Analysis, Systematicity and the Transcendental in Hermann Cohen's System of Critical Idealism

by

B Tyson Gofton

A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Philosophy
University of Toronto

© Copyright 2013 by B Tyson Gofton

Abstract

Analysis, Systematicity and the Transcendental in
Hermann Cohen's System of Critical Idealism

B Tyson Gofton
Doctor of Philosophy
Graduate Department of Philosophy
University of Toronto
2013

The dissertation provides a systematic, critical analysis of Hermann Cohen's *System of Critical Idealism*. The first chapter establishes Cohen's reading of the a priori of Kant's Transcendental Aesthetic as founding the possibility of intuition in mathematics, rather than the possibility of mathematics in intuition. The second chapter then investigates the problem of the unity of the transcendental object, or, more specifically, the conditions under which the intelligible predicates of the functions of judgment can be applied to an objective unity.

Chapters three and four compare the idealist responses of Salomon Maimon and G.W.F. Hegel to the problem of objective unity. Both Maimon and Hegel attempt to provide a logic (and a manifold of reality) grounded in the Spinozistic principle of determinability. Ultimately, this leads to the conflation of the totality of intuition with the domain of the intelligible thereby reducing Kant's infinite judgment to a positive assertion. Cohen, however, rejects this solution, and insists that the manifold of reality (or the real continuum of calculus) is the product of continuous thinking. Cohen's principle of production implies an indeterminably determinable manifold, thus providing the intelligible foundations for the eventual set-theoretic foundation of arithmetic and analysis.

The final chapter of the dissertation investigates the consequences of Cohen's innovation for the prospects of systematic idealism as a framework within which normative, theoretical and aesthetic claims may be raised and justified. Since logic does not determine a priori the structure of the intelligible whole, Cohen cannot assume a convergence between natural and ethical representations. The free production of laws, Cohen argues, is the practice of jurispru-

dence, or, the construction and reconstruction of assertoric statements (universal claims) with the aim of limiting contradictions. If, in the natural sciences, we call this aim truth, in ethics, we call it the good, guided by the idea of the end of humanity.

Dedication

To J.B.,

for all the good times.

Acknowledgements

I would like to thank the University of Toronto for supporting my project and my extended time abroad. I would also like to thank the Centre for Jewish Studies at the University of Toronto for their support of my work. I benefited from the help and hospitality of the Humboldt Universität zu Berlin. My stay in Berlin was made possible through the financial support of the Deutsche Akademische Austausch Dienst. The department of philosophy and the department of Judaic studies at Yale University have provided invaluable financial support and opportunities for professional and intellectual development. I owe a special debt of gratitude to my directors, Robert Gibbs and Paul Franks. Paul Franks especially has supported me in too many ways to count. It takes many hands to complete a project of this scope, and to all those who helped in various ways, I offer my thanks.

Contents

1 Introduction **1**

1.1 Critical Idealism 1

1.1.1 Hermann Cohen and the Marburg School 1

1.1.2 Systematicity 7

1.2 Traditional Epistemology and its Critics 9

1.2.1 Substance and Accident 9

1.2.2 Descartes' Descriptive Skepticism 11

1.2.3 Humean Skepticism 14

1.3 Transcendental Epistemology 17

1.3.1 Ontological Nihilism 17

1.3.2 The Copernican Revolution 20

1.3.3 Mathematics and the Continuum 26

1.4 Post-Kantian Systematic Idealism 31

1.4.1 Summary Outline 36

I Analysis and the Transcendental **38**

2 Formalism **39**

2.1 The Apriority of the a priori 43

2.1.1 The Fischer-Trendelenburg Dispute 43

2.1.2 The Form of Sensibility 48

2.1.3 Regressive Analysis 49

2.1.4 The Formal Determinacy of *Sinnlichkeit* 54

2.2 The Three Levels of the *a priori* 60

2.2.1 The Metaphysical *a priori* 62

2.2.2 The First Transcendental *a priori* 64

2.2.3 The Second Transcendental *a priori* 67

2.3 Experience . . . 73

3 The Transcendental **81**
3.1 Transcendental Analysis . . . 82
3.1.1 The Deduction . . . 82
3.1.2 Cohen's Interpretation of the Transcendental Deduction . . . 91
3.1.3 The Empirical Individual . . . 97
3.2 Transcendental Objectivity . . . 106
3.2.1 Objectivity . . . 106
3.2.2 Critical Metaphysics . . . 116
3.2.3 Transcendental Philosophy . . . 118
3.3 Transcendental Synthesis . . . 122
3.3.1 The Paradoxes of Determinacy . . . 131

II Systematic Idealism **141**

4 The Logic of Reality **142**
4.1 Logic of Reality . . . 143
4.1.1 Maimonian Skepticism . . . 143
4.1.2 The Principle of Reality . . . 152
4.1.3 The Principle of Determinability . . . 160
4.2 Logic of Totality . . . 165
4.2.1 The Principle of Limitation . . . 167
4.2.2 The Principle of Intelligible Totality . . . 176
4.3 Logic of Anticipation . . . 177
4.3.1 The Table of Judgments . . . 178
4.3.2 Platonism . . . 190

5 The Manifold of Reality **198**
5.1 The Real Manifold . . . 199
5.1.1 Infinitesimals . . . 199
5.1.2 The Manifold of Limitations . . . 205
5.2 Origin and Continuity . . . 215
5.2.1 Reality and the Infinitesimal . . . 215
5.2.2 Origin, Reality and the Rational Manifold . . . 227
5.3 Problematic Determinacy . . . 231

5.3.1 The Principle of Production . . . 231
5.3.2 Production and Determinacy . . . 236
5.3.3 NSA and the Infinitesimal . . . 241

6 Logic, Truth and Revelation 246
6.1 Logic and System . . . 247
6.1.1 Skepticism and Spinozism . . . 249
6.1.2 The Logic of Purity (Unity) . . . 256
6.1.3 The Character of System (Truth) . . . 265
6.2 Ethics, Revelation and Religion . . . 276
6.2.1 Ethics and Eternity . . . 276
6.2.2 Aesthetics and the infinite (The Good) . . . 286
6.2.3 Revelation and Transcendental Idealism . . . 293
6.2.4 Revelation and Critical Idealism . . . 302
6.3 Conclusions . . . 318
6.3.1 Post-Cohenian Idealism . . . 318

Abbreviations 320

Bibliography 322

List of Figures

4.1 Aristotle's Wheel 148
4.2 Aristotle's Wheel, Finitely 149

List of Tables

4.1 Kant's Table of Judgments 179
4.2 Cohen's Table of Judgments 189
4.3 Kant's Table of Categories 189
4.4 Two Systems of Logical Principles 197

Chapter 1

Introduction

1.1 Critical Idealism

1.1.1 Hermann Cohen and the Marburg School

The object of this study is Hermann Cohen's philosophical system: the *System of Critical Idealism*.[1] Specifically, we are interested in (a) the so-called "analytic" interpretation of Kant's *Critical Project* that forms the foundation of the Marburg School of Neo-Kantianism, and the implications of this interpretation for (b) the "transcendental" approach that is typical of post-Kantian systematic idealism and for (c) the programme of a systematic philosophy in the post-Kantian tradition. These are the three aspects around which this study is oriented: *analysis*, *systematicity* and *the transcendental*.

Hermann Cohen (1840-1918) was the chief architect of the Marburg School of Neo-Kantianism, which was perhaps the most influential philosophical school in Germany, if not continental Europe, in the second half of the nineteenth century. Broadly speaking, Hermann

[1] The *System of Critical Idealism* consists of three completed works: *Logic of Pure Knowledge* (Cohen 1902b), *Ethics of Pure Will* (Cohen 1907), and *Aesthetic of Pure Feeling* (Cohen 1912). Additionally, Cohen planned, but never completed, a fourth book on psychology, the capstone of the system. Some commentators also urge that we consider *Religion of Reason out of the Sources of Judaism* (Cohen 1919) (Cohen 1995) as an appendix to the system. I think, however, that Cohen is explicit enough to rule out including the *Religion* in a system defined above all through a prohibition on metaphysical commitment.

Cohen's *System of Critical Idealism* is part of the philosophical movement originating in Germany as an immediate response to Kant's critical philosophy, a movement generally referred to as 'German Idealism'. The movement spread beyond German academic circles, and to varying degrees still survives today, especially in continental Europe. The purpose of this study, however, is not to examine the rise (and fall) of neo-Kantianism or the Marburg school in Wilhelmine Germany, nor even to trace the historical development of a particular line of thought from its antecendents (Plato, Leibniz, Kant, Fichte, Hegel, Trendelenburg) through to its philosophical consequences (Rosenzweig, Cassirer, Benjamin, Adorno, Habermas).[2] While Neo-Kantianism's importance in the history of philosophy, and especially its place mediating between contemporary analytic and continental schools of thought, is widely underestimated, this study is directed towards a very different task.[3] Here, I propose a critical examination of Cohen's philosophical system as an articulation of the programme of what I will call *systematic idealism*. That is, I will interpret Cohen's system of critical idealism as a response to a specific set of philosophical problems arising from Kant's proposed revision to traditional epistemology, and the subsequent worry that Kant's solution is inadequate or incomplete.

While Cohen's system of idealism shares many similarities to earlier manifestations of post-Kantian idealism, the origin and motivation for these concerns is quite different from the more familiar projects of Fichte, Schelling and Hegel. In particular, Cohen — and many other Neo-Kantians — were concerned by what appeared to be a resurgence in metaphysical speculation, particularly in the Hegelian school. The call for a return to Kant represents, for

[2] The standard work on the history of Neo-Kantianism is K}ohnke's *The Rise of Neo-Kantianism: German Academic Philosophy between Idealism and Positivism* (Köhnke 1991). Neo-Kantianism's importance is not to be felt so much in the specific school that it may have engendered, but rather in its approach to metaphysics and epistemology. In assimilating the theory of categories (and thus the a priori conditions of metaphysics) to the analysis of scientific practice, Neo-Kantianism effectively made ontology a part of metaphysics. Metaphysics is no longer — as it was for Kant and perhaps even Kant — a philosophical problem, but rather a theoretical problem bound to the problem of scientific method: investigation, analysis and justification. In this sense, the tradition of French philosophy of science (Meyerson, Duhem, Bachelard) that is the background of Quine's attack on the analytic/synthetic distinction in "Two Dogmas of Empiricism" (Quine 1953) is Neo-Kantian in spirit.

[3] Andrea Poma's *Yearning for Form* (Poma 2006) provides a good survey of some of the ways in which Cohen's legacy relates to issues in contemporary continental philosophy. At the other end of the spectrum, Helmut Holzhey provides a spirited defence of the relevance of Cohen's epistemological project for contemporary philosophy of science in his commentaries on Cohen's philosophy of science, and in his own work inspired by the Marburg programme (Holzhey 1986a), (Holzhey 1986b).

Cohen, a return to a more modest, sober examination of the formal conditions of the possibility of scientific knowledge.[4] That is, Cohen aims to recover the "critical" spirit of Kant's project without thereby arriving at the "speculative" philosophy of Hegel, and the metaphysics implied by a particular reading of Hegel's logical project.[5]

One auxilliary claim of this study is that, in addition to the historical interest of Marburg

[4]The standard reference for Hermann Cohen's philosophy is Helmut Holzhey's *Cohen und Natorp* (Holzhey 1986a). However, the best study of the development of Hermann Cohen's logic is Geert Edel's indispensible *Von Erkenntniskritik zur Erkenntislogik* (Edel 1989). A number of other studies (primarily doctoral theses) in which Cohen's philosophy figures in part or in whole are available in German, but these vary in quality. One excellent — and in my opinion essential resource — is David Gawronsky's study of the problem of reality in Hermann Cohen's critical system: *Das Urteil der Realität in der Philosophie Hermann Cohens* (Gawronsky 1910), which attempts to justify the logical concerns of Cohen's project (and especially the concept of continuity) within the framework of the new set-theoretic foundation of number. While Cohen's more famous student, Ernst Cassirer, carried out a similar project in the third section of *Substanzbegriff und Funktionsbegriff* (Cassirer 1910), Cassirer was less concerned with preserving Cohen's systematic framework than with providing a fully formalized concept of function, i.e., one that is entirely free from ontological or organicist interpretation. There are a few other collections of German papers that deal with aspects of Cohen's philosophy, although primarily from a historical or philological point of view; among these are *Hermann Cohen* (ed. Helmut Holzhey) (Holzhey 1994). Two French language studies provide helpful insight into the Marburg program: Henri Dussort's *L'école de Marbourg* (ed. Jules Vuillemin) (Dussort 1963), and Jules Vuillemin's *L'héritage kantien et la révolution copernicienne* (Vuillemin 1954). In English, the secondary literature is considerably more limited. Andrea Poma's *The Critical Philosophy of Hermann Cohen* (Poma 1997) is the only major work on Cohen's philosophical work available in English, and provides a comprehensive study of Cohen's philosophical contribution, but remains essentially historical and hermeneutic in its orientation. A few English language collections of papers on Cohen are available, including *The Ethics of Hermann Cohen* (ed. Robert Gibbs) (Gibbs 2006) and *Hermann Cohen's Critical Idealism* (ed. Reiner Munk) (Munk 2005). These do not, however, amount to a comprehensive or reliable account of Cohen's philosophical project.

There is also a considerable literature on Hermann Cohen's religious philosophy. This is especially true in English, since *The Religion of Reason out of the Sources of Judaism* (Cohen 1995) is the only major work by Cohen that is available in English. Unfortunately, much of this work disregards or has little interest in the systematic aspects of Cohen's program, and therefore sheds little light on the topics at hand.

[5]Cohen appears to have accepted Trendelenburg's metaphysical reading of Hegel's logic without criticism. (See especially Trendelenburg's *Die Logische Frage in Hegels Logik* (Trendelenburg 1843) and alo the lengthy objection in *Logische Untersuchungen, 2. Auflage* (Trendelenburg 1862).) For more on Cohen's debt to Trendelenburg, see Ernst Wolfgang Orth's *Trendelenburg und die Wissenschaft als Kulturfaktum* (Orth 2001).

Neo-Kantianism, especially for the development of analytic philosophy in the twentieth century, Cohen's system can lay claim to philosophical interest in its own right. In particular, I hope to show that Cohen's approach to a general cognitive logic (as opposed to a symbolic algebra, or "logistics") has significant advantages over competing systems of post-Kantian idealism. This should be of interest in particular to those who still find philosophical appeal in the project of systematic idealism, but who are perhaps suspicious of the presuppositions of synthetic logic (Fichte) or dialectical logic (Hegel).[6]

As we will see, Cohen's *Critical Idealism* includes the metaphysical anti-realism, scientism, anti-psychologism, and formalism that we expect from (at least some readings of) Kant's critical project. However, it also adds important elements of constructive fallabilism and paraconsistency that we expect from the logics of post-Kantian idealism. The combination of these elements, however, is achieved without (a) strengthening the underlying hypotheses of the synthetic unity of apperception (as, for example, in Fichte), and without (b) proposing a logical totality through which all determinations are realized analytically (as, for example, in the logic of Salomon Maimon or Hegel).[7]

[6]One contemporary champion of the Marburg school is Michael Friedman. See especially his *A Parting of the Ways: Carnap, Cassirer and Heidegger* (Friedman 2000). While Friedman focuses primarily on Cassirer as an examplar of a critical mode of the philosophy of science, the philosophical foundation of Cassirer's work are given their systematic justification in Cohen's work. Although Friedman is not interested in the systematic aspects of the Marburg programme (and, in truth, neither was Cassirer), many of the arguments marshalled by Friedman in favor of a return to Cassirer's project apply to Cohen as well, all the more so if we admit the importance of some aspects of a systematic programme for the purposes of a unified system of human knowledge. The more general return to Kantian or German Idealist themes (in an undoubtedly Neo-Kantian spirit) to be found in the work of John MacDowell (Macdowell 1994) or Michael Thompson (Thompson 2008) speaks to a need to consider more carefully the implications of logical formalism and its implications for epistemology. The Neo-Kantian dimension of this problem is undeniable.

[7]Cohen's formalist interpretation of consciousness and its products is investigated in **Chapter 2**. Cohen's "dialectical" logic and its genesis will be contrasted with that of Maimon and Hegel in **Chapter 3**. Cohen is particularly opposed to the organicist conception of logic that is common to Fichte, Schelling and Hegel. The conception of logic as intrinsically teleological and organically structured may be understood as a reaction against the limitations of Kant's conception of (real) empirical knowledge as exhausted by the domain of determinate (numerical) constructions, or, as Friedman has argued in *Kant and the Exact Sciences* (Friedman 1992a), set theory, and a return to an Aristotelian conception of the whole as the genus of the particular. The Marburg "analytic" interpretation of Kant may be thought of as a return to Kant's central claim that "logic" (or what Kant calls 'mathematics'), as the foundation of empirical knowledge, begins essentially in the aggregation of parts. In effect, the Marburg Neo-Kantian programme attempts to provide an epistemological justification for what would eventually become the set-theoretic foundation of mathematics as the expressive language of natural science. This project would eventually be carried out more successfully by Natorp (Natorp 1910) and Cassirer (Cassirer 1910) than by Cohen himself.

The importance of these aspects of Cohen's philosophical system are best appreciated in the light of the many criticisms that have been aimed at the general project of post-Kantian idealism, both in continental and analytic circles. For example:

- The elimination of the subjective moment of receptivity and its phenomenological justification from the critical apparatus allows Cohen's system to avoid becoming involved in the philosophy of consciousness and any metaphysics that this may imply. The system of critical idealism thereby avoids many of the criticisms aimed at the programme of systematic idealism in the twentieth century: subjective adequation (Heidegger), non-identity (Adorno), self-presence (Derrida), totalization (Levinas), etc.

- The methodology employed in the development of Cohen's *Critical Idealism* remains committed to reflective self-limitation (the method of critique: analytic, dialectic and doctrine of method) outlined by Kant's *Critique of Pure Reason*. That is, the logic of critique aims to limit from within the paramaters of its own construction the domain of its applicability. While elements of this method may remain within other idealist projects (Fichte's *Anstoß*, Hegel's *Aufhebung*), the systematic impulse of idealism tends to erase this aspect of Kant's project in favour of systematic completeness: the genetic derivation of the parts from the whole.

- The dialectical logic that forms the core of Cohen's realization of the critical and systematic projects is a flexible instrument that is adapted to the needs of providing a foundational structure for a systematic idealism, but remains nonetheless bound to the factical grounds (here, the fact of a scientific culture) of transcendental analysis and the inherent contingency of such a starting point. Since the structure of the logic develops solely out of the in-principle unlimited productivity of thought, it is neither bound by the limitations of empirical representation, nor by the need to demonstrate the internal consistency of its results. So, for example, Cohen's logic does not commit us to an intuitionist or

constructivist mathematics, as Kant's transcendental logic does,[8] nor does it commit us to the dialectical resolution of contradiction, as the logics of Fichte and Hegel do.

We will return to a more detailed examination of some of these issues at the end of this study.

The central question of this study is therefore the following: What are the consequences of an analytic interpretation of the a priori for transcendental justification and a system of transcendental concepts? The answers that I propose are as follows. First, an analytic interpretation of the production of epistemic representations — including judgments — implies the presupposition (a) of an a priori rational manifold (i.e., the concept of number is intellectual, and not intuitive, as it is for Kant (Friedman 1990)) and (b) an a priori criterion of relational determination, i.e., a principle of continuity. Neither of these are provided by Kant's limited interpretation of logic (a subset monadic logic, plus quantification and identity (Friedman 1990)), and must therefore be added to the apparatus of transcendental investigation, perhaps through a reinterpretation and expansion of transcendental logic. Second, the analytical approach to the representations of consciousness invalidates the argument of the transcendental deduction; at the same time, this strategy also disallows the logics of determinable individuals proposed by, for example, Maimon, Fichte and Hegel. As a result, the general strategy for securing the objectivity of transcendental claims — the correlation of empirical individuals and rational unity — cannot be realized in a theoretical logic. Instead, Cohen proposes a systematic connection between the domains of knowledge, rather than a systematic connection within logic itself.

In order to make explicit (a) the nature of Cohen's interpretation of the Kantian philosophy and (b) the scope of the consequences of this interpretation, considerable attention will be paid to the range of options for interpreting the transcendental doctrine (and in particular the Transcendental Deduction), and the special epistemic claims that it raises. Providing an adequate orientation to the problem will therefore require an unusually long preliminary in-

[8] I take it to be an open question whether or not Kant's logic actually commits us to intuitionism or not. Since the writing of this study, I have changed my personal view on Kant's commitment to intellectual synthesis. This does nothing, however, to change the interpretation to which Kant's philosophy of mathematics was — and still is — generally subjected. The early interpretation of Kant as an intuitionist (especially by Maimon (Maimon 1790)) is still common to nearly all contemporary interpretations of Kant's philosophy of mathematics: see especially *Kant's Philosophy of Mathematics: Modern Essays* (Posy 1992).

vestigation, which is the task of this chapter. The argument presented here is quite general in character, and will, for the most part, have to stand without the complete justification that it perhaps deserves. However, since most of these claims are uncontroversial, and intended only to frame the subsequent investigation, considerations of economy may be allowed to prevail.

1.1.2 Systematicity

There are a number of ways in which we might characterize philosophical investigation as *systematic*. We might think that an investigation is systematic because it is (a) encyclopedic, that it covers all (relevant) topics of philosophical investigation (**content**). We might think that it is systematic because of (b) an internal unity (or consistency) in the method of philosophical investigation applied to different philosophical problems (**method**). We might also think that it is systematic because (c) different areas of philosophical investigation (logic, ethics, epistemology, etc.) share common structural or organizational principles (**structure**). Finally, we might think that a philosophical investigation is systematic because it is some or all of these things. [9] It is systematicity in the last and richest sense that is at issue here.

The general programme of the systematic idealists is to construct a method of philosophical investigation that achieves systematicity in the content, method and structure of knowledge, and Hermann Cohen's system of critical idealism is, as we will see, very much a part of this philosophical programme. Throughout this study, I will use the term "systematic idealism" to refer to a set of philosophical projects that developed in the nineteenth and early twentieth centuries,[10] and which aim to construct a maximally systematic method of philosophical

[9] The *locus classicus* for the articulation of the problem of systematicity is Heidegger's *Schelling's Treatise "On the Nature of Human Freedom"* (Heidegger 1985). My own understanding of the possible configurations of systematic approaches to philosophical investigation is close to Heidegger's, and shares many of the same concerns of grounding, justification, explanation and intelligibility. Other important considerations of systematicity in post-Kantian philosophy include Beiser's *The Fate of Reason* (Beiser 1993) and *German Idealism: The Struggle against Subjectivism* (Beiser 2002).

[10] In hindsight, it is easy to read much of the history of early modern philosophy as engaged in a kind of systematic idealism, and the development of the history of philosophy in the twentieth century — from Hegel and Erdmann to Brunschvieg and Cassirer — is an expression of this general interpretation of the history of modern philosophy as a development towards a systematic and idealist understanding of the problems of philosophy. This assessment does not withstand closer scrutiny. For, the formative questions that made systematic philosophy —

investigation based on the reflective principles of Kant's Critical Project.[11]

For the post-Kantian systematic idealists, systematicity is not merely a characteristic of a certain philosophical persuasion, but a methodological imperative for any philosophical investigation worthy of the name. This radical position is most notably adopted by the familiar faces of the early nineteenth century — Fichte, Schelling and Hegel — but it remained a central dogma in continental circles until the early part of the twentieth century; indeed, in many respects, the programme of a systematic idealism still shapes the agenda of what is today called "continental philosophy".[12] The reasons why the systematic programme came to dominate academic philosophy are many and varied. Developing a better understanding how the programme arose and explaining its near total domination of continental philosophical investigation for more than a century is an important task for a critical understanding of the history of philosophy, and especially for our understanding of the kinds of philosophical projects that are still being pursued today.[13]

---

and especially systematic idealism — such an attractive position in the nineteenth century had not yet come to be fully articulated or fully appreciated until much later.

[11] The term "German Idealism" is conventionally used to isolate a group of projects that emerged in response to Kant's critical philosophy in the early nineteenth century. However, there are many post-Kantian systematic idealists who are not German (e.g., Brunschvieg, Duhem, Bachelard), and many post-Kantian German philosophers who are not systematic idealists (e.g., Herbart, Helmholtz). The term 'systematic idealism' instead designates the family of projects for which systematicity and idealism are intrinsically related as necessary responses to the problem (and project) posed by Kant's Critical Philosophy.

[12] There are some notable exceptions to the general dominance of systematic idealism in the nineteenth century, both inside and outside of academic philosophy. Hermann Lotze's ecclecticism (especially in his "Greater" *Logik* (Lotze 1884), for example, is an anomoly amid the generally systematic ambitions of post-Kantian German academic philosophy. Or again, we might think of Friedrich Nietzsche's radically anti-systematic, anti-idealist method of investigation as a reaction against the generally systematic tenor of the "German spirit" in the nineteenth century. We do not need to share Nietszche's analysis of the causes of this dominance (bratwurst and beer, apparently: "German cuisine quite generally — what doesn't it have on its conscience! Soup before the meal (in Venetian cookbooks of the sixteenth century this is still called alia tedesca); overcooked meats, vegetables cooked with fat and flour; the degeneration of pastries and puddings into paperweights! Add to this the virtually bestial prandial drinking habits of the ancient, and by no means only the ancient Germans, and you will understand the origin of the German spirit—from distressed intestines" (Nietzsche 1967).) to share his general diagnosis of the Zeitgeist. Indeed, many of the classic movements in so-called "continental philosophy" arise as (often critical) responses to the systematic programme.

[13] Understanding the transition from the more modest proposal of Kant's *Critical Philosophy* to the full-blown project of systematic idealism has been the object of a number of important studies in the last decades, including Dieter Henrich's *From Kant to Hegel: Lectures on German Idealism*, (Henrich 2003) Frederick Beiser's *The Fate of Reason*, (Beiser 1993) Karl Amerik's *The Fate of Autonomy* (Ameriks 2000) and Paul Frank's *All or Nothing* (Franks 2005).

1.2 Traditional Epistemology and its Critics

1.2.1 Substance and Accident

The traditional model of knowledge, inherited from Aristotle, is at once rationalist and empiricist, realist and idealist. The structures of reflective knowledge (logic) are taken to be indicative of the structure of reality (the logos) and vice versa. Whether we have access to truth by way of empirical investigation (sense perception), or by way of the reflective investigation of consciousness and language, the truth to which we have access is one and the same. The presupposition of this epistemic model is that the form of epistemic representations (logic) and the form of things in themselves (the logos) is the same. That is, Aristolian logic (monadic logic plus quantification) is isomorphic to the structure of Aristotelian ontology (substance, accident, first and second genus); truth is the adequacy of subjective representation and a correlated objective being: *the principle of adequation*. The principle of adequation rests on two further principles. For, it requires not only that the general nature of reality be intelligible to humans (*the principle of intellgibility*), but also that elementary structures of this intelligibility conform to the grammatical structures of the paradigmatic sentences of the syllogism (*the categorematic principle*). The central claim of Aristotelian epistemology is therefore that the empirical individuals of intuition correspond to an intelligibly structured order of beings; this structure is reflected in the fundamental logic of the syllogism; logic itself is ontotheologic in the Heideggerian (or Kantian) sense (Thomson 2000). But now, while a general principle of intelligibility seems unproblematic, at least as a heuristic principle, it is much harder to see, from a contemporary perspective, why we should endorse the categorematic principle. After all, why think that the elementary structures of (Indo-European) grammar reveal deep features about the structure of reality?

The durability of the Aristotelian model of knowledge is in part to be explained by the primacy of ordinary language for the representation of knowledge claims, at least until the time

of the revolution in modern science.[14] For, the grammatical interpretation of subject and predicate relations in terms of logical and ontological implication has some plausibility provided that we do not stray beyond the paradigmatic sentence types required by the syllogism. Insofar as our epistemic claims are restricted to ordinary objects of empirical experience and expressed in the ordinary language suited to syllogistic interpretation, Aristotelian logic — and so too its implied ontology — remains adequate to the task of coordinating (basic) objective reference. Indeed, our naive experience of objects and their properties suggests that we have a more or less immediate access to particulars as individuals (i.e., we perceive individuals without the need for explicit cognitive reflection), and these individuals are bearers of empirical properties (expressed in space and time), which can be reflectively abstracted as essential or accidental properties. Certainly, problems of vagueness and ambiguity inherent in natural language make the successful application of the Aristotelian epistemic paradigm to ordinary experience problematic in a number of ways, but these sorts of problems were widely — if not universally — believed to be soluble through the precise semantic determination of essential predicates and their rigorous application.

While there were many pressures that led to the ultimate demise of Aristotelian epistemology, foremost among these was undoubtedly the rise of mathematics in its application to natural physics.[15] Whereas Aristotelian physics is concerned primarily with the motion of individuals (*energeia*, as motive powers that can be ascribed to the individual), modern mathematical natural physics was not (or not primarily) expressed in terms of individuals and their properties.[16] For example, the Galilean laws of motion describe the motion of objects generically; that is, motion varies according to a function of space with repect to time (e.g., $v_2 = v_1 a t^2$)

[14] The case of Euclid's *Elements* is interesting. For it is far from clear how exactly the demonstrative proofs of the *Elements* are to be interpreted through the traditional knowledge schema. The problem persists even in Kant's time, and Kant's interpretation of the demonstrative aspect of the Euclidean method is central in the (now famous) dispute between Hintikka (Hintikka 1967) and Parsons (Parsons 1969) over the (logical) nature of Kant's philosophy of mathematics.

[15] The canonical study is, of course, Cassirer's *Substance and Function* (Cassirer 1910), but Cassirer's monumental study, *The Problem of Knowledge in the Philosophy and Science of the Modern Era* provides an impressively synoptic account of the slow demise of Aristotelian epistemology (Cassirer 1920).

[16] However, Newton's *Principia* is surprisingly still couched in terms of substances and motions (Newton 1803).

and does not depend on any properties of individuals, but only on a general property of bodies as modifications of matter. Indeed, the corpuscular theory of matter (Democritus, Epicurus, Hobbes, Cartesian vortices), suggests that knowledge of the (universal) laws of motion is prior to the knowledge of individuals, not, as Aristotle supposed, the other way around. The priority of the individual in the explanation of natural phenomena is undermined by the new paradigmatic expression of natural science: the law of nature.

While there is no single identifiable moment at which the tension between Aristotelian epistemology and the claims of modern physics became insuperable, one decisive moment in the development of the problem is Descartes' *Meditations* (Descartes 1984).

1.2.2 Descartes' Descriptive Skepticism

In the *Meditations*, Descartes advances two different types of skepticism. Most philosophical attention is focused on the first type, raised in the first meditation (Cartesian skepticism, external-world skepticism). There, Descartes advances the possibility that all of our relations to the external world are illusory, and we cannot distinguish between reality and the unlikely but possible scenario of that our perceptions are hallucinations induced by a malicious demiurge. But there is another, more potent form of skepticism raised in the second meditation. There, Descartes famously argues that we know the mind better than we do the body, because there is a difference in the ways in which we have access to the reality of each. On the one hand, we know objects through the speciation of generic concepts (i.e., the monadic relation of genus and species). So, traditionally, thought is interpreted as a mode of access to particulars through the combination of generic concepts and speciating properties. However, what appears to be given in sensation consists neither (a) of generic concepts nor (b) of intrinsic properties. As the methods of empirical observation demonstrate, we seem to have empirical access not to real particulars (and their generic concepts) but to sets of relational properties expressed as magnitudes: polyadic quantifications of space and time.

This difference between representational modes (intelligible speciation and empirical

description), however, results in a difference within the mind — a gap between sensible and intelligible forms — that traditional logic is unable to bridge.[17] For, the logic of the syllogism proposes a system of monadic relations between (a) subjects (substances) and (b) predicates (accidents). However, the properties available to empirical observation consist of (polyadic) quantifications of space and time. So, for example, if we describe an apple as red, what we in fact describe is a consistent qualitative impression across a three-dimensional surface. The complex relation between the empirical unity and sets of space-time points cannot be expressed syllogistically, however, since the expression of a relation between sets requires a polyadic logic. Although Descartes does not frame the problem in terms of the expressive limitations of syllogistic logic, he does draw the relevant conclusion: if thought consists essentially of monadic relations, the empirical individual — perceived as a quantification of serial relations of space and time — is not the immediate object of thought.

Descartes' argument in the second meditation is used to motivate rationalism: if knowledge is expressed syllogistically (i.e., in the form of judgment), then it cannot arise from empirical experience. Accordingly, all knowledge arises from innate ideas, not from sense impressions. This does not, of course, mean that knowledge cannot be correlated with our empirical experience. For Descartes, the locus of empirical knowledge is judgment, not experience, and certainly not pure thought. That is, if knowledge of extension is possible, it is because we have access to, for example, impressions of a ball of wax, whose properties we are able to observe under various transformations. We know that the wax is wax, however, not because it is given to us immediately as an empirical individual with empirical properties, but because we are able to abstract from sensible variations through ideas of the mind: mathematics.[18] Thereafter, we

[17] Descartes seems not to have been aware of the problem that his understanding of 'concept' would not permit (even innate) judgment about the polyadic relations of sense. Friedman has argued that Kant noted the problem, and developed the theory of intuition precisely to provide the mind with two sources of formal knowledge ("logic", or the theory of monadic relations, and "intuition", or the theory of polyadic relations and their quantification) (Friedman 1990), (Friedman 1992a), (Friedman 1992b). Plausibly, however, Kant's reason for denying that arithmetic truths cannot be derived from the principle of contradiction is not a product of any explicit recognition of the limitations of monadic logic, but is rather a consequence of his theory of immediate certainty and determinacy. That is, Kant too (despite Friedman's thesis) may not have recognized the inadequace of monadic logic.

[18] So, as Cassirer notes, "every **problem** of cognition already contains a **presupposition** insofar as it — merely

produce a judgment to the effect that the object of our experience is indeed a ball of wax, since it conforms to the predicate 'wax-like', that is, it is malleable, melts when heated, is inflamable under some conditions, etc. While knowledge is grounded in the forms of thought, it is based upon abstraction from its empirical origin, and, more importantly, on the intelligibility of *change* expressed in terms of mathematical functions.[19] Accordingly, judgments of meta-

to be understandable — must lay the foundation of a particular lawfulness of cognition" (Cassirer 1899, 4). Accordingly, the new method of Cartesian knowledge grounds objects not immediately in empirical sense, but in the laws that are derived in the judgment of the general organization of empirical properties:

> The method of mathematics therefore generally shows that reality, achieved as the result of science, means nothing other than the immanent being of law, which already lays the foundation of the original problem of science. (Cassirer 1899, 9)

[19] As Descartes describes this in his *Rules for the Direction of the Mind*, (Descartes 1985) *Rule II* specifies, "We should attend only to those objects of which our minds seem capable of having certain and indubitable cognition" (AT X:362). The *Meditations* might lead us to believe that rational reflection (on innate ideas) must be the origin of all knowledge. However, this would misconstrue Descartes' broader epistemological project: the foundation of empirical knowledge in the (innate) language of mathematics. As Descartes specifies in the scholium to *Rule II*, "out of all the sciences so far devised, we are restricted to just arithmetic and geometry if we stick to this Rule" (Rule II, Scholium, AT X:363). What this means for knowledge, however, is that have only access to "simple" properies of nature:

> If, for example, we consider some body that has shape and extension, we shall indeed admit, with respect to the thing itself, it is one single and simple entity. For, viewed in that way, it cannot be said to be a composite made up of corporeal nature, extension and shape, since these constituents have never existed in isolation from each other. Yet with respect to our intellect we call it a composite made up of these three natures, because we understood each of them separately before we were in a position to judge that the three of them are encountered at the same time in one and the same subject. (Rule XII, Scholium, AT X:418)

From this we might infer that Descartes implies that we have a distinct intuition of extension just as we do of body, i.e., that it is an individual object, such as a ball of wax. However, the *Rules* are much more specific about how knowledge should be acquired. As *Rule XIX* explains: "Using this method of reasoning, we must try to find as many magnitudes, expressed in two different ways, as there are unknown terms" (AT X:468). The reason for this is that while we have intuitive knowledge of particulars, we can only understand our knoweldge of extension. And thus, (*Rule XIV*): "The problem should be re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures. Thus it will be perceived much more distinctly by our intellect" (AT X:438). There seems to be no room for doubt, at least in the *Rules*, that there is little space left for philosophical logic after mathematics has displaced its claims to knowledge.

physics — e.g., of individuals and their essential properties, or of causal relations between them — are only possible as descriptive relations, which are then ascribed to (rational) unities. Thus, the *unity* of the empirical individual and also the *diversity* of its empirical properties are *subjective in origin*, despite the fact that knowledge of objects necessarily implies sensory perception.

The epistemological consequences of Descartes' skepticism is the denial of immediate access to empirical individuals. Real individuals — if there are any — are mediated by the innate structures of thought and the reflective judgments that they allow us to make about the regularities of experience. Only these judgments allow us to come to know particulars. However, in the Cartesian epistemology claims to knowledge of empirical individuals derives its justification not from direct access to their (empirical) reality, but from (a) the infallibility of our cognitive apparatus and (b) the benevolence of a creator who guarantees the reliability of the sensations upon which judgments of experience are based. If Descartes is correct, and we do not have immediate empirical access to individuals or individual properties, then knowledge of the empirical consists solely of (a) a subjectively grounded necessary unity of the individual (i.e., grounded in the syllogistic structure of thought) and (b) subjectively grounded necessary relations (mathematical functions) that ground empirical properties in the individual. Descartes' epistemology is revolutionary in many respects, but the full significance of Descartes' proposal is not realized until it is confronted with another kind of skepticism: Hume's skeptical attack on necessary relations.

1.2.3 Humean Skepticism

Hume's skeptical attack focuses on causality: the idea of a necessary (temporal) relation between individuals. However, as Kant saw, Humean skepticism can be generalized to undermine any kind of relation that claims necessity, including the structure of space and time and the truths of mathematics.[20] As we have seen, the empirical individual rests on a subjectively grounded

[20] Kant claims that Humean skepticism can be extended to include the truths of mathematics. Unless we are willing to endorse Kant's intuitionism, there seems to be little reason to endorse Kant's view on this point. However, even if we do not accept that mathematics is undermined by Humean skepticism, we may nevertheless draw an

claim of necessary rational unity (the real individual) and a subjectively grounded function of necessary spatio-temporal relations (empirical properties). Since the new paradigm of scientific knowledge — modelled after Descartes, Leibniz or Netwon — consists of knowledge of necessary functions (laws) and judgments about them, Humean skepticism not only undermines the traditional concept of causality, but the entire apparatus of modern science. What the threat of Humean skepticism showed Kant is that it is not merely the concept of cause that requires a justification, but the very idea of a necessary relation, and paradigmatically, the idea of the natural physical law.

Now, Hume's positive epistemological doctrine rests on the possibility of identifying particulars (of sensation, or anything else) through their intrinsic properties. For, if individuals are identified through necessary relational properties, then it would turn out to be the case that relations are necessary for (the identification of) particulars, thereby undermining the generality of Hume's dictum (cf. Jessica Wilson's "What is Hume's Dictum and Why Believe It?" (**?**)). To this extent, Hume's (positive) epistemological programme remains dependent on the traditional epistemology of substances and accidents; in fact, his empirical skepticism amounts to the claim that what can be known consists solely of sensory points and their (intrinsic) qualitative properties.

It is central to Kant's response to Humean skepticism that precisely the presupposition of immediate knowledge of particulars (Russell: knowledge by acquaintance) is called into question. Kant's *Prolegomena* focuses on showing how the concept of causality is necessary to scientific representations, but the subtext is clear enough: sensory particulars can only be identified within space and time, which, as an order of necessary relations, must have a subjective origin. Thus, the thesis of the ideality of space and time amounts to a the claim that empir-

equally devastating skeptical conclusion for natural science: that no relation expressed by a mathematical function can be taken to be necessary. It is central to Kant's strategy that the truths of mathematics cannot be separated from the truths of natural physics. However, if we do not accept the psychological or metaphysical aspects of Kant's transcendental logic, there seems to be little reason to think that the *mere form* of a mathematical relation is undermined by the impossibility of its representation in sensibility. Nevertheless, we may still say that all laws of natural science will have the form of a mathematical relation; and if this is so, the consequence of Humean skepticism is still the denial of the "law" of nature as a necessary relation.

ical individuals (including sense data) are only possible objects of recognition and cognition insofar as they are located within a system of necessary (subjective) relations.[21] Accordingly, the claim that sense data are relationally identified in the manifold of intuition amounts to a denial of the Humean claim that sensory particulars can be identified through intrinsic properties. Indeed, if we accept the claims of Transcendental Idealism (and especially the thesis of the ideality of space and time), then it is not so much the Humean claim against causality that is refuted, but rather the claim that empirical particulars are intelligible independently of the relations in which they are given.[22] Kant's Transcendental Idealism, therefore, does not refute Humean skepticism but rather sharpens the problem: if we accept Hume's skepticism then we lose not just knowledge of causal relations, but knowledge of all relations, including those through which empirical particulars are identified.[23]

The price of accepting Kant's thesis of the ideality of space and time is the denial of a direct correspondence between the modality in which particulars are given (i.e., as polyadic

[21]Compare, for example, Kant's addition to the General Remarks of the Transcendental Aesthetic in the B edition:

> For confirmation of this theory of the ideality of outer as well as inner sense, thus of all objects of the senses, as mere appearances, this comment is especially useful: that everything in our cognition that belongs to intuition ... contains nothing but mere relations, of places in one intuition (extension) alteration of palces (motion) and laws in accordance with which this alteration is determined (moving forces). (KRV; AA B66)

[22]Sellars' argument (in *Empiricism and the Philosophy of Mind*, see (Sellars 2000)) against the intelligibility of sense data as intrinsic individuals (and especially Russell's knowledge by acquaintance) has the same structure as Kant's anti-Humean strategy in the *Prolegomena*. Kant's relational claim against knowledge of intrinsic properties is generalized by, for example, Maimon's claims that even colour properties are known through a relational system of differences (as, for example, a colour wheel, which could not only be "inverted" — as the standard argument runs — but rather "rotated" arbitrarily, as it is merely a system of differences, not a system of intrinsic determinations) (Maimon 1790), (Maimon 1794).

[23]If it were the case that Kant's arguments — either in the *Critique of Pure Reason* or in the *Prolegomena* aimed to refute Humean skepticism, then they would have to establish the following claim: Necessary relations such as cause and effect are necessary objective relations. However, the epistemological transformation implied by the *Critique of Pure Reason* shows that such a claim can neither be proved nor disproved, but rather results from the (epistemically necessary) hypostasis of a priori structures of intelligibility. The most that the *Critique of Pure Reason* is able to show is that necessary relations are a condition of the intelligibility of any claim regarding the objective manifold; that is, that even simple claims about appearance are grounded in necessary relations, without which they would not be intelligible. Accordingly, Kant's epistemic programme cannot refute Humean skepticism. At most, Critical Philosophy can show that Humean skepticism is a justified criticism of dogmatic metaphysics, but fails to establish the justification of its own epistemic claims. Ameriks provides a helpful taxonomy of the various interpretations of Kant's idealism and the degrees of skepticism to which it must lead (Ameriks 2000).

relations) and an onto(theo)logic of substances and accidents. This is precisely what Kant's term 'appearance' designates: that the way in which particulars are given is formally incompatible with the ontological substrate assumed by traditional epistemology. If empirical knowledge is to be possible at all, it must be knowledge of appearances, that is, knowledge of a system of relations. But if such knowledge is to be possible at all, it cannot consist of the adequate correspondence between subjective representation and objective reality. Rather, Transcendental Idealism demands a new kind of epistemology, one that takes up the challenge of showing what it means to say that a relation between relations is "true".

1.3 Transcendental Epistemology

1.3.1 Ontological Nihilism

As we saw, Cartesian epistemology teaches that neither particulars nor properties are given in empirical sensation. Rather, the method of empirical observation and its mathematical description demonstrates that the intelligibility of empirical perception consists solely of relations. Accordingly, the origin of all possible knowledge of necessary empirical relations must have a subjective (or innate) origin in judgment and its (mathematical) articulation. Skeptical empiricism, however, teaches us that no relation (causal or otherwise) with a subjective origin can ever be shown to be adequate to its object on empirical grounds, since no such relation can ever be shown to be necessary in anything more than a subjective sense. However, since empirical individuals (the particulars of sense) cannot be identified and reidentified (recognized and cognized) except through spatio-temporal relations and changes in these, neither empirical individuals nor their intrinsic (qualitative) properties are intelligible on the basis of merely empirical (sensible) perception. Accordingly, it is not just the traditional metaphysics of causality that are in jeopardy, but even — or especially — the possibility of cognition.

The combination of Descartes' property skepticism and Hume's relational skepticism gives rise to a potent form of generalized skepticism: nihilism. If Descartes is right, then

empirical knowledge has the form of space-time relations; but if Hume is right, these relations have no justification beyond a mere rational dogmatism. If knowledge consists of the subjective representation of the necessary structures of objective being, then knowledge appears to be impossible. If we accept the skeptical arguments that lead to Descartes' rationalism and those that lead to Hume's empiricism, then knowledge in the traditional sense is impossible. For, the empistemological validity of the principle of adequation, it seems, rests on an assumption about the nature of empirical perception that must be denied by "rationalists" and "empiricists" alike; it must either be accepted as a matter of dogmatic faith (as Jacobi would propose in his *David Hume on Faith, or Realism and Idealism* (Jacobi 1787)), or the traditional model of knowledge as metaphysical, alethic and formal adequation must be abandoned.

The tension between two competing claims reaches a symbolic climax in Newton's universal law of gravitation. While Galilean kinematics had already offered a mathematical conception of a law of nature, it suffered from two crucial limitations. First, it isn't always true to our naïve observation of bodies (feathers in fact fall more slowly than bowling balls, regardless of the law of falling bodies).[24] Second, the Galilean laws remain laws of *bodies* and therefore seem to be compatible, with some adaptation, with an Aristotelian view of a physics of individuals, at least insofar as it remains a generic law of the motion of bodies (*energeia*). Newton's law, on the other hand is not only *universal* (within the paramaters of early modern observation), but it also does not require the concept of a body; rather, the law of gravitation is concerned with magnitudes of *mass*. What is more, Newton replaced Galileo's kinematic notion of *acceleration*, a property of bodies, with the dynamic notion of *force*, a relational property of mass in general, not one of bodies as particulars.[25] Newton's law, therefore, not only proposes

[24] In order to consider factors such as wind-resistance, we must consider resistive forces, which already moves us beyond a merely kinematic conception of body and requires a generalized theory of the way that bodies act upon one another. While this may be expressed in terms of causes and effects in Aristotelian terms, the spirit of the Galilean laws is to provide a mathematial law of the motion of bodies. It's unclear how a mathematical law of causes and effects could be interpreted.

[25] One can (and we generally do) treat spherical bodies as a point-mass. This, however, is to simplify the calculation, not because a point-mass is the "essence" of the extended body. The complete calculation of gravitational force implies a calculation of the reciprocal force exerted between every mass-point in two bodies. Treating bodies as (approximately) point-masses simplifies this calculation and remains reasonably accurate for objects that are sufficiently far apart.

(as Galileo's already had) something like a metaphysical substrate to mathematical phenomena (an algebraic law of nature), but, more importantly, it suggests that such laws may hold with necessity without referring to individuals or their intrinsic properties at all.[26] In other words, metaphysics may be a property of mathematical expressions — relations between magnitudes — that seem to have no relation to the logic or the metaphysics that had, traditionally, been thought of as the foundation of all possible knowledge. Knowledge of the essential nature of things may not consist of knowledge of individuals, but knowledge of the essential nature of nature itself: the whole of natural being through a calculus of the continuum of difference.

According to Kant, Humean skepticism — in its strengthened form — shows that the gap between appearance and (absolute) reality cannot be bridged. For, Kant denies the Humean premise that knowledge may accrue by habituation, since, in the absence of a subjective order of relations, there would never be any regularity to which we could become habituated.[27] Instead, Kant endorses the rationalist premise that subjective origin is the condition of intelligibility in general; minimally, this is the thesis of the ideality of space and time. But, the Humean claim just is that the objective validity of representations of relations cannot be demonstrated, since this would imply that something relational would have to be given empirically. However, this contradicts the empiricist thesis that sensible particulars are identified exclusively through intrinsic properties. Accordingly, if relationality is the condition of intelligibility, then all knowledge is nothing but knowledge of the structure of subjective representation, or appearance. Whatever correspondence may exist between subject and object, or appearance and reality, can never itself be represented at pains of vicious circularity and vacuous dogmatism.

The consequence of Humean skepticism for traditional epistemology, Kant concludes, is a dilemma: either our epistemic representations are vacuous (concepts), or they are unintelligible (intuitions): "without sensibility, no object would be given to us, and without understand-

[26] Cf. Cassirer in *Substanzbegriff und Funktionsbegriff* (Cassirer 1910, Chapter 7).

[27] For example: "The strict universality of the rule is therefore not any property of empirical rules, which cannot acquire anything more through induction than comparative universality, i.e., widespread usefullnes" (KRV; AA A91/B124). Or again: "**empirical derivation** ... cannot be reconciled with the reality of scientific cognition *a priori* that we possess, that namely of **pure mathematics** and **general natural science**, and is therefore refuted by the fact" (KRV; AA B128).

ing none would be thought" (KRV; AA A51/B75). If we embrace the first horn of the dilemma, then all appearances are in fact necessary, but they tell us nothing about the nature of reality. Such knowledge consists merely of knowledge of possible appearance; knowledge of *reality* as the correspondence between appearance and intrinsically-typed entities would then rest, as it does for Leibniz, on a miraculous correspondence between subjective representations and (unknowable) objective reality. If we embrace the second horn of the dilemma, then individual representations are unintelligible, since all that can be represented is a succession of unconnected events.[28] It is not just Newtonian mechanics that must be abandoned, but the possibility of coherent experience even of ordinary objects.

1.3.2 The Copernican Revolution

Kant's solution to the dilemma is what he calls Transcendental Idealism and Empirical Realism. The label is not particularly helpful, since what it suggests is a kind of hybrid position, or, as it was often interpreted, as a degenerate form of subjective idealism. Kant's characterization of the position as well as that of his critics often fails to highlight the crucial moment of the Copernican revolution. For, it is not the case that subjective form (appearance) somehow becomes the measure of the adequacy of a correspondence between individuals — which would be the precise inversion of the traditional model of knowledge, Protagoras' "Man is the measure of all things" — but rather that knowledge will no longer be expressed in terms of adequation at all.[29] Indeed, the primary casualty of the "Copernican revolution" is the principle of adequation

[28] For example:

> If every individual representation were entirely foreign to the other, as it were isolated and separated from it, then there would never arise anything like cognition, which is a whole of compared and connected representations. (KRV; AA A97)

[29] *Prima facie*, Kant endorses the traditional doctrine of truth: "the nominal definition of truth, namely that it is the agreement with cognition with its object, is here granted and presupposed" (KRV; AA A58/B82). However, the doctrine of Transcendental Idealism teaches just the opposite, since "the object itself is regarded merely as appearance" (KRV; AA A36/B53).

The characterization of Kant's position as "Transcendental Idealism and Empirical Realism" has some truth to it, at least to the extent that the *Critique of Pure Reason* delivers a doctrine of metaphysics.[30] However, the

itself.[33] We can no longer maintain the (Aristotelian) claim that appearance is in some sense adequate to reality. We can, at most, say that appearance is in some sense *like* reality. This "likeness" is informed by, but not determined or demonstrated by, the subjective structures of representation, i.e., the functions of judgment.[34] And so, after the Copernican revolution, truth no longer consists of the adequacy of subjective and objective states of affairs, but of the very activity whereby a correspondence between dissimilar orders of entities is established in the first place. That is, we are no longer interested in the conditions of the identity between individual representations and their objects, but in the functional relation between (loosely speaking) *sets* of representations and *sets* of objects.[35]

---

philosophical innovation of the *Critique* is not the ontology that will ultimately be proposed in the *Metaphysical Foundations of the Natural Sciences* or in the *Metaphysics of Morals*, but the method whereby any possible ontology is to be justified: the method of critique.[31] Accordingly, Kant's idealist successors do not take themselves to inherit Kant's metaphysics, but rather the methodology of *critique*; they are "critical philosophers".[32] Ultimately, the method of critique will be applied well beyond the boundaries of the "transcendental" (i.e., the possibility of objects as such) to include the theory of action, the theory of subjectivity, and, after Hegel, the theory of history, God and "spirit".

[33]In fact, Kant still endorses the principle of adequation as the nominal definition of truth (KRV; AA A59/B84). However, it is clear from the context that the adequation of an object and its subjective representation is understood to be a *metaphysical* proposition that can neither be proved nor disproved, hence the denial of any logical principle of truth (and the rejection of Aristotelian epistemology). The task of the *Critique of Pure Reason* is thus to correlate concepts and appearances (judgments) in order to infer truth in the absence of adequate (metaphysical) grounding; this just is the task of the analogies of experience, i.e., the coordination of the object of ascription (the thing in itself) with subjective representations (continuous appearances).

[34]Kant emphasizes this point especially strongly in the consideration of the Analogies of Experience:

> An analogy of experience will therefore be only a rule in accordance with which unity of experience is to arise from perceptions (not as a perception itself, as empirical intuition in general), and as a principle it will not be valid of the objects (of the appearances) **constitutively** but only **regulatively** ... What must be remembered about all synthetic principles and especially noted here is this: that these analogies have their sole significance and validity not as principles of the transcendental use of the understanding but merely as princples of its empirical use, hence they can be proven only as such; consequently the appearances must not be subsumed under the categories per se, but only under their schemata. (KRV; AA A180/B222)

It should be noted, however, that the point here is that (transcendentally) appearances are subsumed under mathematical principles; Empirical Realism is a *metaphor*, through which we treat objects "as if" they were intelligible, when, in fact, they are not (**?**). Any questions that might arise about the object of reference of transcendental judgments (i.e., judgments that employ the categorial forms) must therefore take into account the following limitation: the determination of objects through their phenomenal marks (rather than their "real" marks or their spatio-temporal functions) is nothing more than a *façon de parler*; it is an *analogy*. If Kant's transcendental philosophy includes a doctrine of reference, it is essentially as a *heuristic*, and cannot be (transcendentally) well-founded.

[35]Or, as Natorp notes in his *Logische Grundlagen der exacten Wissenschaften*, "it is a wholly new stage in reflection, not to inquire into some particular object, but to ask after the laws whereby this and any object of science is first formed into an object" (Natorp 1910, 10). We can grasp a central hermeneutic problem for the

Above we noted that the principle of adequation depends on two further principles: the principle of intelligibility and the categorematic principle. After the Copernican revolution, the principle of intelligibility can no longer be directed toward the reality objective being, but can, at most, serve as a heuristic principle for the (finite) construction of intelligible forms of appearance. Accordingly, the principles of synthesis are transformed into regulative principles: the idea of the world-whole. That is, the possibility of knowledge rests on the rational conviction that the totality of the manifold (the "natural" world) ultimately falls under the syllogistic structures of intelligibility. This principle is merely regulative, however, since the world-whole is not a possible object of cognition, i.e., it has no determinate representation in intuition.[36]

Similarly, the ontological implications of the categorematic principle can no longer be

---

interpretation of the *Critique of Pure Reason* in the following question: 'how many objects are there?' On the one hand, there seems to be only one rational object, the 'transcendental object', which implies only one (absolutely) real object: 'the' *Ding an sich selbst*. However, transcendental synthesis makes possible *arbitrary* combinations of objects, and thus an infinite number of possible modifications of the manifold (although, Kant appears to make significant assumptions about what combinations are in fact possible). There are, therefore, an infinite number of possible continuous appearances. However, there is only a finite number of empirical individuals, that is, of unities that may be brought to apperception. Accordingly, if the problem of the transcendental doctrine is that of coordinating sets, three questions must first be answered:

(a) What sets are being coordinated?

(b) How many objects are in each set?

(c) What is the nature of the correlating function? Is it a one-to-one, one-to-many, many-to-many, or many-to-one relation? Is it a functional relation, or merely a 'metaphor', or an intelligible 'closeness of fit' that tolerates several interpretations?

