannot be gradually constructed, but the number of neurons suddenly explode in an early stage; then as it gradually decreases, the neural network is systematically arranged. It is very similar to the construction of an arch in which, at first, a lot of rocks are piled up and after that rocks at the center are removed to construct an arch. Here the number of rocks suddenly increase at first and then it decreases. Some biologists concentrate on the first increment of information, and point out that it is found also in phylogenetic processes, known as C-value paradox of DNA contents. Note that an arch structure cannot be built up if one gradually piles up rocks, except for the center.

Also, the behavior of a self-referential system suggests the relationship between death and life and/or a general mechanism of senescence. Many biologists generally regard senescence as a kind of system failure arising from local failure in specific subsystems. If it is enlarged as a nonlocal failure, the system becomes ill and entails death. Here death is regarded as a result of the

local failure's prevailing, and the cause of local failure is defined independent of a living system itself. There is a life/death dualism.

However, a self-referential system shows that life cannot be separated from death due to the effect of finite velocity of observation propagation. If the hamming distance equals zero, it implies death. Therefore, living can be regarded as the failure of the process leading to death, for a whole system to proceed approaching to the improvement of self-contradiction (i.e., the hamming distance = 0) or death. As long as a system does not die (i.e., self-contradiction is not executed), a system lives (i.e., self-contradiction is perpetually generated and is to be removed at the same time). The sudden increase of the approximated rule numbers suggests an evolution in the phylogenetic process or the peculiar information generation like the appearance of a cancer cell in the ontogenetic process. The appearance of cancer is here not accidental, but is a consequence of disequilibration in a self-referential system.

The self-referential system will be examined in detail as a model of both phylogenetic and ontogenetic processes if we take the finite velocity of observation propagation into consideration. It can prove the evolution toward self-organized criticality [8]. The system is also constructed in another way, but the essence of a system, the disequilibration process, is a common feature in both constructions.

5. CONCLUSION

We estimate the effect of finite velocity of observation propagation in time evolution of an interactive system. We show that if we accept the finite velocity of observation propagation, the formal description of a system falls into infinite regression. Infinite regression itself cannot be programmed in principle, but it can be expressed as a fixed point in terms of Lawvere's theorem of a fixed point. We regard the fixed point itself as a structure of "time," and define the self-referential system that realizes one-to-many type mapping.

Whenever we can describe an interaction, we must ignore that it takes a finite time to measure the spatial information (i.e., input sent from other components). In other words, any formal system by which we describe an interaction involves the assumption of infinite velocity of observation propagation. Any formal languages can be expressed as a specific category, topos, in which mono (epi in dual category), products (coproducts), equalizer (coequalizer), pullback (pushout), terminal (initial), and power can be defined. Most of these definitions are deduced from the definition of limit (colimit), and it is a consequence of the assumption of infinite velocity of observation propagation.

Therefore, formal descriptions implicitly involve the assumption of infinite velocity of observation propagation in the form of a projective (inductive) limit and we cannot construct finite velocity of observation propagation by the solution of the degeneration of limit. Our construction, called the self-referential system, using a fixed point, is a peculiar but unique method to express the aspect of finite velocity of observation propagation.

A self-referential system demonstrates evolutionary time evolution in which the behavior of a system sometimes looks to be attracted into a stable fixed point like the limit cycle, but after that it looks like it was flipped from that point, and this sequence is perpetually repeated. This is because a self-referential system involves the mechanism generating self-contradiction in the form of a fixed point, and a system itself manages to remove its own self-contradiction; however, it can by no means succeed. Self-contradiction implies the autonomic generation of perturbation and/or information. This is a typical mechanism demonstrating perpetual disequilibration.

However, many discourses, of which the essential mechanism of living systems are regarded as self-reference differ in detail, but are alike in regarding a paradox as a new value or state embedded in hyper-logic in living systems. Therefore, they accept that paradox must be removed in living systems, and that paradox is represented due to the difference between the observer's and the living system's logic. Our stance is intrinsically different from theirs. To understand living things is to accept a paradox itself, not to solve a paradox. That is why we can demonstrate perpetual disequilibration or evolutionary generating processes.

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