These are difficult questions to answer. Fortunately, we do not need to know what Kant ultimately thought in order to make sense of what problems and solutions arise out of the principles of his philosophical project.

[36]Indeed, the solution to the cosmological dialectic rests in the application of Transcendental Idealism as a (heuristic) substitute for the empirical principle of intelligibility:

> Nothing is really given to us except perception and the empirical progress from this perception to the other possible perceptions. For in themselves, appearances, as mere representations, are real only in perception, which in fact is nothing but the reality of empirical representation, i.e., appearance. To call an appearance a real thing prior to perception means either that in the continuation of experience we must encounter such a perception, or it has no meaning at all. (KRV; AA A493/B521)

straight-forwardly endorsed after the Copernican revolution. Nevertheless, Kant does not simply abandon Aristotelian logic and its metaphysics. Rather, the a priori concepts of the understanding are taken to be necessary (cognitive) conditions of the intelligibility of *unity* in empirical representation.[37] Accordingly, the hypostasis of a priori structures (space and time, the categories, transcendental unity) forms a hypothetical metaphysics that is warranted only as the intelligible condition of objective epistemic claims.[38] The consequence of the Copernican revolution is therefore a transformation of the primary modality of epistemic claims. It is no longer the case that knowledge claims are expressed in terms of the adequation between subject-predicate pairs and substance-accident pairs. Indeed, the mathematical revolution in physics suggests that syllogistic judgments have little, if anything, to do with knowledge of nature, which rather consists of complex systems of relations of the continuum of space and time.

The crucial moment of the Copernican revolution, however, is the concept of *unity* and its transcendental expression: *reality*. For, the possibility of the synthesis of the noumenal and the phenomenal rests on the fundamental homogeneity of the unity of the empirical and intelligible object; indeed, "all judgments are accordingly functions of unity among our representations" (KRV; AA A69/B94). The expression of this synthesis is the transcendental predicate 'reality', which affirms that the empirical individual (i.e., the unity of an intuition of a particular) subsumes the intelligible unity of the predicate; that is, that the cognitive condition

[37] That is, as we have noted, the categories of relation are schematized as "analogies of experience"; this analogy is conditioned by (a) the irreconcilable structures of polyadic quantification and subject-predicate expressions and (b) the commitment to the (intrinsic) necessity of certain combinations of synthetic principles (e.g., judgments of substance), which are not directly warranted by experience. The former condition is satisfied by the a priori interpretation of geometrical construction through the categorial forms (paradigmatically, through the construction of the series). The second condition is satisfied by the (perhaps a priori) commitment to the intelligible unity of the individual, i.e., its continuity and its boundaries. Both aspects of this solution are extremely problematic from the point of view of post-Kantian systematic idealism.

[38] That is:

> It is permissible to take refuge in opinion concerning the actuality of the object, which opinion, however, in order not to be groundless, must be connected as a ground of explanation with that which is actually given and consequently certain, and is then called an **hypothesis**. (KRV; AA A770/B798)

of the application of an intelligible predicate to the manifold is the unity of empirical intuition, or the fact that a priori intuitions indeed apply to empirically given intuitions. The judgment of affirmation (i.e., the predicate 'reality') therefore requires not only the schematic quantification of a judgment of quantity (i.e., the quantification of the serial order of space or time), but furthermore the (antecedent) logical unity of the intuited individual, without which there would be no intelligible unity to which the predicate applies. The possibility of a cognitive synthesis therefore rests on a bold claim about the original unity of apperception and the objects that may appear within it.

The peculiarity of Kant's transcendental epistemology is this tension between the empirical manifold (appearances as such) and the empirical individual (appearance).[39] The Dialectic

[39]It is not easy to discern the nature or the extent of Kant's metaphysical commitments. Partly, this is because Kant is not always explicit about just what the object of knowledge actually consists in. Indeed, Kant fails to make explicit the distinctions between intuition as such (i.e., the a priori forms of intuition, or the subjective representation of space and time), the object of empirical laws, or appearance as such (i.e., the manifold of objective, or real space-time) and the intuition of a particular appearance (i.e., the empirical object).

The differences between these representations is made explicit through the science which applies in each case, and the unity the corresponds to the science in question. The science of intuition as such is mathematics: arithmetic and geometry, i.e., the science of possible constructions of the a priori manifold. The unity of the a priori structures of space and time is guaranteed by the fact space and time may be thought of as given totalies, i.e., that we may quantify a priori across the whole of space or time. The science of appearance is natural physics, i.e., the science of those configurations of space that occur in time, or the laws of space-time. The unity of the empirical manifold is guaranteed by the transcendental unity of apperception, and, more specifically, through the continuity that is thought to be an essential feature of this transcendental unity. The science of appearance as such is the empirical description of particulars, i.e., descriptive (empirical) concepts. The unity of the appearance of a particular is provided by the synthetic activity of the imagination, and, by hypothesis, draws on the implicit structures of the functions of judgment. How we understand the role of unity (and thus the place of the synthetic) in Kant's project depends on which of these descriptive sciences (empirical objects or empirical laws) we take to be explanatorily primary.

For the purposes of this study, I take the view that Kant is committed to the rehabilitation of the metaphysics of substance and cause for the purposes of the descriptions of natural physics. This implies a metaphysical reading of *Critique of Pure Reason*, although not one which commits us to transcendental realism. Rather, the Copernican revolution implies that our descriptions must employ concepts of substance and cause, even though these categories cannot be explicated in terms of the manifold of space-time.

of Pure Reason reveals the tension between the logical mode of intuition (serial order) and the logical mode of the understanding (syllogism). However, Kant's doctrine of synthetic cognition conceals another dialectic: the dialectic of the totality and the individual. This dialectic arises within intuition itself — rather than in the empirical interpretation of intuition — as the tension between the infinite a priori totality of the form of intuition (space and time) as infinite *quanta continua* and the unity of the finite empirical individual as it is present in apperception. The Kantian thesis is that the unity of the empirical individual is ultimately grounded in the synthetic activity of the imagination (KRV; AA B136). However, the problematic nature of the Kantian view arises precisely in this apriority of the synthesis of the imagination: how can totality (the *totum reale* as whole, i.e., as the manifold of space and time) and the individual (the *totum reale* as part, i.e., as the empirical individual, its continuity and its boundaries) both be a priori forms of intuition? Conversely, how can logical totality (with its manifold of rational differences) and the logical unity of the synthetic function both be conditions of one and the same understanding? These problems are not resolved by the *Critique of Pure Reason* — not even in the resolution of the Mathematical Antinomies. Their solution, however, is crucial to the post-Kantian transformation of transcendental logic.

Now, Kant was certainly not the first to propose changes to the traditional epistemology. What is peculiar to the *Critique of Pure Reason*, however, is the consistency with which the project is carried out. For, Kant devotes as much attention to the critical restriction of the application of the structure of synthetic judgment in the Dialectic of Pure Reason. Accordingly, it is not merely the metaphysics of nature that must be revised, but also those other classical branches of metaphysical speculation: rational psychology and theology. Kant recognized that the new model of knowledge in mathematical physics implies a complete transformation of the modalities of knowledge, one that generally undermines traditional claims about the nature of the subject, the cosmos, and God. That is, Kant's new "transcendental" epistemology proposes an entirely new *method* of investigation, one based on the dialectical nature of the manifold of the continuum and the syllogistic forms of representation. That is, the *Critique of Pure Reason*

does not simply reject the metaphysical objects of traditional epistemology, but transforms them into cognitive conditions (transcendental conditions) of the intelligibility of reality.

Even if Kant was not himself aware of the depth of the transformation implied by Transcendental Idealism, critics and commentators were quick to point to the generally skeptical conclusions that follow from the proposals of the *Critique of Pure Reason*. And so, while Kant's stated ambition is to put metaphysics on the sure path of a science, it is not at all clear, at least in the *Critique of Pure Reason*, how the critical apparatus of the philosophy of nature will yield similar results in (philosophical) anthropology and theology.

1.3.3 Mathematics and the Continuum

The rise of mathematical physics transformed the language in which truths about ordinary objects were expressed. Just as important as the transformation of the language in which epistemic claims are framed is the object implied by mathematical physics. Classical (Aristotelian) physics is concerned with particulars and their interactions. This model of investigation is well-suited to classical epistemology, with its syllogistic logic and substance ontology. After the Copernican revolution, however, it is no longer the particular which is the proper object of physical inquiry, but rather appearance as such. That is, natural physics is not the investigation of objects or bodies, but the investigation of the manifold of space and time, or, what is the same, the real continuum ($\mathbb{R}^4$): "Nature".

The new physics requires, as we have seen, a different mode of expression and a different mode of justification. However, it also requires a new object: the real continuum. With this new object, a new problem is introduced into logic, metaphysics, and epistemology, and ultimately mathematics: the problem of the continuum.[40]

[40]Of course, it was Leibniz, not Kant, who introduced the continuum as the ultimate object of philosophical investigation, and Leibniz who gave the continuum its first precise formulation. While the *Monadology* remains committed to the traditional epistemology of adequation (pre-established harmony) and an ontology of substances and accidents (monads, perceptions), Leibniz nonetheless already recognized that every truth would be a functional relation. For Leibniz, this functional relation does not take the form of a system of polyadic relations of the continuum (as it does for Newton or Kant), but of a monadic relation between an objective time-index and the complete concept of the monad, or its "substantial form". The Leibniz-Wolff school is famous for endorsing the

It was not just logic and epistemology that laboured under the burden of the Aristotelian legacy. Mathematics, too, was shaped by the particular project of the Aristotelian school. This burden is perhaps nowhere more explicit than in the doctrine of the infinite and the infinitesimal. Aristotle famously declares that there is no actual mathematical infinity: *infinitum actu non datur*.[41] Indeed, this dogma was still actively enjoined when, in the 1880's, Georg Cantor began arguing for an actual infinity of actual infinities (i.e., the transfinites $\aleph_n$). However, Aristotle distinguishes between different types of infinities, of which some may be potential, and others are straightforwardly impossible. In the *Physics*, Aristotle identifies (among others), two essential types of infinities: infinite numbers and infinite extensions. Furthermore, number and extension may be thought of as infinite in two different ways: as an additive infinity or as a divisive infinity.

Aristotle's claim is that there is no actually infinite number, but there is a potentially infinite number. The reason for this is that for any (actual) number that can be given, another (potential) number that is greater than it can be found, simply by adding one to the actual number. However, Aristotle also claims that there is neither an actual infinite extension, nor a potential infinite extension. In other words, all (particular) extensions are finite. Conversely, there is no actually divisively infinite number, for every numer reaches a point at which its division (without remainder) leaves only the unit (the number 1), which cannot be further subdivided (into units) without remainder. In other words, the least number is finite. Extension, on the other hand, has no actually infinitesimal part, since for any actual extension that can be

view that all of logic — including mathematics — could be derived from the principle of contradiction. This may be interpreted as an overly optimistic claim about formal logic, or, for Leibniz as the expression of his understanding of fundamental ontology. For Leibniz, the monad is a (real) subject that has at most one proper (real) predicate: its complete concept. There is exactly one true predication of a monad, and that is the complete (and infinite) set of states (perceptions) that it will go through according to the law (or fact) of its appetition. While the foundation of the calculus in the concept of order may ultimately require polyadic logic, its ontological foundation requires only monadic logic: there are substances (monads) and their unique predicates (complete concepts). If, however, we wish to say that the complete concept can or must be explicated in terms of an infinite set of perceptual states (i.e., the complete concept is in fact a polyadic predicate), then the principle of contradiction will not suffice. This, of course, would give Leibniz a strong motivation to recover the concept of findal causes through which the series of perceptual states becomes intelligible as a function (i.e., where the whole precedes the parts) rather than as a set (where the parts precede the whole).

[41] Aristotle, Physics

found, there is another (potential) smaller part which results from the subdivision of the actual infinitesimal extension.

If Aristotle's position seems somewhat obscure, at least given our contemporary sensibilities, it can be clarified through two observations. First, Aristotle is interested in the existence of particulars; this is the primary ontological presupposition underlying his doctrine of the infinite. This applies both to number and to extension. Indeed, this explains why there is not even a potentially infinite extension. For, the concept of an infinite extension implies the existence of a particular that is greater than all possible particulars (including itself), which is a contradiction. Similarly, there can be no infinitely divisible number, because only whole numbers are proper particulars. Rational numbers, for Aristotle, are ratios between whole numbers; that is, they are relations, not numbers. Second, the modality of which actuality and potentiality are quantifications is time, which, for Aristotle, conditions the existence of the particular. Aristotle denies the actuality of infinite numbers because the larger infinite number does not exist — yet. And so, while the concept of an infinite number is, for Aristotle, well-defined, it cannot be said to exist actually since the condition of existence is the operation through which a particular is constructed. The same consideration explains why there can be no actual infinitesimal extension, although its concept is well-defined and it clearly exists as part of the actual extension in question.[42]

[42] Aristotle's concept of number is implicitly *syncategorematic*, since for every x that can be given, there is y that can be found that is larger than it: $\forall x \exists y, y > x$. The syncategorematic expression of number requires quantifier dependence, i.e., polyadic logic. Aristotle, however (the question of his logic aside) considers numbers to be particulars, since his epistemology is restricted to *categorematic* cognition: i.e., thoughts of particulars, and not reflection on the domain of possible existents (Platonism). Accordingly, the first quantification ($\forall x$) is interpreted temporally, since there is no universal quantification across all possible numbers, but only particular numbers that could be enumerated in some order (this is an interpretation that Kant shares with Aristotle). The second quantifier ($\exists y$) is subjected to two interpretations, depending on whether the progression of numbers is given (*datum*), as in division, or is merely a possible (further) construction from a particular number (*dabile*), as in composition (or enumeration). Aristotle therefore conceives of the next iteration as (a) merely possible in time as a modality of being (as in division of magnitudes or enumeration) or (b) impossible (as in numerical division beyond unity and in the composition of magnitudes). As we will see, Kant's account is more sophisticated, because his conception of the pure form of intuition (or the concept of number) through the category of totality allows the unrestricted quantification of the existential quantifier in, for example the application of the category of totality to space or time. Neither the totality of space nor time may be represented (i.e., the existential quantifier must be interpreted ideally, not empirically, as the fact the existence of a determinate y), however the totality of possibly existing determinations (all places in the order of space or time) may be thought through the "idea" of space or time. Thus, Kant affirms the syncategorematic conception of number as an idea, whereas for Aristotle it remains a merely

Freed from the logical, ontological, and methodological priority of the particular, physics was able to take up a new object, which up to that point had remained an object of metaphysical speculation: the totality of space and time. For example, while Cartesian dualism retains the substance ontology of Aristotelianism in the reification of extension, Descartes' extension is no longer, as it was for Aristotle, always the extension of a particular, but is instead the extension of the whole of which particulars are no more than modifications.[43] Cartesian metaphysics — despite its inadequacies — makes it possible to characterize the totality of physical extension: laws of nature are just properties of the extended substance. Similarly, Leibniz's ontological approach to physical being denies the existence of space as a real relation, but nonetheless reifies time as a totality in the complete concept of the monad; for Leibniz, final causes are not only possible, but necessary if temporal relations are to be understood as intrinsic properties of monadic entities. Although both of these projects are, in different ways, deeply indebted to the legacy of the Aristotelian project, they point towards the new object of physics: the totality of space and time.

However, it was the mathematics of the infinite and the infinitesimal that ultimately showed the inadequacy of the Aristotelian conception of the infinite. For example, the infinitesimal method for calculating the tangent of a curve at a point consists of assuming that very small intervals of a curve may be (a) thought of as finitely valued for the purposes of calculating the differential quotient, even though (b) the punctual contact of the curve and its tangent implies that the interval has no extension (i.e., has a zero value).[44] Applied to the cal-

possible cognition within the domain of an objective time in which the cognition of a (larger or smaller) particular number is possible. We will see this in greater detail below when we examine Kant's solutions to the mathematical antinomies.

[43] In this sense, Spinoza is a philosopher of the continuum as well. For, while he lacks an adequate concept of order in the *Ethics* (Spinoza 1992) (Spinoza thinks of extension not, as Descartes does, in terms of a polyadic system of ordered relations, but in terms of monadic relations of cause and effect), he is nonetheless committed to the view that the original object of knowledge is the whole, of which intelligible individuals are nothing more than modes.

[44] The concept of the infinitesimal was long considered to be problematic, if not outright contradictory, due to the fact that the infinitesimal is treated as (a) finitely-valued (b) zero-valued and (c) as of negligible value, depending on the requirements of the calculation. This, for example, is Berkeley's main objection to the infinitesimal in *The Analyst* (Berkeley 1754). The concept of the infinitesimal was also criticized for implying the (actual) existence of an infinitely small quantity. Indeed, infinitesimal quantities were widely thought to be finitely determinate (for example, Isaac Barrow (Barrow 1735) and de l'Hôpital (De L'Hôpital 1716) both held this position), despite the

culation of instantaneous displacement (velocity), the concept of the infinitesimal suggests that there an *actual* least magnitude of displacement that correlates to a zero-valued interval of a displacement curve. The value of the ratio of the least magnitude of displacement to the least magnitude of time is the differential quotient of the curve. Physically interpreted, however, this implies a purely punctual velocity, or a velocity with no corresponding displacement.

Whereas the infinitesimal method assumes the existence of least values in order to calculate the tangent (i.e., the differential quotient), the limit method assumes that the tangent may be approximated by calculating the slope of cords that cut the curve across successively smaller intervals. As the intervals of the intersection of the cord approach zero, the calculation of the tangent becomes increasingly accurate. However, the precise determination of the differential quotient at any point — and thus also the determination of the derivative function — assumes an infinite approximation. The mathematical interpretation of the limit method implies actually infinite calculation, even if this calculation can never be finitely realized. However, if the limit method is physically interpreted, it implies that every subdivided section of the manifold implies an (actual) infinity. That is, within every finite part of the continuum, there is another complete (i.e., actual infinite) continuum, and within every finite part of this continuum, there is another complete continuum, and so on *ad infinitum*.

As a mathematical technique, the calculus quickly achieved results that were wide-ranging in implications and applications.[45] However, the physical interpretation of the calculus — and its mathematical object — was beset with paradoxes.[46] On the one hand, the

contradictions implied by this interpretation of the infinitesimal.

[45]One of the most important early applications of the Calculus was in the calculation of ballistic trajectories, and the Calculus was widely taught at military acadamies.

[46]It is a fact of no small importance — even to our subject here — that the continuum (and the Calculus of the continuum) was initially interpreted physically. This is true not only of Newton (and his fluxions) but also of Leibniz. Even in Kant, it appears to be the case that the manifold of space-time has dynamical properties. This is revealed especially in the Analogies of Perception and in the *Metaphysical Foundations of Natural Science*, where Kant does not begin with a mathematical conception of a space-time function, but rather with the theory of the motion (of matter). Indeed, in the Transcendental Exposition, Kant claims that time provides the foundation for a theory of motion, not for a theory of arithmetic; this just is because the content of time (*Zeitinhalt*) is reality, which is not merely a modification of space, but a real relation. The post-Kantian turn to a (neo-)Aristotelian conception of space as an organic totality is scarcely a deviation from the underlying mechanistic view of the continuum. Indeed, one of the principle innovations of the Marburg school (anticipated, of course, by Bolzano (Bolzano 1950), if not widely publicized) was to reinterpret Kant's concept of function according to what is now

infinitesimal interpretation of the calculus implies the existence of real-valued differentials for zero-valued intervals of a function. Physically interpreted, this suggests that the continuum of reality may be thought of as (a) moving and (b) accelerating, even for zero-valued intervals of displacement. In other words, velocity and acceleration appear to be properties of the physical manifold, not properties of particulars in motion.[47] On the other hand, the limit interpretation of the calculus implies the existence of an infinite number of intervals within any finite interval. Physically interpreted, this suggests that every real-valued interval of the physical manifold of space and time contains within itself *another* infinitely sub-divisible manifold, and so on. In other words, even a finite displacement implies a displacement across an infinite interval, i.e., the real continuum contained within any finite interval.[48]

1.4 Post-Kantian Systematic Idealism

The *Critique of Pure Reason* stands at the cross-roads between the skepticism of the empiricists and the new epistemology implied by mathematical physics. On the one hand, Hume and Locke raise skeptical concerns about the possibility of justifying the objectivity of relational concepts. On the other hand, Descartes, Leibniz and Newton propose that (empirical) knowledge is only possible if it is expressed in terms of just those relations challenged by skeptical empiricism. Kant's solution, as we have seen, is not to provide a metaphysical justification of

---

the more-or-less modern conception of a (real) function as a relation between two variables of the real order. Eliminating metaphysics from the Kantian account includes reforming the Kantian conception of the "manifold" and its latent dynamic properties.

[47] Aristotelian physics is grounded in bodies (substances) and their properties, and thus spatial and temporal extension appear to be the concatenation of particulars. Accordingly, change (and thus velocity and acceleration) arises from the concatenated activity of individuals. However, the physical interpretation of the method of integration of a function across an interval suggests that displacement (or magnitudes) may be generated out of the infinite concatenation of zero-valued displacements with real-valued instantaneous velocities (or accelarations). In other words, it may appear to be the case that the physical manifold of extension is the *result* of the dynamic manifold of reciprocal accelerations, or force.

[48] Before Cantor had opened the "paradise" of the transfinite, Leibniz had in some sense already discovered its possibility in the labyrinth of the continuum. While Leibniz lacked the vocabulary to express Cantor's transfinite, he clearly understood that every finite interval of the continuum contains a complete continuum within itself (i.e., there are uncountably many ($\aleph_0$) parts of every real-valued segment of a continuous, real topology). Although both Kant and the systematic idealists were aware of Leibniz's labyrinth few seemed to have posessed the conceptual resources to appreciate the depth of the problem.

relations (causal or otherwise), but to reframe the question of truth and justification in terms of transcendental conditions and the hypothetical assertability of metaphysical (a priori) commitments.

The monumental significance of the *Critique of Pure Reason* for the history of philosophy is due to the fact that it takes on not one, but two different tasks. First, in displaying the formal dualism between the polyadic totality of spatio-temporal intuition and the monadic structure of individuals, Kant establishes a dialectic that is the logical completion of Hume and Locke's relational skepticism. The Dialectic of Pure Reason thus serves the critical task of at the same time undermining the dogmatism of rationalism and nullifying the skepticism of the empiricists.[49] Second, the *Critique of Pure Reason* establishes, in the Transcendental Deduction, a new method for justifying claims to truth that is intended to completely supercede traditional notions of truth and justification. The *Critique of Pure Reason* is therefore a watershed moment, announcing the end of one era in philosophical investigation, and the beginning of another.

However, this dualism in the *Critique of Pure Reason* is not without a certain hazard. For, if the second task is taken to be a failure, then the *Critique of Pure Reason* announces the end of one form of justification, but does not provide a satisfactory alternative. Indeed, this is precisely how the project of the *Critique of Pure Reason* was interpreted by most post-Kantian idealists. Even if Kant's successors found the results of the analysis of our representations (intuitions, concepts, judgments) to be convincing, and even if they found the skepticism of the dialectic to be compelling, they were not, as a rule, satisfied by the results of the Transcendental Deduction. While the reasons for this are many and varied, the consequence remains the same: the Kantian project achieves its critical or destructive ambition, but does not establish a reliable alternative. As a result, the state of the justification of knowledge after the *Critique of Pure Reason* appears to be worse than it was before. For example,

(a) The *Critique of Pure Reason* completes the skepticism of Locke and Hume by undermining

[49] That is, the skepticism of the empiricists leads either (a) to an equally dogmatic metaphysics of empirical particulars or (b) to cognitive nihilism.

the traditional epistemology in a general way. The Dialectic of Pure Reason diffuses a number of problems in the classical metaphysics of nature, but also undermines the traditional foundations of metaphysics, and with it, a mode of argumentation and justification that had widely served as the foundation for other rational sciences. Thus, the *Critique* seems to lead inevitably to a wide-ranging skepticism — if not nihilism — about the "reality" of God, Man and World, and thereby destabilizes many different forms of knowledge.

(b) While Kant attempts to recover the syllogism within the *Critique of Pure Reason*, it is clear that role of Aristotelian logic — and the epistemic profile of judgment in general — has been radically transformed. The introduction of "transcendental logic" and the coordination between the syllogism and the principles of mathematical construction in the Schematism points towards a new (polyadic) logic of relations, but does not explicate its internal structure or its application in the justification of knowledge claims. Perhaps despite Kant's intentions, the epistemological revolution of the *Critique of Pure Reason* destabilizes the concept of logic and its place in the method of philosophical investigation.

(c) The *Critique of Pure Reason* undermines the metaphysical concept of the intrinsic individual and places a new object at the centre of philosophical inquiry: the continuum. However, the concept of the continuum is beset with a number of perplexities, starting with the concept of the infinite and the infinitesimal. Furthermore, while the concept of the continuum is clearly a result of the thesis of the ideality of space and time, and thus is a central component of Transcendental Idealism, it is not at all clear how this central piece of the Copernican revolution is to be transposed into other domains of knowledge, or if such a transposition is possible at all. Despite Kant's continuation of the critical project in the *Critique of Practical Reason* and the *Critique of the Power of Judgment*, the systematic role of the continuum in determining the nature of knowledge and reality is never fully determined, and the internal structure of knowledge as a human enterprise is destabilized.

These are by no means the only problems that arise from the *Critique of Pure Reason*

or the critical project in general. However, they do highlight the scope and significance of the disturbance that the *Critique of Pure Reason* necessarily created, and help to some extent to guage the immediate and indelible imprint of the work in Germany and beyond.

Even if Kant's immediate successors took the project of the *Critique of Pure Reason* to be a failure, they also saw in the remedies to these limitations the foundation of a new, speculative approach to epistemology that takes up the skeptical moment of Kant's transcendental solution only to transform it into the foundation of all possible knowledge: speculative knowledge of the absolute:

(a) The Copernican revolution undermines the traditional foundations not just of knowledge of Nature, but also of knowledge of Man and God. The critical project, however, suggests the possibility of finding a new way to secure the foundations not just of physics, but of many, and perhaps all different types of knowledge.

(b) Mathematical physics unifies the objects of its study in a relational continuum. This, in turn, suggests that the continuum may serve as a model object not just for physics, but for other domains as well. Indeed, under the correct interpretation, the continuum may serve as a single, unified object through which Nature, Man and God may be understood as related aspects of a single unified totality: the continuum of reality.

(c) Transcendental logic suggests a deep connection between syllogistic reasoning and mathematics, but does not fully articulate what this connection might consist in. If the transcendental method can be extended beyond the domain of natural physics, then transcendental logic — or some modification of it — may be taken to provide a unified methodology for philosophical investigation as a whole.

The new science of systematic idealism will be a science that provides a new epistemological justification not just for natural sciences, but for the whole of human thought. That is, the **content** of human knowledge will be shown to be systematic in nature. The systematic nature of human cognition will be demonstrated through a more complete interpretation of the manifold

as the domain of finite cognition. That is, the **object** of human knowledge will be shown to be systematic in nature. Finally, the logic through which the articulations of the manifold are to be understood will take up where Kant's transcendental logic ends; it will provide a logic that incorporates not only the serial ordering of the manifold and the syllogistic structure of reason, but also provides for the a priori unity of the empirical individual and the teleological nature of human thought in all of its expressions. That is, the **structure** of human knowledge will be shown to be systematic in nature.

Of course, a great deal more work would be required to demonstrate that some or all of these factors were decisive contributors for the development of any one particular philosophical system. However, for our purposes, this rough sketch serves to orient the programme of systematic idealism within a complex of related problems. While the *Critique of Pure Reason* showed the *possibility* of a radically new epistemology, the project appeared to be incomplete. It is not surprising, therefore, that post-Kantian systematic idealists attacked the problem with optimism and enthusiasm. In hindsight, it may be easy to dismiss the programme of systematic idealism as hopelessly optimistic. Nevertheless, the lessons that can be learned — about the nature of logic, about the nature of (transcendental) reality, or about the nature of objective thought — are not only the cautionary lessons of speculative overreach. Rather, the project of systematic idealism sheds light on the nature and structure of conceptual determinacy, objective justification and systematic validity that are as relevant today as they were when Kant first announced the demise of traditional epistemology.

Whether the programme of systematic idealism succeeded or not, it provided *the* model for philosophical research for more than a century. Indeed, the project of the systematic justification of the constructions of cognitive representation is an important objective for many contemporary philosophical programmes, even those that claim no precedents in the speculative logic of Maimon, Fichte, Schelling or Hegel.

1.4.1 Summary Outline

This study of Hermann Cohen's *System of Critical Idealism* has two parts.

In the first part ("Analysis"), we will examine Cohen's formalist interpretation of the doctrine of transcendental cognition and the implications of an analytical interpretation of the *Critique of Pure Reason*. The first chapter of this part will focus on Cohen's analytic interpretation in his *Kant's Theory of Experience* (**Chapter 2**, "Formalism"). The second chapter will focus on the interpretation of the transcendental and its implication in Kant's Transcendental Deduction and for the concept of a transcendental logic (**Chapter 3**, "The Transcendental").

In the second part ("Systematic Idealism"), we will turn to the implications of the formalist interpretation for the concept of the transcendental and the realization of a systematic idealism. The first chapter of this part (**Chapter 4**, "Logic of Reality") will explore the 'rational' *logic of totality* developed by Maimon and Hegel through a system of four constitutive principles. On the basis of these principles, I will offer a reconstruction of Hermann Cohen's *logic of origin* as this is presented in his *Logic of Pure Knowledge* (Cohen 1902a). We will see that whereas other prominent systematic idealists embrace the continuity (reality) of the *totum reale* as a fact of consciousness or thought, Cohen understands the problem of continuity as one that must be resolved within logic itself, through an a priori construction of the continuity of the manifold. The second chapter will focus on the doctrine of the rational manifold of difference implied by a logic of determinability, that is, a formal interpretation of mathematics (**Chapter 5**, "Manifold of Reality"). In this chapter, we will also compare Cohen's formulation of the rational manifold with that of Maimon and Hegel. Here, a closer consideration of the problem of constructing continuity a priori will show the affinity between Cohen's *logic of anticipation* and the formalist construction of the continuous manifold in the methods of Dedekind and Cantor.

The third chapter of the second part will examine the nature of Cohen's *System of Critical Idealism* as a system of transcendental anticipations: the anticipation of the *unum*, the *verum* and the *bonum*. Since Cohen's logic does not provide the systematic objectivity of Kant's Transcendental Deduction or Hegel's *Objective Logic*, the possibility of knowledge, for Cohen, can

only be secured through a systematic approach to human knowledge, or the system of sciences (**Chapter 6**). This chapter will also examine some of the anti-skeptical resources of Cohen's system, in particular its reponsiveness to criticisms of the totalization of human cognition in classical German idealism in connection with the distinction between reason (scientific idealism) and faith (theological realism).

Part I

Analysis and the Transcendental

Chapter 2

Formalism

The focus of this chapter is Cohen's three "stage" (*Grad*) interpretation of the a priori and the related theory of two types of a priori: the *metaphysical* and the *transcendental*.[1] Cohen's interpretation of the transcendental method as the regression from empirical judgments to the forms of intuition terminates with the forms of sensibility — adjacency and succession — and the system of principles — the schemata of the categories.[2] Crucially, however, Cohen takes adjacency and succession to be purely formal relations, which are both necessary and sufficient for the foundation of mathematics. That is, whereas Kant's reconstruction of the manifold implies that serial ordering arises from both rational and sensible components (in the Schematism), Cohen takes the relations of mathematics to be pure products of the activity of thought, already implicit in the Transcendental Aesthetic. As we will see, this interpretation of

[1]Hermann Cohen's Kant interpretation was first expressed in his *Kant's Theory of Experience* (*Kants Theorie der Erfahrung*), published in 1871. The success of *Kant's Theory of Experience* led Cohen to complete a cycle of Kant's three *Critiques*: *Kant's Foundation of Ethics* (*Kants Begründung der Ethik*) (Cohen 1910) and *Kant's Foundation of Aesthetics* (*Kants Begründung der Aesthetik*) (**?**). In the years between the printing of the first and the second edition of *Kant's Theory of Experience*, Cohen also published two important studies related to the history of idealism: *Plato's Doctrine of Ideas and Mathematics* (Cohen 1878) and *The Principle of the Infinitesimal Method and its History* (Cohen 1883). For an excellent study of the transformations of Cohen's conception of the role of logic, see Geert Edel's *From Erkenntniskritk to Erkenntnislogik* (Edel 1989).

For the purposes of this study, we will consider Cohen's Kant interpretation only in its mature expression, that is, in the second edition of *Kant's Theory of Experience* and in *Logic of Pure Knowledge*.

[2](Cohen 1871a) (Cohen 1885) (Cohen 1902a) The most concise formulation of the Marburg interpretation of the transcendental method is found in Dimitry Gawronsky's dissertation (under the direction of both Cohen and Natorp), *The Judgment of Reality and its Mathematical Presuppositions* (*Das Urteil der Realität und seine mathematischen Voraussetzungen*):

> The general **logical** form of the transcendental method can be described as follows: a fact, which one takes as given, whose being one presupposes, is taken to be a problem insofar as we ask after its explanation, that is, after those conditions that make this fact possible, which contain the source of its essence and its existence. (Gawronsky 1910)

One might object, along Kantian lines, that the transcendental method cannot discover the source of objectivity *with respect to their existence*, i.e., its real ground. Indeed, this description seems only to have understood the Transcendental Analytic, and little else besides. This, indeed, is the general problem of "transcendental arguments" insofar as they attempt to derive metaphysical (realist) conclusions from epistemological (idealist) premises. While Gawronsly writes as though his interpretation applies to Cohen's broader project, Cohen abandons the transcendental method after the second edition of *Kant's Theory of Experience* precisely because the presupposition of the being of "a fact, which one takes as a given" contains a number of paradoxes that, as we will see, the Cohenian solution cannot resolve directly in a theory of cognition (or knowledge).

the fundamental structure of sensibility has decisive consequences for the interpretation of the Critical Project.

The second significant transformation of the Critical Project arises from Cohen's view that the Kantian term 'experience' (*Erfahrung*) refers not to the perception of ordinary objects, or even perception at all, but rather to the representation of laws of nature, paradigmatically the mathematical representations of natural physics. Whereas Kant takes the explanations of natural physics to employ the same metaphysical language (substances, causes) as the expression of naïve experience, Cohen understands the language of physics to be essentially the language of mathematical functions. That is, whereas for Kant the possibility of physical knowledge implies judgments about conceptual unities (substances and their properties), Cohen takes the fundamental structures of conceptual representation (i.e., the pure concepts of the understanding, the categories) to be functional relations of space and time. Thus, whereas for Kant the paradigmatic expression of natural physics is that of *force*, a necessary relation between two *bodies*, for Cohen the paradigmatic expression of natural physics is that of *motion* as a possible *function between time and space* as real variables. Whereas Kant understands substance to be the subject to which predicates of force are applied, Cohen understands substance itself to be nothing more than the reification of functions; this is the view ultimately consolidated by Cassirer (Cassirer 1910).

The immediate consequence of Cohen's view is that the categories are no longer thought of as constrained by the expressive powers of syllogistic logic. Rather, the categories are thought of as conceptual *translations* of the System of Synthetic Principles, that is, as relational structures of a manifold of serial differences. This, in turn, implies that the Schematism is not a structural analogy between heterogeneous forms of logical construction (philosophical logic, mathematics), but rather a translation between two different types of mathematical expression: one phrased in terms of conceptual "categories", the other phrased in terms of mathematical functions. For Cohen, this implies that the structures of empirical knowledge are not only a priori, but are also "analytic." That is, what the Transcendental Deduction shows is not that

synthetic a priori constructions are possible, but rather that a priori conceptual constructions are equivalent to the (analytic) constructions of mathematics.

Here, then, are the two principal theses of Cohen's interpretation of the *Critique of Pure Reason*, as this is to be found in *Kant's Theory of Experience*:

Formalism The formalist interpretation of the *Critique of Pure Reason* assumes that the form of sensibility (adjacency, succession) permits the rational construction of a serial order of differences, which does not depend on the real condition of sensibility. This interpretation implies that the serial order of the manifold (i.e., the a priori forms of intuition) arises solely from a faculty for spontaneous construction, rather than from the interaction between a faculty for spontaneous construction (thought) and a faculty for givenness (sensibility). As we will see, this implies that there is no real or modal distinction between faculty for concepts and the faculty for empirical representation.

Analyticity The analytic interpretation of the *Critique of Pure Reason* assumes that the schematic comparison of pure concepts of the understanding and the principles of construction implies the complete interpretation of the logical form of intuitions (mathematics) in terms of the the logical form of conceptual construction, and vice versa. This, in turn, implies that the faculty for concepts has the expressive power to model the continuum of empirical experience; intellectual synthesis of polyadic relations must be possible. Accordingly, the assumption of an analytic comparison between a priori concepts and the a priori manifold implies that the task of the Transcendental Deduction is not to illuminate the possibility of synthetic knowledge, but rather to demonstrate the translation of the language of conceptual representation into the language of mathematical representation. The criterion of construction is thus not a criterion of truth, but rather of validity.

The consequence of Cohen's two central theses is what I will call *formal monism*. This is not a metaphysical thesis, nor even an epistemological thesis, but is rather a thesis about the representational structure of knowledge claims: i.e., logic and mathematics. It implies that

there is at most one fundamental form of intelligible representation; all forms of representation are merely modes of this single original form of representation: *Erkenntnislogik*. This thesis, we will see, directly contradicts Kant's formal pluralism.[3]

2.1 The Apriority of the a priori

By the time Hermann Cohen put forward his own interpretation of Kant's *Critique of Pure Reason* in 1871, the return to Kant was already well under way. On the one hand, Kant's views had been widely appropriated as the basis for investigations into cognitive psychology, most notably by Herbart and Helmholtz. On the other hand, the scholarly interpretation of Kant had already become a well-established part of the academic scene, even though neither its methods nor its results had achieved much sophistication. As we will see, the contemporary debate was focused on the question of the meaning of Kant's claim that space and time are ideal forms of representation. In reality, the question that preoccupied the "second-wave" Kant interpretation (that is, the first wave of neo-Kantianism) is no different to that which preoccupied his immediate critics (and champions): whether there is any way at all for Kant to claim to be a transcendental idealist without merely being a subjective idealist.

2.1.1 The Fischer-Trendelenburg Dispute

Cohen's first foray into the contemporary philsophical debate was an intervention into the notorious Fischer-Trendelenburg dispute.The dispute centered around the interpretation of the doctrine of the ideality of space and time. Today, the debate is best known for introducing Trendelenburg's "neglected alternative", i.e. the view that space and time can be both subjec-

[3]Indeed, it has often been claimed that the Marburg interpretation of the Critical Project implies a variety of Leibnizianism or Logicism. This is true to the extent that both Leibniz and the Marburg school are committed to formal monism, and therefore accept the implications of this thesis for the epistemological role of conceptual construction. However, with respect to the fundamental orientation of method and systematic construction, the Marburg school embraces Kant's skeptical result and hypothetical method in metaphysics. As we will see, Cohen's understanding of the production of the manifold (i.e., the infinitesimals) is even less theoretically and ontologically committed than Kant's doctrine of intuition.

tive and objective.[4]

The intelligibility of Trendelenburg's position — and Fischer's attempted refutation — depends on how we understand 'subjective' in the context of the Transcendental Aesthetic, and, accordingly, the kind of necessity that can be thought of as attaching to the claims of the Metaphysical Exposition of the Transcendental Aesthetic. For Kant, the *Critique of Pure Reason* begins from a *transcendental perspective*. This implies the replacement of the principle of adequation and its transcendent metaphysics with the (dialectical) model of subjective construction: "only the cognition that these representations are not of empirical origin at all and the possibility that they can nevertheless be related *a priori* to objects of experience may be called transcendental" (KRV; AA A56/B81). The transcendental perspective, therefore, finds objective truth in the possibility of the formal unity of distinct modes of representation. The transcendental perspective may be constrasted with the *empirical perspective*. From the empirical perspective, a priori subjective constructions (the categories, the a priori forms of intuition) are hypostasized as transcendent realities, which we (empirically) just think of as hylomorphic unities of form and matter. The empirical perspective is necessary to alethic claims, and thus to scientific (and metaphysical) realism; however the metaphysical hypostasis of subjective structures that conditions its intelligibility is vulnerable to the Humean skeptic. The claim of the *Critique of Pure Reason* is that the metaphysics of empirical realism derive a methodological justification from their role as conditions of the possibility of objectively oriented judgments. However, this methodological justification does not extend to the objective reality of the hypostasized structures, for this would amount to transcendental realism, i.e., dogmatic metaphysics.

Accordingly, in arguing for the objective reality of space, Trendelenburg assumes that space and time — as forms of appearance — are "merely subjective", that is, they are *bloß Scheine* that are to be interpreted as subjective images; these in turn correspond to an (objective) transcendent reality, which can only be a *Ding an sich selbst*. Kant, Trendelenburg claims, has shown in the Transcendental Aesthetic that space and time are necessarily subjective, since they

[4](Trendelenburg 1840) (**?**) (Trendelenburg 1869) For a more recent treatment of the problem of the 'neglected alternative', see Graham Bird's "The Neglected Alternative: Trendelenburg, Fischer and Kant" (Bird 2006a).

are necessary to subjective representations (appearanaces). However, this does not rule out the possibility that there is an objective order of space-time relations, which, while not perceived directly, nonetheless corresponds to the subjective structure of space-time relations.

In response to Trendelenburg's proposal, Fischer argues that the strategy of the Metaphysical Exposition is to show that space and time have a property that *beings* cannot have. Whereas concepts represent parts of the manifold ("*Theilvorstellungen*") under general properties, space and time are not "*Gattungsbegriffe*", but rather totalities. Thus, while space is the whole "in" which every possible object is subjectively represented, no concept is a species of (or inheres in) space.

> My proof goes: space and time are something that no concept is: the **whole**; thus space and time are not concepts. Concepts are something that space and time can never be: "**partial representations**" ("*Theilvorstellungen*"); thus space and time are not concepts.
> This proof, I say, is precisely Kant's.[5]

The result of this argument is supposed to be that space and time are not concepts — which refer to objects — but rather pure intuitions, the conditions of the subjective representation of concepts, as a kind of subjective container (concept of the whole) for the total unity of objects.[6] Since space and time are not properties of objective being, but rather only of subjective representation, space and time are necessarily and exclusively subjective.[7]

---

[5](Fischer 1870, 32) Compare §7 and §11 from the 'Doctrine of Elements' in the *Jäsche Logic*.

[6]Formally, Fischer's argument resembles what Karl Ameriks calls Reinhold's "short argument" for idealism. Ameriks outlines three ways in which we may argue that knowledge of the thing-in-itself is impossible:

> (a) the *concept* of such things in any form is in principle incoherent; (b) the *knowledge* of such things in any form is in principle incoherent; (c) the specific nature of *our theoretical* way of knowing is such that we are left with the capcity for knowing only certain kinds of features. (Ameriks 2000, 127)

For Kant, the argument for (transcendental) idealism is something like (c), whereas Ameriks attributes (b) to Reinhold. Fischer's argument appears to be of the stronger form (a); for, the concept of space and time as objects is incoherent, since they cannot be "*Theilvorstellungen*." Accordingly, it is *conceptually incoherent* to think of them as objects at all. And since they are not objects, they are (by exclusive disjunction) intuitions.

[7]Fischer correctly points out that space and time are not features of concepts. However, the problem that Trendelenburg raises can also be formulated quite generally as the possibility of an objective order of relations; indeed, the problem had already been so formulated by Hermann Andreas Pistorius in his review of J. Schulze's 'Erläuterung zur Kritik der reinen Vernunft' (Pistorius 2007). Pistorius, following Leibniz, points out that the objective order of space and time need not have the same structure as the subjective (or phenomenal) order, but need only instantiate an analogous fact, just as every monad "mirrors" the totality of the universe. The only response

Cohen's intervention in the Fischer-Trendelenburg dispute (Cohen 1871b) takes sides neither with Trendelenburg, nor with Fischer, but rather points to the distinction between the "merely subjective" and the a priori, shifting the terms of the debate from the question of how to interpret the ideality of space and time to the question of the status and origin of the a priori. For, Trendelenburg's proposal — and also Fischer's rebuttal — rests on two fundamental errors. First, Trendelenburg has misunderstood the Transcendental Aesthetic, whose purpose is not to demonstrate the ideality of space and time, but rather its apriority. For, the task of the Transcendental Aesthetic is to show that space and time are (a) necessary to the possibility of empirical representation and (b) that they are not empirical in origin. Thereby, Kant has demonstrated that a structure with a subjective origin is necessary for empirical representation; that is, space and time are a priori. Second, Trendeleburg has misunderstood the structure of the Critical Philosophy, where the demonstration that a representation is necessarily a priori is sufficient to show that the question of its transcedent existence (or non-existence) is not a well-formed question; existence (reality) is a property of an empirical object, which an a priori structure can never be. Indeed, the doctrine of the ideality of space and time arises from the thesis of transcendental idealism, which Kant does not (and cannot) argue for directly in the Transcendental Aesthetic, but rather arises as a response to the Transcendental Dialectic as the "discipline" of Pure Reason.[8]

---

to this — still deeper — challenge is to reply that the ordering of space and time are not necessary features of our sensibility, since they are a priori necessary to the possibility of cognition, and thus must be provided by a faculty for form. This does not rule out the possibility that the world is in fact composed of monadic components that in fact instantiate analogous relations (perhaps through pre-established harmony). However, we cannot consistently ascribe this fact to the objects of experience, not can we legitimately speculate about objective metaphysics. A committment to the view of either Trendelenburg or Fischer implies a form of Transcendental Realism, and thus falls afoul of Kant's method, but the possibility that transcendent reality is thus and so cannot be ruled out — nor, perhaps, should it be.

[8]The demonstration of the apriority of space and time is already achieved by the Metaphysical Expositions of Space and Time. However, Kant extends the analysis of the Transcendental Aesthetic in the Transcendental Exposition of Space and of Time in order to show that space and time are not merely a priori, but are also transcendental, i.e., necessary to the possibility of judgments of experience. However, the demonstration of the Transcendental Exposition anticipates a position that cannot be fully explicit until the principles of construction are introduced in the Transcendental Analytic. *A fortiori*, the claim in the conclusions of the Transcendental Aesthetic — the transcendental ideality of space and time — must also be seen as an anticipation of a position that becomes intelligible only once the structure of transcendental judgment has been introduced. Taken on its own, the argument of the Metaphysical Exposition cannot show anything more than a subjective necessity. If we take the argument of the Transcendental Deduction to be an "analytic" argument (in the style of the *Prolegomena* from the "fact" of

Cohen points out that neither Trendelenburg nor Fischer have adequately appreciated the special significance of the a priori in Kant's Critical Philosophy, nor its relation to metaphysics. For, apriority is not a metaphysical property, but rather a property of epistemic claims. To say that a claim is a priori is to say that it is grounded in the productive structures of a spontaneous subject. Humean skepticism has taught us, however, that the truth of a priori claims cannot be demonstrated in the terms of the traditional epistemology. Accordingly, the *Critique of Pure Reason* proposes a new epistemological foundation, one based on the transcendental structures of empirical representation: the categories and the synthetic principles; only formal a priori structures provide the intelligible ground of objects, and objects can claim no (transcendental) reality beyond their intelligibility. The empirical interpretation of the logical structures of epistemic representations results in (epistemically qualified) metaphysics, and critical philosophy outlines the limits and legitimacy of this interpretation in the Doctrine of Method.

Indeed, when Cohen once more takes up the debate in *Kants Theory of Experience* (Cohen 1871a) (Cohen 1885), he correctly points out that space and time are not "merely" subjective orders of representation, but transcendental orders of synthetic representation: the synthesis of the specific conditions of sensibility and the rational conditions of understanding. Accordingly, they are subjective-objective or "transcendental". Thus, "'the transcendental-subjective' means, on the one hand, full objectivity, and at the same time no less the claim of an exclusive subjectivity, since there is no higher objectivity than the apriority of intuition recognized in the formal character of sensibility" (KTE2; CW I.1:170). The substance of Cohen's interpretation of the Critical Project, and the cornerstone of the Marburg School Philosophy rests in this claim: that the apriority of intuition is to be recognized in the formal character of sensibility as an ordering function.

---

geometry to its transcendental conditions, it remains unclear why we would need to ascribe this necessity to the subject (and not the matter!) of intuition. The ideality of space and time as intelligible structures is, accordingly, demonstrated by the Schematism, not by the Metaphysical Exposition. If, however, by 'space' we only mean the fact that sense data appear "spread out" — if space is just the phenomenal property through which relations of order are made phenomenally self-evident — then the ideality of space and time can be demonstrated by the Metaphysical Exposition. In this case, however, the a priori form of space and time (space-iness and time-iness) is not really a formal property at all, but just a condition of the phenomological discriminability of order. This is a considerably weaker claim than Kant is ordinarily taken to be making in the Transcendental Aesthetic.

2.1.2 The Form of Sensibility

Unlike his predecessors, Cohen does not undeterstand the doctrine of the a priori to be a psychological, logical or metaphysical thesis.[9] Rather, the determination of the a priori is the determination of a positive content: *Erkenntniskritik*.[10] This body of knowldge is not an epistemology (*Erkenntnistheorie*), but rather a set of laws of consciousness that determine the boundaries of knowledge by providing an a priori determination of reality, that is, the space of all possible true predications of relations. The *Erkenntniskritische* content of the analysis of experience consists of the formal conditions of intelligible objects as such, and in this capacity, they are the pure or formal conditions of real, objective consciousness.[11] Starting from the fact of experience, the critical analysis of knowledge (*Erkenntnis*) uncovers the formal grounds of all possible judgments of truth: the principles of construction of valid theorems of knowledge.

Cohen's interpretation of the a priori depends on two principal theses. First, he claims that Kant's argumentative strategy in the metaphysical and transcendental expositions of space and time is a regressive analysis to the *formalist representation of formal conditions*. Both Ex-

[9]Kant's *Critique of Pure Reason* lends itself to naturalization in terms of 19th century science. For, the concept 'function' can readily be interpreted in naturalistic terms, provided, of course, that we still have a teleological conception of nature. And, indeed, the concept of the continuum as the (mathematical) representation of an organic whole is common in Kant's time, and is clearly expressed both by Leibniz and Newton. It is not hard to detect the implicit physicalism of space-time in the *Critique of Pure Reason* (especially in the Analogies of Perception), nor should it be particularly difficult to understand why Kant thought that his solution to the Third Antinomy was satisfactory: it just is possible to conceive of the whole series of causal modifications of space-time as organically structured. The tension in Kant, of course, is that 'Nature', strictly defined, is just the manifold of space-time relations, even though Kant seems to entertain a more robust conception of 'Nature' at many key points in the system. The a priori form of intuition allows us to show — at most — that all laws of appearance will necessarily have the form of functions of the continuum of space and the continuum of time. Physics must be contingent with respect to mathematics (KRV; AA B165), and the mathematical is thus not a sufficient condition for the necessity of a teleological conception of nature. The Marburg revival — nearly a century later — is antipathic to the Aristotelian conception of nature. Accordingly, any attempt to interpret the functions of judgment as continuous with the (organic) continuum (as was, perhaps, the case with Leibniz), is for the Marburg school, merely the reification of organic teleology into the mathematical foundation of nature.

[10]This is the term that Cohen endorses in his introduction to *The Principle of the Infinitesimal Method and its History*. In the *Logic of Pure Knowledge*, however, he would eventually abandon the term in favour of *Erkenntnislogik*, which, unlike the method of *Erkenntniskritik*, is not constitutively bounded by the empirical conditions of its application (i.e., through some fact of culture). For a detailed textual analysis of the changes in Cohen's view, see (Edel 1989).

[11]To what extent this project differs from that of the *Critique of Pure Reason* will be made explicit through an investigation of the transcendental (see **Chapter 3**) and the rational logics proposed in response to apparent limitations in Kant's construction of transcendental synthesis (see **Chapter 4**).

positions (Metaphysical and Transcendental), Cohen maintains, refine and deepen the concept of apriority, ultimately grounding the a priori in the formalism of mathematics. Second, Cohen claims that the distinction between feeling (*Empfindung*) and sensibility (*Sinnlichkeit*) tracks the distinction between the *factum* of sense and its constitution as a pure a priori form (*Funktion, Handlung*). These two claims, as we will see, dovetail in his interpretation of what he calls the three "levels" (*Stufen*), "degrees" (*Grade*) or "meanings" (*Bedeutungen*) of the a priori.

2.1.3 Regressive Analysis

In defending the claim that the argument of the expositions is regressive, Cohen draws on two pieces of evidence. First, there is Kant's claim from the *Prolegomena* that the synthetic method of the *Critique of Pure Reason* also permits of an analytic presentation, the model of the *Prolegomena*, which also seems to adopt a regressive analysis to forms of thought that cannot have an origin in empirical evidence alone. Second, Cohen believes that the "analytic regression" that Kant thus implies is actually carried out in the analysis of the Transcendental Aesthetic, first in the "regression" of the axioms of intuition, and second in a regression from the Metaphysical to the Transcendental Exposition. The axioms of intuition do indeed suggest a progressive limitation of what space and time could be, culminating in their determination not as *concept* but as *intuition*. The "conclusions" drawn in the Transcendental Exposition are, accordingly, construed as an "escalation" (*Steigerung*) of the analysis, further differentiating the content of the concept of an a priori from its roots as a psychological and subjective foundation of cognition through the ultimate foundation of intuition in *mathematics*. Thus, for Cohen, the Transcendental Exposition shows how formal mathematics constitute the absolute intelligibility of intuitions. Indeed, Cohen argues, by connecting the structure of the a priori to the formal structures of mathematics, Kant has definitively overcome the traditional distinction between subject and object, and reconstrued the meaning of "objectivity" as (subjective) necessity and universality.

Accordingly, Cohen understands the mathematics of serial orders (arithmetic, and ulti-

mately analysis) to be not only the formal foundation of *subjective consciousness*, but also the formal legitimation of *experience* as the science of nature, i.e., of the manifold of space and time. Indeed, Cohen believes that the Kantian argument connects the possibility of *intelligibility* to a correct interpretation and reconstruction of the a priori, and, especially, the a priori forms of space and time as a serially ordered continuum. This interpretation of the Transcendental Exposition is extremely problematic from a hermeneutic perspective. Nevertheless, it is decisive for Cohen's interpretation of the transcendental doctrine as a formal logic, and for the logicism and idealism ultimately embraced by Cassireer (Cassirer 1910).

The Analytic Method in the *Prolegomena*

Let's begin by considering the evidence from the *Prolegomena*. Cohen cites the well-known passage in which Kant addresses the methodological differences between the *Critique of Pure Reason* and the *Prolegomena*:

> Here then is such a plan subsequent to the completed work, which now can be laid out according to the analytic method, whereas the work itself absolutely had to be composed according to the synthetic method, so that the science might present all of its articulations, as the structural organization of a quite peculiar faculty of cognition, in their natural connection. (Prolegomena; AA 4:263)

The question now is: what could Kant possibly mean when, in the *Prolegomena*, he describes its method as "analytic"? On Cohen's interpretation, the "analytic method" implies a regression from the "fact of experience" or a "fact of consciousness" to the original formal grounds of thought: the a priori. The "synthetic" method, on the other hand, is just that part of the doctrine which considers the real conditions of the application of the a priori, i.e., the System of Synthetic Principles. On this interpretation, analysis is concerned with the problem of form, and synthesis with the problem of the production of representations.

This is in some sense a plausible reading of what Kant has it mind, since it tracks Kant's explicit disinction between the analytic and synthetic methods in the *Jäsche Logic*,[12] and this

[12] The distinction is quite explicit:

> **Analytic** is opposed to **synthetic** method. The former begins with the conditioned and grounded

distinction seems, at least at first, to cohere with the distinction between analytic and synthetic judgments, a canonical if problematic distinction in its own right.[13] However, we need to be careful about the context in which the term "synthetic" appears. For, Kant here refers to a "method" through which the whole material of the *Critique of Pure Reason* is presented, as compared to that of the *Prolegomena*. What does the method of analysis yield in the *Critique of Pure Reason*, and how should this be compared to that of the *Prolegomena*? On a narrow reading of what "synthetic" implies, we may be led to believe that Kant only has in mind the *analysis of form*, that is, the regression to the pure forms of intuition and understanding, independently of the (synthetic) question of their unification, that is, the Deduction and the Schematism. However, in the *Critique of Pure Reason*, "analytic" considerations apply quite broadly to the problem of synthetic cognition; first, in the Transcendental Analytic, which includes the Transcendental Deduction, and secondly in the Analytic of Principles, which includes the

---

> and proceeds to principles (*a pricipiatis ad pricipia*), while the latter goes from principles to consequences or from the simple to the composite. The former could also be called **regressive**, as the latter could **progressive**.

However, Kant adds the following note:

> Analytic method is also called the method of **invention**. Analytic method is more appropriate for the end of popularity, synthetic method for the end of scientific and systematic preparation of cognition.

See the *Jäsche Logic*, §117.

[13] The word-pair "analytic-synthetic" is the subject of signigficant polemical debate in the 20th century, albeit one oriented around the an interpretation of a distinction endogenous to the philosophy of language, not to the problematic of a systematic idealism. Thus, it is the object of criticism by both Quine ("Two Dogmas") (Quine 1953) and, more proximate to Kant, Bennet ("Kant's Analytic") (Bennet 1966). However, both Quine and Bennet seem to be targetting views in the analytic philosophy of language that confuse the analytic method with analytic definitions, much as Cohen seems to confuse the two with his search for an *a priori* a priori in the *Critique of Pure Reason*.

Schematism and the System of Principles.[14]

So construed, the word-pair 'analytic/synthetic' yields a fundamental ambiguity: between analytic and synthetic *methods*, and between analytic and synthetic *definitions*. For, on the one hand, the distinction may be thought of as designating a specific mode of inquiry: one that inquires after purely intelligible forms (the a priori). On the other hand, it may be thought of as designating a specific mode of the concepts thus defined: one for which merely formal relations are necessary to the intelligibility of relations between concepts. This ambiguity is perhaps nowhere more explicit than in the Analytic of Principles, which is concerned primarily with the "systematic representation of all synthetic principles of pure understanding" (KRV; AA A158/B197). There, it is clearly the case that what we are concerned with is a priori forms — the synthetic principles of the understanding. But it is also the case that these "a priori principles" are not such as to allow the a priori intelligibility of the relations between concepts; that is, the product of the method of *analysis* may be a synthetic definition.

[14] This usage of 'analytic' and 'synthetic' is also clarified in the *Jäsche Logic*. Here Kant distinguishes between "analytic" and "synthetic definitions" in §100 of the Doctrine of Elements:

> All definitions are either analytic or synthetic. The former are definitions of a concept that is **given**, the latter of one that is **made**.

We can leave aside, for the moment, our perplexity at the claim that a concept is "given", and point to Kant's subsequent claim, here in §101:

> The synthesis of concepts that are made, out of which synthetic definitions arise, is either that of **exposition** (of appearances) or that of **construction**. The latter is the synthesis of concepts that are made **arbitrarily**, the former the synthesis of concepts that are made empirically, i.e., from given appearances as their matter (*conceptus factitii vel a priori vel per synthesin empiricam*). Concepts that are made arbitrarily are the **mathematical** ones.

Although it is difficult to pin down what Kant (or the editor of the *Jäsche Logic*) has in mind, we can at least point out that analytic "definitions" are of concepts that are "given"; candidates for analyticity, therefore, must be very limited, and include perhaps only the categories and the concept of space and time as infinite magnitudes. Compare this with Kant's characterization of analytic judgments as "judgments of clarification" and synthetic judgments as "judgments of amplification" with respect to their logical content (KRV; AA A7/B10). It is difficult to argue that the 'analytic/synthetic' word-pair has any fixed denotation in Kant's usage throughout the *Critique of Pure Reason*. This is of special significance in considering Kant's philosophy of mathematics, where the distinctions between analyticity and syntheticity sometimes refer to the structure of concepts (as made or as given (a priori)), sometimes to the character of the implied cognition (true by definition, as the common notions of Euclid's *Elements* or true ostensively, as the axioms of geometry) and sometimes to the character of judgment (true in virtue of the cognition of the concepts (clarification) — analytic or synthetic — or true in virtue of a mediate cognition (judgment, and thus amplification). The analytic/synthetic distinction is a thorny thicket indeed.

Accordingly, a synthetic component of the method of *critique* is required: dialectic. It is central to Kant's argument that dialectic is constitutive of the possibility of experience. And it is this synthetic moment — dialectic and its discipline — that is absent in the *Prolegomena*. The analytic method proposed in the *Prolegomena* therefore consists of an analytic regression to the components of synthetic consciousness: the characteristic of space and time, the table of categories and the system of principles. This, however, is an analysis of synthetic consciousness in potentiality, or transcendentally. For, the actuality of cognition depends constitutively on the real condition of sensibility, and thus on the determination of actuality with respect to a relative condition. To put a finer point on it, "analytic" objective cognition is no such thing, for Kant, as it remains merely an analysis of the conditions of possibility of synthesis (the transcendental a priori). The "actuality" of cognition is synthetic, and is subject to an ungrounded condition — sensibility.[15]

On Cohen's understanding of the method of transcendental analysis, the ultimate goal of the analysis of experience is to isolate that in experience which is independent of the *synthetic* moment of cognition as a psychological, metaphysical and contingent condition of experience. The analysis to purely analytical forms of reality (i.e., to formal mathematics) thereby overcomes the contingency of the subjective moment of the *factum* by grounding it in the universal (and timeless) constructions of mathematics.[16] Cohen's regression to the *analytical* formal ground of determinate intuitions therefore suggests that the indexical moment of actuality is not

[15]Nevertheless, Kant does allow for discursive cognition (*cognitio discursiva*) in the *Jäsche Logic*, and seemingly also in the *Critique of Pure Reason*, even though this contradicts the famous adage that concepts without intuitions are empty, and intuitions without concepts are blind. Indeed, it seems that Kant is slightly overstating the case in order to make the point that objective knowledge requires both components. However, we may use concepts merely discursively (i.e., without empirical representation), and some mere intuitions have cognitive content (i.e., they provide certain knowledge!): the axioms of geometry. Kant's famous dictum is thus to be taken with a grain of salt.

[16]Rolf-Peter Horstmann (Horstmann 2008) calls this an"*a priori* a priori", since Cohen is proposing an analysis of cognition that is purely formal, that is, which need not suppose that anything has ever been cognized or ever will be. It would be more perspicuous to say that Cohen interprets the task of transcendental analysis as uncovering an analytic (i.e., given) a priori concept of the synthetic (i.e., order), since Cohen is interested in what can be known about the determination of cognition solely through analysis, that is, without taking into consideration the synthetic (and therefore alethic) structure of a priori consciousness in Kant's transcendental analysis. Thus, Cohen has elided Kant's distinction between analytic definitions (those which are given, as for example the pure concepts of the understanding), and synthetic definitions (those which are made, i.e., determinate concepts of number and their combinations (schemata) in empirical concepts).

just irrelevant to the a priori truth of individual constructions, but, strictly speaking, falls outside the range of transcendental investigation, since it is not susceptible to a priori foundation.

2.1.4 The Formal Determinacy of *Sinnlichkeit*

But how is it possible to understand the determinacy of individual intuitions a priori? Cohen takes up Kant's distinction between the real and the formal conditions of intuition in the first edition of *Kants's Theory of Experience*: "The **possibility** in appearance, that the manifold, which is offered only through intuition, can be intuited in an ordered way, this potential is called form" (KTE1; CW I.2:42). *Prima facie*, Cohen seems to be following Kant in his distinction between the form of intuition and its matter: "I call that in the appearance which corresponds to sensation its matter, but that which allows the manifold of appearance to be intuited as ordered in certain relations I call the form of appearance" (KRV; AA A20/B34). However, Cohen takes a bold step beyond Kant's position in the *Critique of Pure Reason* and gives 'form' a much broader interpretation than Kant seems to have intended. Kant's intention in the transcendental aesthetic seems to have been to uncover the a priori form of intuition, or that in intuition which provides the formal structure of the manifold. On Cohen's interpretation, however, Kant uncovers the systematic formal foundations of "pure sensibility" as the condition of appearance. And so, "Kant admits a pure intuition alongside empirical intuition. This pure intuition is the other moment of sensibility. Sensibility is no longer just mere feeling or feeling-intuition [*Empfindungs-Anschauung*]. There is a pure intuition, and also a pure sensibility" (KTE2; CW 1.1:110).

If the a priori forms of intuition are space and time, what then are the a priori forms of *Sinnlichkeit*? For Cohen, the form of sensibility is determined a priori by the formal mathematics of adjacency and succession: geometry and arithmetic.[17] The connection between the

[17]Note that there is no obvious connection between adjacency and succession and the concept of order. The reason for this is that we require the concept of ancestor (and quantification of ancestors) in order to derive an order from the one-place relation of adjacency or succession. This is precisely Friedman's point in his (now well-known) distinction between monadic and polyadic logics (Friedman 1990). Cohen is assuming that the concept 'number' (or better, the logic necessary to the construction of an indefinite serial order) is implicit in the form of

forms of intuition and the mathematics of the manifold is explicitly drawn by Kant himself. However, it appears that for Kant, the connection between the a priori forms of space and time and the forms of mathematical representation is a specifically "transcendental" question, and it is for this reason that it is treated in the Transcendental Exposition. That is, adjacency and succession are the condition (the a priori form of space and time), and mathematical synthesis is the conditioned (the synthetic construction of transcendental cognition). For Cohen, however, the relation of the metaphysical exposition to the transcendental exposition is that of contioned and condition; it is the transcendental exposition that uncovers the formal foundation — geometry and arithmetic — that is the (again, analytical) formal condition of the possibility of the continuum of space and time.[18]

Cohen's foundation of the forms of appearance in formal mathematics has metaphysical import to the extent that we follow Kant in interpreting the a priori as "metaphysical." For, it is not the apriority of adjacency and succession (and thus the sensible condition) that validates the application of the categories to objects of experience. Rather, it is mathematics itself that provides the formal (and now a priori!) foundation of the possibility of objects in general. Accordingly, mathematics is the (formal) "metaphysics" of sensibility, and thus of the possibility

sensibility. This too seems to be Friedman's claim. For Cohen, however, this will eventually force a dilemma: either abandon the ideality of space, or concede that the ground of intuition is intelligibility (and thus logic) not sensibility itself. Cohen adopts the second option, as we will see.

[18]There is significant debate about how, exactly, the a priori form of intuition is to be interpreted. The view advanced by Michael Friedman (Friedman 1990), (Friedman 1992a), (Friedman 1992b) suggests that intuition is a source of formal knowledge. That is, adjacency and succession — as conditions of polyadic relations and polyadic representation — are formal conditions that are independent of the understanding. This suggests that Kant is not just a representational pluralist, but is committed to a plurality of faculties for form: sensibility itself has formal properties that cannot be interpreted merely through the functions of judgment. Others (for example, Lisa Shabel (Shabel 1998), (Shabel 2003), (Shabel 2004)) interpret space and time as providing significant (mereo)topological properties such that the Metaphysical Exposition alone provides the necessary and sufficient conditions for the objectivity of Geometry. Although I held a different view when I began this study, I now think that adjacency and succession are not, in pure intuition, forms of order, but interpretations of the ordering principle of the Schematism (number) such that the purely intellectual order of number is phenomenologically distinguishable. This implies not only that Friedman and Shabel are mistaken, but also that Cohen is mistaken in attributing any formal content to adjacency and succession in their definition in the Transcendental Aesthetic. In other words, the Transcendental Aesthetic merely shows us how intelligible order can be imagined by us (as successive or as synchronic but spatial), but does not — at least not without the Schematism of number — provide an intelligible rule of ordering. I fear that both Friedman and Shabel will eventually end up confirming Cohen's view (that mathematics has a logical foundation) if they also want to preserve the claims (a) that there is a single faculty for intelligible form and (b) that its products are ideal.

of consciousness in general. This "metaphysical" interpretation of the mathematical can, for example, be seen in Cohen's view that all objects of experience are ultimately justified through their foundation in mathematical forms:

> I define form thus: that it designates that which is independent of experience in order to determine the object of pure intuition, to determine the appearance of geometrical science, and finally, by means of this type of object, to vindicate the objects of the appearance of feeling itself.[19]

However, Kant's claim in the *Critique of Pure Reason* is not that transcendental logic affirms the absolute reality of the object, but rather that it provides the conditions under which metaphysical predicates are truth-apt, that is, a synthetic justification of reality as a necessary predicate of all alethic claims. Cohen, however, now makes a much stronger claim: the determinacy or reality of objects is a purely formal determination, one that requires no further justification than a pure foundation in analytic mathematics. Cohen's understanding of the a priori goes both too far (it claims to vindicate objects!) and not far enough (it cannot legitimate the hypostasis of the purely rational).[20]

Now, in order to make this reading of Kant's intentions in the Transcendental Aesthetic plausible, Cohen must introduce a distinction between 'subjectivity' in its meaning as phemone-

[19](KTE2; CW 1.1:155) As we will see in the next section, however, in *Kant's Theory of Experience*, Cohen interprets "metaphysics" to be the first level of analysis (i.e., the "Metaphysical Exposition") of the a priori. Ultimately, the a priori is "transcendental" (formal, analytic). This interpretation, however, is not consistent with Kant's view of the a priori as such as the legitimate rational ground of empirical judgment. Cohen later came to understand the error in the interpretation, but favoured instead the (Platonic) language of the *hypothesis* (the laying-of-foundations, or substratum of knowledge), which has fewer unfortunate pre-critical resonances.

[20]Note that in interpreting the structure of the serial order as a priori, Cohen must thereby interpret the concept of order as "given", i.e., as analytic, rather than as synthetic (or "made"). This is the result of Cohen conflating the analytic method with its results: analytic or synthetic definitions. For Kant, it is clearly the case that all mathematical concepts are synthetic concepts. There is no "analytic" mathematics, even if there is an intellectual (discursive) synthesis of number in arithmetic. For Cohen, however, mathematic is "analytic" just to the extent that it does not involve sensibility — or psychology — directly for its realization. Of course, this was never Kant's position. Mathematics, for Kant, is synthetic because its concepts are always constructions of a concept of order that is no analytic, and objectively valid because only synthetic cognitions with an empirical correlate (i.e., concepts of extensive magnitudes) are objectively valid. This limits 'mathematics' strictly interpreted to geometry. All other forms of what we call 'mathematics' are parasitic on the original science of empirical magnitudes: geometry. Cohen's misinterpretation is understandable, since Kant's usage of the analytic/synthetic distinction is multi-valenced and he is rarely explicit about how it ought to be interpreted in individual cases. (See, for example, the egregious case of B16, which is all but unintelligible unless one keeps in mind that there are two uses of 'analytic' and 'synthetic' in play.)

menal immanence (*das Gegebene*) and 'subjectivity' as the pure or original (*rein, urpsrünglich*) act or action (*Act, That, Handlung*) of thought. If 'intuition' means the faculty for appearances, then, Cohen claims, "the act of intuition itself is called form, the form and method of pure intuition" (KTE2; CW 1.1:155). This "act" of intuition is, for Cohen, the a priori. It can also be called 'the pure', where "the pure is the order of feeling-material [*Empfindungs-Materie*]" (KTE2 CW 1.1:159). Thus, empirical intuition is a synthesis consisting of two parts: the *material* or *phenomenal* aspect of sensibility and the *form* or act of pure sensibility: the determination of the pure manifold of determinables (i.e., the "analytic" concept of the order of adjacents and successives). The former is the object of psychological investigation and phenomenological or ontological speculation. However, it is the latter — the pure activity of thought manifest in the a priori serial order of sensibility — that is the legitimate content of a *transcendental investigation*, since this investigation is not interetested in the mere "*factum*" of consciousness, but rather in the possibility of consciousness as such, that is, in a priori consciousness.[21]

To solidify the distinction between (mere) determinacy and the a priori determinability of sensibility, Cohen introduces a terminological distinction between 'awareness' (*Bewußtheit*) and 'consciousness' (*Bewußtsein*):[22]

> It may be useful to make a distinction in the concept of consciousness with the following aim: so that the individual eminent ways in which the unity of consciousness presents itself psychologically can no longer be confused with the meaning of consciousness reserved for the *erkenntnisskritische* problematic. To such an end, I make use of the expression 'awareness' as opposed to 'consciousness', even though Kant did not use this expression. (KTE2; CW 1.1:207)

Indeed, in the *Logic of Pure Knowledge*, Cohen goes much further: "**Awareness is myth; consciousness is science**" (LRE; CW 6:424). Broadly speaking, this distinction tracks the Leib-

[21] While Kant does allow the possibility of an intellectual synthesis of number concepts (especially in the case of arithmetic), Cohen takes it to be the case that just this intellectual synthesis is the intelligible content of cognition. There is, as a result, no role remaining for intuition or empirical imagination for the objectivity of cognition. Accordingly, Cohen's interpretation of intuition as empirical representation is already destined to terminate in mere phenomenalism.

[22] Compare this with Leibniz's claims about qualitative and quantitative consciousness: "It is well to make a distinction between *perception*, which is the internal state of the Monad representiong external things, and *apperception*, which is the consciousness or the reflective knowledge of this internal state" (G VI 600 (D.211; L411).) Or also: "I would prefer to distinguish between *perception* and *being aware*. For instance, a perception of light or colour of which we are aware is made up of many minute perceptions of which we are unaware" (NE 134).

nizian distinction between "clear" and "distinct" ideas, that is, between the psychological immediacy of continuous magnitudes (apperceptions, or *quanta continua*) and the reflective reconstruction of the same in terms of logical abstracts (conception, *quanta discreta*). Whereas this distinction had specific cognitive implications for Leibniz in his analysis of reflective thought, the distinction here refers only to the psychological or empirical dimension of experience (the individual) and its transcendental foundation (the universal). Accordingly, Cohen also reads this distinction between the psychological and the epistemic into the distinction between empirical and transcendental characterizations of consciousness and its content.

Through this interpretation, however, Cohen has effected an unexpected inversion in the meaning of an essential distinction in Kant: that between form and material as the distinction between form as the determinable, and thus the possible (the transcendental) and sense as the determinate, and thus as the actual (the empirical). For Kant, these are two perspectives or interpretations of the activity of cognition; the former refers to the pure possibility of an objective judgment; the latter refers to its realist interpretation as a judgment of experience. According to Cohen, however, "'material' means the determinable in general, 'Form' its determination" (KTE2; CW 1.1:173). The "determinable in general", however, can only refer to the a prioir form of sensibility: the serial order of mathematics. The "form" which is opposed to this (intensional) structure is the activity whereby sensibility (as an a priori serial order) is determined as an intuition. Accordingly, mathematics (or better, the theory of number or the continuum) is the "matter" of cognition, and functions of space and time are its "form". This, however, seems to be precisely the opposite of what Kant had intended.[23]

[23] It is frequently pointed out that the Marburg Neo-Kantians provide a Leibnizian interpretation of Kant's *Critique of Pure Reason*. This is true in the following sense. Leibnizian epistemology rests on the claim that all knowledge derives from the intellect (ultimately, the principle of contradiction, and thus a logicism), and sense is just confused or obscure intellect. For Kant, on the other hand, intelligibility derives from the intellect, but knowledge derives from intuitive representations. For, only intuitive representations are both (a) intelligibly structured and (b) discriminable. So, whereas Leibniz thinks that only intellectual synthesis provides determinacy, Kant thinks that intellectual synthesis is indeterminate, and only intuitive representations (and paradigmatically, extensive magnitudes) are both determinate and descriminable. Cohen's interpretation is Leibnizian in the sense that the discriminability of intellectual synthesis is assumed. He does not take Kant's fear that merely intellectual concepts are somehow "obscure" or indeterminate seriously. For Cohen, however, sense experience is not so much obscure intelligibility as it is completely irrelevant to epistemic foundation. The connection between sense and intelligibility is one that Cohen will struggle to reestablish, but with little success.

For Cohen, it is feeling (*Empfindung*) that is the empirical matter or subjective moment of the determinacy of sensibility in general. Thus, feeling is synonymous with the given as the ungrounded (and purely subjective) moment of experience. 'Sensibility' (*Sinnlichkeit*) on the other hand, is the a priori relational material of the determinable in general, and, as an a priori form, it is, like intuition, an *act* of consciousness, and therefore susceptible of a transcendental characterization. Thus, for Cohen, the nature of the a priori manifold may be inferred from mathematics: geometry and arithmetic. For, these axiomatic disciplines are grounded in a theory of the a priori determinable continuum and the possible constructions of its infinite system of relations. This would be a plausible reading of Kant's intention were it not for Kant's regular insistence that experience is given as *appearances*, whose psychological substrate is at most a matter of metaphysical hypothesis. For Kant, the nature of sensibility cannot be further determined than what can be inferred from empirical consciousness. However, Cohen's regression to mathematics as the description of "pure sensibility" blurs the boundary between form and content by suggesting something quite un-Kantian, but, conversely, characteristic of Kant's antecedents (Leibniz) and his successors (Maimon, Fichte, Hegel): *formal content*.

2.2 The Three Levels of the *a priori*

Cohen's analytic interpretation of the transcendental method meets its first — if not decisive — challenge in the interpretation of the Transcendental Aesthetic. For, it is here that Kant is most explicitly engaged in a problematic form of "analysis" that must necessarily produce heterogeneous results. On the one hand, what Kant proposes is an analysis of the form of sensibility — space and time. On the other hand, neither space nor time are pure concepts; that is, they are pure *intuitions*. As we saw from our consideration of the Fischer-Trendelenburg dispute, it is not easy to reconstruct what the difference between concepts, intuitions and pure intuitions may be. Fundamentally, the problem of the analysis of the manifold is that the a priori form of the manifold (as either adjacency or succession) is that of a logical condition, and therefore a transitive relation; its actuality, however, is a serial order of intransitive relations.[24] For Kant, the realization of the serial order is always a result of synthesis, i.e., through the application (minimally) of the synthetic principle of quantity (i.e., through the successive addition that is the concept 'number'). However, as we also learn from the resolution to the Antinomies of Pure Reason, the products of synthesis are always determinate.[25] Thus, if there is such a thing as a pure intuition, it is neither the sum of the parts of the manifold (i.e., a *compositum ideale*), nor

[24] That is, if we interpret the order of space and time (as Cohen does) as an a priori order of serial succession, everything in space is adjacent to everything else; its just a question of how adjacent. However, real objects are only actually adjacent to one other object. That is, formal succession is transitive, real succession is intransitive. Here again, we put our finger on the problem with intellectual synthesis, for Kant. The indeterminacy inherent in arithmetic is resolved by counting fingers, or by otherwise subordinating the (mere) succession of number to the (real) adjency of objects. This is not the same as subordinating arithmetic to geometry, however. This would imply that a Euclidean-Eudoxian theory of proportions is the basis of a reliable arithmetic (Sutherland 2006). However, when describing counting, Kant turns to objects, not magnitudes. We are still counting mental acts, but these mental acts (and their enumeration) are "stabilized" by objective objects. Obviously, counting using an abacus does not imply determinate magnitudes (and thus, does not rely on the theory of proportions), but makes use of an objective, determinate order of being to coordinate an intellectual synthesis. It is for this reason that intuition aids the intellectual synthesis of number: by rendering the transitive intransitive through real objects.

[25] Kant's claim in the resolution of the Antinomies is not that our representations must be finite, but rather that they must be determinate. From this, we may infer that they are also finite only if we think that Kant's concept of number (like Aristotle's) is confined to finite particulars. This is the most plausible interpretation of Kant's view, since he seems to think of number *syncategorematically* (i.e., in the Schematism, number is just the concept of successive addition), and from this it follows that infinite number (and thus the quantification across all possible places in the order of space or time) yields a syncategorematic concept of space and time (even if the whole of space is supposed to be given as a pure intuition). However, it is crucial that we keep in mind that the resolution of the Antinomies relies on determinacy, not finitude. Finitude may follow from determinacy, but is not the resolution to the Antinomies itself.

the appearance of the whole of space and time (i.e., a *totum reale*), but something in between.[26] The ambivalence of pure intuition as an *act* that is poised mid-way between parts and wholes, between ideality and reality, is the central pivot of the transcendental doctrine.

However, Cohen's interpretation of the form of sensibility as an a priori (analytic) serial order implies that pure intuition is nothing more than the act of determining a pure order of relations. That is, *sensibility* has the form of an (infinite) series of parts (the *compositum ideale*), and *intuition* has the form of the determination of the series of parts as a totality (the *totum reale*). Sensibility is related to intuition as arithmetic is related to analysis; that is, just as analysis consists of *functions* of ordered series, intuition consists of *functions* of sensibility. For Cohen, therefore, the *totum reale* that is a "pure intuition" just is some determinable function of the manifold of serial relations of sensibility. The task of science just is the determination of this original function. Whereas Kant takes intuition to be the decisive representation in which concepts become determinate, Cohen's interpretation of the serial order as analytic has as its result the claim that the totality is already determinate (through the concept of number) and what is required is merely the determination of the function (not the underlying order) of the continua of space and time.

Now, the distinction between awareness and consciousness allows us to make our first distinction in degrees of apriority. First, the a priori may be thought to represent originality (*Ursprünglichkeit*) with respect to our awareness. So construed, we have first-personal, empirical-phenomenological evidence of the necessity of a particular form (for example, space) as a condition of the representation of an object. However, while this method may permit of a certain reduction, insofar as the possibility of an individual experience is resolved into the question of experience as such, this analysis begins from, and remains essentially tied to, a psychological, metaphysical analysis of consciousness. It is, accordingly, called the 'metaphysical a priori'. Second, the a priori may be thought to represent originality with respect to our consciousness. On this construal, we are no longer interested in the fact of psychological consciousness, but

[26](KRV; A438/B466) Cf. **Chapter 3** for more on this distinction.

rather the form of consciousness as the possibility of representational unity. Thereby, the object of the reduction to formal elements isolates not psychological facts, but psychic processes or acts of consciousness. These forms constitute not objects themselves, but the universal conditions of objects as such. As a result, they are what Cohen calls the 'transcendental a priori'.

2.2.1 The Metaphysical *a priori*

The first 'level' of the analysis of the a priori is what Cohen calls the "metaphysical a priori."[27] First, to say that a concept is metaphysical is, for Kant, to say that it is a priori, that is, it need not (and perhaps cannot) be realized in intuition, since it just is a condition of the same. Metaphysical knowledge, therefore, consists of just those a priori concepts that can be shown to be *necessary conditions* of a particular type of judgment, that is, features of the pure potentiality of real cognition. And so, for Kant, space and time and the categories can be called "metaphysical" concepts because it can be shown — a priori — that both are necessary to the possibility of discursive cognition; without either the a priori of intuition or the a priori of the understanding, synthetic judgment would be impossible. And so, as Cohen notes, what is often thought of as a "transcendental condition" can also legitimately be called a concept of metaphysics: "Knowledge that a concept is a priori Kant calls: *metaphysical*" (KTE1; CW 1.2:36). Knowledge of metaphysics, then, is exhausted by knowledge of apriority, that is, of the fact that certain concepts are necessary to all of our judgments of experience.

Cohen's taxonomy of the a priori derives from the headings added to the 'Transcendental Aesthetic' in the B edition: the Metaphysical Exposition and the Transcendental Exposition. (These titles are also conventionally used — on Kant's suggestion — to distinguish between the Metaphysical Deduction and the Transcendental Deduction, although the parallel is not exact.)

[27] As Rolf-Peter Horstmann points out, it's very hard to make sense of what it would mean to say that there are different kinds of a priori. (Horstmann 2008) After all, it seems that the a priori is only shown to be truly a priori, for Cohen, if it can be resolved into the norms of formal mathematics. However, this very restriction on apriority suggests that what Cohen means (or perhaps should have said) is that these are in fact three levels of analysis or degrees of the characterization of the a priori. That is, the predicate applies to the analysis, not the a priori form or concept. In any event, Cohen's mature philosophy retains only the latter of the three, which suggests that we needn't take talk of different levels of apriority too seriously.

In the B edition, the 'Metaphysical Exposition' enumerates four claims about the epistemic conditions of the pure intuition of space (or time).

1. Space is not an empirical concept.

2. One can never represent that there is no space.

3. Space is not ... a general concept of relations of things in general.

4. Space is represented as an infinite **given** magnitude. (KRV; AA A:23-4/B:38-9)

As Cohen points out, the first three criteria are negative; they explain the ways that space cannot be. Furthermore, the fourth criterion may also be interpreted negatively, since the assertion of the infinity of space can hardly be taken to be positive claim but instead serves to negate the claim that we could ever encounter the boundary of space in intuition.[28] It is from this final argument — the endlessness of space — that Kant concludes that space cannot be a concept, but must instead be an a priori intuition.

However, the necessity upon which the arguments of the 'Metaphysical Exposition' is based is not fully explicit. Kant has begun the *Critique of Pure Reason in media res*, and has not at this point made available the analytical materials that allow us to distinguish between different types of possibility — logical, metaphysical, actual.[29] As a result, the necessity (can not, is not) of these demonstrations may rest on nothing more than a psychological ground: subjective necessity. Indeed, this is just the claim raised against Kant by many of his interpreters. However, for our purposes, we need only recall that Trendelenburg's argument for the "gap" in

[28]As we will see below, when considering the problem of determinacy, a great deal hangs on the question of whether intuition consists at every moment of the totality of the manifold of sensibility, or just those elements that constitute a particular appearance.

[29]Some even deny that Kant posesses a concept of something that is logically possible but metaphysically impossible. I think this is an overinterpretation of Kant's claim about the co-dependence of intuition and concept. There seems to be room, in Kant, for discursive cognition of concepts that creates intelligible entities for which there is no possible empirical object. Of course, these could not be objects (since they cannot be in space or time), but we would be begging the question against logically possible entities that are not embodied if we just assume that to be means to be a body. I do not think that Kant intended to go so far, so I am prepared to allow a domain of logically possible entities of which we can have no objective knowledge. This, however, just follows from the fact that they are not possible objects of experience, so the epistemic consequence is trivial.

Kant's proof rests decisively on interpreting this necessity as a *subjective* necessity that is not specifically connected to the possibility of cognitive objects. Cohen, in his own interpretation of the 'Metaphysical Exposition' essentially concedes the point to Trendelenburg: "[The metaphysical a priori] is the first step toward the a priori: that we discover it in our own mind [*Geist*] and, isolated it in a conscious experiment, as carried in things. This **Fact of Consciousness**, that space adheres to all of our representations, makes it a priori" (KTE1; CW 1.2:13/KTE2; CW 1.1:104).

However, we note already that in conceding the point to Trendelenburg, Cohen has not abandoned the argument. For, the metaphysical (psychological) a priori is not the a priori itself, but only the first "step" on the way to discovering the true a priori. If the initial analysis of the a priori is psychological in character, this is because it is provisional in character. The "metaphysical" a priori is thus not an independent type of a priori, but merely an initial, psychological criterion that must be satisfied by any candidate for a priority. Thus, Cohen understands metaphysical a priori as follows:

(a) The a priority of space and time means originality. (KTE2; CW 1.1:198)

Here, originality means only that space and time are constitutive of the possibility of subjective awareness; although the way in which this is so remains opaque at this point.

2.2.2 The First Transcendental *a priori*

The distinction made above between subjective awareness (*Bewußtheit*) and consciousness (*Bewußtsein*) now bears further fruit for Cohen's analysis. For, if the discovery of the metaphysical a priori is intrinsically reflective and psychological, it is thereby a characterization of the conditions of awareness, and not the conditions of consciousness. It is only by discovering the significance of this subjective necessity for the possibility of consciousness as such that we will arrive at a characterization of the "true" a priori: the transcendental.

Cohen's claim is that the analysis of the form of space must abstract from the metaphysical question of the appearance of space (phenomenal space) and uncover the formal condition

of sensibility that makes it possible to realize space as a determinate, but nevertheless pure intuition (formal space, or what we would now just call order). It is this difference between the phenomenal (aesthetic) appearance of space and its formal characterization that Cohen designates as the difference between a *metaphysical* and a *transcendental* a priori. To uncover the transcendental a priori, Cohen turns to the question of the form of space. For, on the one hand, we may conceive of space as a concept: the formal representation of an infinite continuum.[30] However, we may also understand space to be a pure intuition: the procedure whereby the continuum is constructed.[31] The difference, as we have seen, is that a pure intuition of space is only possible through the conditions of sensibility, that is, *Sinnlichkeit* as the condition of intuition. We already found this key characterization of space in the previous section; it is *adjacency*. So, for Cohen, the first level of transcendental apriority uncovers the fundamental formal condition of space and time in the basic forms of sensibility:

> b) space and time as forms of sensibility [i.e., adjacency and succession - BTG], and thus as conditions of cognition, amount to apriority." (KTE2; CW 1.1:209)

That is, space and time are a priori because the form of sensibility that is their formal condition is a priori. These relations — adjacency and succession — are essentially the object of a priori science, that is, of the possibility of the synthetic a priori.

Thus, the first transcendental a priori is the formal condition of the a priori form of intuition: space. This (for Cohen) just is formally grounded in the condition of sensibility: adjacency. It is through the form of adjacency that the a priori concept of space (an infinite continuum) becomes a pure intuition: an infinite, n-dimensional system of relations of adjacencies. Whereas the formal concept of space can represent no more than the mereological concept of containment (that is, of inherence), the addition of the form of sensibility — *rela-*

[30] While Kant insists that space and time are not concepts but pure intuitions, we seem nevertheless to have a concept of space and time. Moreover, the Mathematical Antinomies assume that we can also have an idea of space and time (i.e., an indeterminate representation of the fully determinate concept). We are not stepping outside of the Kantian domain in referring to a concept of space or time.

[31] For Kant, pure space and pure time appear to be either; (a) pure forms of the imagination, in which case they are just the bare (unsynthesized) concept of an order (expressed as adjacency and succession) as Friedman must interpret it; or, (b) merely phenomenological characterizations of the discriminability of an intellectual synthesis of order (i.e., that we can tell 3 from 4 because they are next to each other) as Cohen appears to interpret it.

tion — implies an order, or better, a system of calculable relations. Now, it is easy, from a contemporary point of view, to lose sight of the fact that the logic that Kant assumed was a monadic logic. It did not afford a description of polyadic relations. However, the mathematics of functions is such a polyadic "calculus", and presupposes a formal representation of such relations. For Cohen, then, logic and mathematics are thefore understood to be two asymmetric aspects of intellectual cognition. Mathematics is part of logic, but is the reality of logic insofar as it is realized (a) intuitively or (b) in the object of cognition. Indeed, the schematism relies on the analogy between the constructability of a continuous function of variables (intuition) as a series of discrete values. Accordingly, Cohen's claim that Kant has discovered a new source of lawfullness through the form of sensibility can be made intelligible as the claim that the *form* of sensibility provides the law of order (or better, indeterminate order such as the syncategorematic conception of the infinite), a representational determination that is alien to monadic logic.[32]

Thus far, Cohen's interpretation of the second level of the a priori as the form of sensibility is a plausible, congenial reading of Kant's strategy in the 'Transcendental Aesthetic'. The label that Cohen has applied — the 'transcendental' a priori — can also be understood in a way that is sympathetic to Kant's intentions. For, the a priori form of space and time are essentially characterized by relations of adjacency and succession respectively, and it is these real relations that make the schematism of the understanding possible. In this context, the Transcendental Exposition can be construed as a comment on the extension of the a priori form of intuition into the domain of the pure concepts of the understanding, and specifically, as an anticipation of the materials that will ultimately be developed in the Analytic of Principles.

[32] We may say that Cohen has anticipated Friedman's claim (Friedman 1992a), that natural science requires polyadic relations, and these are not provided by logic alone. However, Cohen takes order to be, loosely speaking, a product of a cognitive act, and thus, strictly speaking, it should be a part of logic. This conclusion will ultimately force Cohen to concede that space and time are a part of what he will thereafter call '*Erkenntnislogik*' (as opposed to the 'mere' logic of the syllogism). Cohen never gives up the (Kantian) commitment that the logic of order is tied to the reality of objects. His *Principle of the Infinitesimal Method* and his *Logic of Pure Knowledge* remain committed to the task of understanding the condition of the application of mathematical form to objects as "logical" conditions of knowledge. As we will see, this leads to a robust interpretation of continuity, not as a formal criterion of the reals, but as an epistemic condition of the intelligibility of objects generally. We will have more to say about this later.

This is true, insofar as it goes. However, Cohen does not understand the "transcendental" a priori to be an anticipation of the development of a system of relational concepts. Rather, he understands the investigation of the form of sensibility to be foundational for the possibility of pure intuition. In other words, we must investigate the relations implied by sensibility in order to account for sensibility at all, and this must be an a priori investigation if it is to demonstrate the "metaphysical" (again, as an a priori concept) import of sensibility for the determination of objective reality, that is, to save Kant from the charge of merely subjective idealism. Decisively, for Cohen, "adjacency and succession ... are the content with which Kant equipped the forms of sensibility" (KTE2; CW 1.1:213). For Cohen, adjacency and succession are not real conditions, but content. And, what is more, this content is capable of an a priori description. By interpreting the form of sensibility — adjacency and succession — as also the content of sensibility, and thus ultimately the content of space and time, and the content of intuition itself, Cohen has opened the door to a formal interpretation of the content of sensibility: a formal interpretation of the given.

2.2.3 The Second Transcendental *a priori*

There is an implicit ambiguity in both of the first two stages of the analysis. For, construed as a *psychological* regression in the metaphysical a priori, or construed as a reduction to psychic processes in the first level of transcendental priority, an apparent connection to psychology and its methods remains. If the reduction of metaphysics to the formalism of *Sinnlichkeit* reveals a set of relations that determine the construction of the pure forms of intuition, the "form" remains ambiguous between a psychological interpretation and formalist interpretation. Construed as a process of psychic construction, form may be reducible to physio-organic processes, as Herbart proposed, or to drives (*Triebe*) as Fichte proposed (Fichte 2005). This ambiguity arises because "the two earlier levels of the *a priori* designated the appearance of the identity of the a priori with the innate" (KTE2; CW 1.1:214). Accordingly, a further level of analysis is required, one that resolves the latent ambiguity of the form of the a priori into a "formal" (i.e., logical) form.

This is the reduction of the a priori to mathematics.

The first "stage" of transcendental analysis shows the form of sensibility to be the constitutive principle of space and time. These forms are, as we have seen, the basic *relations* of adjacency and succession. This form, however, can only be protected from a metaphysical (psychological) interpretation (perhaps as a real order of sensibilia, or as organic processes of perception) if the form of space and time themselves can be given a formal justification as purely intelligible — and thus purely ideal — structured relations. The second stage of transcendental apriority is provided, then, by the reduction of the sensibility to the systematic science of relations of order: geometry and arithmetic. Cohen's intention here is not, or at least not primarily, scientistic. Rather, Cohen intends to avoid a static interpretation of the a priori. For, such an interpretation seems to entail either significant commitments to fundamental (innate) structures or faculties of human cognition, or simply to metaphysical structures that determine the ideality of cognitive activity. Instead, Cohen aims to focus our attention on the fact that thought consists, for Kant, of "functions", and these functions are unifications that make possible the intelligibility of plurality through unity. As Cohen insists, "unifications are acts, not drives, "originally produced concepts", not innate "mechanisms"" (KTE2; CW 1S.1:252). This strategy is not unsympathetic to Kant's approach in the *Critique of Pure Reason*. There, Kant aims to show that logical features of the representation of real relations provides forms that condition the possibility of real and intelligible judgment generally. That is, the task of the schematism consists in isolating the acts of consciousness (functions or unifications) that permit the correlation of systems of real relations (intuited relations of adjacency or succession) with rational unities (logical relations). These turn out to be functions in intension, or principles of construction.

For Cohen, however, the only way to save the real relation of sensibility from an inevitable reduction to psychologism (i.e., to a theory of processes or physio-organic functions of the brain) is to claim that the real relations of adjacency and succession have an adequate a priori representation, that is, that there is a formal representation of the form of sensibility.

This means that adjacency and succession, as the formal conditions of sensibility, must be a priori intelligible, and further, that this intelligible relation is constitutive of the possibility of intuition *as a form of cognition*. In other words,

> (c) the apriority of space and time means that they are, as pure intuitions, the formal or constitutive conditions of experience. (KTE2; CW 1S.1:214)

That is, the adequate formal description of space and time is not just a necessary condition of pure intuition, but a "*constitutive condition of experience*". Or, the empirical determinacy of a particular intuition is in some way constitutively dependent on the formality of the basic relations of adjacency and succession. And so, the science of the a priori forms of appearance just is the a priori science of relations of adjacency and succession, and this just is what Kant investigates in the Transcendental Exposition: mathematics as the "conclusions" that one draws out of the Metaphysical Deduction. But what does this mean?

For Kant, mathematics is not a purely formal enterprise. Rather, principles of construction allow the formal representation of relations that may, intuitively, at least, be indeterminate (although their synthesis is surely determinate). However, these systems of relations only support thetic claims if they are ultimately claims (whether fully justified or not) about truth and being. And so, for Kant, mathematics is essentially a *real* discipline for the classification of intuitive magnitudes, that is, it implies empirical magnitudes just as a condition (a) of real determinacy and (b) intuitive certainty. This is the condition of mathematical formalism having "meaning" (or what we might also call alethic and semantic validity). However, Cohen's interpretation of the Transcendental Exposition suggests that mathematics provides the systematic determination of the transcendental possibility of appearance in general, not the product of the schematic comparison of pure intuition and formal constructions. That is, Cohen understands the formalism of mathematics to be prior to — and thus independent of — the conditions of intuition. In other words, the real relation of adjacency — exteriority — and the real relation of succession — asymmetric, intransitive relations of the descriminability of determinacy — are not constitutive of the possibility of mathematics, but rather, these relations between real mag-

nitudes (or objects) are possible only through the forms of mathematics implicit in the functions of cognition.

Ultimately, this is where Kant and Cohen diverge irreconcilably. For, Cohen does not understand the task of the Transcendental Exposition to be a *correlation* between fundamental forms of real relation and logical forms of activity, but rather the *identity* of formal sensibility with the rational construction according to principles outlined in mathematics, and central to Kant's strategy in the schematism. Indeed, for Cohen, "sensibility is preservered as the fundamental condition of experience through mathematics" (KTE2 CW 1.1:239). It is therefore mathematics that provides the decisive defense against skepticism, against the psychologistic reduction of Herbart or Helmholtz, and against the explicitly subjectivist logic of Fries (Fries 1828) or a subjectivist reading of Hegel's *Science of Logic* (Trendelenburg 1843). It is able to do so because it preserves the special sense of 'sensibility' as, fundamentally, a formal determination of consciousness that cannot be further reduced, and it does so by resolving sensibility into the formalism of mathematics.

The results of Cohen's two-stage regression to the transcendental a priori can be summarized through what he terms the "Copernican Transcendental". Like Cohen's regression (metaphysical, transcendental), the Copernican Transcendental is a "double" inversion. The first inversion is the familiar Kantian inversion: the intelligibility of objects is dependent on a priori forms of understanding. The second inversion, however, is much more ambitious: the possibility of experience (the a priori) is itself constitutive of the objectivity *and reality* of objects. For Cohen, the alethic and semantic claims of judgments of experience are not grounded in the synthetic actualization of representations (empirical intuition), but merely in the a priori conditions of this realization: it is form all the way down. That is, truth and being are provided by the a priori. And this is Cohen's understanding of the "Copernican" strategy of the *Critique of Pure Reason*:

> First: objects are only possible through the possibility of experience.
> Second: objects made possible by experience have objective reality. (KTE2 CW 1.1:143)

Here, however, Cohen understands not "objects of cognition" over and against "objects of intuition", but objects as such. For, there are no "mere apperceptions", no "continuous images", no "merely phenomenal" objects in his "theory of experience". There are only formal objects, for it is formal objects that are represented in "cognition" (*Erkenntnis*) and its is these objects alone that are the subject of science.

Prima facie, this is a distinctively Leibnizian conception of reality: every formal form just is objectively real, although not every objectively real form is actual. Cohen's claim, then, is that the form of experience is a priori — and this includes the determinacy of intuition. This means, for Cohen, that the possibility of knowledge does not depend constsitutively on the real condition of sense, but only on its formal constructability. That is, knowledge is "analytic" through the purely formal relations of mathematics: alethic and semantic claims must therefore be satisfied within the paramaters of formal mathematics — a mathematics that is purely "analytic" and requires no synthetic realization as warrant for claims to objectivity and reality.[33]

Now, it is important to keep in mind that this interpretation of the two "stages" of the a priori is intended to be an interpretation of the Transcendental Exposition and its relation to the Metaphysical Exposition. This is a contentious reading to say the least. For, when Kant refers to geometry in this context, it is explicitly directed toward the explanation of "synthetic a priori cognitions," or more specifically, what "alone makes the **possibility** of geometry as a synthetic a priori cognition possible" (KRV; AA B41). Ultimately, for Kant, it may be the case that the forms of space and time derive from the rational structures of the categories, or are at least related to them in a systematic way. This may in turn have as a result that the forms of appearance must ultimately conform to some kind of intelligible structure, even if that structure is not already given explicitly (distinctly) in intuition.[34] And further, this belief in the unity of the grounds of feeling and the grounds of reason may be what make "rational faith" in transcendental philosophy and its restricted metaphysical claims justifiable. To this extent,

[33]Cohen will nonetheless understand the construction of mathematical forms as "synthesis". However, this synthesis does not invoke the sensible condition, and thus may be synchronic.

[34]This is at least one way of understanding the unity of the principles in the Transcendental Ideal.

it seems that Cohen's interpretation of the Copernican revolution is not entirely unjustified. However, as we anticipated above, Cohen's interpretation of the transcendental exposition as the formal foundation of the metaphysical exposition effectively inverts Kant's strategy. Rather than understanding the contingent reality of intuition as that which provides the real ground of an a priori form, Cohen understands the contingency of sensibility itself to be something that is wholly intelligible through the formalism of a theory of number: we are but a step away from the logicism of Russell and Couturat interpreted as the formalism of objects of possible experience.

Whereas the sense of the Copernican revolution is an inversion of the dependence of intelligibility — knowledge of form is a priori — Cohen has accepted a much stronger interpretation, and one that is usually a feature of subjectivist readings of Kant: that even the structure of objects of possible knowledge is provided by a priori form.[35] It is not just *objects of cognition* that depend on the a priori conditions of cognition generally ("objects are only possible through the possibility of experience"), but that there is no other "object" to which the object of cognition could be compared. In other words, the actuality (objectivity) of objects is already guaranteed just in virtue of the fact that they are so constructed: "objects made possible by experience have objective reality." The analytic foundations of the object — in the case of spatial objects, geometry — just is the real and formal foundation of the object.

[35]For a more detailed account of these two interpretations of the Copernican Revolution, see Graham Bird's *The Revolutionary Kant* (Bird 2006b).

2.3 Experience

According to many interpreters — Cohen included — experience is that fact of consciousness which, once conceded, yields the necessary materials from which the a priori conditions of just this experience may be derived. In other words, if one accepts the possibility of something like experience, then one must concede the necessity of something like the analytic content of this experience. This construal of experience is common to those who read Kant's argument — and especially the Transcendental Deduction — as primarily an anti-skeptical argument, one that *regresses* from a common concession (that we have something like experience) to a disputed origin of some aspect of this experience (a priori cognition).[36] The consequence of rejecting the fact of experience would be, so the argument goes, to endorse a skepticism much stronger and much less defensible than the more moderate skepticism about the a priori origins of thought.

Cohen's interpretation suggests instead that experience is not, primarily, a *cognitive* problem, one dependent on a faculty psychology, or even, as it may be for Kant, a faculty psychology construed as a problem of formal interpretability, but rather a problem of representation. Cohen is not concerned with *how* it is that minds represent (cognizing, "*Erkennen*") but *what* is cognized (knowledge, "*Erkenntnisse*").[37]

[36] The subject of "Transcendental Arguments" has been the subject of some significant debate. To what extent arguments of this type (whatever it may be) can be ascribed to Kant is itself a matter of dispute. See especially *Transcendental Arguments* (Stern 1999) for an overview. Even if there are such arguments from conditioned (the fact) to the condition (the a priori) in Kant's *Critique of Pure Reason*, none of the conditions thus derived (the a priori) are metaphysical facts in the ordinary sense. Kant calls them 'metaphysical' by which he only means that they are hypostasized as a condition of empirical judgment. However, their hypostasis requires an additional premise and a different epistemic attitude: the metaphysical committment of Empirical Realism. Neither the premises of Empirical Realism nor its epistemic attitude can be derived through some "transcendental argument." Thus, the connection to Kant — if there is any — is tenuous at best.

[37] 'Erkenntnis' actually has a four-way ambiguity in German, as Cohen points in the Introduction to the *Logic of Pure Knowledge*. It means,

Judgment a formal decision within the juridical process;

Cognizing the process of cognizing;

Cognition the individual product of cognition;

Knowledge the system of cognitions that forms a science.

The problem of "experience" is accordingly the question of the place of facticity in the *Critique of Pure Reason*.[38] Is 'experience', as Cohen argues, the given from which the analysis of the a priori alone becomes possible? Or is it rather a facticity that needs to be accounted for *within* the system, resting on a surd of unintelligible determinacy within the very heart of the analysis of the form of sensibility? On Cohen's interpretation, as we have seen, the regression from the fact of experience to the formalist forms of apriority precludes the possibility of retaining a moment of unintelligibility within the foundations of representation. This is clear, first, in Cohen's rejection of "awareness" as a legitimate term in the analysis of *Erkenntniskritik*. Furthermore, the foundation of the determinacy of sensibility in mathematical formalism excludes the moment of sensible dependence from any account of the real conditions of cognition. As we will see, Cohen's analytic approach to the fact of experience has two important and closely related consequences.

First, if the analytic regression from the "fact of science" must regress to a homogenous set of grounds (analytic forms, in the sense that they are "given" (as in the *Jäsche Logic*), even if they (paradoxically!) require intellectual synthesis), then the a priori forms of intuition (space and time) and the a priori forms of the understanding (logic, mathematics) cannot be different in

While Cohen's view certainly diverges from Kant's, it is not easy to determine the extent to which they really differ in their interpretation of the nature — and especially the epistemic significance — of experience. Indeed, one source of the difficulties in interpreting the *Critique of Pure Reason* is that Kant's project seems to be to explain the formal character of cognition through an abstraction from cognitive activities in particular cognitive agents (transcendental judgment, or judgment in potentiality) to the necessary conditions of experience, of which empirical science may be some or all of the content — which it is supposed to be is not entirely clear, but we have good reason to think that Kant thought that at least part of our ordinary experience can and should be preserved. The *locus classicus* of the problem is the category of substance, which, for Kant represents an intelligible unity (thought of as ontological), whereas Cohen — guided instead by the Anticipations of Perception — is more interested in the conditions under which it is possible to predicate persistence. Accordingly, for Cohen, the category of substance has a functional interpretation that does not readily reduce to a monadic concept (which, of course, it must do for Kant, since the substance is always the bearer of natural-physical predicates, not the space-time function of its persistence, which could only be expressed through a polyadic logic).

[38]By 'facticity' I mean (a) that which is given, and thus (b) is contingent with respect to intelligibility, and accordingly (c) is intelligible, but not necessary, as a fact of empirical knowledge. For Kant, objects are facts of experience whose necessity is to be grounded in the (ultimately necessary) laws of natural physics. For Cohen, on the other hand, physics itself (the fact of science) is the relevant fact to be grounded in the necessary structures of intelligibility. Thus, whereas the product of the *Critique of Pure Reason* is a *Metaphysical Foundations of Natural Science*, where what is implied is the metaphysics of *the Nature*, Cohen's system of laws are only necessary with respect to the actual world — or better, our actual science of Nature. Accordingly, *Erkenntnislogik* provides the necessity of science in general, not the necessity of the laws of Nature; for Cohen, we inhabit a world of possibility that is much wider than that envisioned by Kant.

kind; they are at most different expressions of a single mode of formal (rational) expression. The immediate consequence of this is that there is no real or modal distinction between the faculty for appearances and the faculty for concepts. Whether we interpret Kant's faculty dualism metaphysically (Paton 1936), epistemically (Strawson 1966) or merely formally (Allison 2004), the fact remains that this distinction is constitutive of the transcendental problematic and the solution that Kant proposes.[39]

The second consequence of Cohen's strategy is that the specifically synthetic character of the transcendental subject — as both spontaneity and passivity — cannot be fully interpreted. The reason for this is that the unified subject is, like the a priori forms of intuition and understanding, nothing but a mathematical expression where the subject (y) is nothing other than the functional unity of a complex expression (i.e., a composite of functions of judgment) articulating a particular index in space and time (x): $y = f(x)$.[40] Strictly speaking, the transcendental unity of apperception is, for Cohen, nothing more than the unity of the categories, that is, the fully determined function that corresponds to a determinable object of cognition. Just as Cohen's "formalist formalism" is supposed to resolve the problem of the subject-object opposition into a question of the formal representation of objectivity, so also must it resolve the problem of the dual-aspect (or dual-essence) subject into the same formalization. Within such a formalism, the subject cannot be legitimately described either as spontaneously active or as passively

[39] Above, I claimed that for Kant there is (minimally) a formal (or functional) dependence of *real* mathematics on the functions of judgment (i.e., syllogistic logic and its critique). This, however, does not mean that there is any epistemic priority of one over the other in purely discursive cognition. Nevertheless, they remain modally distinct for Kant insofar as the forms of construction cannot be realized in discursive consciousness except through concepts provided by the understanding. There is, then, a modal distinction between the functions that generate concepts and the functions that generate mathematical concepts (i.e., concepts + principles of construction). Here, talk of "functions" may be interpreted to have varying degrees of epistemic and metaphysical commitment, depending on how one reads the *Critique of Pure Reason* and how one weighs its prospects. For our purposes, however, we need only note that none of these distinctions survive in Cohen's analysis, where "formalist" formalism precludes shading function-talk with pscyhologistic or metaphysical implications.

[40] One might observe that the identification of the subject (y) with a functional unity that represents the determination of its determinability (i.e., the intelligibility of an empirical state) is an essentially Fichtean strategy (Fichte 2005). This is true, for what it is worth. However, it is plausibly also the case for Kant, at least in any instance of knowledge. What Cohen does not endorse is the Fichtean subjective idealism: that the states of the subject are produced by the intellect, which essentially returns Kant to the Leibnizian view. While it is often pointed out that the (Marburg) Neo-Kantians draw some lessons from Fichte (particularly with respect to the intuition of cognitive activity), the analogy is much looser than is often supposed to be the case.

receptive; or better, the question of the (metaphysical) locus of activity is irrelevant to the question of the formal structures implied by cognition. Rather, these two aspects of the subject are at most "scientific abstractions" that allow us to separate the contingent fact of science from the *Erkenntniskritische* task of providing a formal a priori foundation for knowledge claims.

And so, it seems that facticity — and with it the phenomenal interpretation of first-personal consciousness — escapes entirely from Cohen's account of the transcendental. Whereas Kant's strategy in the *Critique of Pure Reason* seems to rely on the irreducible facticty of consciousness (that is, judgments of modality, especially actuality), Cohen has simply rejected the problem of the individual — and thus also the lurking problem of affection. The distinction between "consciousness" and "awareness" allows Cohen to distinguish his own project from those of other interpreters of Kant, but it also thereby excises an important moment from the Kantian analysis. As we have seen, this moment is the facticity of sensibility. It is this unintelligible moment of sensibility that constitutes the asymmetry of adjacency as exteriority, and succession as an asymmetric condition (i.e., what allows us to count our fingers, rather than merely counting in our heads). This moment is necessary not just to the distinction between the faculties of intuition and understanding, but more importantly to Kant's strategy in the "negative" conceptualization of contingency through the judgment of actuality. It is because intuition contains an irreducible conditioning relation that determinations of temporal actuality (*mutatis mutandi*, determinations of exteriority) are possible at all. By bracketing the problem of facticity within the structure of transcendental critique, Cohen has avoided the problems of its metaphysical interpretation, but has also discarded the critical solution to the paradoxes of representation that it offered.

Now, before we dismiss Cohen's interpretation as simply missing the point, there are a number of advantages to his approach that are worth keeping in mind. First, the duality of the transcendental unity of apperception — as the locus of plausibly metaphysical "forces" or "causes" — is a thicket of hermeneutic problems, and possibly also of some covert meta-

physical committments.[41] Ultimately, judgments about the "nature" of the transcendental unity of apperception — that is, specific characterizations of its active functions — cannot rest on a first-personal evidentiary basis.[42] Furthermore, there is no legitimate inference from the *Erkenntniskritische* recognition of formal unity to any kind of essentialist dualism, interpreted either in psychological or metaphysical terms. That means that neither "two-aspect" nor "two-world" interpretations have any traction, since there is no distinction that needs epistemic or ontological justification. While Cohen is not denying that there may be aspects of the "thing-in-itself" that we may not know about (that is, if we even allow ourselves to think in these terms), he does deny that we are able to raise valid claims about whether or not we have such knowledge. For, claims about knowledge that cannot be structured as knowledge claims are not just alethically invalid (as they are for Kant), but are formally invalid. Thus, claims about things as they are in themselves are not just meaningless, but are not even intelligible.

Above, we saw how representations can be taken to refer to objective forms in one of two different ways: either through formal isomorphism (Descartes) or through some kind of conditional dependence (Hume). Maintaining both modes of reference leads to one of two different dilemmas: property skepticism and relational skepticism. Kant's solution to the problem of is to follow Leibniz in making reality a rational predicate, that is, to say 'x is real' is in fact to say 'x is assertable of some intuition'; this is neither Descartes' conception of reality (as both rational and empirical), nor Hume's (as merely empirical). Unlike Leibniz, however, Kant al-

[41] The problem of affection has a long and distinguished lineage (Jacobi 1789), (Adickes 1924), (Prauss 1974), (Van Cleve 1999); so too does the physio-organic interpretation of the concept 'function'. Cohen's interpretation, however, brackets the problem of the application of cognition (it is, so to speak, "hardware-neutral") and the metaphysics of the subject. It is, therefore, better suited to applications as an epistemology (or philosophy of science) than it is to applications in the philosophy of mind, except insofar as it provides abstract structures for interpretation through cognitive science.

[42] Kant regularly and vigorously objects to any conception of affective access to consciousness as such, but he does not exclude the possibility (and indeed, seems to presuppose it) that we can be aware of the activity of the subject. Indeed, this seems to be presupposed by the idea of enumerating our fingers. When we count fingers, we are not actually counting fingers, but those cognitive acts through which fingers are apprehended. Thus, awareness of our cognitive acts appears to be a condition not just of counting, but of knowledge generally. This is not, however, "intellectual intuition" in the sense of God's creative intellect: that whereby he produces plurality from mere unity. In any event, Cohen's interpretation has already intellectualized the synthesis of order; whether order arises from counting "thirteen round dollars" or whether it arises from the composition of an extensive magnitude is irrelevant if we are only concerned with the fact that representation *as such* requires order. The claim is weaker, but also much broader in scope.

lows that some parts of thought may not have a priori determinable grounds; moreover, even if the forms of intuition are a priori intelligible, their intellection depends on their presentation in intuition — we are intuitive cognizers, not intellectual cognizers. More specifically, Kant claims that judgments of actuality arise when form is presented in intuition for which we cannot provide an intelligible ground. Insofar as an intuition is brought to apperception, it must be intelligible. That does not, however, mean that we can articulate its intelligibility — even if the imagination has already (unconsciously!) provided the law of the composition of the particular. Indeed, it seems to be the judgment of actuality that allows us to orient ourselves with respect to (universal) time, i.e., within the system of necessary laws of nature. It is because something is presented in intuition, and yet cannot be reduced to a deliberate act of representation, that we know that we inhabit a present time of which we are not the author. Here, the negation of rational assertability, the concept of actuality, makes it possible for forms of thought to be determinate with respect to their occurence in time.[43] Thus, purely negative judgments of conceptual determinacy are possible from the transcendental perspective. However, the possibility of this negation rests on the evidence of sensibility; there must be *something* in intuition that fails to be interpreted in order for judgments of actuality to become possible. This, however, means that the transcendental judgment of actuality is only a theoretical possibility in the absence of the direct evidence of some unintelligible sensory evidence. Kant must therefore appeal directly to the evidence of sensibility in order to avoid the problem of indeterminacy.

As I have presented Kant's position here, the formal analysis of experience yields the a priori forms of representation as the pure potentiality of cognition. The actuality of thought, however, requires the realization of synthetic cognition either explicitly or implicitly through the direct or indirect involvement of the real conditioning relation sensible particulars in the manifold. Thus, for Kant, the contingency of the *fact of consciousness* is present at most negatively for the pure possibility of synthetic cognition, but is constitutive of the actuality of expe-

[43] Since all possible real predicates must be predicable of space, all real predicates are possible simultaneously. It is only the modal judgment of actuality that allows us to claim that a particular configuration of the manifold obtains now or not.

rience. For Cohen, on the other hand, the formal analysis of experience yields the a priori forms of representation as the *actuality* of consciousness; actuality — like reality — is nothing more than a formal moment of the analysis of cognition. The contingency of the *fact of consciousness* is therefore an *extra-logical* problem, the problem of subjective awareness: a problem for empirical psychology. The problem of providing a rigorous grounding for the knowledge claims of science, however, is taken to be answered by the reflective formalism of an analytic a priori. That is, for Cohen, there is but one ground of science (not two): its timeless intelligibility, or the Platonic form of an (anticipated) "final" science of actual Nature.

For Kant, sensibility serves as the boundary between transcendental possibility and empirical actuality, and is therefore the crucial point of inflection between the purely formal analysis of thought and the empirical analysis of metaphysics, both of which are contained within the methods of transcendental critique. For Cohen, however, the distinction between '*Bewusstsein*' and '*Bewusstheit*' is invoked to separate the phenomenological or psychological dimension of sense experience from its formal grounds. However, this distinction does not track the distinction between the transcendental and the empirical. For, as we have seen, the unity of the transcendental and the empirical rests on the immanent feeling of real condition. Cohen's understanding of the distinction between formal and empirical feeling is designed specifically to eliminate the contingency of the latter through the formal determinacy of the former. In other words. The reduction of awareness to consciousness — like the reduction of the fact of science to its formal mathematical grounds — must eliminate every trace of the singular, and with it the possibility of reconstructing the original determinacy of actuality. Moreover, the method that Cohen proposes in *Kant's Theory of Experience* owes its plausibility to the initial *Faktum* of experience. Within the context of a science of formal foundations, the givenness (*Gegebenheit*) of the fact of science cannot be explained. Rather, it invokes the conditioning relation of real reference that leads in the first instance to the problem of Cartesian skepticism and disjunctivism.

And so, while Cohen has — correctly — focused the attention of Kant's interpreters on

the status of intuition as the objective unity of appearance and intelligibility, his own refusal to admit the data of sensibility as relevant to the determination of form leads the interpretation to embrace *both* problematic aspects of the theory of logical analysis. On the one hand, Cohen seems to have no non-dogmatic account of how the reconstructed forms of objects (their "vindication") is able to account for their facticity. There is, then, nothing "in" sensibility over and above that which can be accounted in terms of the formally structured activity of the "act" of intuition. However, because of this, Cohen's solution to problem of isomorphism (that is, the Leibnizian symmetry of intuition and concept) leaves no avenue for distinguishing between actuality and mere possibility.

Whatever its independent merits may be, whatever role it may have played in shaping the Kant reception, Cohen's analytic interpretation of the transcendental in *Kant's Theory of Experience* cannot be deemed an adequate interpretation of Kant's method of *critique*. Cohen's mathematical formalism comes very close to restoring a Leibnizian conception of the formal foundations of both concepts *and* appearances, and in so doing restores the problem of formal indeterminacy (which Kant's conception of intuition resolved) within logic. However, Cohen reads the transcendental dialectic as a prohibition against precisely the kind of speculative metaphysics that make the Leibnizian approach possible. Instead, Cohen's method of analytical regression comes much closer to Descartes' method in its attempt to analyze apodictic truth from the contingency of mental representations. As such, Cohen also faces the problems of property skepticism that arise from Descartes' new method of science. The analytical theory of pure foundations may have appeal — especially as a way of responding to interpreters such as Bona Meyer or Helmoltz — but it also seems to be incapable of accomodating Kant's critical strategy and the crucial role played by the indexicality of intuition. In between the publication of the first and the second editions of *Kant's Theory of Experience*, Cohen became aware of these limitations.

Chapter 3

The Transcendental

3.1 Transcendental Analysis

3.1.1 The Deduction

The centrepiece of Kant's transcendental investigation is not the doctrine of the a priori, but rather the extended argument of the Deduction of the Pure Concepts of the Understanding.[1] Now, the Transcendental Deduction defends the claim that judgments of the possibility of an object are possible a priori. This means that we make judgments about whether or not particular modifications of intuition are valid combinations of the sensible faculty in a way that is logically (not chronologically) prior to the presentation of that configuration through outer sense (or inner sense). The validity of these combinations is established merely by considering the form of the representations through which objects are cognized (intuitions, concepts), or, more precisely, by considering the formal principles that are necessary to the construction of different types of representation and their synthesis in cognition. Now, in order for the Deduction to succeed, it must be the case that transcendental judgment is not only possible (i.e., the claim of the Transcendental Deduction), but is also (a) necessary and (b) universal.[2]

Now, the claim to necessity (a) arises as a claim about the nature of the relations of the faculties in synthetic judgment; more precisely, it is a claim about the formal relations between different types of representations and the principles that constitute them. We will see what this entails below. The claim to universality (b), on the other hand, is not a claim about the genetic

[1] If the Deduction succeeds, then Kant has achieved the impressive feat of grounding a new fundamental approach to epistemology: Transcendental Idealism and Empirical Realism. The central place of the Deduction in the *Critique of Pure Reason* may also serve as a decisive hermeneutic principle: The structure of Kant's transcendental investigation must be understood in such a way that the Deduction succeeds, or comes as close as possible to succeeding as we can manage.

[2] In the B Introduction, Kant defines the a priori as follows:

> **First**, then, if a proposition is thought along with its **necessity**, it is an *a priori* judgment; if it is, moreover, also not derived from any proposition except one that in turn is valid as a necessary proposition, then it is absolutely *a priori*. **Second**: Experience never gives its judgments true or strict but only assumed and comparative **universality** (through induction), so properly it must be said: as far as we have yet perceived, there is no exception to this or that rule. Thus if a judgment is thought in strict universality, i.e., in such a way that no exception at all is allowed to be possible, then it is not derived from experience, but is rather valid absolutely *a priori*. (KRV; AA B3)

dependence of one type of representation on principles embodied by another faculty (i.e., the dependence of space and time on the functions of judgment, and then on the synthetic unity of apperception). Rather, it consists of a claim about the equivalence between intelligible and empirical structures in the representation of the object of experience. That is, it shows that every possible empirically directed combination of a priori concepts can be realized in the manifold of intuition, regardless of whether that particular configuration is given as an empirical individual (i.e., it is synthesized by the reproductive imagination) or whether it is a merely possible object of experience (i.e., it is synthesized by the productive imagination).

Now, the argument of the Transcendental Deduction defends the claim that empirical individuals (the empirical object) would not be intelligible if they did not fall under the the principles of the understanding, and ultimately under the principle of the synthetic unity of apperception.[3] That is, every intuited object is intelligible. But even if the argument of the Transcendental Deduction succeeds, Kant still needs to be able to show that transcendental judgments are possible for configurations of the manifold that may never be given at all. That

[3]The A and B editions have the same general strategy, although they differ in many places. In the A edition, Kant is primarily concerned with the fact that the plurality of the manifold of extension necessarily appears within the (instantaneous) unity of time, i.e., that all intuitions may be construed from the point of view of inner sense. Accordingly, unity is necessary to the apperception of an intuited object, and this intuition correlates with the unity of the apperceiving subject. The argument of the A edition is problematic, and it will not be the focus of our attention here.

In the B edition, however, Kant's argument is concerned with combination; for, any combination is necessarily a product of the spontaneity of the subject, and must (ultimately) derive from the unity of the functions of judgment and the unity of the self-activity of the subject. Here is the claim:

> The **combination** (*conjunctio*) of a manifold in general can never come to us through the senses, and therefore cannot already be contained in the pure form of sensible intuition; for it is an act of the spontaneity of the power of representation, and, since one must call the latter understanding, in distinction from sensibility, all combination, whether we are conscious of it or not, whether it is a combination of the manifold of intuition or of serveral concepts, and in the first case either of sensible or non-sensible intuition, is an action of the understanding, which we would designate with the general title **synthesis** in order at the same time to draw attention to the fract that we represent nothing combined in the object without previously having combined it ourselves, and that among all representations **combination** is the only one that is not given through objects but can be executed only by the subject itself, since it is an act of its self-activity. (KRV; AA B129)

is, it must be shown that the a priori principles of the understanding can always be realized in the manifold of intuition insofar as they correlate to modifications of space and time. This claim is not defended by the Transcendental Deduction, but rather by the Schematism.[4]

The argument for the validity of transcendental judgment can therefore be understood to unfold in four distinct steps.

(a) Kant argues that there are necessary and universal conditions (a priori principles) of intuitions, i.e., the a priori forms of intuition, in the Transcendental Aesthetic.

(b) Kant argues that there are necessary and universal conditions (a priori principles) of concepts, i.e., the categories, in the Metaphysical Deduction.

(c) Kant defends the claim that synthetic perception is possible in the Transcendental Deduction.

(d) Kant argues that every serial ordering of the manifold corresponds to a possible configuration of intelligible forms (unity) in the Schematism, and also in §26 of the B Deduction.

Still, even if the argument of the Transcendental Deduction (broadly construed) is valid, it still does not show that the conclusion is true. For, the soundness of the argument depends on the truth of its premises: specifically, that there are objects of experience. This, however, is not a transcendental claim, and will need to be treated separately.

The a priori Necessity of the Categories

The first claim — the necessity of a priori judgments of objects — amounts to the claim that a transcendental judgment (a judgment of the pure, i.e., subjective in origin, possibility of the

[4]The crucial claim of the Schematism is that the concept 'number' "generates" time (and accordingly, also space). Without this claim, Kant cannot guarantee that space and time are intelligible. While there may be grounds to argue that, in the A edition, Kant thought that he only needed to subsume intuitions under the general form of time (i.e., that intuitions are given *alongside* the synthesis of the imagination), the B edition makes clear (in the concept of combination) that every intelligible relation must be the product of the activity of the intellect, if not the understanding itself.

object) implies a specific attitude towards the possibility of a judgment of experience; accordingly, it forms the core argument of the Transcendental Deduction. The task of demonstrating the necessity of the categories is not found in the Transcendental Deduction itself, but in what Kant (in the B edition) calls the 'Metaphysical Deduction'. This argument — the response to the *quaestio quid facti?* — must show that the categories are necessary to the possibility of judgments of experience.

Kant's strategy in this section is obscure, and has, understandably, provoked a great deal of debate. However, we can understand the justification of the Metaphysical Deduction as analogous to that of the Metaphysical Exposition in the Transcendental Aesthetic. Now, the argument of the Metaphysical Expositions of Space and Time offers intuitions as empirical evidence, and then shows that space (or time) cannot be removed from the representation of the intuition. Similarly, the Metaphysical Deduction may be thought of as taking the table of judgments as empirical evidence, and then claiming that these judgments are not possible in general without the application of the concepts. As Kant argues in the first section of the "Clue", "we can ... trace all actions of the understanding back to judgments, so that the **understanding** in general can be represented as a **faculty for judging**" (KRV; AA A69/B94). And since "all judgments are accordingly functions of unity among our representations" (KRV; AA A69/B94), it follows that all (pure) concepts of the understanding are just functions of unity. Accordingly, "the functions of the understanding can therefore all be found together if one can exhaustively exhibits the functions of the unity of judgment" (KRV; AA A69/B94). However, if the argument of the Metaphysical Deduction were the same as that of the Metaphysical Exposition, we would expect Kant to argue from judgments to concepts, not the other way around; that is, we would expect Kant to show that we could not judge without employing the "functions of the understanding."

However, the argument of the first section of the "Clue" merely identifies the categories through the table of judgments. The claim that these are necessary (if not yet legitimate) is only defended in §10. There, Kant traces *both* the judgments and the categories back to the general

functions of synthesis (KRV; AA A77/B103). Accordingly the necessity of the concepts of the understanding arises from the fact that the understanding is generally construed a faculty for synthesis; and so, "**pure synthesis, generally represented** yields the pure concept of the understanding" (KRV; AA A78/B104). Whereas the Metaphysical Exposition (of Space and Time) claims a rational or psychological necessity as evidence of the apriority of space and time, the Metaphysical Deduction appears to raise a claim about the origin of synthesis generally. The challenge represented by the argument, however, is the question of how, precisely, it is to be interpreted. For, on the one hand, it is not clear whether the justification for this argument is (a) a merely formal consideration of the structure of syllogistic logic and its origin in the synthetic faculty, or whether (b) the necessity of the categories depends on a psycho-dynamic account of the interaction of the faculties. On the other hand, while the Metaphysical Deduction (if it succeeds) may be taken to show the co-dependence of judgment and concepts in a general way, this does not constitute a justification of just those judgments (or categories) that Kant has identified. Accordingly, the question of the completeness of the table of judgments (or the categories) is as much a question about the nature of the table of categories as it is a question about the apriority of the pure concepts of the understanding.[5]

We will not resolve the question of which interpretation is most plausible on the Kantian account. It should be noted, however, that the justification of the necessity of the categories requires a further justification either (a) through the demonstration of their completeness, or (b) through a more detailed psycho-dynamic account of cognition.

Transcendental Possibility

The core argument of the Transcendental Deduction aims to defend the possibility of a transcendental judgment as the synthesis of intuition and the understanding. That is, it must show how it is possible to unite the *pure form* of intuition with the *pure form* of the categories, or

[5]The question of the completeness of the table of judgments exercised not only Kant's successors (especially Hegel), but also more recent interpreters, such as Reich (Reich 1992) and Wolf (Wolff 1995). Much of the question, however, seems obscured by the assumption that Kant has a very narrow construal of logic. I'm not convinced that this is a defensible interpretation, Kant's comments in the "Clue" notwithstanding.

(as we have now seen) the pure form of synthesis. The structure of the argument derives from Kant's doctrine of apperception, which holds that an empirical individual is a possible object of cognition if, and only if, it may be represented (a) in intuition (b) by the understanding (c) for a unified consciousness of the intuition and the understanding, i.e., the transcendental unity of apperception. Accordingly, the empirical individual is (logically) conditioned by the transcendental judgment of the possibility of the cognitive object, which implies the following three conditions.

1. Objects of experience must be represented in space and time if they are to be intuited; therefore they must conform to the pure forms of intuition.

2. Objects of experience must be represented as concepts if they are to be thought; therefore they must conform to the pure forms of the understanding.

3. Objects of experience must be represented as both intuitions and concepts; therefore the manifold of space and time must conform to the pure forms of the understanding (i.e., the Schematism) and must be represented (transcendentally) as unified for a single judging subject (i.e., the synthetic unity of apperception).

These three conditions are articulated in §17 of the Transcendental Deduction, "The principle of the synthetic unity of apperception is the supreme principle of all use of the understanding":

(a) The supreme principle of the possibility of all intuition in relation to sensibility was, according to the Transcendental Aesthetic, that all the manifold of sensibility stand under the formal conditions of space and time.

(b) The supreme principle of the possibility of all intuition in relation to the understanding is that all the manifold of intuition stand under conditions of the original synthetic unity of apperception [i.e., the unity of the functions of judgment — BTG].

(c) All the manifold representations of intuition stand under the first principle insofar as they are **given** to us, and under the second insofar as they must be capable of being **combined**

> in one consciousness; for without that nothing could be thought or cognized through them. (KRV; AA B136)

The Transcendental Deduction thus results in the following problematic hypothesis: if the experience of empirical individuals is possible, then this implies a possible transcendental judgment, which in turn requires (a) the a priori forms of intuition (b) the schematic conformity of the manifold of intuition to the forms of the pure concepts of the understanding and (c) the reflective unity of both the inuition and the concept in the transcendental unity of apperception. The "rational faith" of the Critical System is buttressed by the fact of experience (that we do in fact experience objects as given), and also by the fact of natural science (in particular Newtonian mechanics). If we accept the argument of the Transcendental Deduction, then we cannot reject transcendental judgment and endorse our empirical beliefs; accordingly, we are committed to the transcendental doctrine: "the categories contain the grounds of the possibility of all experience in general from the side of the understanding" (KRV; AA B167).

Transcendental Universality

One of the problems of interpreting the Transcendental Deduction (§§20-21) is that Kant's designation of the argument itself is somewhat ambiguous. Indeed, as I have reconstructed the argument above, the complete argument of the Transcendental Deduction is not achieved in the section titled "The Transcendental Deduction", but rather depends on preliminary arguments in the Transcendental Aesthetic, the Analytic of Concepts (especially the Metaphysical Deduction), and the argument of the Schematism.

Now, in the B Deduction Kant adds an explicit consideration of the problem of the "strict universality" of the Transcendental Deduction. If the Metaphysical Deduction (§§9-12 in the B edition) demonstrates the "complete coincidence [of the categories] with the universal logical functions of thinking" (KRV; AA B159), and the Transcendental Deduction (§§20-21 in the B edition) demonstrates "their possibility as *a priori* cognitions of objects of an intuition in general" (KRV; AA B159), it still remains to be shown in what sense the categories are applicable

to the manifold a priori such that every objective concept (i.e., each individual category) can be realized a priori as a possible modification of the manifold. Whereas the Transcendental Deduction shows that the categories are valid for any object that is an object of experience, the task of defending the categories in their general a priori applicability must show the universality of the pure concepts for the (productive and reproductive) imaginative synthesis of the manifold.[6] That is, Kant must be able to show how it is possible to apply the categories not just to *actual* experience in general, but to merely *possible* experience. Accordingly, in §26, "Transcendental deduction of the universally possible use of the pure concepts of the understanding in experience", Kant proposes what appears to be a *second* Transcendental Deduction: the demonstration of "the possibility of cognizing *a priori* **through categories** whatever objects **may come before our senses**, not as far as the form of their intuition but rather as far as the laws of their combination are concened, thus the possibility of as it were prescribing the law to nature and even making the latter possible, is to be explained" (KRV; AA B159).

The strategy of the "second" Deduction is to show explicitly the way in which the synthesis of the manifold of intuition (space and time) is dependent on the pure concepts of the understanding. That is, Kant makes explicit that the condition of the combination of the manifold is the application of the categories: "all synthesis, through which even perception itself becomes possible, stands under the categories," since the categories just are the conditions of the possibility of connectedness (synthesis) in general. The argument is short, since all it does is to make explicit Kant's claim from §15, "On the possibility of a combination in general", which

[6] At the end of the Analogies of Experience, Kant makes a claim that appears to show how the synthetic principles (up to the categories of relation) are necessary to show that all possible concepts can be realized in the manifold:

> Our analogies therefore really exhibit the unity of nature in the combination of all appearances under certain exponents, which expresses nothing other than the relation of time (insofar as it comprehends all existence itself) to the unity of apperception, which can only obtain in synthesis in accordance with rules. Thus together they say: All appearances lie in one nature, and must lie therein, since without this *a priori* unity no unity of experience, thus also no determination of the objects in it, would be possible. (KRV; AA A216/B263)

That this is possible requires the arguments of the System of All Principles of the Understanding. The problem does not emerge in its full detail in the A Deduction, where Kant supposes that time and its continuity — rather that synthetic unity of apperception — is the fundamental condition of apperception.

traces *any* combination (including the intelligible combination of serial order in the manifold) to the activity of the understanding.

However, there are two important consequences of this second argument. First, there must be a determinate correspondence between (a) the manner in which the a priori manifold of space and time are constructed (the pure form of sensibility) and (b) the pure concepts of the understanding (especially the mathematical categories: quantity and quality). Second, the nature of this correspondence determines the laws of appearance (and thus the laws of nature) in a very general way: *every* possible modification of the manifold (i.e., every possible object of experience) must conform to the general laws of combination that account for the possibility of constructing the serial order of the manifold (i.e., the Schematism).

Now, the argument of §26 does not show how all of the categories are implicated in the a priori construction of the manifold. Rather, Kant only claims that the possibility of the unity of the manifold of space in the intuition of an object corresponds to the category of quantity (presumably the category of unity), and that the possibility of the unity of the manifold of time in the intuition of a continuous object (the freezing of water) depends on the categories of relation. That is, "**I determine everything that happens in time in general as far as its relation is concerned**" (KRV; AA B163). Here, however, the determination of temporal order does not take place through the affirmative moment of the categories of relation (subsistence-inherence) but rather through its negation: "the occurence itself, as far as possible perception is concerned, stands under the concept of the **relation** of **effects and causes**" (KRV; AA B163).

Even if we accept the argument of the Transcendental Deduction, and even if we accept Kant's account of the dependence of the synthesis of the manifold on the concepts of the categories, the Deduction is still in some sense incomplete. For, Kant's arguments in both Deductions (§§20-21 and §§26-27) only justify the universality of the categories in general. Even if the necessity of the categories is taken to be demonstrated by the Metaphysical Deduction (i.e., the derivation of the categories from the table of judgments), we still may wonder (a) if the derivation of the table of judgments is legitimate, or (b) if all of the categories are really

necessary to the synthesis of objects, or merely some of them. In order to show that it is the case that all of the categories are *both* necessary and universal, Kant must demonstrate that the pure concepts of the understanding can be mapped onto the manifold of intuition a priori. Kant must therefore deliver principles of construction which allow the translation of a priori conceptual structures (the categories) into constructions of the serial manifold if the argument of the Deduction is to be complete. Moreover, if all of the categories really are necessary to the synthesis of objects in experience (and not just the affirmative judgments: generality, reality, substance and possibility), then it seems that Kant must also show that the synthesis of the manifold of intuition (and not just the synthesis of an individual intuition!) is not possible without the contribution of *all* of the categories.[7]

3.1.2 Cohen's Interpretation of the Transcendental Deduction

As we have seen, Cohen's interpretation of the method of transcendental investigation unfolds in three steps. First, we investigate the psychological necessity of the form of a particular kind of representation (metaphysical a priori). In the Transcendental Aesthetic, this corresponds the psychological necessity of space and time, as this claim is defended in the Metaphysical Exposition of Space and of Time. The genuinely transcendental aspect of this a priori, however, is only established (so Cohen believes) through the Transcendental Exposition, where, second, the formal necessity of an a priori structure is demonstrated independently of its psychological presentation (first transcendental a priori). Third, the analytic nature of an a priori structure is demonstrated, paradigmatically through its foundation in the formal sciences of mathematics (second transcendental a priori). Here is how Cohen formulates the method in *Kant's Foundation of Ethics*:

> Experience is given; the conditions upon which its possibility depends are to be discovered. If the conditions that make the given experience **possible**, make it possible in such a way that they may also be declared to be *a priori* valid, that they

[7] Clearly, the third moment of judgment (singularity, limitation, community, actuality), etc. cannot lead to any determinate construction, since it is only the idea of the totality implied by the negation of the application of the predicate in a particular instance, i.e., that which is cognized without the assistance of concepts or general marks.

> may be described as strictly necessary and of unrestricted generality, then these conditions are to be designated as the constitutive characteristics of the concept of experience, and from this concept may be deduced that which lays claim to the epistemic status of **objective reality**. That is the whole task of transcendental philosophy. (KBE; CW II:24)

If Cohen's intepretation of the transcendental method provides an interesting (if misguided) approach to the Metaphysical and Transcendental Expositions of the Transcendental Aesthetic, the interpretation provides more problematic results when applied to the Analytic of Concepts. For, there Kant refers, as we have seen, to a "Metaphysical Deduction" and a "Transcendental Deduction". The application of Cohen's interpretive scheme suggests the following interpretation of the strategy of the Analytic of Concepts. First, the investigation of the "Metaphysical Deduction" must establish the psychological necessity of the categories. Second, that the investigation of the Transcendental Deduction must establish the universality of the categories (a) by investigating the formal necessity of the categories (b) by grounding the formal necessity of the categories in the universality of logic. Quoting A. Stadler, Cohen takes it to be the case that the Analytic of Concepts "deduces the category; the Analytic of Principles deduces the categories" (KBE 58; Die Grundsätze der reinen Erkenntnisstheorie in der Kantischen Philosophie, s. 53). Cohen modifies the claim, since "in fact, the system of principles produces the categories" (KBE; CW II:59). That is, the Transcendental Deduction defends the claim that all forms of intuition are derivable from the categories (necessity); the Analytic of Principles must defend the claim that the categories are produced by the System of Principles. The problem, however, is that this does not appear to be Kant's strategy.

First, the "Metaphysical Deduction" attempts to answer the *quaestio quid facti?* That is, it attempts to derive the forms of pure conceptual representation from the functions of judgment. The derivation of the categories from the functions of judgment — while problematic — appears, like the Metaphysical Exposition, to claim the necessity of a formal structure (the functions of judgment) for the possibility of a type of representation: concepts. While the justification of the claim that the functions of judgment are necessary to the possibility of pure concepts of the understanding may be psychologically interpreted (i.e., the functions are inter-

preted as empirical psychological activities with metaphysical implications), the result of the argument is supposed to be that the functions of judgment provide the necessary *forms* for the construction of concepts, not that they provide the necessary psychological conditions for the representation of concepts as such. Neither does it seem to be the case — as some have contended — that the Metaphysical Exposition derives the structure of space from the "fact" of geometry and the Metaphysical Deduction derives the structure of logic from the "fact" of a logic of judgment. Rather, for Kant, the strategy of the Metaphysical Deduction appears to be to defend the categories by showing that the abstract structures implied by the categories are already widely accepted — in the table of judgments. Thus, Kant's strategy is to derive (a) a logic of judgment and (b) a the categories from a single set of cognitive structures.

Second, the Transcendental Deduction attempts to answer the *quaestio quid juris?* That is, it attempts to justify the application of the categories to a heterogeneous representational form: intuitions. That is, as we have seen, it asks whether or not the categories are necessary to the possibility of the apperceptive cognition of intuitions. The analogy to the Transcendental Expositions of the Transcendental Aesthetic should be clear: both are concerned with cognition. But whereas the Transcendental Expositions are concerned merely with an investigation of the possibility of the transcendental (i.e., intuitive mathematics), the Transcendental Deduction is concerned with the *justification* of the necessity of intuitive mathematics. Cohen, however, understands the task of the Transcendental Deduction to be a formal analysis of the results established in the "Metaphysical Deduction". This, however, misses the essential difference between "metaphysics" (as the a priori condition of a cognitive representation) and "the transcendental" (as the synthetic conditions of the unification of heterogeneous representations). That is, Cohen takes the task of the Transcendental Deduction to concern essentially (empirical) conceptual representations, whereas for Kant, it is clearly oriented towards demonstrating the necessity of a priori combination in general.

Third, the Transcendental Deduction does not establish the universality of the a priori forms of the understanding by grounding them in logic (or mathematics). Rather, the claim that

the conceptual forms of the understanding are universal is established through the Analytic of Principles. There, the principles of construction are intended to demonstrate the transcendental equivalence between the categories and modifications of the manifold. Cohen, however, understands the task of the Analytic of Principles to consist fundamentally of the formal foundation of the a priori forms of concepts (the categories) *in the analytic structures of mathematics*. That is, whereas Kant appears to be arguing that the categories may either (a) be used to interpret the manifold or (b) be projected onto the manifold by the productive imagination, Cohen takes it to be the case that the manifold (i.e., mathematics, or relations of serial order) may be used to interpret the categories. That is, the ultimate (formal) foundation of the categories is not the "pure logic" of the syllogism, but the logical structures of mathematics. For Cohen, then, the core argument of the Transcendental Deduction is found in the Analytic of Principles, not the Transcendental Deduction. For, it is only through the demonstration of the universality of the categories (i.e., their foundation in formal mathematics) that it can be shown that the categories are a *universal* condition of the possibility of empirical concepts. That is, that the mathematical can produce all possible empirical concepts, i.e., all representations necessary for claims to empirical knowledge.

If Cohen's interpretation of the Transcendental Aesthetic is problematic, then his interpretation of the Analytic of Concepts can only be described as disastrous. For, his interpretation of the Transcendental Deduction shows just how completely the doctrine of judgment in the *Critique of Pure Reason* has been misconstrued. For, whereas Kant is concerned primarily with the problem of the synthesis of heterogeneous forms of representation — however this heterogeneity may be interpreted — Cohen's interpretation is possible only if we deny that there is any plurality of representations. That is, it depends constitutively on the claim that mathematics is the common formal ground necessary to the construction of epistemic representations, without distinction between the forms of intuition and empirical concepts.[8]

[8]Cohen's misconstrual of the strategy of the Deduction may be explained by the following limitation of Kant's Deduction. The concept 'number' provides the form of combination necessary to interpret the structure of the manifold through the categories. However, Kant's concept of number is that of successive addition: $x_i = x_{i-1}+1$. While Kant may believe that the structure of enumeration derives from the category of unity (i.e., from the rational

Although Cohen claims to be returning to the Kantian problematic — that is, to be avoiding the excesses of the post-Kantian Systems of Idealism — his own interpretation, as we have seen, leads directly to one of the primary theses of the Systematic Idealists: that the distinction between representational forms is, at most, a conceptual distinction, and therefore the serial order of intuition must be represented through an intellectual synthesis, i.e., one that is not subject to the real condition of sensibility. Cohen's interpretation differs from the dominant models of the post-Kantian Systematic Idealists in the following way: Whereas the difference between intuitions and concepts is of psychological and ultimately logical significance for Maimon, Fichte and Hegel, Cohen's interpretation of the transcendental method suggests that the difference between intuitions and concepts is only relevant for the preliminary analysis of the a priori (the metaphysical a priori); the foundation of intuitions and concepts in mathematics (the second transcendental a priori) suggests that there is no significant *logical* distinction between intuitions and concepts for the purposes of a science of the a priori, i.e., a transcendental philosophy.

As we saw above, one of the central problems addressed by Systematic Idealism is how to account for the reality of judgments. There, we addressed the problem as a question of the possibility of empirical determinacy: what are the conditions under which we can claim that the concatenation of possible modifications of the manifold correspond to a real possibility? This question is concerned primarily with the nature of the serial manifold and its possible combinations, i.e., with the nature of *rational plurality*. In the context of the Transcendental Deduction, however, the problem is sharpened in the following way: The argument of the Transcenden-

quantification 'all ∀'), this is not the case. For the Deduction to succeed, it must be the case that the categories succeed in generating the serial order through which appearances are to be interpreted. It seems, therefore, that (a) the concept 'number' is necessary to the success of the deduction (b) it cannot derive from intuition (since it is supposed to subsume intuition, not derive from it) (c) it cannot derive from the categories themselves. This leaves us with an apparent lacuna: the concept of number as successive addition is intellectual in origin, but not grounded by the categories. How this can be a piece of "logic" (general or transcendental) is unclear. Cohen, however, just takes it to be the case that the concept of number (which he has already assumed is a priori, as the form of sensibility) is necessary to the derivation of the categories as intelligible "names" for functions of the mathematical continuum. While this is not a bad way to think of the categories, it puts (at least from Kant's perspective) the cart before the horse. For, Kant takes himself to be justifying the categories as pure concepts, whereas Cohen just assumes that the concept of order is fundamental (somehow) and uses it to ground the "functions of judgment." This is clearly not Kant's argument, but a novel — if misguided — interpretation of the Transcendental Analytic.

tal Deduction depends on the fact of experience, i.e., that there are empirical objects that are brought to apperception. Accordingly, the Transcendental Deduction introduces the problem of the conditions under which reality may be a predicate of an object of experience: what are the conditions under which we can claim that the plurality of the rational manifold correponds to a real (i.e., possible empirical) individual? This question is concerned primarily with the relation between the serial manifold and the analytic unity (the pure of "given" concept 'unity') of the empirical individual, i.e., with the problem of *rational unity in real plurality*.

In the Introduction we saw how the rational manifold is the paradigmatic object of post-Kantian Systematic Idealism. With the Transcendental Deduction, the problem of accounting for the empirical individual — or rational unity in real plurality — becomes the central problem for the unified logic of Systematic Idealism. This problem arises because of a tension between two claims: that (a) the central theoretical object of Systematic Idealism is the totality of the manifold, and that (b) the legitimacy of the categories depends on their applicability to an empirical individual. Accordingly, the problem for post-Kantian idealists is how to account — a priori — for the possibility of a rational unity within the rational manifold. Whereas the problem of reality concerns only the compossibility of the manifold (i.e., combinations of plurality), the problem of rational unity concerns the following difficult question: How do we understand the rational unity of the empirical individual if individuals are just quantifications of the rational plurality of the totality of space and time? That is, how can the parts of a rational whole also be thought of as real unities?

The question of the intelligibility of the empirical individual, however, is a problematic question from the point of view of Cohen's interpretation. As we saw in the **Chapter 2**, Cohen's interpretation of experience as propositions of natural science leaves the empirical object with a problematic, probably psychological status within the structure of the *Critique of Pure Reason*. As we have just seen, this interpretation of experience as scientific experience is necessary to the plausibility of Cohen's interpretation of the strategy of the Transcendental Deduction as the formal vindication of empirical concepts in mathematical structures (and indeed, their a

priori validation in the Analytic of Principles). However, this interpretation also suggests that the empirical individual has no significance from the point of view of the logical foundation of the transcendental doctrine. That is, the empirical unity of the individual is not a question of significance for a transcendental investigation whatsoever. Accordingly, the problem of the real individual does not even arise. Before we can assess the significance of this gap in the Cohenian account of the transcendental doctrine, however, we must establish the significance of the empirical individual for the justification of transcendental judgment in general.

3.1.3 The Empirical Individual

Now, the core argument of the Transcendental Deduction consists of simple entailments. If empirical individuals appear, then they appear in space and time. If empirical individuals are apperceived, then the form of space and time conforms to the constructive principles of the categories. If empirical objects are apperceived, then they conform to the synthetic unity of apperception. In order for the core argument of the Transcendental Deduction to satisfy the claims of a transcendental epistemology, however, some auxilliary premises are required. First, it must be the case that empirical individuals actually are given in experience. Without this premise, it may be the case that a transcendental judgment is necessary to the possible of a judgment of an empirical individual, but there simply are no empirical individuals, and therefore no transcendental judgments. Second, it must be the case that the representations of a synthetic judgment are object-oriented in a particular way. That is, if the actual intuition implied by a transcendental judgment does not refer to external reality, but only to the subject of representation, then no judgment of experience is implied but only a judgment of perception. Judgments of experience, therefore, imply a specific attitude towards the representations of intuition: Empirical Realism. However, this points to an important question for the Transcendental Deduction: what part of the empirical individual is a priori, and what part is given? Is it only the objectivity of the object a priori (i.e., its in principle possibility as an intelligible object) or is the factical unity of the empirical individual also a priori (i.e., its unity as an intuited object)? Considered another way,

we may ask, why do the parts of an intuition seem to cohere? Is the coherence of an appearance as an empirical individual an empirical fact or an a priori construction of transcendental judgment?

How we answer this question decides a great deal about how we understand the transcendental doctrine. If we think that the task of the transcendental doctrine is to explain only the intelligiblity of objects in general, not their a priori possibility as empirical individuals, then we may accept that objects may appear to be unified (or not) as a contingent fact about our intuitive representations. That is, the unity of the empirical individual is immanent to the representations of intuition, and is not a condition of the objectivity of judgments. This, indeed, is Cohen's interpretation of the fact of experience in *Kant's Theory of Experience*. The difficulty with this position becomes apparent when we consider the empirical realist attitude that is necessary to judgments of experience. Without this attitude, a transcendental judgment merely implies that some modification of the manifold is (a) intelligible through concepts and (b) constructible through concepts; that is, judgments of perception and judgments of experience are not distinguished without the realist attitude. However, with the realist attitude, the subject of a judgment (i.e., the intelligible object) is hypostasized as the subject of ascription of judgments of experience. That is, we have now hypostasized a (psychological) unity as objective. How could such an objective unity possibly be justified if it rests on a merely empirical feeling of unity?

But now, what the Transcendental Deduction (if it is successful) shows is only that the *manifold* of intuition must be intelligible as a unity. A fortiori, every modification of the manifold must be intelligible as a unity, since it is just a modification of the unity of the manifold. However, this does not explain how an object comes to be intuited as an individual rather than as a set of scattered points, but rather only requires that these scattered points appear in a single manifold whose form is compatible with the synthetic functions of judgment and apperception.[9]

[9]It may not at first be obvious what the problem of the empirical individual ultimately consists in. The claim is that objects are intuited as unities. For example, we do not intuit a plurality of space-time points arranged chairwise. Rather, we just intuit a chair, and it is only through cognition (apprehension, reproduction, recognition,

If we follow Cohen's interpretation of the fact of experience and deny that the unity of the empirical individual is relevant to the argument of the Transcendental Deduction, then we are conceding that intuitive unity (i.e., the phenomenological or psychological unity of the object) is not an a priori condition of the objectivity of judgments. Transcendental judgments are objectively valid because they show the a priori possibility of reconstructing any serial modification of the manifold through concepts, not because they show that these judgments are necessary to the possibility of their apperception. However, if this is so then the subjective constructions of transcendental judgment do not account for the empirical reality of the unity of an intuited object; rather, the coincidence of an empirical unity and an intelligible unity is contingent; it is explained by principles of empirical psychology: either through the spontaneous activity of the imagination or through passive receptivity to real individuals. That is, transcendental judgment provides a defence of the a priori possibility of the intelligibility of

---

cognition) that we come to know the chair as *also* a determinate modification of space-time, and therefore as falling under certain laws of intelligible objects. The question, however, is this: how does the chair come to be intuited as a chair, rather than, for example, as a legs-and-a-seat, or as a chair-table-cutlery appearance? That is, even if the imagination "reproduces" unconsciously the unity of the appearance of an object, how does this "blind faculty of the soul" pick out individuals in the manifold of sensibility, and, what is more, do so a priori?

The problem is that the individual appears to be a question of the *content* of our representations (and our judgments), not a question of its *form*. While the transcendental investigation can only concern itself with the question of form, the existence of a necessary content (the individual object of all possible experience) appears to be a presupposition of the argument of the Transcendental Deduction. The question therefore is: is there an a priori form of the content of experience?

In the B Deduction, Kant appears to concede that the empirical unity of the individual is not constructible a priori. Consider, for example, the following passage from § 26:

> The pure faculty of understanding does not suffice, however, to prescribe to the appearances through mere categories *a priori* laws beyond those on which rests a **nature in general**, as lawfulness of appearances in space and time. Particular laws, because they concern empirically given appearances, **cannot** be **completely derived** from the categories, although they all stand under them. Experience must be added in order to come to know particular laws **at all**; but about experience in general, and about what can be cognized as an object of experience, only those **a priori** laws offer instruction. (KRV; AA B165)

This returns us once more to the following suspicion: the unity of the empirical object is not explained, but only presupposed by the argument of the Transcendental Deduction. That is, it is not the unity of the object that becomes intelligible through the application of the categories, but only "nature in general."

continuous modifications of the manifold, but is not concerned with the nature or genesis of the unity of empirical individuals. Here, transcendental judgment confers objectivity on empirical judgments only in the sense that these judgments are (a) necessary to the intelligibility of a given object and (b) account for the intelligibility of all possible intuited unities, whatever these may be. This interpretation implies a "rational nihilism", or the concession that empirical individuals are not objectively intelligible. Rather, what is made intelligible through a transcendental judgment is the mere possibility of object-structured concepts. The judgment that these are indeed objective (i.e., that they are analogies of experience) is not defended.[10]

[10]Clearly, more is supposed in the manifold of space and time than first appears to be the case. In the Analogies of Experience, Kant appeals to the concept 'matter', as though this (at a minimum, mechanical) concept followed naturally from the combination of judgments of quantity and judgments of quality. However, nothing in the Deduction suggests that this is so. Neither is there anything in the Axioms of Intuition or the Anticipations of Perception to justify the claim that the continuum of intuitions is necessarily mechanical. Indeed, Kant even has trouble explaining how it is the case that the intuitive representations of a boat that appears to be floating down a river fall into the right order. If even the basic continuity of the manifold of space and time is at issue, what are we to think of the claim that its parts are dynamically related? No such claim follows from geometry as the science of magnitudes, nor even from kinematics (as the science grounded by time). For, nothing in kinematics prohibits two real motions from passing through each other. Only a judgment of substance makes this possible.

However, if *all* of the categories are necessary to the synthesis of the manifold, then either (a) it seems that intuition necessarily falls under mechanical laws (substance, causality), or (b) all of mathematics consist of judgments of community and necessity. If (b) is the case, however, then the dynamical categories play no role in the synthesis of the pure form of intuition (the continuum of order), but only in the synthesis of empirical intuition (the true concept of Nature). The problem now is that we should require two deductions. One to demonstrate the validity of mathematical categories as applied to the mathematical manifold of pure intuition (and kinematics, or analysis with two real variables), and one to demonstrate the validity of the dynamical categories as applied to the real manifold of empirical intuition. However, it's not clear how the latter could be possible at all, since we have no way — on the basis of appearances alone — of telling whether two points moving straight toward each other pass through each other (i.e., are mathematical and ideal) or rebound (i.e., are substantial and real).

The better strategy for Kant appears to be to abandon the claim that the dynamical claims are necessary to the synthesis of intuition. In this case, however, their legitimacy is not secured by ordinary experience, but only by the science of appearances, i.e., by natural science. There may nonetheless be a deduction of the dynamical categories available (as they appear to be combinations of the mathematical categories in any event), but this is an argument that Kant does not provide, and indeed cannot provide in the Deduction (at pains of making every judgment of

If, now, we think that the task of the transcendental doctrine is to explain not just the a priori possibility of intelligible objects, but also the a priori possibility of the rational unity of the empirical individual, then it must be the case that Kant can account for the empirical individual. There are two ways to account for the unity of the empirical individual a priori. First, Kant may be claiming that the truth of the argument of the Transcendental Deduction is conditioned by a factical empirical individual; that is, it is only the case that Kant's demonstration succeeds if it is actually the case that intuition is provided with empirical unities. In this case, the empirical unity of the object — and not just its empirical content — is given. Second, Kant may be claiming that the truth of the argument of the Transcendental Deduction is conditioned by apperception. That is, the initial entailment of the Transcendental Deduction is only satisfied by objects that are brought to apperception, since only objects that are brought to apperception have empirical unity. In this case, the empirical unity of the object is subjective in origin, and only its empirical content is given.[11]

---

natural science necessary to the possibility of intuition). Certainly, later systematic idealists take the view that the only way to secure the empirical individual is to exploit precisely this ambiguity in the Deduction and argue that mathematics itself is a kind of physics.

[11] It is difficult to say which of these alternatives Kant intends, or if his position on this point is even consistent (diachronically and synchronically). At times, Kant suggests that empirical individuals can appear without the spontaneity of the subject being involved. For example:

> objects can indeed appear to us without necessarily having to be related to functions of the understanding, and therefore without the understanding containing their *a priori* principles. (KRV; AA A89/B122)

Or also:

> Appearances would nonetheless offer objects to our intuition, for intuition by no means requires the functions of thinking. (KRV; AA A90/B123)

However, Kant also expresses an apparently opposite opinion, here from the "Clue":

> The same function that gives unity to the different representation **in a judgment** also gives unity to the mere synthesis of different representations **in an intuition**, which, expressed generally, is called the pure concept of the understanding. (KRV; AA A79/B104)

From the context, it is not entirely clear whether Kant means that the "mere synthesis" of different representations includes the "analytic" synthesis of appearances, or only the synthesis of a transcendental judgment. However, as we saw above, the concept of a combination of the manifold (and thus also the unity of the plurality of sensibility) is taken to imply the activity of the subject, at least in the context of the B deduction:

The first case gives rise to the question: how do we explain the factical unity of the empirical object? For, while the synthetic unity of apperception accounts for the intelligible unity of the apperceived object (through the functions of judgment), it does not also account for the empirical unity of the intuited object. Accordingly, the results of the *Critique of Pure Reason* must be taken to be problematic (i.e., hypothetical) in nature. That is, it is not merely the system of metaphysics (i.e., the hypostatis of the a priori) that is the product of a hypothetical judgment; rather, the structure of transcendental judgment as the justified application of categories to intuitions is problematic, and depends on the empirical (psychological or phenomenological) thesis that empirical individuals are indeed given.

This interpretation has some plausibility due to the fact that Kant is concerned with the problem of empirical (and moral) chaos in the *Critique of the Power of Judgment*. There, Kant attempts to establish precisely what we would expect from an empirical approach to the individual: that we can make *reflective* judgments of real individuals (as beautiful or rational). However, the concern of the *Critique of the Power of Judgment* is not to establish the *unity* of

> The **combination** (*conjunctio*) of a manifold in general can never come to us through the senses, and therefore cannot already be contained in the pure form of sensible intuition; for it is an act of the spontaneity of the power of representation, and, since one must call the latter understanding, in distinction from sensibility, all combination, whether we are conscious of it or not, whether it is a combination of the manifold of intuition or of serveral concepts, and in the first case either of sensible or non-sensible intuition, is an action of the understanding, which we would designate with the general title **synthesis** in order at the same time to draw attention to the fract that we represent nothing combined in the object without previously having combined it ourselves, and that among all representations **combination** is the only one that is not given through objects but can be executed only by the subject itself, since it is an act of its self-activity. (KRV; AA B129)

The result of this is well known: "The **I think** must **be able** to accompany all my representations." This, however, applies whether we are concerned with judgments, concepts, or merely intuitions: "That representation that can be given prior to all thinking is called **intuition**. Thus all manifold of intuition has a necessary relation to the **I think**" (KRV; AA B131). That is, even the (mere) intuition of the object has a necessary connection to the activity of the subject, for without this, it would not appear as a unity.

We cannot attempt to resolve the (many) interpretive problems of the Transcendental Deduction here. Our only task is to see at least some of the complications and possible answers in order to establish at least part of the range of options that are available to post-Kantian systematic idealists, including Cohen. However, it is worth noting that both positions may draw on a textual basis; however, the claim of the B Deduction asserts a strong point about the possibility of unity (empirical or transcendental), and therefore argues in favour of the idealist interpretation of the problem of the empirical individual, i.e., that the *genesis* of the empirical individual as a unity is subjective in origin.

the empirical individual, but rather its a priori intelligibility as (a) conceptually structured or (b) teleologically structured. That is, the arguments of the *Critique of Pure Reason* still depend on the fact that an empirical individual is given; it shows at most that the (free) activity of the imagination is compatible with the structure of determining judgment. While this modest interpretation of the project of the *Critique of Pure Reason* is compatible with a contemporary pragmatic skepticism, it is hard to reconcile with Kant's attempt to put metaphysics on the "sure path of a science."

The second case presents a different problem. For, it seems that if we take empirical unity to be conditioned by apperception, then as soon as we concede that there are objects the unity of which we are conscious, then we have conceded not only the series of entailments implied by the deduction, but also the subjective origin of the empirical unity of the object. But if this is the case, then it seems that the claims of the Transcendental Deduction are vacuous. For, it now seems to be the case that empirical individuals are merely products of the activity through which they are brought to apperception; the "experience" of the individual therefore merely amounts to its construction through subjective structures — notably through the unification of the categories in the synthetic unity of apperception. Indeed, even if we take the view that the empirical unity of the individual is a product of the activity of the imagination, this activity is only intelligible (according to the *Critique of the Power of Judgment*) insofar as it conforms a priori to the functions of judgment. That is, the unity of the individual is subjective in origin whether it is taken to originate in apperception, or whether it is taken to originate in the (a priori intelligible) activity of the imagination.

Although Kant frames part of the preliminary discussion of the Transcendental Deduction in terms of empirical objects (e.g.,), the argument itself (as, for example, in §17) does not depend on any claims about the unity of the empirical object or its origin. Rather, the problem of the Transcendental Deduction is to show that "the manifold in a given intuition also necessarily stands under categories" (KRV; AA B143), and this is shown by arguing for the necessary conformity of different types of representation to the a priori principles (i.e., forms) of (a) intuition

(b) the understanding and (c) the transcendental unity of apperception is necessary to the possibility of objects of cognition. Thus, the argument of the Transcendental Deduction requires that cognition consist of a variety of different types of representations (intuitions, concepts, apperceptive unity) and to each type of representation, a formal principle that is necessary to that type of representation (serial order, functions of judgment, unity of the functions of cognition). The claim of the Transcendental Deduction may then be understood as the (merely formal) claim that the principle of the empirical representation of the object (serial orders of adjacency and succession) can be constructed only through the logical forms provided by the functions of judgment (the categories), and the categories in turn can only be constructed through the fundamental function of unity, i.e., the formal expression of diversity in unity.

If we follow Cohen, then the Transcendental Deduction only aims to show that the categories provide a universal language for the expression of modifications of the manifold, modifications that are ultimately grounded in the combinatoric of a mathematics of pure order; these quantifications of the manifold are intelligible as unities through mathematics, but need not correlate to any kind of intuitive unity. In this case, "objectivity" consists solely in the universality of the language, i.e., that all possible quanitifications of the manifold may be expressed through the categories.[12] If, on the other hand, we think that the Transcendental Deduction aims to show that the categories provide a mechanism for making epistemically justified claims about the structure of reality (i.e., a metaphysics of the a priori), then there appears to be a gap in Kant's proof. For, the empirical unity to which objectivity is ascribed is either (a) contingent or (b) merely subjective in origin. Both cases yield a skeptical conclusion.

This dilemma raises an important question about the nature of the claim to objectivity

[12]Note that this will place restrictions on applied mathematics. While we may think of mathematics as constructing formal systems of many different types (many discrete, many with unusual topologies), if Cohen believes (a) that the categories are necessary to the possibility of objective representation and (b) categories are particular or particular types of mathematical functions, then it follows that only a limited subset of functions is actually applicable to the interpretation of Nature. Faced with non-Euclidean geometries and relativity, we have no choice but to concede that the "categories" (e.g., the particular function necessary to represent an object as a substance) is historically contingent. This, indeed, is precisely what Cohen does. For Kant, however, substance (as a pure concept) grounds mathematics (not the other way around). This puts Kant in a much more difficult position with respect to the changing modes of natural scientific representation. Indeed, one might argue that the categories were obsolete as soon as they were coined.

that is necessary to the broader claims of the Deduction. In order for empirical claims grounded in transcendental judgments to be objectively valid, there must be an underlying empirical unity to which the (metaphysical) claims of our conceptual constructions can be ascribed. This need not be the unity of a metaphysical monad (i.e., a intelligible form); however, it must be a unity that is (a) a priori intelligible (as provided for by the Schematism) and (b) a priori unified. The last claim — that empirical unities are unified a priori — is not provided by the argument of the Transcendental Deduction.

3.2 Transcendental Objectivity

3.2.1 Objectivity

The problem of empirical individuals has led us to the following question: what does the objectivity of the transcendental claim consist in? How we answer this question determines what we think the purpose of the Transcendental Deduction is, and accordingly how successful its argument will be.

First, we may think that Kant intends to ascribe objectivity to the unity of the object. If, on the one hand, the empirical unity of the object is given, then it is immanent to intuition, either as a product of the imagination or as a product of receptivity. In the latter case, the intelligibility of our claim to objectivity (e.g., that forces are exerted between bodies through the law of gravitation) then depends on hypostasized unities that have no a priori explanation. Accordingly, the claim to the a priori validity of judgments of experience depends on a metaphysics of unities (perhaps a monadology?), which can only be dogmatic in character. Let's call this interpretation *empirical objectivity*, because objectivity (i.e., the real subject of ascription) is empirical. If, on the other hand, the empirical unity of the object is a priori (a necessary, perhaps teleological function of the imagination), then transcendental logic implies a method for deriving or generating the unity of the individual from mere principles of construction. Kant's explicit discussion of transcendental logic (the Schematism and the System of Synthetic Principles) does not provide such a method, but it is conceivable that such a method could be developed. Let's call this interpretation *logical objectivity*, because the real subject of ascription is logical (or ideal), not empirical (or real). We may also think that Kant intends to ascribe objective claims to the unity of the manifold, not the unity of the empirical individual. If this is the case, then the claim to objectivity is not a claim about empirical particulars, but a claim about the nature of empirical representation, or appearance as such. Accordingly, the Transcendental Deduction does not deliver a claim about the objectivity of the object, but about objectivity reconceived in a way that is independent of empirical particulars. Let's call this interpretation *transcendental*

objectivity, because the real subject of ascription is transcendental (i.e., the unity of the manifold of intuition), and neither purely ideal (since it is synthetic) nor empirical (since it cannot be realized in any particular intuition).

Empirical Objectivity

The first case, *empirical objectivity* supposes that the object of intuition is given as a unity. Accordingly, the objective unity to which an intuition refers is mind-independent. While the pure concept of this objective unity is the transcendental object ($= X$); the pure "variable" tracks an objective object — the thing-in-itself — which is the ultimate object of reference. Accordingly, intuitions — whether they are brought to cognition or not — have an intrinsic objective reference in the unity of the appearance of the object.[13]

This view gains some support from the *problem of affinity*, which arises in the A Deduction. There, Kant appears to suggest that an a priori condition of the possibility the unification of the intuited object lies in the manifold of sensibility itself: "The ground of the possibility of the association of the manifold, insofar as it lies in the object, is called the **affinity** of the manifold" (KRV; AA A113). That is, while we do not directly sense unity, there is an aptness of fit between the constructive rules of the understanding and the relative disposition of the data of sense experience such that the reconstruction of the latter by the former is a priori possible, even if it did not in fact take place. Notice, for example, Kant's use of the subjunctive to indicate that the unity of the intuited object is not (necessarily) a product of the activity of the understanding:

> we say that we have cognized the object if we have effected synthetic unity in the manifold of intuition. But this is impossible if the intuition could not have been produced through a function of the synthesis in accordance with a rule that makes the reproduction of the manifold necessary *a priori* and a concept in which this manifold is united. (KRV; AA A105)

The use of the subjunctive appears to leave open the possibility that the intuited object is an objective unity, the unity of which is given in the affinity of the manifold, but cognized (i.e.,

[13] This, for instance, is Feder's position in *On Space and Causation* (Feder 1787).

actively synthesized) only insofar as the object is brought to cognition, or is represented through concepts.[14]

Although the problem of affinity is not reproduced in the B Deduction, Kant still at times appears to suggest that subjective unity (i.e., unity from synthesis) may be distinct from

[14]This would, however, be a contentious reading of Kant's intent in the A Deduction. For, at the conclusion of the Deduction, Kant summarizses the argument as follows:

> The objective unity of all (empirical) consciousness in one consciousness (of original apperception) is thus the necessary condition even of all possible perception, and the affinity of all appearances (near or remote) is a necessary consequence of a synthesis in the imagination that is grounded *a priori* on rules. (KRV; AA A123)

Here, Kant appears to be claiming that no intuition could even be *(ap)perceived* if it were not brought to the original unity of apperception. That is, the unity of the object in intuition (insofar as it is apperceived) arises from the original unity of apperception.

Part of the difficulty of interpreting Kant on this point arises from his fluid definition of 'cognition'. As we have already seen, Kant at time speaks of the cognition of (mere) intuitions, and at times speaks of cognition as though it were necessarily an activity of the understanding. Here, for example, Kant points to the ambiguity implicit in his use of the term:

> Synthesis in general is, as we shall subsequently see, the mere effect of the imagination, of a blind though indispensable function of the soul, without which we would have no cognition at all, but of which we are seldom even conscious. Yet to bring this synthesis **to concepts** is a function that pertains to the understanding, and by means of which it first provides *cognition in the proper sense*. (KRV; AA A78/B103, my italics)

Indeed, Kant goes further and suggests that the mere synthesis of the manifold alone does not yield cognition (in the proper sense):

> The first thing that must be given to us *a priori* for the cognition of all objects is the **manifold** of pure intuition; the **synthesis** of this manifold is the second thing, but it still does not yield cognition. (KRV; AA A78/B103)

Finally, Kant adds the following consideration on the nature of cognition in the B edition:

> We cannot **think** any object except through categories; we cannot **cognize** any object that is thought except through intuitions that correspond to those concepts. Now all our intuitions are sensible, and this cognition, so far as its object is given, is empirical. Empirical cognition, however, is experience. Consequently **no *a priori*** cognition is possible for us except solely of objects of possible experience. (KRV; AA B165)

Accordingly, cognition is "through" intuitions, but intuitions are not cognized unless this "through" is in fact a *correlation* of a conceptual construction and an intuition (i.e., an empirical analogy). Even if this ambiguity is less evident in the B Deduction, where Kant has made explicit the dependence of the intuition of the object on the synthetic unity of apperception (and not merely the "original" unity of apperception), Kant's use of the term 'cognition' in other parts of the text is unreliable.

objective unity (i.e., given unity):

> This is the aim of the copula **is** in them: to distinguish the objective unity of given representations from the subjective. For this word designates the relation of the representations to the original apperception and its **necessary unity**, even if the judgment itself is empirical, hence contingent. (KRV; AA B142)

Here Kant suggests that the copula is essentially a relation (judgment) of the representations (i.e., intuition of an object, or appearance) to "original appecerception and its necessary unity." Once more, this suggests a unity that is in some sense objective and not a mere product of the imagination, grounded in the same rules as the activity of the understanding. The claim, then, is that intuitions have objective unity prior to or independently of the synthesis of the imagination or a conceptual schematization. That is, intuitions are objective because they refer immediately to their causal antecedents, and these causal antecedents are objective unities, i.e., minimally, they produce configurations of sensibility that fall under the *a priori* rules of the affinity of the manifold. Indeed, Kant goes further and claims (here in §26), "the lawfulness of things in themselves would necessarily pertain to them even without an understanding that cognizes them" (KRV; AA B164). Once more, it appears that lawfulness (in perhaps a very robust sense indeed) is attributable a priori to either (a) the manifold or (still more problematically) (b) things in themselves.

Accordingly, we might think that Kant's project is bound to faculty dualism because only a psycho-dynamic account of cognition allows us to account for the metaphysical duality of the subject as both a passive subject (sensibility) and a spontaneous subject (intellect), that is, as an explanation of the two "sources" of experience (receptivity and spontaneity) and the peculiar way in which intuition tracks objective unities. Certainly, this interpretation was widely held by Kant's critics (Jacobi) successors (Fichte) and commentators (Herbart), and is still defended by some today. On this metaphysical interpretation, intuitions are objective because they are representations of objective (empirical) reality. That is, intuitions are, as configurations of sensible states, causally dependent on mind-independent antecedents. Intuitions therefore represent through the inference from effects (intuitions) to causes (empirical objects).

Concepts, on the other hand, are objective because they are representations of objective (transcendent) reality through their coordination with the empirical object represented by the intuition. That is, concepts are adequate to a transcendent object (the thing in itself), on which it is causally independent. We may note, in passing, that the causal theory of representation is beset with problems, particularly the problem of perceptual error.[15] However, these (antique) philosophical concerns are not the ones that should motivate us to reject this reading. For, the metaphysical interpretation implies precisely the kind of epistemological model that Kant was at pains to deny. Indeed, the project of the *Critique of Pure Reason* is not merely to replace the dualism of subject and object with the dualism of concept and intuition in an epistemology of adequation. Rather, the nature of Critical Philosophy undermines, as we have seen, the general apparatus of adequation and its logical and metaphysical apparatus.

Whatever textual or hermeneutic reasons may be offered to favour such an interpretation as Kant's authentic intention, are outweighed by the systematic consideration that the Critical Project must not replace one form of metaphysics (substance, adequation) with another (sensibility, reason), a requirement of which Kant himself was well aware. If the Critical Project is to have any philosophical importance, it is not merely as a heterodox metaphysical system, but rather as a revised epistemology.

Logical Objectivity

If we do not think that the unity of the intuited object is mind-independent, then we must provide an account of the subjective genesis of the unity of the object. Now, the synthetic unity of apperception, the categories and the schemata together provide the conceptual and constructive materials from which the unity of the object can be constructed.

[15] Causal theories of knowledge experienced a short-lived renaissance in the '80's, particularly through the work of Jerry Fodor (**?**), (**?**). However, modern accounts (even though now enriched with information theory) have encountered a number of insoluble problems. Foremost among these is a difficulty distinguishing between natural states and what philosophers of mind like to call 'intentional states', or those natural states that are "semantic" or carriers of information. Idealist theories, like that of Kant and Cohen, however, permit of extensionalist readings in which the content of a mental state supervenes upon — but is not casued by — natural phenomena. Consider, for example, Timothy Williamson's influential account in *Knowledge and its Limits* (**?**).

For example, in Fichte's systematic idealism, the problem of the unity of the object plays a central role. Following Jacobi (Jacobi 1789), Fichte rejects Kant's (purportedly) metaphysical account of the receptive subject and offers instead a form of idealism in which the transcendental subject alone is constitutive of both conceptual and empirical representations. Whereas Kant accepts that the content of intuition (sensibility) is given — at least with respect to the constructions of the understanding — Fichte does not distinguish between two "sources" of knowledge, and instead insists that all representation, whether empirical or conceptual, is to be understood as resulting from the simple activity of the subject. Accordingly, Fichte, unlike Kant, cannot accept that the appearance of empirical unity is to be explained through the (unconscious) activity of the imagination. Rather, he must be able to provide an account of how this unity comes about, since these facts themselves must be subjective in origin if they are to be explained at all. Accordingly, Fichte must provide a way of understanding how the empirical unity of appearance is possible a priori. His solution in the *System of Ethics* is to propose the doctrine of drives (*Triebe*), that is, a priori or perhaps innate functions of thought which constitute a priori the unity of (especially organic) unities. While it is difficult to see how Fichte's doctrine of drives could ever terminate in anything other than a degenerate subjective idealism, it serves as an important point of departure for Hegel's attempt to resolve the problem of "a priori" empirical unity.

Now, Hegel's *Phenomenology of Spirit* provides a close study of the problem of the unification of empirical appearance and its conceptual representation. However, it can serve as no more than a prolegomena to a system of absolute idealism. For, it does not provide a systematic explanation of the unity of appearances (or its conceptual representation), but rather starts from a factical standpoint where both objective and subjective unities are given. However, the project of the Doctrine of Being in the *Science of Logic* is precisely to explain how empirical unity is constituted by the very possibility of the conceptual representation of an object. Hegel's transcendental investigation of the concept of the object takes the form of a genetic account of the "concept" through its dialectical stages of progressive quantitative determination. While

this account is often thought of as an "organicist" account, Hegel's dialectical logic need not be understood as implying panpsychism or a metaphysics of substantial forms. Rather, Hegel's contention is that the construction of the concept of the object implies a complex succession of manipulations of the manifold of intuition; the appearance of organicism arises from the infinite complexity of these relational structures.[16] This manipulation of the manifold structures the material of sensibility such that it forms a qualitative unity. Accordingly, the (construction of the) concept of the object is constitutive of the appearance of empirical unity.

Whereas for Fichte, the unity of appearance emanates from the subject through a system of "drives", which are thereafter made intelligible, Hegel takes the construction of the concept of the object to be a dialectical interaction between concept and intuition (or quantity and quality). That is, the concept of the object structures the manifold of intuition, but succeeds in doing so only because the perceptual material (i.e., the manifold of being) is disposed to be so structured. In other words, the construction of the object is a determinate cognitive achievement, but one that is objective in the sense that (a) it is universal and commands necessary assent and (b) it guarantees the (empirical) objectivity of the object. Thus, for Hegel, the two criteria of objectivity fall together in a single dialectical logic: the *Objective Logic*.

Transcendental Objectivity

One of the difficulties associated with interpreting Kant's strategy in the Transcendental Deduction is, as we have seen, the ambivalence between appearance and appearances as such, or between intuition and pure intuition. The problem arises in two different ways.

First, it is difficult to determine whether the plurality of representations in the manifold

[16] That is, one may think of Hegel's concept of the continuum (ultimately, 'being') as a reciprocally determining whole — a kind of totalization of the Aristotelian *entelechy*. The (reciprocally affecting) determination of part of the continuum thus implies the affection of the whole. While, superficially, this may appear to import organicism and teleology at the very beginning of Hegel's *Logic*, I think it is more plausible to think that Hegel departs from a Kantian conception of the manifold as (in the Analogies of Perception) already a *dynamical* manifold. The plasticity of thought thus lends itself to functional structures of unlimited complexity (a possibility that Kant does not exclude; witness the Second Antinomy). The logic that Hegel has in mind is that of a particular kind of substance (or thought) that has a very narrowly conceived set of properties closely connected with a holistic conception of the continuum. To Hegel, however, this is not metaphysics (whatever we might think), but rather logic, as a configuration of necessary conceptual structures.

of intuition denotes (a) the plurality of spatial parts or points that constitute an individual form or (b) the sequence of individual (uninterpreted) "intuitions" that constitute the dynamic object (or the appearance as such). Now, Kant's argument in the A Deduction relies on the concept of time. For, (a) the plurality of an intuition has a necessary unity through the (instantaneous) time in which it appears, i.e. the unity of the time-point. Furthermore, (b) the unity of the sequence of individual forms in time is a necessary condition of the possibility of recognition, i.e., what Kant (in the A Deduction) calls the "original unity of apperception." If it is obvious why the unity of spatial parts in time is necessary, it is less obvious why it should be the case that modifications of space have unity through time. For, even if the time-series is ascribed to a single apperceptive origin (the subject), there is no guarantee that the sequence of intuited forms occurs in the same sequence as the time-series. The claim that appearances are unified in time is not an a priori claim about the nature of time, but about the nature of its contents. To justify this claim would require, as we have seen, either (a) a strong criterion of continuity (such as the one that Cohen attempts to provide; see **Section 5.2**) or (b) a strong criterion of the objective affinity of the manifold, which lends itself to the "empirical objectivity" reading considered above.

Second, in both the A and B versions, the core argument of the Transcendental Deduction claims that the a priori forms of intuition fall under the categories and the synthetic unity of apperception as a condition of bringing the intuited individual to apperception. The B Deduction adds the (crucial) claim that all intuited forms are products of synthesis, whether conscious (the understanding) or not (the imagination). This allows the B deduction to explain the necessity of unity in time; for, all intuitions — including those in time — fall under the unity of the categories just because the categories (as the pure concepts of synthesis in general) are necessary for the synthesis of the *manifold*. This still does not establish that intuitions are necessarily continuous, but it does show that they are a priori intelligible. However, to show that the "given" unity of the empirical magnitude is also a priori intelligible (i.e., that it is a bounded, continuous quantity) is more than the Transcendental Deduction (broadly construed)

seems to deliver. This, at least, is the judgment of Kant's most significant successors.

One way to respond to this uncertainty is to suppose that Kant's argument in the Transcendental Deduction need not vindicate the empirical object at all; if the unity of the empirical object is a merely intuitive occasion for judgment and not a condition of its objectivity, then the judgments that arise are ascribed not to the object itself, but rather to the principles through which the object is constructed as an object, or what Kant calls the general principles of the manifold or general laws of nature.[17] The claim, then, is that — despite the *Metaphysical Foundations of Natural Science*, and despite most of Kant's examples — Kant does not intend for us to ascribe properties to individual appereances, but rather to appearances as such. That is, all that Kant's epistemology will be able to provide for is knowledge of the general laws of the manifold. Empirical individuals, therefore, have merely psychological significance, and may be disregarded for the purposes of knowledge.

This is, of course, the interpretation implied by Cohen's reading of the *Critique of Pure Reason* in *Kant's Theory of Experience*. As we will soon see, it is by and large the same account that we will find in the *Logic of Pure Knowledge*.[18] This is an appealing interpretation for the following reason. First, it greatly simplifies the argument of the Transcendental Deduction, almost to the point of triviality. For, the Transcendental Deduction now only claims that since appearances must appear in space and time, and since space and time are synthetic, then all appearances are products of the understanding, even if unconsciously. Second, this

[17]For example, Kant defines 'nature' in the *Critique of Pure Reason* as follows:

> by "nature" taken substantively (*materialiter*) is understood the sum total of appearances insofar as these are in thoroughgoing connection through an inner principle of causality ... if one talks about the "things of nature," then one has in mind a subsisting whole." (KRV; AA A419/B446)

Or again, in the conclusion to the Analogies of Experience:

> There are therefore certain laws, and indeed *a priori*, which first make a nature possible; the empirical laws can only obtain and be found by means of experience, and indeed in accord with its original laws, in accordance with which experience itself first becomes possible. Our analogies therefore really exhibit the unity of nature in the combination of all appearances under certain exponents. (KRV; AA A216/B263)

[18]Cohen does provide a logical account of the individual — and its relation to intuition — in the *Logic of Pure Knowledge*; however, his conception of the individual is (deliberately) antinomic and unstable.

interpretation conforms to a reductionist-physicalism that denies the ontological significance of "medium dry goods" and grounds existence claims ultimately in claims about universal laws of nature. Accordingly, concepts such as 'substance' and 'cause' are replaced by concepts of 'force' and 'function'.[19] Furthermore, it lends some credibility to Cohen's interpretation of the general strategy of the Deduction.

The difficulty with this interpretation, however, arises when we consider the metaphysical implications that are supposed to follow from the *Critique of Pure Reason*. For, Kant does not offer the apparatus of the theory of cognition (i.e., the Transcendental Aesthetic and the Transcendental Analytic) in isolation. Rather, the project of the *Critique of Pure Reason* is to deliver a method whereby the metaphysical claims of reason become methodologically justified as hypothetical claims. The hypostasis of subjective structures (i.e., Empirical Realism) is warranted within the context of judgments of experience alone, but it is necessary to consider it part of the apparatus of Kant's Transcendental Philosophy if we want to explain one of the longest sections of the *Critique of Pure Reason*: the Dialectic.[20]

[19]The ultimate realization of this interpretation is perhaps Cassirer's *Substance and Function* (Cassirer 1910), which traces the demise of substance ontology and its replacement with systems of mathematical functions through the transformations in physics and mathematics from Galileo to Dedekind and Cantor. Although my understanding of the general epistemological problem of modernity is similar to Cassirer's, my reading of Kant is less indebted to Cohen than Cassirer's is, and accordingly, I see the challenges of post-Kantian idealism in a different light.

[20]We can distinguish between "transcendental arguments" and "transcendental logic" as follows. Transcendental logic, on the one hand, is a theory about relations between representations or the principles of their construction. This special relationship may be explained in a number of different ways, but has the following characteristics: (a) it allows us to raise claims to objective truth without committing to the truth of metaphysical claims underlying the conceptual interpretation of reality (e.g., substance ontology, causality), (b) it allows us to direct these claims to other cognitive agents as normative claims. The transcendental investigation of cognition, however, does not yield any metaphysical conclusions. These only arise from the method of critique (analysis, dialectic, method) and thus are more properly part of "critical philosophy".

On the other hand, transcendental arguments — as these are advanced by, for example, Strawson — claim that certain representations entail the existence of their objects. These arguments are typically regressions from some (usually idealist) epistemic claim to another (usually realist) metaphysical claim. The argument form differs from modus ponens only to the extent that it implies a theoretical commitment to additional premises that condition the possibility of the representation; the justification for metaphysical claims follows from these premises. Generally, Kant does not use transcendental arguments, since the transcendental — strictly speaking — only concerns relations between representations. However, there are some apparent exceptions. For example, Kant argues in the Refutation of Idealism that the form of time depends necessarily on a prior given: the persistent. From this, we

3.2.2 Critical Metaphysics

In the Preface to the B edition, Kant announces the project of the *Critique of Pure Reason* as, if not precisely a metaphysical project, at least a project necessary to the advancement of metaphysics: "criticism is the preparatory activity necessary for the advancement of metaphysics as a well-grounded science, which must necessarily be dogmatic" (KRV; AA Bxxxv). However, in the introduction to the *Critique of Pure Reason*, Kant claims,

> Transcendental philosophy is here the idea of a science, for which the critique of pure reason is to outline the entire plan architectonically, i.e., from principles, with a full guarantee for the completeness and certainy of all the components that comprise this edifice. It is the system of all principles of reason. That this critique is not itself already called transcendental philosophy rests solely on the fact that in order to be a complete system it would also have to contain an exhaustive analysis of all human cognition *a priori*. (KRV; AA A13/B27)

We can provide some insight into what Kant might have thought of the problem by considering the following passage from the Doctrine of Method:

> All pure *a priori* cognition, by means of the special faculty of cognition in which alone it can have its seat, constitutes a special unity, and metaphysics is that philosophy which is to present that cognition in this systematic unity. Its speculative part, to which this name has been especially appropriated, namely that which we call **metaphysics of nature** and which considers everything insofar as it **is** (not that which ought to be) on the basis of *a priori* concepts, is divided in the following way.
>
> Metaphysics in this narrower sense consists of **transcendental philosophy** and the **physiology** of pure reason. The former considers only the **understanding** and reason itself in a system of all concepts and principles that are related in general, without assuming objects that **would be given** (*Ontologia*); the latter considers **nature**, i.e., the sum total **of given objects** (whether they are given by the senses or, if one will, by another kind of intuition), and is therefore **physiology** (though only *rationalis*). (KRV; AA A845/B873)

Metaphysics, so Kant claims, is "that philosophy which is to present that cognition in this systematic unity." However, metaphysics may be divided into its properly "transcendental" component — which "considers only the understanding and reason itself" — and into its "dogmatic" component — which considers the totality of empirical objects. While the former is

may infer that something exists as a condition of the possibility of representing anything as "in time." Although any metaphysical interpretation of "the persistent" is contentious at best, the argument form does depend on prior (non-transcendental) commitments about the nature of representation.

exhausted by a set of *constructive principles*, the latter implies (a) the hypostasis of principles as hypothetical structures of intelligibility and (b) the substantiality of (i) Nature (the sum total of given objects) and (ii) the subject, as the origin of the spontaneous constructions of the understanding.

The question now is, what is the range of "transcendental philosophy"? Does it include transcendental logic (as a *canon* of judgment (as in mathematics) or as an *organon* (as in natural physics))? And if it does include transcendental logic, does this logic include the empirical unity to which our judgments are oriented — whether this unity is a priori or a posteriori? Furthermore, does "transcendental philosophy" (if indeed it is an *organon*, i.e., can anticipate objects of possible experience a priori) does it also include the Transcendental Dialectic and its solution — the determinacy of empirical representation (sensibility) — as a *discipline*, or is transcendental logic merely a *doctrine*? To summarize, a transcendental philosophy consists of (a) a system of principles for the construction of objects (a canon of judgment), and (b) the doctrine of the necessary and universal relations between different types of representations (a canon of understanding), through which (c) an empirical unity (the intuited object) is coordinated with a rational unity (the unity of the functions of judgment). The (a) system of principles are derived from the investigation of the a priori conditions of cognition: the Transcendental Aesthetic (and especially the Metaphysical Exposition), the Analytic of Concepts (and especially the Metaphysical Deduction) and the Analytic of Principles. The doctrine of (b) the relation between types of representations is the task of a transcendental logic, i.e., the Schematization of the table of judgments. However, the fact of (c) a coordination between an empirical and a rational unity is only presupposed by the method of the Deduction. Indeed, what the Deduction shows is merely this: "that objects of sensible intuition must accord with the formal conditions of sensibility that lie in the mind *a priori* is clear from the fact that otherwise they would not be objects for us" (KRV; AA A90/B123). But why believe that there are "objects" in the relevant sense? And what can we really infer from this fact?

3.2.3 Transcendental Philosophy

So far, I have argued that transcendental philosophy is essentially a claim about (a) a special type of relation between representations that demands (b) the coordination of an empirical and a rational unity. The characteristic position of post-Kantian idealism assimilates the two moments of the transcendental doctrine — Transcendental Logic and the Transcendental Unity of Apperception — into a single doctrine: a speculative or dialectical logic that (a) gives the conditions of objective representation and (b) includes the conditions of empirical and rational unity. This new logic — synthetic, speculative, or dialectical logic — forms the basis of the project of a systematic idealism that will complete the project of the Copernican revolution, but which will compensate for the weaknesses of the Kantian account.

Transcendental philosophy is commited to the following theses:

(a) There is a set of rational principles (space and time (or just number as the concept of order), the categories, transcendental judgment, synthetic unity of apperception) that are ultimately grounded in the synthetic activity of the understanding. This is the thesis of (epistemic) subjective idealism, or what Kant calls 'Transcendental Idealism'.

(b) Transcendental judgments (judgments of the a priori possibility of an intuitable unity) are logically and rationally necessary for judgments of experience.

(c) Intuited individuals (given unities) stand in a special relation to the totality of possible sense impressions (transcendental totality), which alone confers *objectivity* on empirical judgments. This thesis implies:

   (i) a plurality of types of subjective representations; this is the thesis that judgment is always in some sense a synthesis of representations (regardless of their origin), or the thesis of representational pluralism;

   (ii) an explanation of the unity of empirical representation with respect to transcendental totality; this is the thesis that transcendental judgments provide the warrant for the

> hypothetical hypostasis of a priori principles as *metaphysical* principles, or what Kant calls 'Empirical Realism'.[21]

My claim, then, is that in order to qualify as an instance of a transcendental philosophy, a philosophical project must endorse all of these theses.

If a philosophical project fails to endorse (a), then it will fail to qualify as an *idealist* project in the relevant sense. For, the systematic idealism of *klassische Deutsche Philosophie* is not concerned with metaphysical anti-realism, not even as it approaches subjective idealism (perhaps in Rheinhold, Fries or Fichte), but it does claim that all possible knowledge is grounded in subjective principles, not in empirical sensibility.

If a philosophical project fails to endorse (b), then it will fail to qualify as a *rationalist* project in the relevant sense. For, the Kantian proposal rests essentially on the claim that the content of experience is intelligible solely through the subjective principles claimed in (a), even if the Principle of Sufficient Reason does not warrant Transcendental Realism. So, for example, the Lockean empiricist may also endorse the claim (a) that all of our representations are grounded in subjective principles but nevertheless deny that they constitute knowledge in the relevant sense.

If a philosophical project fails to endorse (c), then it will fail to qualify as *objective* in the relevant sense, that is, as an Empirical Realism. For, the Kantian proposal rests essentially on the claim that our cognition of ordinary empirical unities is rationally justifiable. This justification, however, requires an a priori vindication of the unity of the empirical individual. If we deny that this is a possibility the result is either — as Maimon points out — empirical skepti-

[21] When we think of the Transcendental Deduction as providing an anti-skeptical conclusion, this is the thesis that provides for the legitimacy of realism: the a priori objectivity of judgment. However, this thesis is also the one that was most heavily revised by post-Kantian systematic idealists. For, Kant does not seem to provide a satisfactory account of what the relation between empirical and rational unity consists in. If we think that 'Transcendental Arguments' deliver an anti-skeptical payload, then it seems that this result will only be achieved if we are prepared to endorse this thesis, and therefore to find a way to provide an a priori account of singularity (and not just particularity!) with respect to a totality of a priori possibility. This the task to which Hegel's *Objective Logic* is dedicated, and we may also understand Maimon's logic (especially in the *Essay on a New Logic* (Maimon 1794)) as an attempt to realize this project. These attempts provide object lessons in the difficulty of providing such an account.

cism or rational dogmatism. If we deny the first part of the thesis (c.i), then we have effectively abandoned the principle of adequation, since there is now only a single form of representation that can be considered. As we have seen, however, Kant does not intend to reject the principle of adequation, but merely to reinterpret it from a transcendental point of view, i.e., as the formal adequacy of pure intuitions and pure concepts. If we deny the second part of the thesis (c.ii), then we have effectively abandoned the prospect of ascribing properties to parts of the manifold as intelligible unities. As we have seen, Kant requires this (either through the affinity of the manifold or through some other concept of the givenness of empirical unity), and post-Kantian idealists were sensitive to the requirement. Without this claim, the a priori principles of construction can only be ascribed objectivity in the sense that they provide for the universality of construction. This criterion, however, delivers only formal validity, not the stronger claim of objective validity raised by the Kantian proposal, and certainly not enough to provide a warrant for metaphysics construed as a "science."[22]

On the other hand, transcendental philosophy need not be committed to the following (metaphysical) theses:

(1) Transcendental judgment implies a system of innate knowledge structures.[23]

(2) The synthesis of judgment implies a psycho-dynamic account of the faculties.[24]

[22] Accordingly, for the Marburg School, the science of metaphyiscs becomes in some sense identical with the history of science, since metaphysical hypotheses are not independent of the theoretical framework of a particular empirical hypothesis. To say that metaphysics is just a part of the history of science (or any other domain of culture, for that matter), shows how closely the Quine-Duhem thesis is related to the Neo-Kantian project of the Marburg School.

[23] Kant accepts neither the Cartesian claim that conceptual structures (and perhaps even mathematics) are innate (despite Plato's *Meno*), and argues that they are "epigenetically" derived from functions of judgment, if they are acquired at all. Fichte's theory of "drives" appears to restore an element of innatism to the rationalist project that would have been unacceptable not just to Kant, but to the Leibniz-Wolff school as well. While Hegel may have absorbed the organic/teleological assumptions of the Fichtean account within the broader project of a logic of the "concept", Hegel, at least, makes an effort to avoid unnecessarily positing innate determinate concepts — ultimately the last resort of the dogmatic rationalist.

[24] Strictly speaking, transcendental logic need not even be concerned with the problem of temporal finitude. For,

> As pure logic it has no empirical principles, thus it draws nothing from psychology (as one has occasionally been persuaded), which therefore has no influence at all on the canon of the understanding. It is a proven doctrine, and everything in it must be completely *a priori*. (KRV; AA A54/B78)

Indeed, the problem of human cognition as a limitation for infinite construction only arises in the Antinomies, i.e., where the synthetic principles are employed as an organon in the "anticipation of experience;" it is a dialectical

(3) The objectivity of judgment implies a (hypothetical) metaphysical commitment to a common sense ontology.

Now, some or all of these theses may actually be a part of Kant's realization of the Critical Project, and they are certainly part of many variations on or interpretations of the project. We do not, however, need to decide the question here. Rather, it is sufficient to claim that these theses are not an essential part of the claim of a Transcendental Philosophy. This is because — as I have tried to show above — the central claims of the Transcendental Deduction (broadly construed) may be satisfied without having to decide the question (a) of the origin of the subjective principles of construction, (b) of the mechanisms of synthesis (i.e., they are "hardware-neutral"), or (c) of the status of the a priori within a Critical Metaphysics.

Under this proposed scheme, the philosophical projects of Maimon, Fichte, Fries, Schelling, Hegel and Husserl all qualify as instances of a transcendental philosophy.[25] However, Hermann Cohen's *System of Critical of Idealism* rejects both (c.i) and (c.ii), and thus cannot lay claim to being a transcendental philosophy in the sense that I have just outlined.

problem of the application of pure reason (and its critique), not a problem of transcendental logic as such. Nevertheless, the claim of the limitation of the human standpoint is decisive for (Kant's realization of) the problem of empirical individuals, which does play a crucial role in Kant's doctrine of the system of principles. Other forms of systematic idealism, however, attempt to reconstruct the empirical individual (its continuity and its boundaries) from within a purely a priori logic.

[25] We might also add to this list Heidegger, at least in *Being and Time*. There, the analysis of Dasein is, strictly speaking, a piece of transcendental philosophy.

3.3 Transcendental Synthesis

As we saw in the introduction, the transformation of the object of knowledge from the empirical particular to the manifold is contemporaneous with the philosophical elaboration of the problem of the mathematical continuum. Now, the classical problem of the mathematical continuum may be formulated as four philosophical paradoxes:

(a) How can a finite magnitude be constructed out of infinitely small magnitudes?

(b) How can an infinite magnitude be constructed out of finite magnitudes?

(c) How can there be a magnitude that is greater than all other magnitudes, and yet which is still a magnitude?

(d) How can there be a magnitude that is smaller than all other magnitudes, and yet which is still a magnitude?

The first two paradoxes we recognize from Aristotle's denial of actual infinities (see page 27): the additive infinity of number and the divisive infinity of magnitude. The third and fourth paradoxes we recognize from Aristotle's denial of possible infinities: the additive infinity of magnitude and the divisive infinity of number. But if Aristotle's position avoids these paradoxes for the purposes of considering individuals, its solutions are not adequate when we turn to the consideration of the totality of space and time, i.e., the manifold of reality. For, in considering the totality of space and time, we are attempting to quantify an infinite magnitude. That is, we seem already to have conceded that *infinite magnitudes* are at least conceivable.[26]

[26]To put it another way, that pesky universal quantifier must be interpreted as *unrestricted*, not just as limited by what we think of as the "smallest" number ('one') or the largest magnitude ('really, really big'). Then, however, our expression of the infinite ($\forall x \exists y, y > x$) and the infinitesimal ($\forall x > 0 \exists y > 0, y < x$) must be interpreted not as ranging across the domain of particulars, but across the domain of positive "number". Indeed, this is already the case for Aristotle, who merely claims that we must always think a particular number or magnitude (*datum*), but there is always another number that is greater or a magnitude that is smaller, which, however, is merely possible (*dabile*). Even in the *Elements*, however, Eudoxus claimed that magnitudes in space must have general numerical properties ("Magnitudes are said to **have a ratio** to one another which are capable, when multiplied, of exceding one another" (Elements, V.D4): $\forall x \exists y, x \cdot y > x$). As a general theory of magnitudes, the Euclidean-Eudoxian theory of ratios treats magnitudes not as elements of holistic constructions (polygons, circles, etc.), but as having

Furthermore, the concept of the algebraic variable implies a numerical unity that may become infinitely small. That is, we seem already to have conceded that *infinitesimal numbers* are at least conceivable, even if it is not the determinate object of thought: the mathematics of the continuum must be a calculus of the possible, not just the actual.

The philosophical problematic may thus be reformulated as two related questions about the nature of the relation between the finite and the infinite ($1 : \infty$) and the infinitely small and the finite ($0 : 1$):

(a) What is the determinate (or determinable) relation between the finite and the infinitely large?

(b) What is the determinate (or determinable) relation between the infinitely small and the finite?

In the context of the Kantian Schematism, sensibility may posess infinite determinacy, but these variations are always presented as finite magnitudes in intuition. ("**Principle of pure understanding**: All appearances are, as regards their intuition, **extensive magnitudes**" (KRV; AA A162).) Understanding, on the other hand, may be infinite in its compossibility, but its realization as an empirical representation is subordinate to the judgment of quantification, and thus to serial construction.[27] The serial construction of the manifold (the determination of the imagination through concepts) therefore requires a determinate rule of its construction (the series of its composition) as a condition of (a) its apprehension and (b) its determinate recognition.

determinate numerical properties (and thus determinate ratios) of their own. Already, the distinction between number and magnitude is suspended, and the idea of the continuum is implied in the indeterminate nature ($\forall\exists$) of the unrestricted quantification.

[27] Kant is quite explicit about the claim that magnitudes must be generated a priori:

> All appearance contains, as regards their form, an intuition in space and time, which grounds all of them *a priori*. They cannot be apprehended, therefore, i.e., taken up into empirical consciousness, except through the synthesis of the manifold through which the representation of a determinate space or time are generated, i.e., through the composition of that which is homogenous and the consciousness of the synthetic unity of this manifold (of the homogenous). (KRV; AA B203)

Indeed, the concept of an extensive magnitude is nothing other than this generation of the whole from the parts: "I call an extensive magnitude that in which the representation of the parts makes possible the representation of the whole (and therefore necessarily precedes the latter" (KRV; AA A162/B203).

Thus, the infinite (and also the infinitesimal) may be given in intuition. ("Space is represented as an infinite **given** magnitude" (KRV; AA B39).) However, the apprehension, reproduction, recognition and cognition of the infinite (or the infintesimal) depend on the synthetic activity of the imagination, and thus on its determination through concepts (or blindly, through principles, as reproductive imagination), and these must be determinate.[28] Accordingly, while the infinite (and the infinitesimal) is actual in (pure) intuition, it is merely possible from the point of view of the understanding, for only finitely determinate objects can ever come to cognition. More precisely, the pure form of intuition (space and time) and its matter (sensibility) may be infinite, but the apprehension of these in the intuition of particulars is always finite.[29]

The Antinomies of the Cosmological ideas arise from the mis-match between the categories and their schemata. That is, if we try to equate the unspecified category of quantity with the unlimited schema of number, we can convince ourselves that we have a concept of an infinite number or an infinite time.[30] Similarly, if we try to equate the unspecified category of quality with the indeterminate schema of reality, we can convince ourselves that we have a concept of the absolute simple. The Mathematical Antinomies, therefore, arise from the following misunderstanding: appearances always fall under the schemata, not under the

[28]Here again, following the strategy of the A Deduction and the headings of its four sections.

[29]Kant is normally interpreted as requiring an intuitive moment in construction; we cannot count very large numbers because it would take too long. I do not think that this is the correct interpretation of Kant however. Kant's limitation of intuition derives not from an intuitionist constraint on finite constructions, but rather from the requirement that every empirical cognition be the cognition of a *determinate* composition. Thus, like Aristotle, Kant thinks that we can only count particular numbers. We can think the infinite ($\forall x \exists y, y > x$) but we may not cognize it for want of any determinate rule of its composition. Whatever the case may be — whether we think Kant is an intuitionist or an Aristotelian — the limitation amounts to the same thing: only determinate particulars may be thought. (Although, if we take Kant to be compositing the real order of space and time, these particulars may be transfinite, since the real order has $> \aleph_0$ elements.)

[30]The category of totality arises from the merely intuitive cognition of the individual, i.e., when no predicate is applied to the subject. If the category of unity has the logical form, '$\forall x, x$ is a y', and if the category of plurality has the form '$\neg\forall x, x$ is a y', then the category of totality has the logical form, '$\forall x, x$ is not a y'. If the predicate of quantity just is (in the general case) a determinate extensive magnitude, then the cognition of the whole of the manifold just is the unrestricted universal quantification of the whole domain of intuition that is *not* limited by any particular magnitude ('is not a y'). But this, Kant wants to say, can never be brought to cognition not because we do not have an infinite intuition (recall, the totality of space is *given*) but rather because there is no corresponding concept of the determinate construction of the object of intuition. We have only an idea (an indeterminate concept) of the infinite ($\forall x \exists y, y > x$), but cannot correlate this indeterminate idea with intuition, even if our intuition is actually infinite (*datum*). Accordingly, the Aristotelian prohibition against actual infinites (and infinitesimals) is retained, but as a limitation of a determinate concept (and thus a limitation in the intellect), not as a defect of our sensible nature.

categories directly, since it is only through the schemata that determinate rules of construction can be provided. The solution to the Mathematical Antinomies (and the Dynamical besides) consists in understanding that the pure concepts can only be applied to appearances, that is to determinate particulars. Forthwith, the paradoxes of the Antinomies are resolved.

According to Kant, the categories of quantity and the categories of quality constitute the "mathematical" categories (KRV; AA B110), and the principles of their schematization are, accordingly, mathematical (KRV; AA A162/B201). Antinomies arise for pure reason with respect to each of these schematic principles. The category of quality gives rise to the antinomy of substance; that is, the problem of the reality of the components of matter: "reality in space, i.e., **matter**, is likewise something conditioned ..., whose absolute totality reason demands; and that cannot occur otherwise than through a complete division in which the reality of matter either dissappears into nothing or into that which is no longer matter, namely the simple" (KRV; AA A413/B440). *Prima facie*, Kant's claim is extremely puzzling. For, *substance* (and thus the simple conception of matter) appears to be the result of the first moment of a judgment of relation; indeed, reality and substance are (at most) related in the concept of matter only through the concept of actuality, i.e., the judgment of modality.[31] However, we can clarify this

[31] We note that in the Schematism, "the schema of substance is the persistence of the real in time" (KRV; AA A144/B183). However, in the *Metaphysical Foundations of Natural Science*, matter is the intelligible subject of natural-physical judgments. Accordingly, "phoronomy", which "considers motion as a pure *quantum* in accordance with its composition" (KRV; AA 4:477), is not predicated as a general property of the manifold as such (i.e., as laws of motion as such), but rather of matter itself. Motion is understood to be a predicate not of *laws*, but of *matter*: "*Matter* is the *movable* in space. That space which is itself movable is called material" (KRV; AA 4:480). Accordingly, dynamics "takes into consideration motion as belonging to the *quality* of matter, under the name of an original moving force, and is therefore called **dynamics**" (KRV; AA 4:477). Matter is, therefore, not a transcendental predicate (i.e., a modification of the predicate 'reality'), but rather the intelligible predicate of judgments of quality *and* judgments of quantity. In other words, 'matter' is a purely intelligible subject, which is therefore hypostasized as a condition of the empirical judgment of space as such. Or, to be more explicit, geometry is the transcendental science of an immaterial manifold; judgments of experience, on the other hand, are judgments of a material manifold.

confusion if we consider that the category of reality consists of an affirmative judgment. That is, it consists of the affirmation of a transcendental predicate (i.e., a modification of the manifold) of an empirical unity, i.e., "matter" or the intelligible subject of judgments of experience. The real question of the second antinomy is thus not, 'Is space (or time) infinitely divisible?', but rather, 'What is the unity of which (transcendental) reality is a predicate?'[32]

We can begin to make some sense of Kant's strategy in resolving the antinomy if we consider a distinction from the remark to the thesis of the Second Antinomy:

> Properly speaking, one should call space not a *compositum* but a *totum*, because its parts are possible only in the whole, and not the whole through its parts. In any case, it should be called a *compositum ideale* but not a *compositum reale*. (KRV; A438/B466)

The *totum* implies a qualitative totality, that is, a whole that is intelligible independently of its parts. The *compositum*, on the other hand, is a quantitative totality, that is, a whole that is intelligible only as the sum of its parts. However, the problem of the second antinomy (reality) does not rest on the distinction between the *totum* and the *compositum*, but rather on the distinction between the real and the ideal. Kant insists that only totality can be real (actual, empirical); the composite is at most ideal (possible, intelligible). But now, affirmative predications (of experience) have an empirical subject, not an ideal subject; that is, we predicate 'reality' of something given, not something constructed. Since composites can only be ideal, it follows that the subject of a judgment of reality *must* be a totality.

Thus, Kant asserts that the ascription of reality to the parts of an appearance is always a predication of determinate (discrete) magnitudes. Here is the proof of the resolution of the Second Antinomy:

> But now although this rule of progress to infinity applies without any doubt to the subdivision of an appearance as a mere filling of space [i.e., to pure intuition], it

We see, once again, the Aristotelian moment of Kantian physics: physical laws are not properties of the manifold of appearances, but of matter: the intelligible particulars of the manifold.

[32]Thus, the reductio of the proof of the thesis yields this absurdity: "nothing at all would be left over; consequently no substance would be given" (KRV; AA A435/B463). In other words, there would be no subject for the predicate. Kant's use of 'substance' here is misleading, and suggests that his use of 'matter' as the subject of empirical judgments is not as innocent as it seems.

> cannot hold if we want to stretch it to cover the mutliplicity of parts already detached with certainty in a given whole [i.e., to empirical intuition], constituting a *quantum discretum*. To assume that in every whole that is articulated into members (organized), every part is once again articulated, and that in such a way, by dismantling the parts to infinity, one always encounters new complex parts — in a word, to assume that the whole is articulated to infinity — this is something that cannot be thought at all, even though the parts of matter, reached by its decomposition to inifinity, could be articulated. For the infinity of the division of a given appearance in space is grounded solely on the fact that through this infinity merely its divisibility, i.e., a multiplicity of parts, which is in itself absolutely indeterminate, is given [i.e., its a priori form], but the parts themselves are given and determined only through this subdivision [i.e., actually constructed in cognition] — in short, on the fact that the whole is not in itself already divided up. (KRV; AA A526/B554)

In other words, Kant's solution to the Second Antinomy is to hold that our judgments of reality always apply to the discrete quantities of a determinate representation, since the intuition of an individual as a totality (*totum*, whole) always precedes its cognition. In affirming the reality of a magnitude, we are always affirming a *quantum discretum* (productive or reproductive composite), which is also a *totum reale* (empirical intuition). While the subdivision of each part of the determinate quantity is surely subdivisible (for it too lies in space and time), this *quantum continuum* is not part of the judgment of experience, but is merely a potential subdivision of discrete segments implied by their appearance in space and time.

Thus, the modal distinction between possibility and actuality is reproduced in the distinction between the *quantum continuum* (*totum reale*) of intuition and the *quantum discretum* (*compositum ideale*) of the concept.

> The infinite division indicates only the appearance as *quantum continuum*, and is inseparable from the filling of space [i.e., pure intuition, phoronomy]; for the ground of its infinite divisibility lies precisely in that. But as soon as something is assumed as a *quantum discretum*, the multiplicity of its units in it is determined; hence it is always equal to a number. (KRV; AA A527/B556)

Transcendental logic is therefore limited to the construction of determinate representations. Accordingly, it is the requirement that all recognitive compositions of the schema be determinate that reintroduces the Aristotelian moment (determinate particulars) into Kantian physics. While the pure manifold of intuition is given as an infinite totality, the cognition of space and time

is only possible through the apperception of empirical individuals, or appearances. Thus space — while it is a whole — is only *cognized* through its determinate parts. The whole is necessary (through the a priori principle of the construction of space) but only its parts are actual, and hence cognizeable.

However, the Mathematical Antinomies raise a number of questions in an entirely different register; and to these, it does not seem able to provide answers. For, the calculus has taught us that it is not just the case that a finite magnitude is infinitely divisible; it is also the case that this finite magnitude consists of an infinite number of parts; infinite series are thus implied by qualitative judgments. Moreover, Dedekind has taught us that numbers exist in a continuous manifold; the (serial) composition of even a finite real magnitude implies an infinity of relations of succession; continuity is therefore implied by quantitative judgments. Now, Kant seems to recognize that both aspects of the infinite continuum affect the principles of synthesis when, in the Anticipations of Perception, Kant notes that both space and time are *quanta continua*. That is, if the possibility of transcendental judgment relies on the (serial) synthesis of an ordered series, then that series is in fact *infinite* even if the magnitude that is to be synthesized is finite. However, in the Mathematical Antinomies, Kant has disregarded this general characterization of space and time as continua, and focuses on the condition of determinate composition.

Indeed, Kant's solution to both the First Antinomy and the Second Antinomy depends on the determinacy of construction; more specifically, on the construction of a finite magnitude as a composite consisting of a determinate number of parts: the *quantum discretum*. This finite construction serves as the basis for Kant's version of the *principle of infinity* and the *principle of continuity*:

The Principle of Infinity The *infinite* is given as a totality (*totum*) in the pure form of intuition, but can only be cognized determinately as a *quantum discretum*.

The Principle of Continuity The *continuum* is real (*reale*) in empirical intuition, but can only be cognized determinately as a *quantum discretum*.

Now, Kant's solution to the problems of the continuum exploits a modal distinction between what is given (*totum*, *reale*) and what can be cognized (*quantum discretum*).[33] The distinction is modal because whereas the objects of givenness are (at least in principle) infinitely determinate (infinite, the infinitesimal), that which can be thought is always finite, i.e., a *modification* of what is given.

However, Kant's solution to the Antinomies rests on the ambivalence of the concept of a discrete magnitude (*quantum discretum*). On the one hand, the discrete magnitude is thought of as a finite *number* as opposed to the additative infinite of number (cf. the principle of infinity). On the other hand, the discrete is thought of as a finite *interval* of the manifold as opposed to the divisive infinite (infinitesimal) of magnitude (cf. the principle of continuity). The Aristotelian heritage of this ambivalence should be easy to see. On the one hand, the *quantum discretum* is understood to be a discrete number: paradigmatically, an *integer*. Insofar as the discrete magnitude is thought of as *integer*, it is a *compositum*, i.e., as a construction from principles (counting). On the other hand, the *quantum discretum* is understood to be a discrete interval: paradigmatically, a *body*. Insofar as the discrete magnitude is thought of as *body*, it is *reale*, i.e., an intrinsic individual.

Kant hints at the problem in the remark to the thesis of the First Antinomy, cited above. There, Kant notes that in the construction of the ideal totality "we cannot appeal to boundaries which would of themselves constitute this totality in intuition" (KRV; A432/B460). That is, in *empirical* intuition, the individual is demarcated through *boundaries*, which are given. The a priori concept of the boundary, however, implies an infinite regress. Indeed, Kant alludes to the problem in a passage on the system of cosmological ideas:

[33] If successive addition (number) is what generates the order properties of space and time as topologies, then it seems that every construction is both continuous and discrete. Kant must be assuming that the synthesis of the imagination (apprehension) is distinct from that of recognition. The latter subdivides a given (but synthesized!) magnitude into a determinate number of parts in order to give a heuristic concept of the (actually infinite) composition of a determinate magnitude. Thus, while we must posess that determinate rule of the composition of any magnitude that is brought to apprehension (insofar as its synthesis is made possible by the imagination), in making conscious the rule of the composition of the magnitude, we are limited by our capacity to form determinate concepts of very large numbers of parts. The superiority of geometry (over arithmetic) in the construction of finite magnitudes is that the latter requires a determinate concept of composition, whereas the former relies on practical precepts (postulates) that make construction possible in the absence of determinate quantitative concepts.

> The synthesis of the manifold parts of space, through which we apprehend it, is nevertheless successive, and thus occurs in time and contains a series. And since in this series of aggregated spaces of a given space (e.g., the feet in a rod), the further spaces, starting with a given one, are each thought of as the **condition of the boundaries** of the previous ones, the **measurement** of a space is to be regarded as a synthesis of a series of conditions for a given conditioned; only the side of the condition is not in itself distinguished from the side lying beyond the conditioned, consequently *regressus* and *progressus* in space appear to be one and the same. (KRV; AA A412/B439)

The problem is that the pure form of space and time simply consists of relations of adjacency (or succession). The interpretation of the quantification of a serial manifold implies the existence of a boundary within a series of serial relations,[34] and a corresponding method for identifying these boundaries.

We can see this, for example, in Kant's definition of continuity in the Anticipations of Perception:

> The property of magnitudes on account of which no part of them is the smallest (no part is simple) is called their continuity. Space and time are *quanta continua*, because no part of them can be given except as enclosed between boundaries (points and instants), thus only in such a way that this part is again a space and time. Space therefore consists only of spaces, time of times. Points and instants are only boundaries, i.e., mere places of their limitation; but places always presuppose those intuitions that limit or determine them, and from mere places, as components that could be given prior to space or time, neither space nor time can be composed. Magnitudes of this sort can also be called **flowing**, since the synthesis (of the productive imagination) in ther generation is a progress in time. (KRV; AA A169/B211)

Here we note two important claims. First, boundaries (points or instants) enclose *quanta continua*. Second, boundaries (again, points or instants) "always presuppose those intuitions that limit or determine them." In other words, an intuition must be determinate — and must be bounded — *prior* to it being taken up in the synthesis of apprehension or the productive synthesis of the imagination. It is only in this way that a determinate intuition can be brought to apprehension as (a) a real individual bounded in space and time and (b) actual within the continuous manifold of space and time.

[34] The problem of constituting individuals in a continuum of serial relations yields a number of well-known paradoxes.

Accordingly, knowledge of particulars implies knowledge of the boundaries of particulars, without which they would not be intuited as individuals at all. Indeed, Kant suggests as much in a footnote to the proof of the thesis of the First Antinomy:

> We can intuit an indeterminate quantum as a whole, if it is enclosed within boundaries, without needing to reconstruct its totality through measurement, i.e., through the successive synthesis of its parts. For the boundaries already determine its completeness by cutting off anything further. (KRV; AA A426/B454)

It is easy to understand why Kant would say something of this nature. For, how would the imagination, as a mere blind faculty of the soul, every know where to begin and end its syntheses unless boundaries are in some sense already given? But then how are boundaries intuited? And how does make the a priori synthesis of a finite magnitude possible? And how is the a priori synthesis of a *boundary* possible, if it is possible at all?

3.3.1 The Paradoxes of Determinacy

Kant's solution to the mathematical antinomies makes explicit a characteristic feature of his transcendental logic: he has replaced the Aristotelian concept of the individual with the (mathematical) concept of the discrete magnitude (*quantum discretum*). The new scientific object is the manifold of space and time: a determinable totality of serial relations, or the infinite continuum. However, that which is determined within this manifold — that is, what is determined by the laws of appearance as such — consists of finite particulars with immanent boundaries. Transcendental logic mediates between the infinite continuum and the finite particular through finite construction, or schemata.

The solution to the mathematical antinomies rests therefore on distinguishing between the principle through which something is constructed, and its actualization:

> reason never needs an absolute totality in the series, because it is not presupposed as a condition as given (*datum*), but it is only added on as something conditioned, which is capable of being given (*dabile*), and this without end. (KRV; AA A512/B540)

Indeed, Kant argues that we need not be concerned about the fact that some constructions appear to be merely *dabile*, since what matters for the method of transcendental logic is merely how precise we wish to be, not how this construction is to be interpreted:

> In neither of these two cases, that of the *regressus in infinitum* as well as in that of the *in indefinitum*, is the series of conditions regarded as being given as infinite in the object. It is not things in themselves that are given, but only appearances, which, as conditions of one another, are given only in the regress itself. Thus the question is no longer how big this series of conditions is in itself — whether it is finite or infinite — for it is nothing in itself; rather, the question is how we are to institute the empirical regress and how far we are to continue it. (KRV; AA A514/B542)

Kant's strategy in the resolution of the Antinomies is deceptive. For, the antinomies arise not with respect to the empirical object, but with respect to the manifold of space and time and the possibility of a priori construction. However, in introducing the antinomies as a problem for empirical cognition — rather than merely for transcendental cognition — Kant can tacitly introduce the concept of the determinate magnitude as a mediating concept.

Indeed, Kant seems to be challenging us to deny that we perceive objects as totalities, and that we measure them by counting (for example) the inches or centimetres of a ruler. However, if this is so, then it is a misdirection. For, the problem of the antinomies arises not with respect to objects as such, but with respect to their a priori construction: i.e., arithmetic and geometry. So, for example, the problem of the Second Antinomy is not whether the object itself is monadically constituted or not, but whether an infinitely determinate spatio-temporal synthesis is possible at all. For (as Kant points out), the regress arises from the concept of space (so to speak) not from the concept of the object, which, as we know, is always the concept of a unity. Similarly, the concept of infinite time does not arise from the "world-concept", but rather from the concept of an infinite series (or, in Kant's example, a straight line that can be indefinitely extended).[35]

[35](KRV; AA A511/B539) In this respect, I agree with Hegel, who says this about the Second Antinomy:

> Furthermore, Kant did not grasp the antinomy in the concept itself, but rather in the already **concrete form** of cosmological determination. In order to get the pure antinomy, and to deal with it in its simple concepts, the determinations of thought must not be taken in their application and com-

However, Kant's confidence in the concept of the determinate individual is misplaced. It is not just that the (mathematical) concept of magnitude is inadequately defined; it is that Kant's resolution of the antinomies depends crucially on the ambivalence of the concept of a discrete magnitude. For the purpose of quantitative judgment — and *a fortiori* in the resolution of the First Antinomy — the concept of the discrete magnitude is thought of as a *composite*; it is the finite enumeration of the (a priori) relations of order: adjacency and succession.[36] For the purposes of qualitative judgment — and *a fortiori* in the resolution of the Second Antinomy — the concept of the discrete magnitude is thought of as a *totality*; it is the immanently bounded particular of appearance.[37] That is, the concept of the discrete magnitude is at once a composite

---

plication with the representation of the world, with space, with time, with matter, and so on, but rather without this concrete matter. (WL1; HW 21:221)

Accordingly, Hegel concludes that Kant's solution ("reason must not **go beyond sense perception**, and must take appearances as they are") ultimately "sets aside the content of the Antinomy itself, and does not arrive at the nature of the **concept** of their determinations" (WL; HW 21:233).

[36]Extensive magnitudes are, in the Axioms of Intuition, explicitly designated as composites:

> No one can define the concept of magnitude in general except by something like this: That it is the determination of a thing through which it can be thought *how many units are posited in it*. Only this how-many-times is grounded on successive repetition, thus on time and the synthesis (of the homogeneous) in it. (KRV; AA A242/B300)

However, Kant appears to have two different criteria for the construction of magnitudes, one based on continuous construction (drawing), the other based on the enumeration of order:

> Mathematics fulfills this requirement by means of the construction of the figure, which is an appearance present to the senses (even though it is brought about *a priori*). In the same science the concept of magnitude seeks its standing and sense in number, but seeks this in turn in the fingers, in the beads of an abacus, or in strokes and points that are placed before the eyes. (KRV; AA A240/B299)

But when Kant clarifies the method of construction in the Axioms of Intuition, he claims that all construction consists of successive composition:

> I cannot represent to myself any line no matter how small it may be, without drawing it in thought, i.e., successively generating all its parts from one point, and thereby first sketching this intuition. It is exactly the same with even the smallest time. I think therein only the successive progress from one moment to another, where through all parts of time and their addition a determinate magnitude of time is finally generated. Since the mere intuition of all appearances is either space or time, every appearance is an extensive magnitude, as it can only be cognized through successive synthesis (from part to part) in apprehension. All appearances are accordingly already intuited as aggregates (multitudes of antecedently given parts). (KRV; AA A163/B203)

[37]The previous citation continues to specify that not all kinds of magnitudes are perceived as composites:

> All appearances are accordingly already intuited as aggregates (multitudes of antecedently given

and a totality.

If the a priori synthesis of a possible object of appearance consists (as a condition of its possibility) of the synthesis of a finite composite, then following two skeptical concerns arise:

> parts), *which is not the case with every kind of magnitude, but rather only with those that are represented and apprehended by us as* ***extensive***. (KRV; AA A163/B203, my italics)

Mere numbers ('13') for example, are not perceived as extensive, but are still "magnitudes", loosely speaking. In order for a magnitude to be more than a mere representation of some successive composition, however, it must also have intensive magnitude; that is, it must be some determinate modification of sense (with an intensive magnitude > 0) if it is to be an object of sense (inner or outer) at all:

> The infinite division indicates only the appearance as *quantum continuum*, and is inseparable from the filling of space; for the ground of its infinite divisibility lies precisely in that. But as soon as something is assumed as a *quantum discretum*, the multiplicity of units in it is determined; hence it is always equal to a number. (KRV; AA A527/B556)

While the former indicates the *infinite determinability* of the appearance (as articulated in the continuity of space), the latter indicates the *finite determinacy* of the appearance as determinately enumerated. Accordingly, we can already see that judgments of quantity imply an actual (finite) construction (*compusitum, quantum discretum*), whereas judgments of quality imply merely possible (infinite) constructions (*totum, quantum continuum*). Indeed, in a note to the thesis of the First Antinomy, Kant insists that we can have intuitions of totalities:

> we can intuit an indeterminate quantum as a whole, if it is enclosed within boundaries, without needing to construct its totality through measurement, i.e., through the successive synthesis of its parts. (KRV; AA A426/B456)

And yet, Kant seems to insist that the distinction between *continuity* and *discretion* is a feature of the distinction between unities and aggregates:

> All appearances whatsoever are accordingly continuous magnitudes, either in their intuition, as extensive magnitudes, or in their mere perception (sensation and thus reality) as intensive ones. If the synthesis of the manifold of appearance is interrupted, then it is an aggregate of many appearances, and not really appearance as a quantum, which is not generated through the mere continuition of productive synthesis of a certain kind. (KRV; AA A170/B212)

And, as we have seen, unity is a condition of the cognition (and even apprehension) of appearances. Accordingly, *all* applications of the categories (including the enumeration of parts) must stand under the synthetic unity of apperception; accordingly, the synthesis of an appearance (including its quantitative synthesis) must be continuous. Indeed, this applies not only to "intensive" magnitudes, but also to number, insofar as the latter must be thought of as a unity if it is to be cognized at all:

> Since there must be a unity grounding every number, appearance as a unity is a quantum, and is as such always a continuum. (KRV; AA A171/B212)

(a) **Cognitive Failure** It appears to be the case that the a priori synthesis of infinitely determinate magnitudes is impossible, since every a priori synthesis consists of a finitely determinate number of parts. (Thus, as Maimon argues in the *Essay on Transcendental Philosophy* (Maimon 1790) (Maimon 2010), the Transcendental Deduction fails, since it is not generally the case that objects of intuition (some of which consist of irrational-valued magnitudes) can be synthesized a priori.)

(b) **Intuitive Knowledge** It appears to be the case that the component magnitudes of the (finite) composite are not synthesized cognitively. Rather, it must be the case that we have immediate (non-cognitive) knowledge of these magnitudes as determinate but unsynthesized quantifications of the manifold of the manifold of space and time. Thus, Kant must introduce an intuitive element (immediate knowledge of a quantized metric space) into the doctrine of judgment.[38]

However, if the a priori synthesis of a possible object of appearance consists (as a condition of its possibility) of the synthesis of a *quantum discretum* as a totality, then the following two skeptical concerns arise:

(a) **Cognitive Failure** It appears to be the case that every a priori synthesis of an object (or its parts) implies an infinite synthesis: the synthesis of a totality. (Thus, as Maimon argues in the *Essay on Transcendental Philosophy*, objects (or their parts) can only be thought at all if they are thought symbolically, that is, independently of the sensible condition and the "flowing" magnitudes of transcendental synthesis (KRV; AA A170/B211).)

[38] Actually, this is just Kant's conception of the axiomatic. That which is immediately certain in intuition, without the mediation of a third representation (i.e., a judgment) is axiomatic. Thus, we have a concept, but the concept is indeterminate and nevertheless immediately satisfied by intuition without the determination of inner sense. One might suppose that this rests on the "feeling of pleasure" of an aesthetic judgment. More plausibly — since this is only the *Critique of Pure Reason* — Kant thinks that one may be immediately aware, for example, of the identity of magnitude of two sides of a square, AB and BC without having a determinate concept of the magnitudes in question, i.e., our concept AB is generic, not determinate, and therefore cannot determine the productive synthesis of the imagination for the purposes of a theoretical judgment. Nonetheless, we are immediately certain of the truth of the general proposition in the absence of any theoretical judgment, but merely through some as-yet undisclosed epistemic procedure of intuitive assertability. In other words, Kant seems to be untroubled by this consequence. Intuitive knowledge is in fact the *paradigm* of knowledge: it is the method of geometry!

(b) **Intuitive Knowledge** It appears to be the case that we have intuitive knowledge of the boundaries of a totality as a condition of knowledge of particulars (KRV; A426/B454). However, the synthesis of a boundary implies an infinite regress, and therefore our knowledge of particulars rests on a given (boundaries) that has no a priori reconstruction.[39]

However, it is not just the concept of the discrete magnitude that is problematic. For, the concept of the empirical correlate of a discrete individual is also not well-defined in the context of the *Critique of Pure Reason*. That is, the concept of the given serves two distinct functions in Kant's transcendental doctrine. For the purposes of quantitative judgment — and *a fortiori* in the resolution of the First Antinomy — the manifold of space and time is thought of as *ideal*; it is the merely possible, indeterminate enumeration of the (a priori) relations of orders of a composite of adjacency and succession.[40] For the purposes of qualitative judgment

[39]Kant introduces the concept of extensions as defined through their boundaries in the Axioms of Intuition:

> The property of magnitudes on account of which no part of them is the smallest (no part is simple) is called their continuity. Space and time are *quanta continua*, because no part of them can be given except as enclosed between boundaries (points and instants). (KRV; AA A169/B211)

However, the Second Antinomy arises from the demand that this boundary have a *ground*:

> since in this series of aggregated spaces, starting with a given space (e.g., the feet in a rod), the further spaces, starting with a given one, are each thought of as the **condition of the boundaries** of the previous ones, the **measurement** of a space is to be regarded as a synthesis of a series of conditions for a given conditioned. (KRV; AA A412/B439)

Knowledge of particulars, however, is possible: "we can intuit an indeterminate quantum as a whole, if it is enclosed within boundaries" (KRV; A426/B454). And yet, as Kant insists in the resolution of the First Antinomy, "there can be encountered **no experience of an absolute boundary**, and hence no experience of a condition as one that is **absolutely unconditioned empirically**" (KRV; AA A517/B545).

[40]In the Transcendental Aesthetic, both space and time are identified as, respectively, 'infinite' ("space is represented as a given infinite magnitude" (KRV; AA A25/B41)), and 'unlimited' ("the original representation **time** must therefore be given as unlimited" (KRV; AA A32/B47)). What is given, therefore, is a totality that precedes all of its modifications. This is true for space:

> parts cannot as it were precede the single all-encompassing space as its components (from which its composition would be possible) but rather are only thought in it. It is essentially single; the manifold in it, thus also the general concept of spaces in general, rests merely on limitations. (KRV; AA A24/B39)

And it is also true for time: "the infinitude of time signifies nothing more than that every determinate magnitude of time is only possible through limitations of a single time grounding it" (KRV; AA A32/B47). The solution to the Second Antinomy, however, makes clear that the manifold of an intuition is not an *explicit* manifold:

— and *a fortiori* in the resolution of the Second Antinomy — the manifold of space and time is thought of as *real*; it is the continuity of a particular of appearance, a *quantum continuum*.[41] That is, the concept of the given is both ideal *and* real.

> Though all the parts are contained in the intuition of the whole, the **whole division** is **not** contained in it; this division consists solely in the progressive decomposition, or in the regress itself, which first makes the series actual. (KRV; AA A524/B552)

In other words, the infinite subdivision of space — the multiplicity of its parts — is thought in space and time, but is not made explicit until the determinate enumeration (necessarily finite, at least for us humans) is realized:

> The infinity of the division of a given appearance in space is grounded solely on the fact that through this infinity merely its divisibility, i.e., a multiplicity of parts, which is in itself absolutely indeterminate, is given, but the parts themselves are given and determined only through the subdivision — in short, on the fact that the whole is not itself already divided up. (KRV; AA A526/B554)

Thus, as we have seen, "one should call space not a *compositum* but a *totum*" (KRV; AA A438/B466).

[41] Now, we might think that appearances are given as bounded individuals; it is only insofar as they are subordinated to the concept of number that they must be synthesized. Accordingly, we could have a qualitative concept (i.e., judgment of the reality of) an object whose composition is not (or is not fully) fathomable to us. The (as Kant says) "appearance as *quantum continuum*" or the "filling of space" may be distinct from the conceptual construction (cognition) of the object as a *quantum discretum*, in which "the multiplicity of units ... is determined" (KRV; AA A527/B555). However, in an addition to the B edition, Kant explicitly rules out this possibility:

> The perception of an object, as appearance, is possible only through the same synthetic unity of the manifold of given sensible intuition through which the unity of the composition of the homogeneous manifold is thought in the concept of a **magnitude**, i.e., the appearances are all magnitudes and indeed **extensive magnitudes**, since as intuitions in space or time they must be represented through the same synthesis as that through which space and time are determined. (KRV; AA B203)

That is, if appearances are to be cognized at all, then they must be synthesized by the productive imagination; otherwise, they fail to be anything more than "mere perceptions", of which we may not even be consciously aware, and which are certainly not objects of cognition (or consciousness).

In an addition to the B edition, Kant specifies — and this is the passage that confused Fischer (see page 45) — that concepts are organized according to the conceptual containment of marks (i.e., empirical concepts, and thus composites), whereas spaces are contained within the whole as modifications of something that can be thought in its totality:

> Now one must, to be sure, think of every concept as a representation that is contained in an infinite set of different possible representations (as their common mark), which thus contains these **under itself**; but no concept, as such, can be thought as if it contained an infinite set of representations **within itself**. Nevertheless space is so thought (for all the parts of space, even to infinity, are simultaneous). Therefore the original representation of space is an ***a priori* intuition**, not a **concept**. (KRV; AA B39)

If the manifold of space and time is ideal (i.e., only given in thought), then the following two concerns arise:

(a) **Rational Failure** It appears to be the case that space and time have no a priori form (i.e., adjacency and succession), since the form of serial relation implies the real condition (difference), without which it is meaningless (i.e., without content). Accordingly, the forms of space and time are not a priori at all, but are contingent on their realizability in a manifold of real differences.[42]

(b) **Rational Knowledge** It appears to be the case that the continuity of space and time is a property of every space. However, this means that the regression of parts — and thus the articulation of the totality as a *compositum* — is ideal. Therefore, through every part of space considered as a totality, we must also know the totality of its rational manifold (i.e., we must think its infinite articulation).

If, however, the manifold of space and time is real (i.e., only given empirically), then the following two concerns arise:

(a) **Rational Failure** It appears to be the case that space and time do not form a *totum* at all, but rather a *compositum reale*. There is therefore no idea of space and time as ideal totalities (since this would imply an infinite concept), and thus no rational object (intelligible subject) of the a priori synthesis of the manifold.

(b) **Rational Knowledge** It appears to be the case that the continuity of space and time are given as a real totality (*totum reale*) in the empirical individual. That is, we have rational knowledge of the real differences of the manifold.

Indeed, the ambivalence of the concept of the manifold of intuition can be seen in the *principle of infinity* and the *principle of continuity*, as these are formulated above. For, the principle

[42] In other words, without a possible real correlate in intuition, every synthetic concept is just a *Hirngespinnst*. The reason why we can treat extensive magnitudes objectively (even if they are a priori) is precisely because they could be given; that is, they could transcend the unity of the subject.

of infinity applies to intuition *a priori*, i.e., to the pure form of space and time, or space as a "given" totality. However, the principle of continuity applies only to empirical intuition, i.e., to an intuition that correlates with a modification of sensibility.

The reason for this is that the problem of additative infinity arises only with respect to quantitative judgments of composites, i.e., those which involve the concept of number, and concern only the enumeration of discrete elements in the composition of wholes from parts. And, as we have seen, the a priori form of intuition — as adjacency or sucession — consists of relations of serial order. The problem of the divisive infinity, on the other hand, arises with respect to qualitative judgments of totalities, and thus concerns the regressive division from wholes to parts. And, as we have seen, the a priori form of intuition — insofar as it can be thought *a priori* — consists of a totality that is subject to limitations; it is an (empirical) *quantum continuum* that is subject to limitations through (empirical) boundaries.

The preceding remarks are intended to show the inadequacy of Kant's conception of the infinite. Kant's (quasi-Aristotelian) conception of the infinite is not only inadequate to the requirements of a philosophical foundation of the calculus, but is also inadequate to his own project of explaining transcendental synthesis. However the two-sided conception of the infinite as possible (or even necessary) in idea and impossible (thus non-actual) as appearance is constitutive of the doctrine of transcendental logic, at least as Kant construes this. If we take seriously the claim that the determinacy of the concept is an essential part of transcendental logic, then it appears that transcendental logic is inadequate to the task not only of the reconstruction of individuals, but *a fortiori* to the task of adequately expressing laws of appearances as such. For, Kant's concept of a priori judgment implies not only the reconstruction of intelligible individuals, but is *limited* to the reconstruction of determinate individuals.[43] For example, the law of gravitation implies an infinite gravitational field radiating from a point-mass. However, this gravitational field is not a candidate for objectivity, for it is not a candidate for a determinate synthesis of the manifold. But if Newton's signature achievement cannot be adequately

[43] Indeed, if the regress of the *compositum* arises for individuals, then even individuals cannot be finitely constructed a priori.

defended, what hope is there for the general project of the *Critique of Pure Reason*?

Part II

Systematic Idealism

Chapter 4

The Logic of Reality

4.1 Logic of Reality

4.1.1 Maimonian Skepticism

As we have seen, the negative aspect of the *Critique of Pure Reason* consists of the perfection of Humean skepticism such that claims to truth grounded in the principle of adequation are no longer possible (see page 15). Insofar as we take the measure of truth to be the correspondence between the form of subjective representation and the form of objective being, the project of human knowledge is degenerate. The positive aspect of the *Critique of Pure Reason* is more problematic. For, Kant's rejection of the traditional epistemology is based on his view that the traditional logic of the syllogism and the formal structures of mathematics are fundamentally distinct. However, Kant does not simply reject traditional logic or its corresponding ontology. Rather, he takes the language of traditional metaphysics — and also the logic of the syllogism — to be integral to the expression of natural-scientific knowledge: paradigmatically, the laws of Newtonian mechanics. As a result, the Critical Project must consist of the rehabilitation of the language and logic of the tradition, but such that these elements of rational cognition are conditioned by, if not subordinate to, the form of empirical cognition: the serially ordered continuum of sense experience. Justifying the methodological limitation of rationalism (philosophy, logic) to the natural manifold (physics, mathematics) is the task of the Doctrine of Method, which arises out of the resolution to the Dialectic of Pure Reason. However, the demonstration that such an equivalence is indeed possible is the task of the Schematism and the 'transcendental logic' that it implies.

However, Kant's successors were not convinced by the claims of the Schematism. Instead, Kant's idealist successors argue that the a priori concept of the manifold and the a posteriori manifold of an appearance cannot be distinct in the way that Kant intends. The most important critic of Kant's transcendental logic is Salomon Maimon, and especially his skeptical attack on the possibility of synthetic judgment outlined in his *Essay on Transcendental*

Philosophy (1790).[1] Now, the Kantian theory of synthetic consciousness claims that figures presented in intuition can be "recognized" through the schematism of the pure concepts of the understanding and the application of the principles of construction (and, following the A Deduction, they must first be "apprehended", "reproduced", and "cognized"). This requires that real figures presented in intuition — geometrical figures in space and arithmetic figures in time — may be constructed a priori, and, through the comparison of empirical intuitions (productive imagination) and a priori intuitions (reproductive imagination), a determining judgment can be made, that is, that an a priori intuition is adequate to an empirical intuition.[2] For Maimon, this comparison is the answer to the *quaestio quid juris*: "can these symbolic concepts also be made intuitive, and thereby acquire objective reality, or not" (VTP; MW 2:48)?

Prima facie, this is just a misinterpretation of the task of the Transcendental Deduction; for, as we have seen, the Transcendental Deduction need only show that it is logically possible for the form of space and time to fall under the categories. However, Maimon understands the scope of the possibility defended by the Transcendental Deduction more narrowly: is it possible for *all* forms of actual intuitions to fall under the synthetic representations of (or the schematic actualization of) a priori concepts? That is, is transcendental judgment possible if the constructions of transcendental logic are limited to determinate finite constructions? As we have seen, the Transcendental Deduction — and thus the defense of the *quaestio quid juris* defends only the claim that schematic interpretation is possible if apprehension is possible; in principle, all synthetic structures must fall under the categories and the synthetic unity of apperception. But this does not mean that any structures actually do so. Maimon's strategy in the *Essay on Tran-*

[1] By now, Maimon's place in the development of post-Kantian idealism is well known. See, in particular, Yitzhak Melamed's "Salomon Maimon and the Rise of Spinozism in German Idealism." (Melamed 2004), Daniel Breazeale's "Der Satz der Bestimmbarkeit: Fichte's Appropriation and Transformation of Maimon's Principle of Synthetic Thinking" (Breazeale 2002) and Peter Thielke's "Getting Maimon's Goad: Discursivity, Skepticism, and Fichte's Idealism" (Thielke 2001). Although Fichte (and Schelling) no doubt play an important role in any full account of Systematic Idealism, our purpose here is to focus narrowly on the mathematical problems arising from Kant's transcendental logic and the Antinomies. Thus our focus will be on Maimon and Hegel, whose work focused especially on the problems that I raised at the end of the last chapter.

[2] The *locus classicus* for Kant's explanation of the faculties and their relations is the Introduction to the *Critique of the Power of Judgment* (Kant 2000). I will assume that the views expounded there are also applicable to the project of the *Critique of Pure Reason*.

scendental Philosophy is to provide counter-examples in order to show that cognition is not possible in all cases. Thus, even if we concede that the Kantian model succeeds in explaining some cases of cognition, the Deduction does not provide a *general solution* to the problem of cognition. As we saw in the last chapter, the generality of the solution depends on the System of Principles, which shows that any a priori empirical concept (i.e., concept of a modification of the manifold) can be realized in the manifold of intuition. Thus, Maimon's objections may be taken to imply the failure of the Schematism to secure the universality of transcendental judgment. However, Kant's insistence that cognition implies determinate composition (i.e., through the productive imagination) to finite constructions — a limitation that is essential to the resolution of the Antinomies — appears to make the *application* of transcendental logic impossible in at least some cases.

On the one hand, intuition provides us with a "presentation" (*Darstellung*) of an infinitely determinate form, i.e., with a manifold of sensibility that is (in principle) infinitely determinate. The understanding, however, is finite in the determinations that it can provide. That is, in order for the "rules" of the understanding to be applied, they must be realized through intuition through a determinate constructive procedure. As Maimon understands this process, however, it is subject to the condition of sensibility: time. Accordingly, the productive imagination — or the presentation of concepts of the understanding — cannot, like the images of intuition, be taken to be "given" synchronically, but must rather be thought of as a *producing* of the image *diachronically* through an intelligible rule, as a representation (*Vorstellung*).[3] That is, as Maimon understands the problem of the composition of extensive magnitudes, "the understanding cannot think an object as having already arisen, but only as arising, i.e., as flowing" (VTP; MW 2:33, cf. AA A169/B211). Thus, according to Maimon, there is a general inade-

[3]Maimon distinguishes between the complete presentation (*Darstellung*) of a concept and our form of finite representation (*Vorstellung*), a distinction that, roughly speaking, tracks the distinction between the reproductive and the productive imagination. This terminology, however, is Maimon's, not Kant's. The question may arise: is this an accurate reading of Kant? I now think that the answer is 'no' — although I admit that I have only very recently changed my mind. Nevertheless, the immediate question is this: how did Maimon understand Kant? Maimon's view, however, does not differ substantially from many current accounts of Kant's philosophy of mathematics (e.g., (Parsons 1984), (Friedman 1990)).

quation between the intuitive proof model (the intuitive certainy of geometry) and the analytic proof model (the general proofs of symbolic algebra and arithmetic) in at least some cases of synthetic cognition. The upshot of this is that synthetic cognition is not possible in all cases where intuitive cognition — and at least some kind of knowledge — seems to be available.

A Mathematical Antinomy: Irrational Roots

Maimon's simplest example arises from a contrast between two different types of roots:

> a root of 2 has a meaning (a number, which, when multiplied by itself, produces the number 2) and is therefore *formaliter* possible. It is also *materialiter* impossible, since here no object (determinate number) can be given. (VTP; MW 6:58)

The number $\sqrt{2}$ has a perfectly intelligible meaning: it is a real number, represented by the symbol $\sqrt{2}$, with a determinate meaning and an important role in the analysis of geometric figures. Moreover, there are perfectly intelligible *operations* for expanding the irrational root such that it has an approximate numerical value. However, this method consists of an infinite approximation. While $\sqrt{2}$ may be a perfectly useful real number, there is no finite algorithm that leads to its determinate arithmetical production, neither is there a convergent series that allows us to estimate its final value.

Does this, now, mean that the number $\sqrt{2}$ cannot be present in intuition at all? Certainly not, since, its is implied by the ratio of the diagonal of a square to one of its sides every time we perceive a square. Moreover, if counting takes place in continuous time (i.e., if counting consists of real magnitudes), then counting from 1 to 2 implies a transition through every "number" in between, including $\sqrt{2}$. Indeed, Kant even uses a claim of this nature to defend our (intuitive) knowledge of the existence of irrational numbers.[4]

[4]That is, the intuition presents us with an image whose magnitudes are exactly determined. The matter of intuition — if not its form — is given immediately, since it correponds to changes in the state of the underlying subject. Thus, irrational-valued relations between real magnitudes can appear in intuition. Kant, in fact, addressed precisely this problem in reply to a query from August Wilhelm Rehberg on our ability to think $\sqrt{2}$ "in numbers". Kant's response is illuminating:

> If we did not have concepts of space then the quantity $\sqrt{2}$ would have no meaning for us, for one

Maimon's worry, rather, is that, as a concept, the number $\sqrt{2}$ could never be actively presented (i.e., presented in productive imagination) through any finite operation. For, the determinate cognition of a number is possible through "the faculty of intuition," which, however, "certainly **conforms to** rules but ... does not **understand rules**" (VTP; MW 6:34-35). Thus, whereas the image of the diagonal of a square is given *immediately* (that is, without any temporal magnitude and outside of conceptual mediation), the concept of that same image — i.e., the line that it is constructed through — may only be thought as a *method for the construction of lines*, or, what is the same, as a temporally mediated construction of the imagination.[5]

However, in the case of irrational roots, the reconstruction of the value of the root is an infinite construction. Indeed, "the series expressing an irrational root must never come to an end, because if it could, the condition would not be satisfied" (VTP; MW 2:77). On the one hand, we have a perfectly useful mathematical symbol: $\sqrt{2}$. On the other hand, we must concede that we have no way of diachronically presenting this very number in intuition. If this is so, then there is no general schema for $\sqrt{2}$, nor any of its multiples. For, our finite understanding must be able to complete the operation of construction in a finite amount of time. And this condition just is the condition of the *quaestio quid juris*?, as Maimon understands this:

> could then represent every number as an aggregate of indivisible units. But if one represents a line as generated through fluxion, and thus generated in time, in which we represent nothing simple, then we can think $\frac{1}{10}$, $\frac{1}{100}$, etc., etc. of the given unit. (AA 11:195-199)

Kant seems to be inclined to deny that we can think of irrational numbers as composited; for, this would imply the representation of number "as an aggregate of indivisible units" or what we might call infinitesimal counting, i.e., it would imply that we can construct the order of the reals. However, in thinking of the construction of $\sqrt{2}$ as a magnitude (of time, or of space) we can know that the number exists, even if we are unable to provide the rule of its composition. Kant's reply is discussed in some of its aspects in (Parsons 1983). While Kant concedes that we have intuitive access to knowledge of the existence of $\sqrt{2}$ (on the supposition that space and time are continuous), Maimon's objection still stands: Kant denies that we can cognize $\sqrt{2}$, despite the fact that we have intuitive knowledge of its existence. This, however, means that our concept $\sqrt{2}$ is not constructible in intuition. Why not? Because we can give no determinate procedure for its composition in the continuum.

[5] As we have already noted, Kant may hold the view that the synthesis of the manifold into an an appearance (individual) is also subordinate to the sensible condition, thus it too can only provide finite quantifications of sensibility. If this is so, then Maimon's concern about the inadequation of appearance and concept has a resolution — both are merely finite approximations. However, this solution has two limitations. First, it leads immediately to empirical skepticism: some (real) forms simply cannot be intuited. Second, Maimon's "physical" antinomies are intended to show that even if the paradoxes of infinity are merely a product of our mode of representation, it still must be the case that the way in which parts of the (sensible) manifold are related to each other display infinite variability.

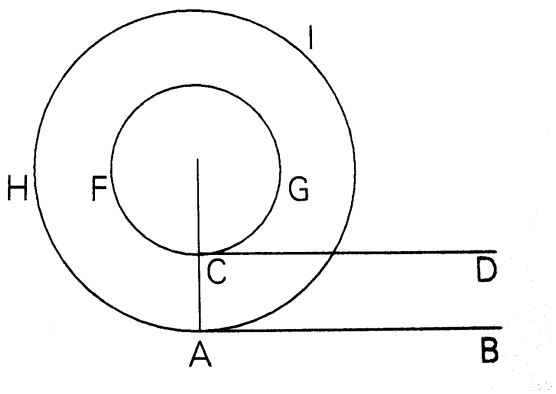
"can these symbolic concepts also be made intuitive, and thereby acquire objective reality, or not" (VTP; MW 2:48)? The answer it seems, is no.

A Physical Antinomy: Aristotle's Wheel

At the end of the 'Short Overview of the Whole Work', Maimon introduces a number of new antinomies that now correspond to our interpretation of the physical world, not just the possibility of its representation. Here, Maimon asks that we grant that the "mathematical antinomies can also be resolved according to Kant's system of sensibility and its forms [i.e., through transcendental logic]" (VTP; MW 2:237). Even if we do so — and we need not — physical antinomies arise that cannot be explained. These antinomies do not affect the possibility of *consciousness*, but rather the object of knowledge of what it is that we cognize: the manifold itself.

Maimon introduces three antinomies, the most important of which is "Aristotle's Wheel" (**Figure 4.1**).

Figure 4.1: Aristotle's Wheel

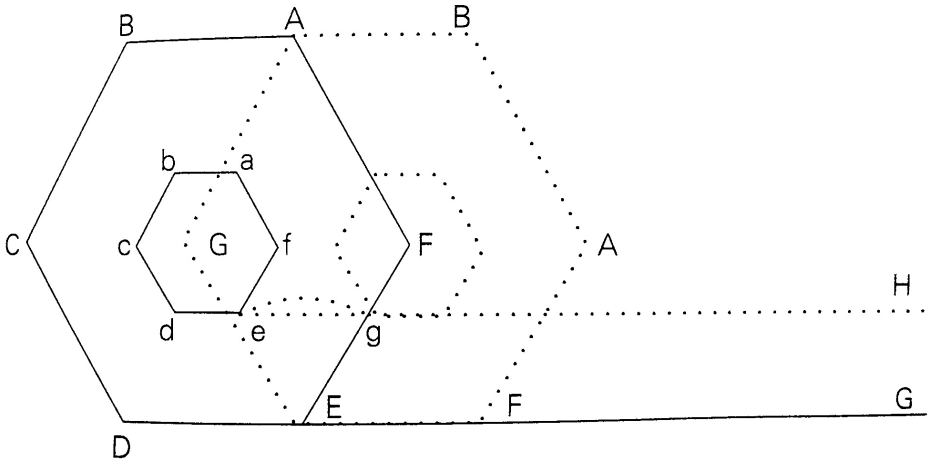


Intuitively, it is obvious that both wheels travel the same distance; they are both attached to the same axle, and therefore start and end in the same place after one rotation. However, we also note that the distance travelled by a wheel is its circumference multiplied by the number of rotations. But given that the wheel rotates only once, how can it be the case that the lengths

travelled by the large wheel (with a larger circumference) and by the small wheel are the same?

The solution lies in recognizing that the wheels, while rotating at the same time, do not have the same angular velocities, and thus one circle or the other must slip if both are in contact with a surface. This can readily be seen from representation of both circles as polygons. For, we see that the length covered by one side of the large polygon is not the same as the length covered by one side of the small polygon (**Figure 4.2**).

Figure 4.2: Aristotle's Wheel, Finitely



We can see that the length of the line dh is composed of the length of each side of the polygon, but must be supplemented by the lengths that it "skips" in rotating. But to show that this is necessarily true for a circle, we cannot rely on a finite demonstration, such as the one just provided. For we must show that this is true for an infinite number of sides (points) along the circumference of the circle. But since the lengths covered and the lengths skipped are proportional to the length of the side, a circle with an infinite number of sides has sides that are infinitely small. What is more, these infinitely small chords of the circle are in a one-to-one correspondence. That is, both wheels have the same number of sides (ultimately, points) and the length of all of the sides is the same: the infinitely small, or the mathematical differential. How, now, should we go about distinguishing which wheel travels farther if the length of each

side is effectively zero?[6]

Maimon's solution is that we must accept an actual infinity as part of the world. His claim is that the only way that the problem can be solved is by taking each point along the line to be an actual infinitesimal, and co-ordinating points along the circumference of the smaller circle with points along the larger wheel. We see then that the points of one circle must be "bigger" than those of the other, even though they are both infinitely small. In other words, the *velocities* of corresponding points are different; accordingly, as each side approaches 0, the points of the larger circle are still intelligibly different from the points of the smaller circle, even if no such difference can be intuitively represented.[7]

By way of elucidation, consider two line segments of unequal length. Suppose that each line is composed of an infinite number of points. Now, the magnitude of these points is taken to be the limit of extension. To derive such a line from an extensive magnitude ab is to infinitely subdivide ab. So, suppose that a point is equal to $\lim_{x\to\infty} \frac{ab}{x}$. Now, this is also true of any other magnitude of arbitrary length, bc; hence, the point is also equal to $\lim_{x\to\infty} \frac{bc}{x} = \lim_{x\to\infty} \frac{ab}{x}$. Considered as mathematical points, the infinitesimal components (points) of ab and bc are the same. Now, suppose that, in generating a line (integrating the function $f(x) = 0$), we compose an infinite number of points with the same direction (i.e., null differential): $\int 0dx$. The question now is: what is the length of the line? Is it ab or bc? In truth, it is neither. For, only a definite integral can have a determinate magnitude: $\int_a^b 0dx$. That is, we must *already know* what length of line is to be generated before we can begin to generate it. However, if we consider the magnitude of $\frac{ab}{\infty}$ to be determinate, and determinately different from $\frac{bc}{\infty}$, even though both are infinitesimally small, then we can show that the lengths of each line segment are determinately different. This, however, is only possible if we accept that there are actual infinities, i.e., that division by infinity can yield a determinate something other than zero. In other words, the

[6]Now, we might think that Maimon's solution is disingenuous: the larger polygon has more "sides" than the smaller one. However, it can be shown that both lines are composed of the same number of points: $\aleph_1$. The two sets of points thus have the same cardinality; however, the functions relating the points in space are different.

[7]It is important to note that Maimon's proposal is *not* that the points have a non-zero extension. Rather, the claim is that they are "differentials", i.e., that the points, while non-extended, represent "instantaneous" motion, or the ration of the infinitesimal difference in space and the infinitesimal difference in time: $\frac{\Delta x}{\Delta t}$.

infinite division of two different lengths yields an actual determinate difference between the points: a velocity. Maimon thus concludes:

> We must therefore admit an actual infinity as the element of the finite (and not merely a mathematical one, i.e., the possibility of infinite division). So this case also give rise to a true antinomy because (through the idea of the infinite divisibility of space) reason commands us never to reach the end of the division of a determinate line so that we finally arrive at an infinitely small part; and yet at the same time reason proves to us that we must actually arrive at such an infinitely small part. (VTP; MW 2:236)

Whereas reason merely "commands" us never to (finitely) think an actual infinitesimal, reason "proves" — now analytically — that there must be some such thing, or else the problem remains insoluble. This gives rise to what Maimon calls a "true antinomy". For, it is not an antinomy of metaphysics (i.e., between representations of the whole and representations of the parts), but an antinomy of physics (i.e., a relation between space and time, or the relation of reality to itself). For, if we deny that there is a true infinitesimal — as reason commands — then the problem has no solution. However, there is manifestly a difference between the velocities of the two circles, one that reason proves to us in considering the problem.[8]

[8]Friedman (Friedman 1992a) claims that we should understand Kant's conception of the manifold as kinematic. This is made clear in Kant's claim in the Transcendental Aesthetic that while the pure form of space provides the a priori foundation for geometry (KRV; AA B41), the pure form of time provides the a priori foundation for "the general theory of motion" (kinematics) (KRV; AA B48). Friedman points to Kant's use of the term *fliessende Größe* in the Anticipations of Perception (KRV; AA A170/B211) as further evidence that Kant thinks time kinematically, since this is the German term for Newton's fluxion. If Kant thinks of time not as a point-set (as Maimon, following the Leibnizian infinitesimal account appears to) but as grounded in (given) motions that are approximated through a standard metric (i.e., in the apprehension of intuition, in which "I generate time itself" (KRV; AA A142/B181)) then Maimon's argument will fail. For, Kant can merely say: there are no absolute points (and thus no number of points) but only absolute motions (in the manifold of intuition). Accordingly, Kant would merely say that Maimon has erred in trying to solve a kinematic problem geometrically.

A number of questions remain. Foremost among these is how reliable Kant's claims in the Schematism are, given the extensive revision to the structure of the argument of the Transcendental Deduction, which focused especially on the concept 'time' and the ground of all possible combination. If we assume that time (in intuition) is not "generated" by the act of bringing sensibility to apprehension, then we must assume that pure intuition, and also the "drawing of lines" in the mind is furnished with a robust set of logical features that are not expressed in Kant's logic. This, indeed, is the core of Friedman's argument in (Friedman 1992a) and (Friedman 1992b). However,

4.1.2 The Principle of Reality

Maimon's *Critical Investigations of the Human Mind* (Maimon 1797) provides a more developed criticism of the *Critique of Pure Reason*, drawing not only on the (partially formed) ideas of the *Essay on Transcendental Philosophy*, but also on Maimon's attempt to provide a response to the limitations of Kantian transcendental logic in his *Essay on a New Logic* (Maimon 1794). Accordingly, the *Critical Investigations* consists primarily of an investigation of the human faculties familiar from the *Critique of Pure Reason* and the other *Critiques*: sensibility, understanding, the will, etc. However, Maimon adds another objection that is perhaps unexpected in a *Critical Investigations of the Human Mind*: an objection to Kant's conception of logic *as such*. Kant, Maimon complains, "outlined no general **criterion of material truth**, which indeed, as I will show, can and should be done" (KU; MW 7:Preface).

Now Kant, it is well known, rejects the possibility that we might be able to provide such a general criterion of truth:

> a general criterion of truth would be that which was valid of all cognitions without distinction among their objects. But it is clear that since with such a criterion one abstracts from all content of cognition (relation to its object), yet truth concerns precisely this content, it would be completely impossible and absurd to ask for a mark of the truth of this content of cognition, and thus it is clear that a sufficient and yet at the same time general sign of truth cannot possibly be provided. (KRV; AA A59/B84)

Kant's point is that the structure of logic derives from the a priori form of judgment, and the a priori forms of judgment — since they are a priori — are independent of any and all material content of thought. The 'laws' of logic, therefore, are revealed by a *Critique of Pure Reason* as the necessary and strictly universal conditions of thinking in general.

However, in the *Essay on a New Logic*, Maimon introduces a distinction between 'logical truth' and 'metaphysical truth': "**logical truth** depends immediately on **representations** and **concepts**; **metaphysical truth**, on the other hand, depends immediately on the **object** it-

Kant appears to have backed away from the position of the A Deduction and strengthened the idealist credentials of intuition in the B Deduction. This would imply that composition from a point-set (i.e., infinitesimal counting) is possible at least for the reproductive imagination, Kant's rejection of infinitesimal counting notwithstanding.

self, and mediately on **representations** and **concepts**" (VNL; MW 5:18). Kant may be right that we cannot provide a criterion of *metaphysical* truth; the transcendental investigation implies that we abstract from the particularity of objects (and thus their existence as transcendent entities) in order to consider them generally. However, Kant's doctrine of judgment depends on the validity of the comparison of appearances and their conceptualization, i.e., on a logical truth: *real* validity. But now, what Maimon's objections from the *Essay on Transcendental Philosophy* have shown is that Kant does not — indeed cannot — provide a general criterion of the validity of the comparison that is a condition of judgment.

'Logical truth' may therefore be understood as the condition under which synthetic judgment is possible at all; that is, a general criterion of the adequacy of an empirical representation (intuition) and a synthetic a priori concept. Accordingly, if the *Critique of Pure Reason* has delimited the domain of the legitimate application of reason itself, it has done so without taking into account the legitimacy of its ancilliary faculties;[9] in particular, *judgment* — and thus logic — are taken to have absolute validity independently of the fact that they must necessarily be oriented towards objects as a condition of a *transcendental investigation*. Maimon, therefore, objects that a genuinely critical investigation into cognition itself cannot simply take logic (judgment) for granted, but must begin 'at the beginning' as it were: with logic itself:[10]

> **Philalethes** My investigation shows that not only **reason**, but also the other **faculties of cognition**, from the point of view of their **relation** to **objects** require a **critique** ...
>
> **Kriton** Where should we begin with such an investigation, since it takes upon itself **logic** as the **science** of the **form of cognition**?
>
> **Philalethes** We must therefore start with **logic**; this will be the first part of our **critique**.[11]

Accordingly, the *Critical Investigations of the Faculties of the Human Mind* is at the same

[9] Another of Maimon's objections to the *Critique of Pure Reason* points out that the positive aspect of the Critique (the doctrine of the faculties) is not justified: "[Kant] assumes (for the purposes of the positive part of his critique) certain **functions of the faculties of cognition** to which he is not entitled" (KU; MW 7:Preface).

[10] Students of Hegel will recognize the substance of Hegel's argument in the "Preface" to the *Science of Logic* in this objection. Although, I am not defending the view that Hegel derives his position from Maimon, the affinity of their positions is difficult to deny.

[11] (KU; MW 7:15) The *Critical Investigations of the Human Mind* consists of three dialogues and a prolegomena. In the dialogues, Maimon speaks through Philalethes, and Kant through Kriton.

time the transcendental investigation of the doctrine of logic elaborated in the *Essay on a New Logic*; that is, whereas the logic of the *Essay on a New Logic* is presented as a doctrine, it is investigated transcendentally in the *Critical Investigations* as the condition of the possibility of objective cognition as such.

Now, the supposition of the *Critique of Pure Reason* is that the structures of logic derive from the functions of judgment, and these functions ultimately derive from the spontaneity of the subject. *Prima facie*, this is an argument for a "first principle" of philosophy in the (perhaps metaphysical) spontaneity of a thinking thing; and, indeed, this is precisely how Kant was interpreted by, for example, Rheinhold and Fichte. However, as Maimon argues, the investigation of the *Critique of Pure Reason* must be an investigation not of pure thought as subsisting in a pure subject, but an investigation of real thinking by real subjects. As Maimon (sarcastically) points out, "the real reason why logic abstracts from the material of thought is — because it abstracts from the material of thought" (KU; MW 7:19).

Accordingly, the first principle of logic — as it is expressed in the *Essay on a New Logic* — provides a definition of logic which places thought at the centre of investigation: "Logic is the science of thought. **Thinking** is the **action** of the subject, whereby, supposing the **identical unity** of the subject in the consciousness of the manifold of thought over and against the object, an objective unity of this manifold may be constituted" (VNL; MW 5:12). Whereas Kant defines the task of logic more narrowly — confining it to the rules of judgment (in general or transcendentally) — Maimon takes logic to be the "science of thought", which includes the constitution of an "objective unity" of a manifold "over and against the object."

This idea is again expressed in Maimon's definition of the fundamental character of thought: "The most general (and therefore indeterminate) function of the cognitive faculty, which alone provides the foundation of its expression, is **consciousness** in general" (VNL; MW 5:15). Superficially, at least, this is an anodyne proposition that even a Kantian could endorse. For, the claim of the Transcendental Deduction (in the B edition) is that consciousness does not arise except through the activity (functions) of the understanding. However, Maimon's

understanding of the scope of 'consciousness' differs from Kant's. Indeed, for Maimon, "if some modification of consciousness (some possible determinate consciousness) is to be called 'representation', then there is not only no representation without consciousness as such, but also no representation without a determinate consciousness" (KU; MW 7:38). That is, it is not just concepts, but also intuitions — anything that is a representation for consciousness — that consist of an determination of the manifold of thought. Indeed, what Maimon understands by "consciousness in general" is not, as it is for Kant, the unity of an individual appearance and its conceptual interpretation, but rather the determinability of consciousness as such; thus the object of logic is not *merely* the individual, but rather "consciousness in general, as the absolute determinable in our faculties of cognition."[12]

In the *Essay on a New Logic*, Maimon introduces the first principle of logical truth through an (apparently) uncontroversial principle: "the first principle of **logical truth** is the **principle of contradiction**" (VNL; MW 5:19). However, as Maimon also points out, "a contradiction ... can only be found between propositions, not, however, between concepts (which can only be posited as opposed to each other)" (KU; MW 7:22). Indeed, in the *Critique of Pure Reason*, Kant explicitly claims that abstract concepts can only be contradictory insofar as they

[12](VNL; MW 5:15) Perhaps the fundamental error in Maimon's interpretation of Kant is that he understands the Kantian doctrine of intuition to consist of a totality (the a priori forms of space and time) of given determinables (i.e., an real point-set topology) that is subject to modifications (intuited appearances); this is true even in the *Essay on Transcendental Philosophy*, where Maimon interprets sensibility as a total manifold of givenness. The determinacy of a particular appearances is, accordingly, always determinate with respect to the totality of which it is a modification. Such a theory of the determinacy of a modification as a relational difference with respect to the "substance" of which it is a modification can be extracted from Spinoza's *Ethics*, and certainly also from the doctrine of the *Monadology* (Melamed 2004). The doctrine of intuition, therefore, becomes the interpretation of determinate individuals (apperceptions, rather than *petites perceptions*) that are modes of a totality that is not necessarily perceived in its totality.

This is not an entirely implausible reading of Kant. However, Kant appears to hold the view that "space" and "time" are constructive abstractions that are real insofar as they are thought of as discrete parts, and ideal insofar as they are thought of as totalities. Brought to cognition in the whole of its infinite continuum, however, it may not be. However, Kant's strategy appears to be unstable, and Maimon may not be far from the mark in designating the nearest approximation of Kant's view that can be coherently realized.

are logical opposites. In order to determine whether or not predicates are incompatible with a subject (or incompatible with each other), we must be able to make judgments about opposition *in appearance*:

> If reality is represented only through the pure understanding (*realitas noumenon*), then no opposition between realities can be thought, i.e., a relation such that when they are bound together in one subject they cancel out their consequences, as in $3 - 3 = 0$. Realities in appearance (*realitas phaenomenon*), on the contrary, can certainly be in opposition with each other and, united in the same subject, one can partly or wholly destroy the **consequence of the other**, like two moving forces in the same straight line that either push or pull a point in opposed directions, or also like an enjoyment that balances the scale against a pain. (KRV; AA A265/B321)

However, Kant does not provide a criterion of "opposition" in the *Critique of Pure Reason*. We may be able to make some sense of the concept of opposition through the dynamics of the *Metaphysical Foundations of Natural Science*, but surely this does not provide us with a general criterion of opposition. Moreover, a general criterion of the *applicability* of a predicate to a subject appears to be necessary if we are to make sense even of simple empirical judgments. That is, we need to have some idea of the ways in which the manifold can be combined before it is possible to make sense of even simple empirical judgments. While it may not be immediately obvious why this is so for judgments of affirmation — which suppose the reality of the predicate — it is much more evident for judgments of negation. For, in order to assert that it is *not* the case that a predicate applies to an intuited object, we must — a priori — have some idea of the conditions under which the predicate does apply, and also the (seemingly infinite) conditions under which it does not. How do we know when a predicate may be negated?

The decisive problem for the Kantian theory of transcendental judgment is therefore the following: How do we recognize the absence of the subject or the predicate? For, it seems to be the case that recognizing that a concept is not instantiated implies knowing the criteria for when this is the case. However, we can also turn the question around and pose it as follows: How do we recognize the *presence* of the subject or the predicate? For, it seems to be the case that in order to recognize an instance of 'cat', one is already implicitly recognizing that it is not an instance of 'dog'. Now it seems that recognizing that a concept is not instantiated implies

knowledge of what is thereby excluded. Generalizing, we can say that recognition (as the parsing of intuitions) requires knowledge of criteria through which non-instantiated concepts may be excluded. Accordingly, even affirmative judgment must have a negative aspect. It is from consideration of simple cases of judgment such as these that Maimon concludes, "there must necessarily be a general criterion of truth, which although not given by logic itself, is presupposed by it, and through which it is not only determined as an *ens logicum* (not a *nihil negativum*), but also as an *ens reale* (not a *nihil privatum*)" (KU; MW 7:40). I will call this the *principle of reality*, since it insists that *reality* is a condition of the validity of logical form in general.

Prima facie, Maimon's proposal appears to be a regression to a pre-critical position. For, as we have seen, pre-critical logic (broadly construed) proposes a metaphysical interpretation of the structures of logic; Maimon's interpretation of the criterion of truth as a determination not only of an object (*ens*) but, moreover, of a *real* object (*ens reale*) appears to restore the pre-critical interpretation of logic. However, we must recall Maimon's distinction between a criterion of logical truth — the relation between representations — and a criterion of metaphysical truth — the relation between a representation and a transcendent object. Indeed, the first principle of logic in the *Essay on a New Logic* shows that Maimon understands logic not — as in the tradition — as also a determination of the objectivity of the object, but rather exclusively with respect to its intelligibility: "**Logic** is the science of the thinking of an object in general, which is indeterminate through its **inner properties** and merely determined through its **relation to intelligibility** [*Denkbarkeit*]" (VNL; MW 5:1). Like Kant, Maimon understands (transcendental) logic to be the science of the principles of the object in general. Moreover, this object is "merely determined through its relation to intelligibility", i.e., through relational properties, and not through whatever "inner properties" it might happen to have.

Maimon therefore concludes that the principle of sufficient reason "does not belong to general logic, but rather to transcendental logic or metaphysics" (KU; MW 5:33). However, it remains a *logical* principle, and, insofar as transcendental logic conditions the possibility of

general logic, then the principle of sufficient reason *conditions* the principle of contradiction. Accordingly, Maimon concludes that only after transcendental logic "has secured the meaning of absolute, intuitively representable reality and negation, then general logic can abstract from all intuition" (KU; MW 7:24). That is, it is only once an intelligible *principle of reality* can be ascribed to a subject-predicate relation that it makes sense to talk of a principle of contradiction at all.[13]

We may now formulate the principle of reality through the following three conditions:

(a) Every real predicate is a modification of the judgment of reality (i.e., is as modification of the manifold);

(b) Not every predicate is predicable of a real (finite) being (i.e., there is no finite *ens realissimum*);

(c) There is a criterion of real (material) opposition and real (material) affinity, such that the real compatability or real opposition of predicates in a particular subject may be determined a priori.

So construed, the principle of reality is one that can be ascribed to Kant. For, Kant surely ascribes to some form of (a); there are no real predicates that are not predicates in some (transcendental) judgment of reality. Second, the doctrine of real opposition (b) is familiar from Kant's pre-critical works. The third condition (c) is more problematic. We have seen that Kant assumes some form of affinity as a property of the manifold. This is expressed in various ways, but most clearly in the fact that the manifold is treated as the matter of experience (*Stoff*), and thus as a quasi-substance with some dynamical properties; this is evident not only in the interpretation of the Analogies of Experience, but also in the *Metaphysical Foundations of Natural*

[13] The principle of reality is also the first hypothesis of Schelling's *System of Transcendental Idealism*:

> It is thus accepted as a hypothesis, that all of our knowledge in general is **reality** ... if all knowledge rests on the adequation of something objective and something subjective (Introduction §1), then the entirety of our knowledge of principles, which are not **immediately** true, derive their reality from something else. (STI; SW 3:353)

This something else is, as we will see, the reality of the whole of consciousness.

Science. Accordingly, the status of the third condition in Kant's doctrine of the faculties is ambiguous. For, the matter of experience — as opposed to its form — is provided by sensibility; it has no a priori reconstruction through the forms of intuition, nor any mediate representation through the pure concepts of the understanding. Rather, it appears to have an ambiguous status as an empirical condition (i.e., a condition of sensibility) which is at the same time a necessary condition of transcendental judgment.

We do not need to resolve this tension in Kant's concept of matter, nor settle the question of whether or not Kant would endorse all of the conditions of the principle of reality. However, we can at least conclude that the problem that Maimon has isolated — the a priority of affinity — is not one that is foreign to the transcendental problematic. Indeed, what the principle of reality shows is that the problem of affinity requires an a priori explanation; moreover, this explanation is fundamental not only to the doctrine of transcendental predication (judgment), but to *logic itself* insofar as the latter invokes the principle of identity and the principle of contradiction. Indeed, if logic does no more than to abstract from real thought, then the determination of the functions of logic must consist of modifications of the principle of reality: logical functions are determinations of reality; conversely, all determinations of reality imply logical functions.

Whereas Kant's transcendental logic appears to be a logic of the schematic analogy of the intuited individual (with a quasi-substantial aspect through the physicalism of the manifold), Maimon's logic has idealized the real moment of sensibility (i.e., the "matter" that is the analogy of experience) as a principle of *logic*. Accordingly, Maimon's logic is not, like Kant's, a logic of the (finite) quantification of a (physical) manifold of (possibly kinematic) particulars, but rather a logic of the a priori determination of an ideal totality (*totum reale*) of differentials (or infintesimal determinables). If we find the implicit physicalism and finitism of Kant's "manifold" of intuition problematic, and if we think that the genuine object of post-critical logical determination is the infinite continuum and not the finite individual, then Maimon's proposal has a number of important advantages over Kant's schematic solution.

If, on the other hand, we are suspicious of the apparently metaphysical aspect of the

logics of post-Kantian systematic idealism, then a number of important questions arise with respect to Maimon's solution. The most important of these is the following: if a principle of reality must be admitted as a condition of the intelligibility (and even applicability) of predicates to an a priori manifold of rational differences, must this principle be expressed as the determination of a real totality (*totum reale*), or is there another way to understand the infinity of the continuum such that every determination of the whole does not imply the determination of its parts?

4.1.3 The Principle of Determinability

As we have seen, the first principle of thought is the principle of reality. If a thought is to count as a representation at all, it must be manifest as a determination of consciousness as the real totality of the manifold. On its own, however, the principle of reality does not specify the conditions under which the reality of subjects and predicates is to be determined — that is, it does not specify how the principle of sufficient reason is to be realized through a manifold of reality. Accordingly, Maimon's logic provides another principle: "the first principle of a real thought determining an object is what I have called the **principle of determinability**" (VNL; MW 5:20). The principle of determinability can be expressed in two forms, one for the subject, the other for the predicate:

The principle of the subject Any subject must not only as subject, but also in itself, be a possible object of consciousness (VNL; MW 5:20);

The principle of the predicate Any predicate must not only in itself, but also as a predicate, be a possible object of consciousness (VNL; MW 5:20).

Superficially, the principle of determinability appears to be a fairly benign epistemological principle. For, both principles appear to claim no more than that a subject or a predicate must be a possible object of consciousness. This should come as no surprise since, as Maimon claims, there is "no representation without a determinate consciousness" (KU; MW 7:38).

However, the principle of determinability also contains a decisive clause. For, a subject is not only a subject, "but also in itself" a possible object of consciousness. For predicates, the condition is the reverse; a predicate is not only in itself, "but also a predicate." What exactly this difference amounts to may not initially be clear. We can, however, take a hint from the different wording of the two principles: the subject may be "just a subject", but the predicates, may be just 'in itself'. However, insofar as thought is 'real', the subject and the predicate must be both 'subject' or 'predicate' and 'in itself'. To make sense of this, we have to go back to Kant. For Kant, the subject of a judgment is always the transcendental object; it is the synthetic product of the functions of judgment, and that to which predicates are ascribed. The subject, therefore, is "just a subject." Predicates, on the other hand, are, as we have seen, modifications of the judgment of reality, and therefore are all modifications of the manifold, i.e., 'in itself' a possible object of consciousness. Maimon's claim in the principle of determinability is that the distinction between the ideality of the subject and the reality of the predicate cannot be maintained. Rather, the subject of every thought — insofar as it is a "real thought determining an object" — must be *at the same time* an intelligible unity (subject) and a sensible composite (in itself). Conversely, the predicate of every thought — insofar as it is a "real thought determining an object" — must be *at the same time* a sensible composite (in itself) and an intelligible unity.

Maimon specifies his taxonomy of concepts as follows:

> **Concepts** are (in opposition to intuitions) either **conditions** or **products** or, (in opposition to objects) **educts** of thought. The former are non-sensible, simple concepts; the latter, on the other hand, (from the point of view of material) are sensible and composited. (VNL; MW 5:42)

Accordingly, in opposition to intuitions (i.e., in a judgment of reflection), the subject is "just" a subject: a simple, non-sensible product of the functions (action) of thought. If, on the other hand, the subject is thought of in opposition to an object (i.e., in an objective judgment), then the subject is an *educt*; the concept thought of as a sensible composite. The same interpretation applies, *mutatis mutandi*, for the predicates of a judgment. If we interpret the distinction between 'mere subject' and 'subject in itself' through the distinction between 'product' and

'educt' (and, similarly, 'mere subject' and 'subject in itself'), then Maimon's claim is that every objective judgment implies that both subject and object are *at the same time* intelligible unity and sensible composite.

Despite its relatively benign appearance, the principle of determinability in fact implies a radical transformation of the Kantian epistemology. For Kant, the judgment is characteristically a function between a unity (the transcendental object) and a plurality (the predicate as modification of the manifold). The unity of thought is possible because the predicates of empirical judgment may be interpreted through the schematic translation of synthetic principles and categories. Maimon's proposal, however, is that the concept — be it subject or predicate — is already a function between an intelligible unity and sensible plurality, at least insofar as it is a "real thought determining an object". This last qualification is important. For, the mere discursive use of concept — for example, in abstract logic — implies the manipulation of purely symbolic subjects and predicates. As such, both subject and predicate are "merely" subject and predicate, or "products" of thought. However, insofar as thought is object-oriented, the objects of our thought and its properties must be modifications of reality, i.e., modifications of the manifold. If subjects and predicates are functions between intelligible unities and real pluralities, then, by extension, a judgment is a function of functions. "Logical truth", therefore, is nothing more than the determination of the unity of this function of functions of the manifold.[14]

Maimon has, in effect, erased the distinction between empirical judgment and the discursive use of concepts. *Prima facie*, this is just a misunderstanding of the *Critique of Pure Reason*. However, the applicability of a predicate requires a criterion of the applicability of (or opposition of) a particular predicate to a particular subject (the principle of reality). The principle of determinability merely advances that the criterion of the applicability of (or opposition of) a particular predicate to a particular subject implies: (a) a general determination of

[14]The principle of determinability is likewise the second hypothesis of Schelling's *System of Transcendental Idealism*:

> It is accepted as a **hypothesis**, that in our knowledge there is a **system**, that is, it is a whole, that supports itself, and agrees with itself in itself. (STI; SW 3:354)

the subject with respect to the determinable, i.e., the manifold of consciousness, (b) a general determination of the predicate with respect to the determinable. i.e., consciousness in general, and (c) a general determination of the compatibility or opposition of (a) and (b). Accordingly, the two forms of the principle of determinability form the two fundamental doctrinal principles of the *Essay on a New Logic* as applied to consciousness in general:

First doctrinal principle Every possible **determination** of the **determinate** is at the same time a possible **determination** of the **determinable** (VNL; MW 5:30);

Second doctrinal principle Every possible **determination** of the **determination** is at the same time a possible **determination** of the **determinable** (VNL; MW 5:31).

Unfortunately, Maimon's formulation of his doctrinal principles is opaque, in the style of Hegel's more extravagant formulations; and the ambiguous use of the term 'determination' is certainly of no help to his readers. However, we can make the following clarifications:

(a) Every determination of the concept of an object — be it a subject or a predicate — is an educt, i.e., it is a composite form.[15]

(b) That which is determined in a determining judgment is called 'the determinate'; it is the subject of a determinate object of real thought.

(c) That which is a determination in a determining judgment is (also) called 'the determination'; it is the predicate of a determinate object of real thought.

(d) That which is determined in any case of real thought is called 'the determinable'. The determinable in general is consciousness as such.

Accordingly, we may reformulate the doctrinal principles as follows:

First doctrinal principle Every possible [composite form] of the [real object] is at the same time a possible [composite form] of the [manifold].

[15] 'Educt' is Maimon's term, borrowed from the language of chemistry, which is the converse of 'product', i.e., the reactants out of which a compound is formed.

Second doctrinal principle Every possible [composite form] of the [real predicate] is at the same time a possible [composite form] of the [manifold].

That is, both subjects and predicates — insofar as they are determinations of a real object of thought — are at the same time determinations of the manifold. So formulated, the principles of Maimon's logic begin to be comprehensible. However, the crucial consequence of these principles does not become fully apparent until we consider a corollary to the principle of determinability:

> (Real) thought in general therefore depends on insight into the relationship between the **determinable** and the **determination** between the members of the manifold to be connected through thought. (VNL; MW 5:28)

This just is the formulation of the principle of sufficient reason in terms of a logic of the real determination of the infinite continuum as a totality, i.e., as a *totum reale*. For, if the concept of a determination of a determinate object (the individual) of consciousness (the whole) implies the determinability of the parts (i.e., an educt, or a *compositum ideale*), then the concept is nothing other than a *function* relating the (infinite) determinability of the individual (the determinate) to the infinite determinability of the whole. That is, a concept is just a function of a part and the whole. Accordingly, if judgment just is a functional relation between the concept of the subject and the concept of the predicate (i.e., the concept of the determinate and the concept of the determination of the determinate), then conceptual predication — the copula — just is a functional relation between two functional relations of a part and the whole. Accordingly, the *determination* of the reality of a predication implies the determination of this "second-order" function of the infinite continuum with respect to itself.[16]

[16] By way of elucidation, we may anticipate the doctrine of the manifold somewhat, and think of the principle of determinability through a mathematical approximation. We saw above that the principle of reality implies a real a determinable difference between real determinations of the manifold. Accordingly, $\frac{ab}{\infty}$ must be determinably distinct from $\frac{bc}{\infty}$. The principle of determinability now interprets the principle of the reality of parts in terms of functions.

4.2 Logic of Totality

If Maimon provides the elements of an extended transcendental logic, the project, as he envisioned it, was never completed. However, the *principle of reality* and the *principle of determinability* became central tenets of post-Kantian idealism, either through Maimon's direct influence, or through the adoption of these principles in Fichte's own attempt to extend transcendental logic to include the objective reality of the object. However, the project would only reach its culmination in Hegel's *Objective Logic*, where Maimon's system of principles would be completed with an adequate conception of the infinite and the totality — both of which are assumed by, but not provided in, Maimon's own logic.[17]

Hegel's logic is not a deductive system, nor is it a system of formal construction from axioms and definitions. Rather, Hegel's logic is a *dialectical system*. To say that it is *dialectical* implies that it is a 'conversation' between modes of representation, that creates a cycle of determinations of progressively refined forms of thought. To say that it is a *system* is to say that the two moments of thought relate to each other; moreover, they relate to each other a priori (that is, they are a presuppositionless 'presupposition' of thought) and immanently (that is, in the act of thinking itself).

Suppose that the manifold of reality consists of an infinite series of (infinitesimal) differences; for the sake of simplicity, let's suppose a one-dimensional manifold, i.e., x. Let the manifold M be the series $\Sigma\Delta x$. If a concept is just a function of the a part of the manifold with respect to the whole of the manifold, then every concept (of subject or predicate) is the *ratio* of the integral of a finite interval of the manifold and the integral of the whole of the manifold:

$$f(x) = \frac{\int \Sigma_a^b \Delta x dx}{\int \Sigma_\infty^\infty \Delta x dx}. \tag{4.1}$$

We will see what this entails in more detail in the next chapter.

[17] Maimon's logic is developed in his *Essay on a New Logic*, which contains a surprising grasp of the limitations of traditional syllogistic logic, and even appears to suggest elements of Boole and Peano's subsequent innovations. These innovations aside, however, the principle of determinability clearly presupposes a conception of the infinite that is also a totality, a *totum reale* derived, evidently, from the Spinozist conception of God as the one substance, subjected to determinate limitations (modes). However, it was ultimately Hegel, whose "philosophy of infinity" provided a way of thinking of the infinite as both composited and holistic. Hegel's brilliant innovation will be the object of our study here: the concept of the boundary (*Grenze*) as the limit of composition and the beginning of qualitative totality. Maimon's insistence on an actual infinity as a requirement of the manifold construed as a point-set topology is finally given its full expression in Hegel's philosophy of quantity.

What, now, are the two moments of the Hegelian dialectic? These may, of course, be considered from a variety of perspectives. The claim that I just raised — that a systematic dialectic is both a priori *and* immament — means that it is present in thought in all of its moments: formal, subjective, universal, and phenomenological. None of these may be privileged above the others.[18] However, I will characterize the two moments as 'Platonic' and 'Aristotelian'.[19] The Aristotelian moment begins with the *finite hypokeímenon* (*das Zugrundlegende*) and attempts to *ground* the *actuality* of the *infinite* in the *compossibility* of the finite, i.e., through a *compositum ideale*, an absolute infinite *in potens*. It is thus the actuality of (finite) thought trying to reach beyond itself (the infinite). But, insofar as it just as this thinking of what is beyond its own finitude, it is at the same time this very beyond. That is, it is also the Platonic moment of ideal intelligibility, *noésis* or the *hypothesis* (*das Zugrundliegende*) through which attempts to *ground* the *reality* of the *finite* in the *determinability* of the *infinite*, i.e., through a *totum reale*, an absolute unity *in potens*.[20] The dialectical determination of thought consists in the systematic

[18]Stekeler-Weithofer — who has also provided several excellent studies of Hegel's mathematics — seems at times to privilege the pragmatic speech-act theory in his interpretation of Hegel's logic. This is not in itself a problem; for, Hegel's logic *is* a pragmatic speech-act theory. However, it is also other things besides, many of which are less easily reconciled with our contemporary expectations about the nature of thought, and thus the nature of logic. We would do well to remember the equiprimordiality of every starting point with respect to the others.

[19]Edward Nelson (Nelson 2005) uses the same coarse taxonomy for exegetical purposes, although I did not derive my own taxonomy from his usage. Roughly the distinction amounts to this:

> The two ways, P[latonic] and A[ristotelian], of regarding numbers lead to different number systems. What is a finite number for P is not necessarily a finite number for A. In contemporary mathematics, the notion of finite is defined in terms of the completed infinity $\mathbb{N}$. There is no clear concept of the finite in terms of which the infinite can be defined as not-finite. One goes in the opposite direction in contemporary, Platonic, mathematics and defines the finite as not-infinite.

What is remarkable about Hegel's mathematics is the attempt to make the Aristotelian conception of "predicative mathematics" immanently (i.e., in thought) equivalent to the Platonic conception of "impredicative mathematics". This distinction echoes the problem Kant addresses in the Mathematical Antinomies, and therefore points to the importance of Hegel's reconstrual of the Antinomies as the central interpretive problem for grasping not just Hegel's philosophy of mathematics, but his conception of the infinite (as both predicative and impredicative) as the central tension of dialectical logic.

[20]This echoes the ambiguity of the two ways in which the world-whole is construed in the First Antinomy. There, we think of the world whole through the category of unity (and thus as the result of a constructive algorith) and through the category of totality (and thus as the idea of the whole set of possible numbers). The difference lies in the interpretation of the quantifier as applying to all cases of number (or magnitude) that have been constructed ($\forall x, x$ is a y, and y is a determinate construction) and the quantification across all numbers in the absence of a determinate predicate ($\forall x, x$ is not a y, where y is a determinate construction).

relation of the Aristotelian (finite) and the Platonic (infinite) modes of *rational compossibility* and *determinability* as a system of immanent grounds (the 'Aristotelian' *hypokeímenon*) and transcendent grounds (the 'Platonic' *hypothesis*). More precisely, it consists in their immanent identity within the finite subject; that is, their ideal identity and real contradiction: the reality and actuality of the idea (*eidos*).

The ideality — and thus the intelligibility — of mathematics lies in the perfection of the Platonic moment of thought. The actuality — and thus the objectivity — of mathematics lies in the perfection of the Aristotelian moment of thought. Thought is *both* of these moments; it is both infinite and ideal, and finite and real. It is both *hypothesis* and *hypokeímenon*.[21]

4.2.1 The Principle of Limitation

Maimon's mathematical and physical antinomies show that an adequate natural-physical interpretation of intuition demands that we reconceive the principle of infinity. As Maimon states explicitly in the *Essay on Transcendental Philosophy*, the antinomies of mathematics and natural physics are only soluble if we accept an actual infinity as a product of the understanding (VTP; MW 2:236). This just is a condition of the intelligibility of natural physical problems, such as calculating the angular velocity of the center of a rotating wheel, or explaining the differences in velocity and displacement of differently sized wheels on a fixed axle in the case of Aristotle's Wheel. Indeed, well before Cantor, systematic idealists were pointing out the necessity of an actual mathematical (and physical!) infinity.[22] What Kant's mathematical antinomies have highlighted, and what Maimon's broadening of the problem of the antinomies has demonstrated, is that the Aristotelian — and thus also Kantian — conception of the infinite is in-

[21] Under 'Platonism', we may also understand the 'formalist' interpretation of mathematical existence; i.e., that all mathematical forms 'exist' insofar as they are well-defined derivations from axioms, whether a constructive proof of their existence can be provided or not. See, for example, (Russell 1903) (**?**). Under 'Aristotelianism' we may also understand the 'intuitionist' or 'constructivist' interpretation of mathematical existence; i.e., that all mathematical forms must be brought to intuitive representation, minimally through time. See, for example, (Brouwer 1981) and (Dummett 1977).

[22] Bolzano, for example, famously attacked Kant's conception of mathematics and argued for the necessity of an actual infinite (Bolzano 1950).

adequate (at least as it was then formulated) for the purposes of understanding (a) infinitesimal composition and (b) the boundaries of limit processes.[23]

The Maimonian conception of the infinite has thus sharpened the aporetic nature of the concept of the infinite (and thus the antinomies) into a general — and thus conceptual — problem. However, it is Hegel, who makes explicit the necessity of a new conceptual determination of the mathematical infinite for a philosophical logic, since "from a philosophical point of view the mathematical infinite is important, because the true infinite has its foundation in it, and it stands much higher than the usual so-called **metaphysical infinite**, from which objections to the mathematical infinite are made" (WL1; HW 21:300-1). Now, Hegel begins his consideration of the infinite in the Objective Logic with the traditional definition of the infinite:

> The usual determination of the mathematical infinite is that it is a **magnitude, beyond which** — if it is determined as infinitely large — **there is none larger, or** — if it is determined as infinitely small — **there is none smaller**. (WL1; HW 21:303)

By the 19th century, however, it had become customary to define the infinite more precisely, in order to avoid ambiguity between the actualist and potentialist interpretations of the (categorematic) infinite. Accordingly, Hegel appends a more contemporary formulation to the first: "or, in the first case is larger, in the second case is smaller, than any arbitrary magnitude" (WL1; HW 21:303). This is a standard definition of the syncategorematic infinite, and is close to the definition used by Weierstrass in his work on limits.

Hegel, however, is dissatisfied with this definition of the infinite. For, such a definition can only be imprecise, since it is determinate only to an arbitrary extent (i.e., at whatever x we decide to stop trying to find larger or smaller y's), but not in its completeness. As Hegel repeatedly insists, mathematics deals only in the determinate, as philosophy deals only in truth. Accordingly, it is not just philosophy, but also mathematics, that requires a new concept of the "true" infinite.

[23] As I have already pointed out, Kant may not have felt the urgency of these problems, since he may have thought of constructions as a posteriori approximations (grounded, of course, a priori) of motions that are not in themselves grounded in a point-set topology, but are originally motions (alterations). This, of course, is compatible with an 18th century conception of the calculus, even if it is difficult (if not entirely impossible) to imagine how such a method could be made to work. Note also that Erret Bishop has shown the possibility of real analysis through merely constructive (what we are, loosely, calling 'Aristotelian' mathematics) in (Bishop 1967) and (Bishop 1985)

To begin, we can make two fundamental distinctions with regard to the infinite. First, an infinite (number or magnitude) may be thought of as *actual* or as *potential*. This distinction should be familiar from the Aristotelian doctrine of the infinite. However, we may also think of infinites as either *categorematic* or *syncategorematic*. The distinction derives from the Aristotelian distinction between terms with categorematic import (and thus with ontological significance), and terms that serve to connect them (operators, quantifiers, etc.). The categorematic infinite defines some (existing) term as infinite, i.e. as larger than all others. (Similar definitions may be provided for categorematic and syncategorematic infinitesimals).

Categorematic Infinite $\exists x \forall y, x > y$.

Syncategorematic Infinite $\forall x \exists y, y > x$.

How we interpret the categorematic and the syncategorematic depends on what we understand the domain of the variables x and y to be, and thus the scope of the quantifiers that are applied to them. For example, under the Aristotelian doctrine of the infinite number, x is taken to range across the domain of elapsed time, i.e., the time in which a particular number has been constructed. However, y, as the successor of x, must range across the domain of possible time, i.e., in the *anticipation* of a number that is not yet constructed. Since y is always the successor of x, we may define the Aristotelian infinites as follows:

Actual Infinite Number $\exists x \forall y, y = x + 1, x \geq y$.

Potential (anticipated) Infinite Number $\forall x \exists y, y = x + 1, x < y$.

The first is plainly a contradiction — hence Aristotle's insistence: *infinitum actu non datur*. The second represents an infinite number that may be thought (as a potential) but can never be actual (i.e., is not constructed in any finite elapsed time).

However, there is also an ambiguous sense in which the potential infinite is 'actual': as the divisive infinite of the finite magnitude. As the divisive infinite, the totality of parts of the finite are present 'in' time insofar as the (bounded) individual is given as an intelligible whole.

This infinity of parts is simultaneously present in temporal actuality. Aristotle's solution — and Kant's also — is to claim that while the infinite parts of the finite magnitude are present in actual time, they do not have cognitive reality as actualized subdivisions. Rather, the (cognitive) reality of the implied parts of the totality is merely potential:

Potential Infinitesimal $\forall x \exists y, y = \frac{x}{2}, x \leq y.$

Here, however, x does not range across the domain of actual elapsed time, but over *space*. However, since x and y both range across a single domain (synchronic construction), the "potentiality" of the infinitesimal is no longer a modality of time, but rather of psychological reality. That is, the potential infinitesimal is actual in time, but is merely potential with respect to the domain of realized cognitive representations.[24]

Kant's resolution of the Second Antinomy exploits precisely this ambiguity in the concept of the infinitesimal:

> If the whole has been empirically given, then it is **possible** to go back **to infinity** in the series of its inner conditions. But if that whole is not given, but rather is first to be given only through an empirical regress, then I can say only that it is **possible** to progress to still higher conditions in the series **to infinity**. In the first case I could say: There are always more members there, and empirically given, than I reach through the regress (of decomposition); but in the second case I can say only: I can always go still further in the regress, because no member is empirically given as absolutely unconditioned, and thus a higher member may be admitted as possible and hence the inquiry after it may be admitted as necessary. (KRV; AA A514/B542)

The problem, however, is that the empirical whole (the bounded appearance as a *quantum discretum*) must be given with all of its conditions, since otherwise it cannot (legitimately) be thought of as a whole at all. However, the cognitive actuality of the infinitesimal cannot be brought to reality, i.e., it cannot be thought determinately, since the synthetic construction of

[24] In the Resolution of the Antinomies, Kant argues that the infinite is always *in indefinitum*, even in the case of space, where the totality is supposed to be given (*datum*) and not constructed (like time, *dabile*). Kant's appeal to the *quantum discretum* is made necessary by the fact that he cannot say that space is thought *in infinitum* (even if it is so given), for then he would be conceding that space is not merely an idea, but also a concept. This would be extremely problematic, given the requirement that we can only represent determinate form; we would then have a (definite) representation of the infinite, and thus the productive composition of the whole (*totum*) would have to be possible. Note that this seems to be necessary for the possibility of the apprehension of wholes, i.e., for their being brought to apprehension, if not necessarily recognized or "cognized" (in the sense of being objects of judgment through general marks).

the very same *quantum discretum* implies an infinite construction, since the real whole cannot be thought synthetically except through the totality of its parts. Accordingly, for Kant, "the quantum is indeed completed [i.e., is actually infinite in itself — BTG], but transcendentally, namely in the **subject**, which gives the quantum a **relation** to a unity [i.e., the transcendental unity of apperception — BTG], only such a determination of the quantum arises as incomplete and merely tainted with a beyond" (WL1; HW 21:305). As Hegel complains, the infinite is thereby explained as "progress to infinity, but only **transcendentally**, i.e., actually [*eigentlich*] represented as subjective and psychological" (WL1; HW 21:306). That is, the real infinite is actual in the transcendental correlation of idea and appearance, but its conceptual representation is an artificial and arbitrary limitation of the representation as a determinate magnitude; it is, therefore, no different to the "arbitrary" form of the limit as it is usually defined.

For Hegel, the failure of Kant's account of the infinite arises from an inadequate conception of the true concept of the infinite, or more specifically, in an inadequate conception of the concept of the limit as the *reality* of number. On the one hand, Hegel, like Maimon, endorses the principle of reality. That is, a magnitude, insofar as it is an object of thought, is a *determinate* object of thought. Accordingly, for the whole of an intuited magnitude (or a magnitude which is thought as a whole), the totality of its parts must also be real determinations of the cognitive subject. That is, the infinitesimal of subdivision is *real* if it is thought at all. However, unlike Maimon, Hegel does not think that we can make the explicit representation of this cognitive reality *actual* through merely symbolic cognition. For, the hypostasis of purely symbolic construction leads to dogmatism, and to transcendental realism. Rather, Hegel accepts the Kantian thesis that cognitive construction (actuality), is determinate and *eo ipso* finite. The possibility of the cognition of objective individuals, however, rests on the possibility of justifying the transcendental identification of the reality of the individual and the actuality of the concept. But now, the Kantian (and Aristotelian) conception of the infinite is not only antinomic, but aporetic. In order to justify the possibility of objective knowledge, it must be the case that the real determinacy of the individual can be correlated with its ideal determinability. This, in turn,

requires a new concept of the infinite as grounded in the actuality of the limit, i.e., in the ideal actuality of limitation as the determination of the individual. That is, we must conceive of the infinite as "the negation of negation, the affirmative, **being**, which has reproduced itself out of limitation" (WL1; HW 21:125). This just is the *principle of limitation*.

Let's begin by defining number as a series, in the style of Frege.[25] Let Ψ_n be the (recursively defined) series,

$$\sum_{0}^{n}(\sum_{0}^{n-1}+1), \sum_{0} = 0. \tag{4.2}$$

This just is the integers, thought of as constituting a *series*.[26] On such an account, the construction of a particular number (expressed as a series), just consists of successive additions — of composition. This is, as we have seen, the *actuality* of the number as a finite composition. However, such a number does not, for Hegel, have *reality*. The reason for this is that the value of Ψ_n for some n is not determinate with respect to the totality of reality. That is, in order for the concept of a number to have reality, it must be a *quantum*, i.e., a determination of the determinable.

Accordingly, insofar as number has reality, the construction of the integers just constitutes a special case of the composition of *unit magnitudes*. [27] We must therefore generalize Ψ

[25](Frege 1884) In this section, I will introduce a certain amount of formalism. This formalism is intended to serve as a shorthand to save both space and time. One might object that Hegel is not entitled to recursive definitions, and still less the (about to be introduced) quantifier dependence (i.e., undefined series containing undefined series). Certainly, Hegel is not entitled to such niceties, given his conception of logic as fundamentally syllogistic. However, Hegel takes himself to be so entitled, and the point here is expository, not critical. If we had to point to a place in Hegel's logic where Hegel's *Objective Logic* "breaks", it would be in the discussion of "Finitude" (WL1; HW 21:127ff.), where Hegel takes it to be the case that the principle of determinability ($\exists\neg p, \neg(\neg p) = p$)) can be generalized ideally, i.e., as the "ought" of idealism. In other words, Hegel identifies 'Idealism' with the claim that *for all* determinations of the determinable, *there exists* a determinate negation of the determinable: $\forall p \exists \neg p, \neg(\neg p) = p$. Here, Hegel's claim exhibits the quantifier dependence that is not provided by syllogistic logic. The assumption of the general claim of determinability is implicit in the first three moments of the *Objective Logic*, where Hegel assumes that in the indeterminate case ('being'), p is indistinguishable from its determinate correlate ($\neg p =$ 'nothing').

[26]Contemporary definitions of the integers are generally framed in set-theoretic terms. However, as should be clear from the *Physics*, Aristotle conceives of number as a series. Similarly, the Schematism shows that Kant also understands number (integers) to be a time series (even if it may be recursively defined), ultimately a *Zeitreihe*.

[27]Talk of 'units' of 'arbitrary magnitude' should raise some red-flags. For, we can only talk of the composition of unit magnitudes if we are *already* thinking of number in terms of a topology, i.e., an ordered set with a metric. These definitions would, therefore, be inadequate for a more general application. Indeed, as we will see, this presupposition is precisely the reason why Hegel's construction of the concept of real quantity cannot be adequate.

as a function of the magnitude of the units that are concatenated in the composition of a real magnitude. Let $\Psi(x)$ be the (functionally defined) relation,

$$\sum_{0}^{n}(\sum_{0}^{n-1}+\frac{1}{x}),\sum_{0}=0, \tag{4.3}$$

such that Ψ just is $\Psi(1)$, i.e., the correspondence between ordinals and integers. Now, $\forall x \geq 1, \Psi(x)$ is a non-converging series; accordingly, it has no finite limit. However, $\forall x < 1, \Psi(x)$ has a finite limit.

However, as we learned from Aristotle — and also the Second Antinomy — the "units" of the composition of the individual magnitude may be arbitrarily small. That is, the unity magnitude is dx, where $dx = \lim_{x\to\infty}\frac{1}{x}$. Accordingly, the construction of a magnitude — even a finite magnitude — implies an *infinite* construction of infinitesimal units. The *actuality* of a number consists of its composition through a constructive algorithm: Ψ_n. However, its *reality* consists of its infinite composition out of real units of the manifold: $\Psi_n(\Psi_m)$. Accordingly, for any really determinate magnitude A, its infinite composition consists of the infinite composition of infinitesimals:

$$A = \sum_{0}^{\infty}(\sum_{0}^{\infty}+\frac{1}{\sum_{0}^{\infty}(\sum_{0}^{\infty}+1)})$$

Accordingly, the determination of x consists of the infinite composition of infinite divisibles, i.e., it is *integration*: $A = \int_{-a}^{+b} dx$. However, as we saw above, the integration of the determinate magnitude A is possible only if we already know the *boundaries* of the magnitude to be integrated, i.e., the interval ab that constitutes the determinacy of the whole as a bounded totality.

Now, the magnitude of appearance is infinite in two different ways. First, it is *synchronically* determinate as *reality*, i.e., as the totality of conditions that must be given as a condition of the appearance of the whole as a determinate (bounded) whole *an sich*. Second, it is *diachronically* determinate as the *actuality* of the whole as a transcendental construction, i.e., as

the composition of the whole out of the sum of its parts *für sich*.[28] The infinite division of the whole of appearance represents the intuitive, qualitative or synchronic totality of the whole; the infinite composition of the whole of appearance represents the conceptual, quantitative, or diachronic totality of the whole. Insofar as a magnitude is real — i.e., is intuited as a real individual — it is synchronically infinite. However, insofar as a magnitude is actual — i.e., is the actuality of the concept — it is diachronically infinite. The possibility of a judgment of experience — i.e., the possibility of transcendental synthesis — depends on the transcendental identification of these two moments: synchronic reality and diachronic actuality as magnitude *an und für sich selbst*.

The concept of a magnitude — and *a fortiori* the concept of the infinite magnitude — therefore invokes two definitions of the infinite. The diachronic concept of infinite compossibility (i.e., the imperfect infinite) must contain in its definition its own supercession. This is the case, for example, in the definition of the *syncategorematic infinite*: $\forall x \exists y = x + 1, x < y$. Accordingly, Hegel will say that the the first definition "contains in itself ... externality", that is, the concept of the infinite contains a number that is outside the scope of the universal quantifier. The synchronic concept of infinite divisibility, however, is "the negation of the same", that is, the negation of externality, or the assertion that $\exists x \forall y = x + 1, \neg(x < y)$. This, however, just is the definition of the *categorematic infinite*: $\exists x \forall y = x + 1, x \geq y$. Here, then, is Hegel's full definition of the infinite: "The infinite quantum rather contains in itself first externality [i.e., $y = x + 1$, which is greater than any actual x — BTG] and second the negation of the same [i.e., $\neg(x < y)$ — BTG]" (WL1; HW 21:306-7).

The determination of the individual (*das Etwas*) — as either finite or infinite — rests on possibility of its real limitation, i.e., on the real, bounded totality. The concept of such an individual, however, consists of its infinite composition. Accordingly, it is not the magnitude itself (i.e., the sum of its parts) which makes the magnitude an individual, but rather the reality of its boundaries: "it is not only the immediate identity of the something with itself, but rather

[28] That, Hegel, like Maimon, takes Kant to endorse the Aristotelian restriction on counting to finite numbers; composition is therefore necessarily finite.

that (immediate identity) through which the something is also that which it is **in itself**" (WL1; HW 21:118).[29] Let's call this the *principle of limitation*.

The Principle of Limitation The individual (X) of the manifold of reality (M) is determinate only insofar as it is a bounded totality. This is made possible through the *dialectical identity* of the individual with its ideal boundary: X is* ($x + 1$ is* x).[30] This just is the assertion of the absolute limit as the identity of the *totum reale* and the *compositum ideale*, i.e., in the absolutely determinate *quantum*. Quantitatively, the ideal boundary is expressed as the claim that a magnitude has determinate limits, which implies:

(a) The *reality* of the *boundary* as a determinate number: $\exists x \in M \forall y \in X, y = x + 1, x \geq y$. This just is the assertion of the categorematic infinite as the absolute determinacy of the real (*reale*).

(b) The *actuality* of the *limit* as a finite composite: $\forall x \in M \exists y \in M, y = x + 1, x < y$. This just is the assertion of the syncategorematic infinite as the absolute determinability of the composite (*compositum*).

(c) The *idea* of the individual as the identity of its limitation (*an sich*) and its determination (*für sich*) : $x + 1$ is* x. This just is the expression of the limit of reality as *continuity*.

Prima facie, the *principle of limitation* just is a contradiction. For, it asserts the "identity" of the limit (x) and the boundary ($x + 1$). However, this identification is not the simple identification of identity ($A = A$), but the "dialectical" identification of synchronic divisibility and diachronic compossibility. That is, it is the ideal unity of the real totality (the perfection of the synchronic totality) and ideal composition (the perfectability of the diachronic totality). We might also say, the dialectical anticipation of the individual is, strictly speaking, a contra-

[29] This is the controversial "Determinate negation" through which A is determined (explicit) as *both* itself ('A') and its negation as determination of the totality ('$\neg X$').

[30] Here, 'is*' designate the Hegelian copula, i.e., the idealized identification of synchronic and diachronic infinities.

diction, but a contradiction that anticipates its own sublation in the perfection of composition, i.e., in the totalization of the diachronic mode of composition.

4.2.2 The Principle of Intelligible Totality

The *principle of intelligible totality* does not introduce a new principle into Hegel's logic. Rather, it is an interpretation of the dialectical copula ('is*') of the *principle of limitation* as a temporally modalized identity that is true only in its perfection, i.e., in the completeness of time itself. As we have just seen, the reality of the composite (i.e., a *compositum reale*) and the ideality of the whole (i.e., a *totum ideale*) consists of the "dialectical" identity of the limit (x) and the boundary ($x+1$), i.e., in a *contradiction*. The sublation of this contradiction is the idea of the absolute unity of the composite and the whole — of reality and ideality — in the *actuality* of form. The *reality* of form is given immediately — or qualitatively — in the bounded individual, i.e., as "something" (*das Etwas*) that is determinate in itself through its limitation. The *actuality* of form, however, implies an infinite diachronic construction. That is, it implies the perfection of time: history. Actuality, therefore, is not given, nor can it be cognized directly. Rather, actuality is anticipated in the *act* of construction itself, as the *constitution* of history through the progressive approximation (actualization, *Verwirklichung*) of form. Through the act of composition, time is realized in its essential character: history as the anticipation of the actuality of the *eidos*.

The principle of intelligible totality, therefore, is the interpretation of the correlation of the Aristotelian moment of formal cognition — its origin in the reality (givenness) of the finite individual, or form as *hypokeímenon* — with the Platonic moment of formal thought — the absolute foundation of the *eidos* in transcendent(al) intelligibility, or form as *noésis*. The principle of totality, therefore, is a *task*: to bring about the actuality of the concept through the reconstruction of the absolute intelligibility of the given.

The Principle of Intelligible Totality The principle of totality implies that the *task* of absolute knowledge consists of the *anticipation* of intelligible totality through the *actualization* of

the concept. Accordingly,

(a) The reality of the totality is *given* absolutely as synchronic determinacy: the *hypokeímenon*;

(b) The actuality of the totality is *produced* in the diachronic determination of the determinable: the *eidos*;

(c) The identification of reality and actuality consists in judgment as the positing of the infinite through the finite: the active anticipation of *noésis*.

The *principle of intelligible totality* implies that we accept the *principle of limitation* as the ground of the idea of the individual. That is, the individual is real insofar as it is given (synchronically) as a determinate totality, and is actual insofar as it is constructable (diachronically) as a the determination of the determinable. Accordingly, the principle of limitation implies (a) the *principle of reality* as a condition of the empirical cognition of the real individual, and (b) the *principle of determinability* as a condition of the transcendental cognition of the actual concept. In other words, judgments of experience — as the schematic comparison of the reality of the empirical individual and the actuality of the intelligible individual — rests on the anticipation of (Platonic) form through the finite recomposition of (Aristotelian) individuals.

4.3 Logic of Anticipation

The consequences of Hegel's principle of intelligible totality are many and varied. These need not detain us here, however. We have explored the constitutive principles of the formal monism of Maimon and Hegel only insofar as is necessary to derive solutions to the fundamental limitations of Kant's transcendental logic. We saw above how Maimon's principle of determinability allows us to interpret the individual totality as a function of the relation of the part to the whole. Hegel's dialectical conception of the infinite — the principle of totality — allows us to understand how such a logic may be realized within a finite subject without resulting in the

Maimonian dogmatism of the infinite symbolic cognizer.

In the following section, I will provide a preliminary analysis of Cohen's *Logic of Origin*. The analysis is merely preliminary because we will not be able to draw out all of the consequences of Cohen's logic until we are able to investigate the doctrine of the manifold — or the infinite continuum — in more detail. This investigation is the task of the next chapter; however, it will rely essentially on the principles developed here.

4.3.1 The Table of Judgments

The interpretation of Cohen's *magnum opus — Logic of Pure Knowledge —* is a difficult task for a number of reasons. First, it carries through the historico-critical methodology of the *Principle of the Infinitesimal Method and its History* and the second edition of *Kant's Theory of Experience*. Second, the logic is presented as a "table of judgments", which is clearly modelled after Kant's table of judgments, but which — in accordance with the demand of *Kant's Theory of Experience* — completes the task of a Deduction of the "categories" (now judgments) through the demonstration of their exhaustive account of the necessary materials for natural science, by showing how the categories are just "names" for functions of the composition of a point-set topology; thus, despite the superficial similarity of the Kantian and Cohenian doctrine of judgment, there are in fact radical differences between the two. Third, the text responds throughout to the logical programme of earlier forms of systematic idealism, and in particular, to Hegel's *Objective Logic*, albeit in very indirect ways. And, as we have just seen, disentangling Hegel's logic of 'quantity' is no simple task.

I will not offer a detailed analysis of the system of Cohen's *Logic of Pure Knowledge* here. However, I will provide a structural outline that interprets Cohen's logic as a transformation of the project of transcendental logic through the reinterpretation of the table of judgments. Here, for the sake of reference, is Kant's original table of judgments (see **Table 4.1**).

Cohen's reinterpreation of the Kantian table of judgments takes place through three distinct transformations:

Table 4.1: Kant's Table of Judgments

Quantity	**Quality**	**Relation**	**Modality**
Universal	Affirmative	Categorial	Problematic
Particular	Negative	Hypothetical	Assertoric
Singular	Infinite	Disjunctive	Apodictic

(a) Qualitative judgment (the forms of pure logic) changes places with quantitative judgment (the forms of mathematical judgment).

(b) The three moments of the laws of thought are no longer structured around the principle of contradiction and the (synthetic) dependence of intuition (which, recall, is banished from Cohen's interpretation), but rather on three distinct, but systematically related laws of thought:

   (i) Origin;

   (ii) Identity;

   (iii) Contradiction;

(c) The table of judgments is reinterpreted along the lines of the "table" of principles, where these represent the constructive principles of representations (or cognitions, judgments, knowledge, *Erkenntnisse*) of mathematical natural science.

Quality and Quantity

The first transformation to the table of judgments implies the priority of the judgments of general logic (the principles of thought) over the quantitative judgments of mathematics (the synthetic principles). More importantly, it implies the priority of a constructable manifold of determinables over the quantification of a *factical* manifold of sensibility. For, Kant's judgments of quantity are only able to appear first in the order of explanation (and thus of apprehension,

reproduction, recognition and cognition) insofar as they take up a manifold that is (paradoxically) already given *as a manifold*.[31] As we will see, however, most post-Kantian systematic idealists take the problem of the manifold not to be a problem of givenness (and thus, perhaps, a problem invoking psychodynamics, or a kinematic conception of sensibility), but rather to be a problem of the determinability of a "set" of perceptual states, perhaps on the model of Leibniz's *petites perceptions* (see **Chapter 5**) and thus a problem of *formal monism*.

Accordingly, Cohen, like Hegel, claims that the qualitative moment of judgment precedes the quantitative moment: "quality leads to **quantity**" (LRE; CW 6:121). However, whereas for Hegel the qualitative moment of judgment is taken to imply an immediate, conceptually indeterminate mode of cognition (intuition), Cohen takes the qualitative moment to be essentially productive (positive) in character. For Hegel, the first judgment is a positive judgment: the "positing" of being as a total determinable. As we have seen, this determinable is a *totality*, rather than a mere whole, since its qualitative moment, or the implicit manifold of its parts, is already implied in the mere positing of being as such. As we will see in a moment, the reason why the qualitative judgment precedes the qualitative judgment is that Cohen and Hegel both understand logic to include the production of intentional content. That is, whereas the Kantian logic of judgment derives its objective orientation from the "given", Cohen and Hegel both understand content to derive from the activity of consciousness, as an act prior to the quantitative judgment, whereby the intentional content of thought may be determined.

The Laws of Thought

Now, following the first transformation to the table of judgments, judgments of quality take precedence over the judgments of quantity in Cohen's table of judgment. Accordingly, the first class of judgments is called the "Laws of Thought" (*Denkgesetze*). These are the original 'thought-acts', or better still, 'representational constructs' through which the content of thought is first produced.

[31] This is, in effect, the requirement that intuitions be provided as already intelligible structures, lest they fail to be constructed, and thus cognized, i.e., the core argument of the Transcendental Deduction.

The Judgment of Origin Posits the original determinable object of thought — which is no object but a "thought-thing", or what Cohen sometimes calls an "aught (*Ichts*)." Accordingly, the judgment of origin is an act of thought, *whereby the original intentional content of any determinate thought is initially posited*. By contrast, the Kantian judgment of affirmation, which asserts the identity of a conceptual representation and a factical modification of intuition (*Vorhandenheit*), has the given as its intentional object. The intentional content of a judgment of experience, for Kant, is always determined at least in part by the givenness of appearances. The Hegelian concept of Being, on the other hand, posits being as a whole; the affirmative judgment of being asserts the identity of the qualitative and quantitative determinacy of the same, i.e., it asserts being as absolute totality. Thereby, the intentional content of any thought is posited in the affirmation of being as the absolutely determinable manifold of thought. The judgment of origin, however, is the purely productive activity of the creation (positing) of a determinable object of thought through the privation of every determination. It is thus a judgment of *privation*, not the affirmation of determinability (or, conversely, the determinate negation of any determinations).[32] Crucially, the judgment of origin does not assert a relation between two representations (heterogeneous or homogeneous), but is itself the production of representational content; this content, however, need not be realized as mental content — it might also be represented through cultural artifacts.

The Judgment of Identity does not derive from the Aristotelian concept of "adequation" (LRE; CW 6:95), nor from the Hegelian conception of the realization of the copula (WL1; HW 21:308), nor from any conception of equality (LRE; CW 6:101), but rather the simple affirmation of the determinable: "A is A and remains A, whenever it is thought. Whenever it is thought, **whenever it is rather represented**; its is thought only as the **one** identity" (LRE; CW 6:96). Identity is therefore not, for Cohen, the affirmation of

[32]Cohen designates the difference by way of a reference to the greek privative particle *mè* versus the negative particle *on*. This distinction is also central to the apophatic theology of *Religion of Reason: Out of the Sources of Judaism* and *Ethics of Maimonides* (Cohen 2004).

identity, but rather the affirmation of the unity of the unique. We see once more that judgment does not consist of a relation between representations, but is rather a productive act, whereby a determinate representation is posited. The judgment of identity is therefore also a positive thought act, whereby the uniqueness of the indeterminate representation is asserted. This does not mean that this representation cannot be multiply instantiated. Rather, it means that whenever it is instantiated, it will be an instance of one and the same representation.

The Judgment of Contradiction Like the judgment of origin and the judgment of identity, the judgment of contradiction does not assert any relation between thought objects or a thought and its correlate, but rather consists of a judgment about the origin: that it is not something other than itself. The judgment of contradiction thus only relates negatively to the positive content of a judgment: "Between A and a not identical A, there is no reconciliation for thought. It must become nothing, or rather be reduced [*vernichtet*] to nothing, so that a judgment about its content can only be completed in this direction" (LRE; CW 6:107). However, the judgment of contradiction does not derive from the principle of contradiction, but itself produces the principle of contradiction insofar as it asserts that everything that is not identical to A is a contradiction of the identity of A. Judgments of contradiction, therefore, may be thought of as the complement of the judgment of identity.

Cohen's table of judgments has a number of remarkable features. First, the three moments of Cohen's table of judgments do not derive from the traditional doctrine of judgment, not even in its Kantian formulation. Second, the three "laws of thought" do not derive from the traditional "axiomatic" principles of thought: the principle of identity, the principle of contradiction, the principle of the excluded middle.

Perhaps most unexpectedly, the laws of thought are laws of the production of representations. That is, they are not comparisons between representations — and subsequent assertions of truth or falsity. Rather, they consist of the production of intentional reference, or that through

which truth is representable, if not represented. As we will see later (see **Section 6.2**), this has important consequences for the systematic architecture of Cohen's system. Whereas the logic of judgment is for Kant, Maimon, and Hegel, at the same time a *doctrine of cognition*, or a theory of mind, Cohen's theory of judgment is not a theory of cognition, but a theory of cognitive representation. It distinguishes itself from earlier projects in the history of logic insofar as it is not (immediately) concerned with the cognitive instantiation of the system of representations that it enables (i.e., psychology). However, unlike some contemporary views of logic, Cohen still understands logic to be a product of the activity of a cognitive subject oriented toward the cognition of the particular; thus it is an *Erkenntinislogik* and not just a symbolic algebra. Even if the products of the logic of origin are not instantiated as mental states, they remain *Erkenntnisse* — cognitive objects — regardless of the medium in which they are recorded and propagated.[33]

As we will see, the different classes of judgment (Laws of Thought, Mathematics, Mathematical Natural Science, Method) define different types of representations, but the three 'attitudes' towards these forms of representation remain constant throughout the derivation of the table of judgments.

[33] In his *The Algebra of Logic*, Couturat writes:

> The fundamental laws of [Boole's] calculus were devised to express the principles of reasoning, the "laws of thought". But this calculus may be considered from the purely formal point of view, which is that of mathematics, as an algebra based on certain principles arbitrarily laid down. It belongs to the realm of philosophy to decide whether, and in what measure, this calculus corresponds to the actual operations of the mind, and is adapted to translate or even to replace argument we cannot discuss this point here. The formal value of this calculus and its interest for the mathematician are absolutely independent of the interpretation given it and of the application which can be made of it to logical problems. In short, we shall discuss it not as logic but as algebra. (Couturat 1914, 3)

Cohen's project is clearly not opposed to that of the formalists. For, both see logic to be a method of unrestricted construction through principles.

The difference between the two projects becomes clear when we consider that Cohen is only interested in cognitive representations, that is, in representations that may be thought of in relation to objective being. In other words, every cognitive representation is a determination of possible reality. The logic of the formalists, on the other hand, is a pure symbolic algebra, which presupposes no specific purpose for its representations. While there are similarities between the two projects, Cohen's logic is at most a sub-species of symbolic algebra, with the added hypothesis that its content is necessarily object-oriented. For many, this will be seen as a devastating limitation in Cohen's logic. Others, however, and in particular those with constructivist sympathies, will see in this extra hypothesis a necessary connection to the content of thought.

The Table of Synthetic Principles

The third fundamental change to the table of judgments is already anticipated by Cohen's *Kant's Theory of Experience* and the interpretation of experience as the fact of science. As we have seen, Cohen's interpretation of the task of the Deduction relates the table of judgments not to the categories (as necessary a priori forms of synthetic cognition) but to the "table" of principles from the Analytic of Principles: "Kant's table of judgments is not oriented towards his table of categories, but towards his table of **principles**" (LRE; CW 6:73). Accordingly, the headings of Cohen's table of judgments are to be interpreted not through the headings of the Kantian table of judgments (or the categories), but rather through the headings of the "table" of principles: The Axioms of Intuition, The Anticipations of Perception, The Analogies of Experience, and The Postulates of Empirical Thinking in General.

We have already seen the first three forms of judgment — origin, identity and contradiction — in outline. However, a few brief comments will still be of some use here. First, Kant's **Anticipations of Perceptions** contains two crucial theoretical developments in Kant's transcendental logic: (a) the concept of the infinitesimal magnitude as a "degree" of affectedness, and (b) the interpretation of space and time as *quanta continua*. *Prima facie*, these are incongruent characterizations of the relation of an intelligible unity (the transcendental object) to an empirical appearance (intuition); the former concerns the *psychodynamic* aspect of affection, whereas the latter appears to concern a property of mathematical manifolds, i.e., the continuity of space. As we might anticipate, however, Cohen's logic rejects the premise that degrees of affection measures a psychodynamic principle of cognition. Rather, Cohen — in *The Principle of the Infinitesimal Method and its History* — interprets the problem of degrees of affection (intesive magnitude) as the problem of *differentials*, that is, as a problem for the mathematics of the calculus. Accordingly, the apparatus of 'reality' and the implied continuity of the manifold of such a judgment is not, in Cohen's interpretation, a judgment of quality at all, but rather the 'original' form of a judgment of quantity. That is, it is the mathematical form of the judgment of origin.

The interpretation of the **Axioms of Intuition** as categories of judgment is relatively straightforward. We have already seen that Cohen adopts 'reality' as the 'original' form of judgments of mathematics; that is, it is the sheer (indeterminate) determinability of a relational continuum, represented categorially as the concept 'number'. Accordingly, the 'identity' of reality is merely the affirmation of a *determinate* number: or, the unity that can be grounded in the unity of the principle of its construction; thus it is the judgment of plurality; this identity is equivalent to the (once again, categorial) production of time in the concept of the (continuous) series.[34] The 'contradiction' of reality is merely the negation of a *determinate* number, or the positing of the manifold as such; this 'contradiction' is equivalent to the (once again, categorial) production of time as the (*in potens*) totality of the manifold: "**Totality in thinking produces that of space.**"[35]

The interpretation of the **Analogies of Experience** is somewhat more complicated. In the A edition, Kant describes the Analogies of Experience as follows: "As regards their existence, all appearances stand *a priori* under rules of the determination of their relation to each other in **one** time" (KRV; AA A176). However, the B edition makes a substantial revision to the general principle of the Analogies of Experience: "**Experience is possible only through the representation of a necessary connection of perceptions**" (KRV; AA B218). Kant's reason for making the change in the B edition is clarified in this addition to the opening of the Analogies of Experience:

> Now in experience, to be sure, perceptions come together only contingently, so that no necessity of their connection is or can become evident of the perceptions

[34] As usual, Cohen's formulations are somewhat cryptic:

> The future contains and conceals the character of time. In the anticipated future the **past** sequences and arranges itself. The past was not first; rather, **the future comes first**, and the past depends on it. From the point of view of the **not-yet** the **no longer** appears. Thus in them emerges both points which form the **series**. Thus emerges in them **the first form of plurality: in the particularization of the past from the original act of the future**. (LRE; CW 6:154)

[35] (LRE; CW 6:202) Here, however, we must be extremely careful. The production of space does not imply the (explicit) production of the totality of the manifold, nor its explicit determination. Rather, 'contradiction' as a form of judgment implies the denial of a determinate number; it is therefore the merely potential determination of a multiplicity (number) that is not at the same time a unity (which would be a judgment of plurality). Space itself is not determined, but rather only the category of space as a manifold that can be given in its totality (i.e., an actual infinity).

> themselves, since apprehension is only a juxtaposition of the manifold of empirical intuition, but no representation of the necessity of the combined existence of the appearances that it juxtaposes in space and time is to be encountered in it. But since experience is a cognition of objects through perception, consequently the relation in the existence of the manifold is to be represented in it not as it is juxtaposed in time but as it is objectively in time. (KRV; AA B219)

This qualification is necessary to the interpretation of the "analogy" of experience as an analogy to a unified *object* (and accordingly matter), and not just to possible configurations of the manifold (i.e., analysis, or the science of alteration, the theory of motion). As we have seen (see page 125), the interpretation of the correlate of experience as object, matter, and ultimately substance is not a relation that is contained in the mere concept of the juxtapositions of space and time (analytical geometry, or calculus), but rather in the implicit physcalism of Kant's empirical realism.[36] For Cohen, the Analogies of Experience are now interpreted as judgments of mathematical natural science; that is, the physicalist interpretation of the analogies of experience is made explicit in Cohen's logic:

> Movement alone would be the dissolution of space in time. However, the relation realized between time and space by movement is not exhausted in this negative achievement. We henceforth recognize the **positive** contribution that thereby participates in persistence [*Erhaltung*]. It is substance; thus it is not absolute; it is the **correlate of movement**. (LRE; CW 6:234)

Here again, Cohen's claim may be somewhat obscure. However, point is that the *mere* relation of space and time produces possible functions of space with respect to time ($f(t) = f(x)$). What makes it possible to think of "persistence" as something durable in time is the judgment of substance. However, judgment is not (as it is for Hegel), the judgment of an absolute identity between the qualitative and the quantitative expression of the concept, but rather the (as yet

[36] Similarly, the apparent requirement that *necessity* be included in the explanation of the objectivity of the object — rather than merely the possibility of conjunctions of the manifold — is the reason Hegel offers for describing the third class of judgments (judgments of relation) as judgments of necessity:

> The determination, to which generality continued, is, as we have seen, the **in-and-for-itself** or **objective generality**, which in the domain of essence corresponds to **substantiality**. It differs from substantiality in that it belongs to the **concept** and therefore not only the **inner** but also the **posited** necessity of its determinations. (WL1; HW 21:335)

indeterminate) claim that movement correlates to *something* that isn't movement.

Once more we see the general structure of Cohen's logic. The 'original' moment of the judgment of mathematical natural science is merely the positing of a functional relationship between space and time (i.e., reality), as the determinable correlate of objective representation: "Substance [has become] a correlate, namely that of movement, in which the reality of the letter [i.e., of the variable x — BTG] enters into the proposition of equality [i.e., the function cognitive function of origin]" (LRE; CW 6:302). Accordingly, the second moment of the logic is the *affirmation* of the unity of the origin (thought) and a mathematical function ($f(x) = f(y)$) in the representation of an objective correlate. This affirmation is, appropriately, called a 'judgment of law', since it asserts the necessity of the function of substance, although it thereby attributes no material or substantial unity to the function in question. Thus, the function of law is the affirmation of an identity between thought and a mathematical function as a determination of reality, such as, for example, the law of gravitation.

Whereas both Kant and Hegel appear to ground the necessity of the individual in the concept, Cohen has already grounded necessity in the judgment of law. The 'contradiction' of the class of judgments of mathematical science therefore grounds the individual in the *negation* of substance: the individual is determined through something that is not (yet) a law; the (function of identity) that constitutes the individual is not a law, but (as the negation of substance) the question: what is this?[37] Indeed, Cohen traces the (historical) origin of the concept to the modern problem of the determination of the life form. Thus, the individual "**is the unity of the natural form, as a life form**, which the organs of the organism perfect" (LRE; CW 6:350). The concept thus rests on the idea of an *internal* articulation of reality.[38] Accordingly, the judgment

[37]Or, more generally, as the negation of the question of being: *ti esti*:

> If the concept means in general the question '**What is?**' and thereby the **laying-of-foundations of being in thought**, then the judgment, which indeed first produces the concept, may be the question '**What is not?**' However, the not implies the *me* (LRE; CW 6:92).

[38]The concept of matter is necessary (a) for the interpretion of the Analogies of Experience in their objectivity (b) for the hypostasis of intelligible forms in the *Metaphysical Foundations of Natural Science*, and (c) for the explanation and interpretation of the First and Second Antinomies. However, in the *Metaphysical Foundations of*

of concept takes the place of disjunctive judgment in the tables of Kant and Hegel; this is because the concept is in-itself contradictory as that which is (a) thought of as a conceptual unity but which (b) fails to have the unity of a law. Like the judgment of totality — which provides the totality of space *in potens* — the judgment of concept provides the structure for thinking the individual without the intrinsic determinacy of 'substance' or 'law', that is, as a contingent individual. Thus, the 'contradictory' nature of the concept must ultimately be resolved into the 'identity' of a law, or remain problematic for thought: "**The concept is a question and remains a question**; nothing but a question. **So too must the answer that it receives be a new question**, and give rise to a new question" (LRE; CW 6:378). Whereas Hegel understands the concept to be the unfolding of the *necessity* of logic, Cohen, on the contrary, understands the concept to represent the open-endedness of thought (i.e., the absence of *Aufhebung*) insofar as it can — and must — think through representations of individuals: "**The concept ... does not want to and may not become complete**. Its unity must eternally remain **open**" (LRE; CW 6:378). Cohen, like Hegel, understands *historicity* to be an essential part of the constitution of our thought (including our 'categorial' concepts); unlike Hegel, however, Cohen does not locate this historicity within the enclosed totality of thought, but rather in the open-ended

---

Natural Science, Kant appears to derive the concept of matter from the concept of repulsive forces in the Dynamics, even though (a) the concept of matter is necessary to the intelligibility of the claims of the Phoronomy, and (b) the forces that constitute matter are actually not justified in the Dynamics, but only in the Mechanics. (Hegel points this out when he takes over the concepts 'repulsion' and 'attraction' not as *metaphysical* concepts of *quality*, but rather as *logical* concepts of *quantity* (WL1; HW 21:202).)

We see again, in the resolution of the Second Antinomy, that Kant's understanding of the problem lies (at least in part) in an organicist conception of the objective unity of the concept:

> To assume that in every whole that is articulated into members (organized), every part is once again articulated, and that in such a way, by dismantling the parts to infinity, one always encounters new complex parts — in a word, to assume that the whole is articulated to infinity — this is something that cannot be thought at all ... Thus only experience can settle how far the organization in an articulated body may go; and even if it was certain to attain to no organic parts, such parts must nevertheless lie within a possible experience. (KRV; AA A526-7/B554-5)

The tension between ""nature" taken adjectively (*formaliter*)" as the "connection of determinations of a thing in accordance with an inner principle of causality" and ""nature" taken substantively (*materialiter*)" as the "sum total of appearances insofar as these are in thoroughgoing connection through a principle of causality" is once again in evidence (KRV; AA A418/B446).

totality that is the concept. That is, whereas history, for Hegel, discloses the necessary structure of the concept, Cohen takes it to be the case that the concept can only complete itself in history through the dissolution of the individual into the universality of law (i.e., when the unlimited judgment is replaced by the judgment of law as a function of identity: 'A is A', or 'Gravity is Gravity'.

The judgments of method — Cohen's interpretation of the **Postulates of Empirical Thinking in General** — will be treated in the next chapter. However, we can summarize the forms of judgment in a table (see **Table 4.2**). For the purposes of comparison, here also is the

Table 4.2: Cohen's Table of Judgments

Laws of Thought	**Mathematics**	**Mathematical Natural Sciences**	**Method**
Origin	Reality	Substance	Possibility
Identity	Plurality	Law	Actuality
Contradiction	Totality	Concept	Necessity

table of categories, or the interpretation of the significance of the functions of judgment for the objectivity of objects in the *Critique of Pure Reason* (see **Table 4.3**).

Table 4.3: Kant's Table of Categories

Quantity	**Quality**	**Relation**	**Modality**
Unity	Reality	Inherence & Subsistence	Possibility — Impossibility
Plurality	Negation	Causality & Dependence	Existence — Non-existence
Totality	Limitation	Community	Necessity — Contingence

4.3.2 Platonism

We have already seen (see **Section 2.3**) Cohen denies the primacy of ordinary objects as the explanadum of judgments of "experience". We saw, in turn, that this interpretation has wide-ranging consequences for Cohen's interpretation of the Transcendental Deduction, and for the Transcendental Analytic in general. However, whatever Aristotelianism remains within Kant's transcendental logic — and thus also in the logics of Maimon and Hegel — is not to be found in Cohen's *Logic of Pure Knowledge*. The reason for this should be clear. If objectivity consists of judgments of experience, then, for Cohen, objectivity is a property not of objects, but of the representations of scientific knowledge. More precisely, objectivity is a property of laws of natural physics. As early as 1877 — in *Kant's Foundation of Ethics* (*Kants Begründung der Ethik*) — Cohen clearly expresses his deflationist view of the objects of ordinary experience: "The object is a fact of laws, not objects in the heavens [*Sterndinge*]" (KBE; CW 2:21).

Nevertheless, Cohen's deflationism is not a nihilism. For, he still grants the existence of a thing-in-itself, the transcendent correlate of our objective judgments. However, the thing in itself is, unsurprisingly, not an object, a 'Ding', but is itself a law, or, more precisely, the lawfullness of the law:

> The conditions of given experience — the synthetic principles — determine the possibility of experience, and the possibility of experience grounds the objects of experience. In other, more modern words: the principle, the **law** is the principle of reality; the law is the thing in itself. (KBE; CW 2:27)

Thus, idealism does not undermine the possibility of knowledge of objects; Kant has, in this sense been misunderstood. Rather, it is Transcendental Idealism that makes objects possible in the first place. For, objects have neither (a) an objective unity that derives (immanently or a priori) from the manifold of determinables, nor (b) an objective existence that is directly validated through the judgment of experience. Rather things exist insofar as — and just insofar as — they are grounded in the constructive principles of thought (mathematics) and the logical forms of judgment that permit the construction of the universal — and thus ideal — laws of the constutition of the individual. Thus, Cohen argues that transcendental idealism — the ideality

of the object in the laws of its constitution — is not anti-realism at all, but rather fundamentally the only realism available:

> Do things not exist? and is the idealist in every sense a skeptic? Absolutely not! Rather, things are, because of and insofar as ideas are. Things are appearances. Are they therefore illusions (*Scheine*)? Absolutely not! Rather, they are appearances because of and insofar as there are **laws**, in which the reality of the appearance is grounded, in which they **consist**. (KBE; CW 2:20)

Whatever we might make of this interpretation of Kant's project in the *Critique of Pure Reason*, this is no Strawsonian descriptive metaphysics.

Accordingly, we see that the Aristotelian moment of post-Kantian systematic idealism has been negated. Cohen's logic — unlike that of Hegel, Maimon, and perhaps also Kant's transcendental logic — does not begin with a real (given) individual to be determined with respect to its determinability (absolute or relative). Rather, Cohen's logic begins with the universal in order to found the individual in these laws. Thus,

> the **individual** itself, in its apparent finitude, counts as a bullwark and as secured content. However, clearly the object ought not simply mean the individual thing, but merely represent it. But that it represents it, that it can and must represent it, this is what is new in the problem and interest of thinking; that it does not merely dissappear into the general; that it, like the **monad**, encloses the whole universe in itself. The new character of the the object depends on such a new appreciation of the individual as the representative of the totality. [39]

The object, therefore, is literally the representative (*Vertreter*) of the objectivity of the totality, i.e., of the system of laws. That is:

> *the fact that it is possible to form concepts of individuals shows the unity of the system of laws that are the ground of the reality of the object and the objectivity of the object itself.*

And so, while Cohen denies that the object has any *ontological* validity, it has *epistemic* unity as the realization of the unity of the laws through which the totality of reality is constructed.

[39](LRE; CW 6:319) "Every singular substance expresses the whole universe in its own way, and in its notion are comprised all its events with all their circumstances. And the whole series of external things" (G II 12) (Leibniz 1989).

Accordingly, we interpret Kantian realism as essentially grounded in the idealization of the concept of the object:

> Thus it can be explained that **Kant** took up the concept of the object in the most important formulation of his highest principle: the conditions of the possibility of experience are at the same time the conditions of the possibility of the object of experience. (LRE; CW 6:319)

The Principle of Pure Individuality

Hegel's *principle of intelligible totality* consists of the temporal anticipation of the "good beyond being" through the finite composition of finite parts. That is, *compossibility* is temporally modalized. Thus, the significance of the dialectical copula 'is*' may be interpreted as the idealization of composition. However, as we have seen, Cohen interprets the sensible condition as *form* in the Platonic sense: *eidos*. Composition, therefore, is not temporally mediated, but is thought absolutely as the *transcendental* or *pure* condition of the possibility of the object. Thus, whereas the central problem of Hegelian (and Maimonian) logic is the actuality of the *determination* of the determinacy of the determinate (given), the problem for Cohenian logic is the reality of the *production* of the product of thought (purity). It is the reality of the individual, which, for Cohen, is problematic from the point of view of pure thought.

As we saw above, Hegel understands the relationship between possibility, actuality and necessity to be a temporal modality; history itself is realized in the immanent relation between the qualitative *hypokeímenon* as the absolutely real and the intelligible *eidos* as the perfection of composition, or the realization of the rational representation of the manifold of reciprocal determinations. However, whereas Hegel understands the completion of the concept to be *immanent* in the object itself (i.e., as its *energeia*), Cohen takes the problem of the completion of the individual to be *transcendent* with respect to intelligibility (*noésis*): thought is ideally correlated with its object, the fact of culture. That is, "**correlation must take the place of immanence**" (LRE; CW 6:236).

For Cohen, "possibility" is nothing other than pure consciousness, i.e., the system of "substance" as a synchronic order of representables: the infinite, unbounded compossibility

of the infinitesimal. That is, "**it is possible, that means: it makes possible new cognitions**" (LRE; CW 6:426). Possibility, therefore, is not an order of temporally modalized construction (factical consciousness), but rather the pure, formal system of the synthetic principles: logico-mathematics. However, the problem of consciousness is, as we have seen, concerned with the object: "**the problem of consciousness is thereafter the problem of the object**" (LRE; CW 6:426). The problem of the object, for Kant, is the problem of *objective* consciousness, or, as we have seen, the problem of the schematic coordination of the empirical individual and its intelligibility in judgments of experience. That is, the objectivity of judgment consists essentially in the relation of transcendental judgment to the factical (and thus objective) nature of feeling as the absolute determinacy of the subject.

For Maimon and Hegel, the problem of the schematic coordination of actuality and possibility is — through the principle of reality and the principle of determinability — immanent within a unified system of determinables: consciousness as such. Here, feeling is no longer really or modally distinct from thought. Indeed, for Maimon, the distinction between absolute determinacy (feeling) and ideal determination (thought) consists in the distinction between a real order of determinables and its symbolic representation (see **Chapter 5**). For Hegel, however, the distinction between feeling and concept is modal; however the modality is not logical but *temporal*. That is, feeling (qualitative immanence) and thought (quantitative transcendence) are absolutely identical; their difference arises only in time. This just is the result of the interpretation of the *principle of intelligible totality*.

From the point of view of an *Erkenntniskritik*, actuality — the fact of culture — does indeed arise with respect to possibility — *noésis* as pure intelligibility. Whatever is actual is in some sense "in" the domain of possibility, insofar as nothing can be actual — there are no "facts" of consciousness — that are not *eo ipso* intelligible. However, from the point of view of an *Erkenntnislogik*, there is no "fact" from which logical analysis begins; rather, *Erkenntislogik* is pure production. What, then, is the purely logical significance of actuality, and how is it related to intelligibility as such? As we saw above, the second moment of judgment is always

the moment of identification; that is, actuality just is the possibility of a possibility. In other words, actuality is pure contingency, not as that which is opposed to necessity, but rather as the individuality of possibility.

Cohen, therefore, understands actuality to be the expression of the individual from within the system of the univeral, i.e., the pure noetic possibility of the contingent. In Kant, for example, the pure contingency of the intelligible is represented by *feeling* (or, in more Kantian terms, sensibility), as that which points toward what is to be understood through schematic reconstruction. From the point of view of *Erkenntnislogik*, feeling is strictly transcendent with respect to the products of thought. The concept of feeling — what feeling too must mean for Kant — is not a fact (being, *Sein*) as it is for Maimon and Hegel, but rather a referrence, or a logical "symbol" (an ought, *Sollen*). Thus, for Cohen, "feeling is an expression, which ought to describe the **direction** of thought in the transfer of its content" (LRE; CW 6:464). Here, we can point to two characteristic features of Cohen's understanding of actuality with respect to possibility. First, feeling is not a part of logic; it is, rather, transcendent with respect to the pure products of cognition: *Erkenntnisse*. Even if feeling is the occasion for thought, thought does not determine a manifold of feeling, but a manifold of pure posits (again, see **Chapter 5**). Second, feeling, in its logical significance, is nothing other than the possibility of "transfer" of content, i.e., the thinking of thought as thought "about" something: intentionality.

Thus, judgments of method are essentially judgments about the (pure) applicability of possibility (intelligibility) to factical cognition, i.e., to actuality as the contingency of the individual. The individual object is not immanent in the pure judgment of possibility, but only in its affirmation as identity: that is, in the assertability of a possibility as actual: this individual "is". But to assert the existence of the possible is to assert the contingent; it is, strictly speaking, to point beyond the domain of what has — as of yet — been explicitly constructed. Thus, we may understand the relation of possibility and actuality as a *correlation*, or, as a possibility that points beyond its pure possibility to its (contingent) actuality. Thus, to return to a passage cited above, "**correlation must take the place of immanence**" (LRE; CW 6:236). The object

of knowledge — the referent of the *Kulturfactum* — is transcendent with respect to the pure products of thought. Actuality, therefore, is the "symbol" of the individual, i.e., a transcendent contingency that is never entirely produced within the system of principles of production. This "never", however, is not temporal in its significance; it is not the time immanent in the compossibility of thought (Hegel). Rather, the anticipation of actuality implies the anticipation of something which lies beyond the system of principles: the individual as *truth*. In the Platonic tradition, this just is the good beyond being (the *hyperbelon*). Accordingly, we may interpret the judgments of method — the final moment of the table of judgments — as the Cohenian version of Hegel's *principle of intelligible totality*. However, whereas Hegel's logic must produce pure univerality (necessity) immanently out of thought, for Cohen the relation between pure thought and its object is a mere correlation: the actuality of the object as a task for consciousness. That is, "**the unity of judgment is the production of the unity of the object in the unity of judgment**" (LRE; CW 6:68).

The principle of pure individuality, therefore, is the interpretation of the correlation of the Platonic moment of formal cognition — its origin in the purity of the universal law, or the *hypothesis* — with the Aristotelian moment of formal thought — the pure individuality of the empirical appearance in transcendent(al) actuality, or form as the *hypokeimenon*. The principle of pure individuality, therefore, is a *task*: to bring about the reality of the individual through the reconstruction of the pure concept of the individual in a total system of laws.

The Principle of Pure Individuality The principle of pure individuality implies that the *task* of pure knowledge consists of the *anticipation* of real totality through the *realization* of the concept. Accordingly,

(a) The actuality of the totality is *given* contingently as the fact of culture;

(b) The reality of the totality is *produced* in the diachronic production of intelligibility: the *hyperbelon*;

(c) The correlation of actuality and reality consists in judgment as the positing of the fi-

> nite through the infinite: the active anticipation of the *hypokeimenon* as the actuality of pure intelligibility.

The *principle of pure intelligibility* implies that we accept the *principle of the infinitesimal* as the ground of the idea of the individual. That is, the individual is actual insofar as it is given contingently as the fact of culture, and is real insofar as it is a pure product of the necessary intelligibility of the universal. Accordingly, the principle of the infinitesimal implies (a) the *principle of origin* as a condition of the pure production of the universal, and (b) the *principle of production* as a condition of the transcendental reality of the concept of the individual. In other words, judgments of experience — as the schematic comparison of the necessity of the intelligibility of the universal and the contingency of the fact of the empirical individual — rests on the anticipation of (Aristotelian) form through the infinite production of (Platonic) individuals: ideas, or the "infinitesimals".

The interpretation of Cohen's logic therefore requires a systematic inversion of the principles of Hegelian logic.

- Whereas Hegelian logic begins with the real individual as an immediate essent (the *hypokeímenon*),[40] Cohen's logic begins with the intelligibility of the universal, i.e., *hypothesis*. The condition of thought is therefore not the determinacy of the totality of consciousness, but the purity of absolute production. Accordingly, the *principle of reality* is replaced with the *principle of origin*.

- Whereas Hegelian logic demands that an object must be determinable with respect to the totality if it is to have actuality, Cohenian logic demands that unity be produced *ab nihilo* if it is to have reality. Accordingly, the *principle of determinability* is replaced with the *principle of the concept*.

- Whereas Hegelian logic determines the individuality of the individual through ideal limitation (i.e., the immanent boundary), Cohenian logic produces the ideal individual through

[40] "The knowledge or knowing which is at the start or is immediately our object cannot be anything else but immediate knowledge itself, a knowledge of the immediate or of what simply **is**" (PhG; HW 9:22).

the infinitesimal. Accordingly, the *principle of limitation* is replaced with the *principle of the infinitesimal*.

- Whereas Hegelian logic anticipates the intelligible totality (the *eidos*) as the perfection of determinacy, Cohenian logic anticipates real individuality (the *hypokeímenon*) as the perfection of production. Accordingly, the *principle of intelligible totality* is replaced with the *principle of pure individuality*.

Table 4.4: Two Systems of Logical Principles

Hegel	**Cohen**
Reality	Origin
Determinability	Concept
Limitation	Infinitesimal
Intelligible Totality	Absolute Individuality

In the next chapter, we will see how these principles are brought to bear in the production and determination of a rational manifold. There, the difference between immanence and correlation will be made more explicit through a comparison of the reconstruction of "continuity" from within Hegel's logic and that which may be ascribed to Cohen's logic.

Chapter 5

The Manifold of Reality

5.1 The Real Manifold

5.1.1 Infinitesimals

The Kantian model of the (a priori) intelligibility of reality rests on two very specific claims about the nature of cognition. First, it is assumed that judgments about serial orders cannot be analytic, but must instead be based on the enumeration, which can only be determinately represented in sensibility.[1] Accordingly, all serial orders (and paradigmatically the orders of space and time) are conditioned by the 'real' condition of sensibility; space and time are a priori serial orders (pure intuitions) with an a posteriori determinability in the empirical experience of the natural manifold (i.e., the manifold of intuition), and in the space and time "filling" properties of the appearance of particulars. Second, it is assumed that rational quantification (some, all, none) is structurally analogous to the fundamental operations of arithmetic, and paradigmatically, to counting. That is, the rational structure of quantification allows us to distinguish between sets whose members are contiguous and those whose members are not. As we saw, we are able to apply the rational categories of quantity to a *quantum discretum* only if that magnitude is antecedently or implicitly judged to be a *quantum continuum* through a judgment of quality. However, purely rational enumeration lacks the expressive power to characterize the (real) differences between sets (i.e., their cardinality), although 'rational' quantifications (all, some, none) are possible. Accordingly, it is only the synthesis of (a) rational enumeration and (b) really conditioned differences that makes the determinacy of empirical concepts possible.

The first assumption implies a constructivist mathematics. For it consists of the denial

[1] I will use the term 'real' here to designate a difference of position in a serial order. For Kant, the manifold of reality derives from the synthetic structuring of (a) purely intelligible orders of adjacency and/or succession (i.e., the logical structure of implication) and (b) the sensible condition of their manifest difference in intuition. The sensible condition is necessary because a pure relation of succession or adjacency between merely rational relata yields a relation that is transitive. So, for example, the regression of time is transitive — i.e., $((t_3 \rightarrow t_2) \vee (t_2 \rightarrow t_1)) \rightarrow (t_3 \rightarrow t_1)$ — and thus there is no merely rational distinction between antecedents — i.e., t_2 and t_1 are not rationally distinct with respect to t_3. An order of *determinable* differences requires the (logical) function of succession, which is intransitive. For Kant, the necessary supplementary condition — which makes the difference between the transitive and the intransitive condition — is *real* or *material* implication, i.e., the sensible condition. Accordingly, I will also call the sensible condition of real determinacy the 'real' condition of cognition (and reality) as opposed to the 'rational' condition (i.e., the a priori) of logical implication. In this context, 'real' does not designate any metaphysical property, but merely a property of ordering.

of an analytic order of differences, i.e., there is no a priori analytic serial order.[2] Significantly, this also commits us to the denial of the unity of philosophical logic and mathematics. For, the latter is based upon an order of real differences that can only be provided in intuition. The second assumption implies that the sequential regression of logical implication (consequent $\rightarrow$ antecedent) provides a structural *analogy* to the real order of empirical conditions.[3] This analogy, however, does not go so far as to provide a formal equivalence between the logical condition and the sensible condition; for, this would imply that logic and mathematics share a common logical structure (a rational successor function), and not just a structural similarity (implication $\simeq$ succession). Nevertheless, the analogy does allow for the schematic interpretation of appearances in terms of synthetic principles. Thus, for example, the subordination of a rationally quantified, (immanently) contiguous, serially ordered sub-set of the continuum to the logical condition of affirmation (i.e., the category of reality), allows us to develop the concepts of relation (substance, cause, community) as correlates to complexes of the synthetic principles.[4]

Maimon's objections — and Hegel's extensions of the same — have shown that, paradoxically, *knowledge* may be impossible for cognizers whose capacity for representation de-

[2]This may not be entirely correct. Kant does seem to think that an intellectual synthesis of order is possible. It would be better to say that the intellectual synthesis of number may not be determinate or determinable, and thus cannot serve as an epistemic surrogate for real construction in intuition.

[3]It may not be immediately obvious why this is so. However, recall that the relation of space may be interpreted either as adjacency (*Nebeneinandersein*) or as exteriority (*Außereinandersein*). This ambivalence is explained by the fact that, in the schematization of space, the series of real conditions may be interpreted either through concepts of quantity (i.e., through rational quantification) or through concepts of quality (i.e., through rational implication). When we interpret space qualitatively, we interpret it as exteriority, that is, we interpret the real condition as implying the exteriority of the antecedent with respect to the consequent — i.e., the emperical object and the appearance of the object (i.e., empirical realism). Thus, even if intellectual synthesis of order is possible, without a real index (magnitude, set of strokes on the paper) to keep track of where we are in the order, or intellectual synthesis may become indescriminable. Even counting to 1 000 000 would require the sequential ordering of number-names, since simply counting 'one, another one, another one, another one, etc.' would not (for intellects like ours) lead to a discriminable enumeration of '1 000 000', since we would be bound to lose track of the count at some point.

[4]The empirical correlate of the concept of substance is the representation of a contiguous section of the space-time continuum under the concept of matter, which is subordinate to a single predicate (i.e., the predicate can be affirmed for every member of the rationally quantified set, even though individualy members cannot be rationally enumerated): e.g., 'This (blob) is a dog' $\rightarrow$ 'All parts of the appearance are (blob).' The empirical correlate of the concept of causality is the representation of a non-contiguous section of the space-time continuum, which is subordinate to a single predicate (i.e., the predicate can be affirmed, but not of all members of the set): e.g., 'This (blob A) is a dog and therefore this (blob B) is a dog'.

pends on the sensible condition. Thus, we are compelled to be empirical skeptics insofar as we accept sensibility as a limiting condition for the construction of serial orders of difference. Indeed, knowledge, if it is possible at all, is not empirical in origin. For, intuition, at least insofar as it is given and cognized a posteriori, is never fully intelligible, and thus cannot be the source of the laws that we count as knowledge. And yet, it seems that we *do* have knowledge. Moreover, it seems that our most secure form of knowledge — constructive geometry[5] — is most affected by the paradox of finite cognition. What is more, these solutions appear to be *analytic* solutions to problems given in intuition. How can such analytic, infinite solutions be offered by a thinking being that is merely a finite cognizer?

Maimon's solution is to propose that while the presentation of the understanding is in principle limited by the sensible condition of temporal finitude, the understanding can operate as though it were infinite, if only through the idea of its infinite determinability. That is, we have an infinite understanding for the purposes of merely symbolic cognition. However, the application of the understanding *in consciousness* is nevertheless subject to the limitations of sensible representation. Accordingly, we must have a mode of cognition that is *merely symbolic*, that is, which merely maps the continuity of experience, but which is not itself the reality of our sensible determinacy. To fulfill this role, Maimon posits the ideas of the understanding, where "an idea of the understanding is the material completeness of the concept, insofar as this completeness cannot be given in intuition" (VTP; MW 2:75). These just are differential components of ideas of reason, or "complete concepts", which cannot be realized in intuition. Accordingly,

> we assume an infinite understanding (at least as idea), for which the forms are at the same time objects of thought, or that produces out of itself all possible kinds of connections and relations of things (the ideas). Our understanding is just the same, only in a limited way. (VTP; MW 2:64)

[5]Euclidean geometry was long taken to be the benchmark for a systematic science; other fields of mathematics — such as algebra and analysis — were not thought to have the same degree of rigour. Whereas the apogogic proof method of analytical geometry seemed adequately defined through intuitive concepts like 'magnitude' and 'equality', algebra and geometry depended on a concept of number — and indeed, on negative, irrational, imaginary, infinite and infinitesimal numbers — that were not well-defined until an adequate theory of serial orders was developed.

The way in which our understanding is limited however, does not restrict the a priori possibility of presentation (*Darstellung*), but only its a posteriori realization through sensibility (i.e., *Vorstellung*).

For Maimon's analytical logic of the manifold, rational differentials — and their schematic, synchronic composition as ideas of reason — are the necessary condition of the intelligibility of sensibility, even if they are really distinct from sensibility itself. By contrast, Bergman describes Maimon's doctrine of differentials as "a doctrine designed to reduce sensibility to silence" (Bergman 1967, 59). This is only partly correct. For, Maimon does not intend to replace intuitions in their role for *consciousness*, nor do rational differentials take over the ontological role played by real differentials as states of the subject, i.e. the matter of intuition.[6] Rather, rational differentials — ideas of the understanding — are posited to supplement the deficiencies of finite cognition for the purposes of infinite, or symbolic, representation. For, as Maimon's antinomies have shown, we may be able to have intuitive consciousness of the truth of some propositions, but knowledge of the same rests on an a priori determination of the manifold. This, however, cannot be finitely achieved without the supplement of rational (symbolic) infinitesimals, or rational differentials. This distinction — between what we can be aware of and what can be known — tracks Maimon's interpretation of the role of intuition and the understanding as faculties in general. However, it can most clearly be seen in his distinction between the "subjective order" of consciousness and the "objective order" of knowledge.[7]

[6] In Leibniz, and also, arguably, in Kant, the 'differential' components of sensibility consitute the real being of nature. That is, the differentials of sensibility are real states of the subject, the interpretation of which is the interpretation of fundamental (absolute) reality. For Maimon, however, the ideas of the understanding are free constructions of the mind; that is, they duplicate the sensible states of the subject for the purposes of transcendental judgment. These 'mathematical' or 'purely symbolic' differentials are the rational manifold, which, for Maimon, stands over and against the real manifold of the sensible subject, but are the domain of its free construction — and thus the absolute (and infinite) determination of knowledge.

[7] The distinction between the subjective order and the objective order is Maimon's. However, in distinguishing between these two orders as two registers of thought — between the symbolic and the factical, between knowledge and consciousness — I am following both David Lachterman and Ernst Cassirer in their interpretation of Maimon (Lachterman 1992) (Cassirer 1920). Here, for example, is how Cassirer frames the problem:

> we must, metaphysically, see the world as the work of a most perfect understanding, and thus for this understanding as fully transparent, or we must, empirically, see the world as a given sum of

First, there is what Maimon calls the "subjective order (with respect to our consciousness)", which may be taken as that which is necessary for cognition :

1. Sensibility (which certainly does not provide consciousness itself, but rather the matter for consciousness).

2. Intuition. The ordering of homogeneous sensible representations under their *a priori* forms (time and space); from this consciousness arises, although certainly no thought.

3. Concepts of the understanding (categories); from this a thought arises, i.e. the reprsentation of a unity in the manifold.

4. Ideas of reason. Totality of the concpets of the understanding. (VTP; MW 2:81)

Second, there is what Maimon calls the "objective order considered in itself", i.e., the order of knowledge:

1. Ideas of the understanding, that is to say the infinitely small of every sensible and of its forms, which provides the matter to explain the way that objects arise.

2. Concepts of the understanding, and

3. Ideas of reason, whose use has already been explained.

We can immediately notice two things. First, consciousness and knowledge both culminate in "concepts of the understanding" (categories) and "ideas of reason". This is because the objectivity of consciousness just is that which arises from the understanding, rather than that

> facts, about which we may specify no claims if and when it allows itself to be explained in terms of general concepts and laws. (Cassirer 1920, 96)

This dualism is both perspectival and ontological in its implications. As Cassirer points out, the differential has, for Maimon, a "*methodological* role" (Cassirer 1920, 100), in allowing for the idealization of givenness by distinguishing conditions of consciousness (the magnitudes of intuition) from the real object of knowledge (the differentials of sensibility). This idealization, however, does not mean that feeling is abjured from *consciousness*; rather, it means only that *knowledge* is no longer grounded in the insensible differences of intuited magnitudes, but rather originates in the intelligible difference of the mathematical differential. Finite cognizers must still make judgments about given objects (i.e., empirical presentations), but may do so through the models of infinite calculability. Maimon's solution to the *quaestio quid juris*, in other words, implies that the foundation of knowledge is not synthetic judgment but the pure principles of mathematical construction.

which arises from sensibility. In contrast, sensibility and intuition are replaced by "ideas of the understanding" in objective knowledge. This is because objective knowledge is not possible where "material completeness" is lacking. Material completeness cannot, however, always be provided by intuitions. As a result, the possibility of knoweldge rests on our ability to supplement or replace the material completeness of the concept with a pure product of the understanding: "ideas of the understanding".[8]

Accordingly, we consider the understanding in *two* distinct ways: as infinite (as idea) and as finite (as intuited image). This yields two orders of intelligibility: *symbolic differentials* and *actual infinitesimals*. And so, for Maimon,

> the solution rests on this: that the understanding can and must be considered in two opposed ways. 1) As an absolute understanding (unlimited by sensibility and its laws). 2) As our understanding, in accordance with its limitation. So the understanding can and must think its objects according to two opposed laws. (VTP; MW 2:227)

For Maimon, then, we must be able to represent ideas in two different ways in order to think them at all. The first, finite form (intuition) is necessary (and sufficient) for the possibility of consciousness. The second, infinite form (ideas of the understanding) is necessary for the possibility of *knowledge*. The first is what Maimon calls a finite or conscious understanding. The second is what Maimon calls an infinite — but limited — understanding, or also "symbolic thought".[9]

---

[8]Bergman is correct when he points out that we must "consider spatio-temporal *content* as objects created by the understanding in accordance with various rules" (Bergman 1967, 59). However, he is misguided in equating ideas of the understanding with laws (Bergman 1967, 59). For Maimon, rules or laws are provided in two distinct ways. Either *factically*, in the complete concept of a form (i.e. an idea of reason) or *relationally* through the concepts of the understanding (i.e. the categories), which, for Maimon, are relational laws of the organization of the manifold. Ideas of reason are the matter of symbolic representation, as the pure representation of an a priori relation, the determinability of the determinate, insofar as this can be known a priori through analysis.

[9]Many commentators have taken the view that Maimon is committed to the (Leibnizian, Spinozistic) position that sensibility is merely the confused product of the understanding (Bergman 1967) (Thielke 2003) (Melamed 2004) (Beiser 1993). The view is primarily grounded in a passage from the *Essay on Transcendental Philosophy* where Maimon seems to prefer the Leibniz-Wolff system over the Kantian (VTP; MW II:63). However, this view — usually called "Maimonean Monism" — assumes that the ideas of the understanding are in fact identical to the differentials of perception. Here, for example, is Peter Thielke:

5.1.2 The Manifold of Limitations

As we saw above, the central task of the Doctrine of Being in Hegel's *Objective Logic* is the determination of the objectivity of the object, i.e., the possible (transcendental) relation between the totality of the manifold (being) and the concept of the individual. Through this determination, the individual is posited as a finite concept that is immanently correlated with its infinite composition as idea. One aspect of the Doctrine of Being that is too often overlooked is that it is first and foremost a philosophy of mathematics, or more precisely, an analysis of the concept of quantity as the finite part of infinite continuum of the manifold. That is, every determinate (subject) is an infinitely determinate totality (in space and time) that is nonetheless a finite modification of an infinite manifold (being). Accordingly, the (infinite) determination of (finite) totalities within the infinite continuum of the manifold is the primary task of the Doctrine of Being; this is not the ontological or metaphysical determination of material reality, but rather the determination of the concept of a finite quantity, i.e., the concept of the discrete individual, or what Kant calls the *quantum discretum*. However, it is Hegel's Doctrine of Being, and the purported philosophy of mathematics contained therein, that has been the object of perhaps the most vigorous attacks on Hegel's system — even from friendly quarters.[10]

> Differentials of perception are to be understood as, in some sense, the *products* of the understanding itself, (Thielke 2003, 117, my emphasis)

since,

> what to the quotidian consciousness appears as merely given is, from a philosophical perspective, a *product* of the rule-governed but not 'rule-understood' differentials of the productive understanding. (Thielke 2003, 118, my emphasis)

Maimon, then, is a Leibnizian, who claims that there is merely a conceptual distinction between the activity of intuiting forms and the activity of the understanding. However, Maimon is explicit that the ideas of the understanding must be posited to compensate for the deficiency of the understanding in representing infinite structures. Thus, it is not the case that our understanding produces sensibility — after all, it is finite with respect to real modifications of sensibility. Rather, through the ideas of the understanding, we may think "as if" we were infinite cognizers, but merely symbolically (i.e., outside of the sensible condition) (VTP; MW II:64). Maimon, therefore, retains central aspects of Kantian dualism — and especially the sensible limitation on the representation of sensible form. Those who wish to retain the subjective monist view must explain the sense in which Maimon synthesizes Spinoza and Leibniz, with the skepticism of Hume and Kant and is not merely a Leibnizian; that is, why is Maimon a dogmatic rationalist *and* an empirical skeptic?

[10] If one adopts an ontological interpretation of the doctrine of quantity (MacTaggart), or if one sees in the dialectic a metaphysic (Trendelenburg), then the peculiar terminology of the Doctrine of Being can only seem to be systematic ontology along the lines of Spinoza's *Ethics*. Indeed, in this case, Hegel's curious theory of the

One response to the general vilification of Hegel's philosophy of mathematics is to ignore the (anachronistic) criticism of Hegel's view, in order to understand it on its own terms.[11] Another response is to take a revisionary approach to Hegel's philosophy of mathematics. This is the route pursued by Terry Pinkard in his 'Hegel's Philosophy of Mathematics' (Pinkard 1981). I would like to briefly consider this proposal as a way of introducing the problem of the a priori construction of the manifold, beginning with an infinite, but implicit, continuum: being.[12]

According to Pinkard, "Hegel should begin with units". By units, Pinkard has in mind "the ones" (*die Eins*), i.e., the components of the one, or the final product of the infinite subdivision of the Second Antinomy (the real continuum). We could as well begin with Dedekind's notion of counting (Dedekind 1909) or, perhaps more perspicuously, with Cantor's notion of the power (*Mächtigkeit*) of a set.[13] The logical presupposition of the methods of enumeration employed by Dedekind and Cantor is a mechanism for maintaining the distinctness of elements, i.e., it is a rational criterion of difference. Now, for Dedekind and Cantor, this distinctess arises

'repulsion' and 'attraction' of 'the ones' that form 'the one' can only be interpreted as an egregiously metaphysical, even physicalist, interpretation of infinitesimal quantities and the continuity that arises from them. However, the reading of Hegel's Doctrine of Being that I have been advancing here evolves out of Maimon's preoccupation with the a priori determinability of the concept. If we now apply these — purely conceptual — problems of a priori determinability to an a priori *manifold*, then many 'Hegelian' consequences appear to follow unproblematically.

[11] This is David Carlson's approach in his 'Hegel's Theory of Quantity', where he claims that Hegel provides "philosophy's most rigorous definition of quantity" (Carlson 2001, 2027). However, given the general consensus that the category of quantity is problematic, and perhaps spurious, in the light of the development of the concept of order, it's not clear that this should be taken to be an achievement of particular merit. The rehabilitation of the concept 'quantity' carries with it significant intuitionist and phenomenological commitments that are not readily defended.

[12] Contemporary constructions of the continuum begin with countable individuals and construct the continuum therefrom, which is what motivates Pinkard's revisionary proposal. However, as we have seen, the foundational concept of Hegel's logic is the concept 'being', which already contains implicitly an infinite continuum of determinables (by the *principle of reality*). The problem for Hegel's philosophy of mathematics (quantity, or the explicit composition of the whole), therefore, is to derive parts from the whole, not a whole from the parts. This, we will see, has devastating consequences for Hegel's ability to achieve anything more than density in the manifold.

[13] (Cantor 1915) The advantage of the Cantorian approach is that we can think of 'the one' as a set with cardinality $\aleph_1$, i.e., as a transfinite or "perfect" infinity of real members ('the ones'). However, this supposes (a) that 'the one' is somehow given as an infinity of enumerable parts and (b) that these parts are given as implicitly related, i.e., as *an sich*, in some kind of point-set topology (or just as a set without any phenomenological topology — yet). Both of these properties, however, seem to be implicit in the positing of 'being.' Accordingly, the problem for a Hegelian philosophy of mathematics is not *producing* the infinite continuum, but rather developing the concept of its explicit determinability through a system of limitations. That is, we need a predicative determination of the determinability of the continuum of real numbers.

from a difference in the way that elements are constructed according to rules, either through the concept of a successor function (Dedekind) or through the construction of a power set (Cantor). As we will see, it is possible to interpret Hegel's view in a way that is broadly consistent with this approach. However, the question is not whether Hegel can construct ordinal numbers, but whether he is in a position to do so at the point when the concept of quantity is first made explicit.

Hegel's logic begins with "**being, pure being**", which, as we know, must be thought of "without any further determinacy," that is "in its indeterminate immediacy," in which it "has no difference within itself" at pains of "losing its purity" (WL1; HW 21:59). However, within this "manifold" of determinability, the real is not the totality, which implies the synchronic composition of the whole. Rather, "the finite is the real existence" (WL1; HW 21:143). And so, rather than beginning with the infinitesimal — as Hegel does in "Quantity" — Pinkard argues that he should have begun with "the one", the countable unit, since "the ones [*die Eins*] are thus **presupposed** over and against each other; — **supposed** through the repulsion of the one [*das Eins*] from itself; **presupposed** as **not** supposed; its being-posited is sublated, they are **beings** over and against each other, and only as related to each other" (WL1; HW 21:186). That is, 'the one' (the unit) is sustained by a system of differences ("they are beings over and against each other, and only as related to each other"), just as the system of serial ordering is a system of rational differences; and, furthermore, 'the one' contains in it 'the ones' (the infinitesimal subdivision of the unit) *in potens*, just as Dedekind derives the rationals from the integers, and the irrationals from the systematic bisection of the rationals (i.e., Dedekind 'cuts') (Dedekind 1909).

The problem with this approach is that it conflicts with both the letter and the spirit of the text. First, let's take a look at the letter. In the introduction to Being-for-itself, Hegel explains the simple nature of being reflected into itself: "Being-for-itself is **first** an immediate being-for-itself, **the one**" (WL1; HW 21:169). Here, Hegel uses the singular neuter article, 'das', to indicate the simplicity of the unit, which is more *unity* than it is unit. Indeed, as being-

for-itself, the one "is the simple unity of itself and its moments" (WL1; HW 21:178). That is, it is a "simple unity", and "the **moments** of being-for-itself are mired in **undifferentiatedness** [*Unverschiedenheit*]" (WL1; HW 21:179). Although the dialectic allows the ones to evolve from the concept of the one, and (conceptual) reflection of this one, "goes over into the **plurality** of **the ones**" (WL1; HW 21:170), the order of the derivation is crucial.

To see why, we need only consider the spirit of the derivation of becoming from 'Being' and 'Nothing' more closely. As Hegel reminds us in a note to 'The Unity of Being and Nothing', "**Nothing** must be opposed to **something**" (WL1; HW 21:60). The reason for this is that "if a determinate content, any determinate existence, is **presupposed**, then this existence, because it is **determinate**, is in a manifold of relations to other content" (WL1; HW 21:65). This just is the *principle of determinability*. To be an an (explicit) relation just is to be something in virtue of being determinate. It is, therefore to be a finite existence, a something with a determinate composition. It would be incorrect to say that Being is not essentially all of its possible determinations. However, it is all of its determinations only implicity, that is, as immediate determinacy (as in a Kantian intuition), not as explicit determinability (as in a Kantian concept). Similarly, "the one" (the continuous magnitude) is opposed to "the ones" (i.e., the units of its subdivision). Indeed, insofar as "the one" is determinate at all, it "is in a manifold of relations to other content," that is, the manifold both external and internal. To be a one just is to be a determinate totality within an infinite continuum.

However, the process of determining numbers (i.e., instances of "the one"), for Hegel, begins not with the continuity of a quantitative continuum, but rather with a merely qualitative continuum: the general relational determinability of something in general, the principle of determinability as the a priori system of determinable relations.[14] While the one precedes the unit ("the ones") in the order of the derivation of the categories, the category of the one is, as a

[14]While Hegel is almost certainly thinking of quantity in terms of an implicit, phenomenological, continuous topology, he is nevertheless making an interesting point. What makes "the one", as "consciousness as such" determinable? Clearly, it is the thinking of differences within thinking itself. Insofar as thinking is a possible state of consciousness, determinations are already implicitly within "the one" (i.e., the determinability of the determinable is already *in potens*, logically, even if no determinate thought has yet been realized.)

qualitative category, merely a concept in potentiality. That is, the concept of the one has no determinacy, and thus cannot determine the concept of the unit of its infinite subdivision. Rather, it is the enumeration of "the ones" that will ultimately provide the conceptual foundation for the explicit category of the one: the infinite. As Hegel insists, "the count is therefore not a plurality **against** the inclusive, limited one, but rather makes this limitation itself, which a determinate quantum is; the many make a number, a two, a ten, a hundred, etc." (WL1; HW 21:243). Thus, while we require a qualitative category of unity in order to be able to think the mere possibility of a unified number; number would remain utterly without determination in the absence of the counting procedures that generate "a two, a ten, a hundred, etc."

The qualitative continuity of the number is accordingly what Hegel calls a "principle", but this is merely the categorial intelligibility of a unity in general, and is not the assertion of a quantitative or determinate continuity within the one. Indeed, the continuity of number is the *third* moment of quantity: the infinite. Again, we see that Hegel's triadic structure is at work in the concept of number: "Discrete magnitude has first the one as principle, and is second the plurality of the ones [*der Eins*], third it is essentially continuous [*stätig*]" (WL1; HW 21:238). While the one remains the "principle" of number, this categorial thought really is "nothing" unless it can be given an internal articulation, and this articulation must come not from continuity, but rather from discrete enumeration. Indeed, "quantity is only concrete unity insofar as it is the unity of **different** moments" (WL1; HW 21:234), i.e., analytical composition. What we require then, is a means whereby a system of differences may be constructed insofar as it relates the the infinite totality of a particular number. For, it is only insofar as strictly constructive procedures can be given for the determination of differences that they may be conceptually interpreted, and correlated with the idea of the number as an infinitely determinate totality. This just is the *principle of intelligible totality*.

Consider, by way of contrast, Kant's approach to the determinability of the manifold. The totality of the manifold is given as an infinite extension in space and time: "Space is represented as an infinite **given** magnitude" (KRV; AA B39). The relationality of this manifold

is a fact of sensibility, i.e., the real condition of sensibility as affection (or, the dependence of sensibility on external or antecedent conditions). Thus, for Kant, the matter of thought is a *given* totality of implicit relations, that are only realized through determinate totalities. For Hegel, on the other hand, arithmetic does not operate on a real, or given manifold. Indeed, "counting is ... no sensible determination, which remains for the *a posteriori* alone according to Kant's determination of intuition" (WL1; HW 21:249). Rather, "arithmetic considers number and its figures, or, better, does not consider them, but operates with them" (WL1; HW 21:245). That is, arithmetic is the manipulation of *purely conceptual* relations. It is a strictly a priori operation. The validity of arithmetic therefore relies not on some (a posteriori) relation between arithmetic figures, but depends instead on a system of strictly a priori, conceptually constructed relations.

While the qualitative "discretion" of the one into the ones is given in the Book I ('Determinacy'), this determinacy is, as I have emphasized, merely qualitative and immediate. Emphatically, "the quantum, **at first** quantity with a determinacy or a limit in general, — is in its complete determinacy **number**" (WL1; HW 21:240). The concrete or conceptual determinacy of number thus relies on the construction of a system of relational differences that is not given a priori or a posteriori, but which must be constructed. Similarly, the continuity of number is given qualitatively as the determinability of unity, but we require a constructive procedure in order to render the difference of individual moments intelligible. Accordingly, for Hegel, "the **first** producing of number is the taking-together of many as such, i.e., that in which a many is posited as only one, — **enumeration**."[15] That is, number itself arises from counting units and their assimilation to a set or collection that we take to be a unity, not from the "principle" of a continuous unity, which is nothing more than the categorial determination of number as such.

The conceptual construction of a system of quantitative determinations is therefore (a) the a priori condition of arithmetic in general and (b) the a priori condition of the general determinacy of the one as a determinate totality. *Pace* Pinkard, it is not the case that the a pri-

[15](WL1; HW 21:247) Parsons argument (Parsons 1984) that the Kantian categories of number harbour an implicit conception of set theory would be better applied to Hegel. Here, Hegel makes quite explicit that the concept of counting implies "a many is posited as only one," i.e., as a collection or set. Hegels direct identification of enumeration with a conception of cardinal sets is much more explicit than Kant's offhand remark in the Schematism.

ori system of construction starts with discrete, continuous magnitudes. Rather, Hegel begins with counting (*Zählen*) as a procedure for the construction of a system of numerical relations. Accordingly, the construction of a system of numbers is the construction of the a priori determinability of unity as such:

> Number is in its principle, the one, willing something externally taken-together in general, a merely analytical figure, which contains no internal connection. Because it it is merely something externally produced, all calculation is the bringing about of numbers [*Zahlen*], a counting [*Zählen*], or **more determinately**: counting together [*Zussamenzählen*]. (WL1; HW 21: 246)

Again, *pace* Pinkard, it just is the case that Hegel begins with countable units and their enumeration as the explicit conceptual determination of number; however, these units are not the integers, but the a priori order of the continuum, a system of determinable "infinitesimals." It is only in and through the identification of procedures of construction with the principles of unity that an infinite system of determinations will be possible, and thereby, the possibility of a determinate concept of continuity.

We have seen, then, that Pinkard's suggestion is mistaken in two different ways. First, it is incompatible with the general strategy of Hegel's dialectical logic, which always begins with the qualitative "principle" of a categorial determination in principle, and then constructs an external representation of the same (being-for-another) in order to identify the immanent principle of being with its formal representation. Second, the consistent application of this triadic scheme clearly shows that Hegel's approach to the construction of number is much closer to the Dedekind-Cantor approach than Pinkard would have us believe. Although what is presented here is merely a sketch, we have good reason to believe that Hegel's general strategy is compatible with a realist approach to number, grounded in formal methods of the construction of integers.[16]

[16]It might be odd to think of a "realist" mathematics as grounded in the formal methods of construction. Recall, however, that Hegel takes the manifold to be give. That which is constructible in the manifold is *eo ipso* real. What is not clear is whether formal existence (i.e., definition from axioms — obviously ones that are related to the manifold) is sufficient to guarantee reality also. There are no doubt counter-examples to Hegel's conception (and irrational roots are the low-hanging fruit here), but it is nonetheless an intelligible, if limited, philosophy of mathematics.

The point here is not merely to show why Pinkard is mistaken to want to renovate the Hegelian proposal. Rather, the exploration of the concept of number in Hegel's Doctrine of Being shows how a system of numbers emerges out of a system of implicit differences: the manifold of "the ones" as the systematic connection of the limitations of magnitude. We have seen that Kant takes the manifold to be an (ideal) given, represented as a totality, and Maimon proposes a system of "ideas of the understanding" that are posited analytically as a system of differences. Hegel, however, according to the *principle of intelligible totality*, understands the determination of a system of numbers to be the (progressive) determination of a system of finite modifications of an infinite continuum. The realization of these finite totalities is grounded in a principle of counting, whose ideal correlate is the quantitative continuum, i.e., the "power" $\aleph_0$ of every bounded subset of the continuum.[17]

While there are many puzzling aspects to Hegel's derivation of the individual totality — e.g., the "repulsion" and "attraction" of "the ones" or the reciprocity of the boundary as "in-and-for-itself" — we do not need to address these issues in detail here. Rather, we can retain the following conclusions:

(a) 'Being' is an implicit infinite continuum of determinables;

(b) Finite real modifications of being are finite quanta of the infinite continuum of the real numbers;

(c) The principle (concept) of the explication of the quantitative continuum of the finite magnitude is the enumeration of the infinitely subdivisible units (infinitesimals) of the qualitative continuum;

[17] This should raise a warning flag. For, if Hegel's conception of subdivision of "the one" into "the ones", then limit of the natural numbers (and thus the number of elements in the composite of "the one") is $\aleph_0$. However, the real continuum has (if the axiom of choice holds) a cardinality of $\aleph_1$ (Cohen 2008). One would have to suppose that transfinite counting (to the next cardinal $\aleph_1$ is possible if we want to believe that Hegel's approach to the progressive division of the manifold (or any magnitude) will terminate in a real ordering. Hegel may, however, be able to derive density — i.e., the claim that every line segment is arbitrarily small (or, equivalently, that every point in the manifold is aribtrarily close to another point also on the line). This may not be sufficient for a rigorous account of real analysis, but it may satisfy less demanding epistemic projects.

(d) The possibility of the bounded totality (the idea of the finite individual) implies the idea of the boundary of the infinite, i.e., that we actually count to the transfinite cardinal that is the actually largest natural number $\aleph_0$.

However, as we saw in the last chapter, the actuality of the infinity of number (i.e., the compossibility of the infinite) is diachronic, and thus merely potential. Even if Hegel's conceptual explication of the concept of number out of the methods of enumeration suggests a formalist or 'Platonic' approach to the infinite construction of totalities, the application of this method of construction is in fact quite limited. For, Hegel — like Kant — insists that the cognition of determinate individuals must (until the end of history and the perfection of absolute spirit) take place through a finite approximation; that is, the finite construction of the transfinite boundary is essentially a paradox whose sublation is at most intelligible through the principle of construction, but which cannot be thought of any determinate individual. For, the transfinite is not *in itself* well defined, but is always just the determination of "another", i.e., it is the "externality" of the immediately given boundary that we can never ascribe directly to any determinate individual — it is never anything more than "another".

Indeed, if we recall the definition of the real magnitude from the previous chapter, we see that determinate magnitudes (A) only arise as explicitly determinate through the (infinite) composition of the limits of magnitude:

$$A = \sum_0^\infty (\sum_0^\infty + \frac{1}{\sum_0^\infty (\sum_0^\infty + 1)}) \tag{5.1}$$

That is, every magnitude is just a composition of the limits of magnitude, i.e., it is the intelligible construction of the *limitation* of a manifold that is already *given*. Whereas Maimon's allows that the ideas of the understanding may be pure posits, for Hegel, the "infinitesimal" is always the limit of an intuitive magnitude. And, while the limit of magnitude may be thought, it is not realized. That is, the *transfinite* limit of real determinacy is not arrived at through mere

composition.[18]

The consequences of Hegel's interpretation of the explication of the manifold (i.e., as being for another) as a system of limits is that elements of reality (that is, the irrationals) remain unintelligible; the real is not rational. The reason for this is that the Hegelian derivation of explicit continuity (the intensive as the limit of extension) produces at most a *dense* manifold, i.e., a manifold that contains the limits of every possible interval of the topology. In other words, every intensive magnitude x is arbitrarily close to another point y that is also in the manifold. However, this does not show that the manifold is *continuous*. To show that the manifold is continuous, Hegel would have to be able to demonstrate that the manifold is totally ordered, i.e., that every intelligible number (including the reals) is also produced by the limit method for the construction of intensive magnitudes. This, however, is not the case. Hegel's constructive approach to intensive magnitudes as the limitation of the space provides only the set of rational numbers (or rational magnitudes, "ones"), not the real numbers — including the irrational numbers and their "bad" infinity. And so, while Hegel can produce the field of rational numbers (as the infinite concatentation of the infinitesimal), this does not produce the irrational numbers, which are not well-defined with respect to Hegel's cosntructive procedure. That is, the continuity of the continuum — and thus the irrational moment of reality — remains unaccounted for.

Now, before we complain that the distinction between *density* and *continuity* is an anachronistic imposition, we should recall that Maimon first introduces the manifold of real determinables as a way of resolving the problem of the construction of irrational numbers, and specifically the composition of $\sqrt{2}$. The problem is that Hegel's derivation of the explicit mani-

[18] To put it bluntly, Maimon does not tell us how the point-set topology of the real manifold is derived — either empirically or rationally. It may, therefore, simply be given. Hegel, however, takes Being to be given. From this, logic must develop the continuum (*qua* principle) into a quantitative order of infinitesimal parts. This cannot, however, be achieved by the continuous subdivision of a magnitude by natural numbers. This yields at most the rationals — and thus density — but not continuity. The reals might well be given, but they cannot be thought of as the limit of determinacy as subdivision, since the irrationals lie between the rational numbers. Bolzano-Cauchy-Weierstrass convergence would nonetheless be possible, but would supposes (a) that the irrationals are given (which we cannot know a priori) and adds yet another dimension of infinite approximation: Cauchy convergence only requires that every convergent series of rational numbers converges on a real number. But this is an infinite convergence within an infinite subdivision, thus a double-idealization of the continuity of reality.

fold provides only the dense set of rational numbers, not the totally ordered set of the reals. That is, we cannot say that $\sqrt{2}$ is "close" to an intensive point of the rational manifold. But since the irrational numbers are not determinately ordered with respect to the set of intensive magnitudes, we cannot say that irrational numbers can be cognized, not even symbolically. Accordingly, the Hegelian concept of the manifold of rational determinables is never fully emancipated from its intuitive origin. For, even though irrational ratios are (surely) given in the manifold of *reals*, the method whereby intensive magnitudes are explicitly constructed as the limits of magnitude produces only the rational numbers.

That this is so can be easily seen merely by inspecting the (explicit) definition of a real magnitude. Through this method of construction, A will *always* be some ratio of integers — $m : n$ — corresponding to the successive generation of progressive approximations of rational determinacy. However, these ratios never converge on irrational values, and thus the Hegelian manifold is never — not even at the transfinite limit of its completion — adequate to the totally ordered manifold of "reality".

5.2 Origin and Continuity

5.2.1 Reality and the Infinitesimal

Continuity and Reality

Above, we saw that the Schematism requires a principle of continuity in order to be successful. It is unclear, however, if or how such a principle is delivered by the Schematism or by the doctrine of the synthetic unity of apperception. Cohen's approach to the problem of reality consists of a reinterpretation of the Anticipations of Perception from the Analytic of Principles, where Kant explicitly discusses the problem of the continuity of reality. Kant's discussion of the Anticipations of Perception is notoriously obscure, but it raises the following claims. First, in order that a judgment of reality be possible of a particular modification of the manifold it must

be the case that the particular part of the (enumerated) manifold in question have an *intensive magnitude*. This magnitude is a positive, non-zero degree of affectedness.

From a superficial consideration of Kant's language, he appears to be making the claim that the judgment of reality implies a representation of the causal dependence of sensibility on a transcendent condition. That is, a judgment of reality amounts to a judgment that the subject is causally affected by an external object, and this just is a condition of transcendental judgment in general. The intensive magnitude thus measures the degree to which we receive this impression of an external object. However, this is a problematic way to interpret the language of affection and the judgment of reality. For, so interpreted, Kant must be advocating a direct realism: the representation of the dependence of sensibility on a transcendent object. This metaphysical position is incompatible with the project of a transcendental idealism. Accordingly, Kant must either have fallen into a contradiction, or we have misunderstood the nature of the transcendental doctrine.

To avoid this dilemma, we may understand 'affection' to refer more generally to the fact of sensible determination. The (causal or logical) antecedent of a sensible determination is not a matter for transcendental investigation. However, it is necessary to the possibility of an a priori representation that the subject be sensibly determined. For, sensible determinacy (and thus reality) allows us to distinguish between objects that can be brought to intuition and those that cannot. In the case of given intuitions, sensible determinacy is implied by the synthesis of an intuition by the reproductive imagination. In the case of productive imagination, sensible determination is implied by the schematization of quantity, which requires enumeration, or the determination of differences within a sensible manifold. Whether actual or merely possible, all empirical representations imply sensible determinacy. But where no schematization is applied in the cognition of a concept, no corresponding determination of sensibility is implied. Thus, these concepts do not imply sensible determination, and therefore reality is not a (logically necessary) judgment for such representations; it is, therefore, not a possible object for a transcendental judgment.

Accordingly, a positive, non-zero degree of sensible determinacy is required as a condition of the possibility of a judgment of reality, and thus of the possibility of a transcendental object. This sensible determination may, of course, imply a logical or metaphysical antecedence of sensibility on an external condition. However, 'external' may be interpreted transcendentally to mean 'represented in space'. It need not immediately imply the causal or logical dependence of sensibility on anything other than the subject.

If the judgment of reality seems to do little more, for Kant, then to require that a time series have sensible content (*Zeitinhalt*), Cohen sees in it the possibility of perfecting the Schematism. In the concept of the intensive magnitude of the Anticipations of Perception, Cohen believes that he has found — if only implicitly — the missing criterion of continuity, which is necessary to the Schematism. For, 'intensive magnitude' is, in the 18th century, frequently used to designate the mathematical concept of an infinitesimal magnitude, as it is, for example, in both Maimon and Hegel. Now, infinitesimal magnitudes were thought to be very special kinds of magnitudes; for, they are, on the one hand, inextensive, but, on the other hand, they also represent real degrees of potential change. For example, the velocity of a point in space may be thought of as (a) a point with no extension and also (b) as a velocity, or a determinate magnitude of potential displacement. Many early mathematicians understood the mathematics of the calculus to operate with these two apparently contradictory conceptions of the differential: velocity is a potential magnitude that is actualized in time, but which has no actual instantaneous extension.[19] The reinterpretation of the doctrine of judgments of reality as a doctrine of *continuity* is possible if we understand intensive magnitudes, or infinitesimals, to form the logical foundation of the concept of continuity. And, indeed, this is precisely the role it played in Leibniz's interpretation of the calculus.

However, in turning to the infinitesimal, Cohen is working against history. For, in the 19th century, the concept of the infinitesimal had fallen into disrepute, and had been (if only incompletely) replaced with the concept of the limit. As Cohen notes, "since **D'Alembert**, math-

[19]The interpretation of velocity in terms of actual and potential magnitudes shows the Aristotelian heritage that is still at work in early conceptions of the calculus.

ematicians have been grounding infinitesimal calculus in the **method of limits**" (PIM; CW 4:2), and it is a trend that showed no signs of waning until Abraham Robinson's *Non Standard Analysis* (Robinson 1996). And yet, Cohen's strategy is to reject precisely this method of limits in order to recover a "logical", prematheматical significance in the "infinitesimal", the "differential" or the "intensive magnitude". This seems to involve not only a rejection of contemporary methods in analysis, but also an anachronistic restoration of at least a certain interpretation of Leibniz's widely maligned infinitesimal approach to the calculus. However, Cohen forcefully claims, "the presupposition of intensive reality" — and thus the principle of the infinitesimal — "is latent in **every** principle, and must therefore be made self-sufficient. **This presupposition is the meaning of reality and the secret of the differential concept**; the logical secret that *Erkenntniskritik* unveils" (PIM; CW 4:28). Thus, according to Cohen, every principle of mathematics — including the simple ordering function of counting — presupposes "intensive reality", or the infinitesimal, which is the "meaning of reality", and a "logical secret" that must be discovered by methods that are, strictly speaking, outside of those proposed by the logicism of Frege and Russell. In other words, the conditions of the possibility of mathematics as such include a concept of "intensive reality" that is not provided by logicism or pure formalism, and must be supplied by the kind of critique of knowledge proposed by the *Erkenntniskritisches* model of *Kant's Theory of Experience*. Accordingly, when Cohen addresses the problem of the generation of continuity from the infinitesimals in his *Principle of the Infinitesimal Method and its History*, he must achieve a number of different goals. He must:

(a) provides a genetic account of the problem of reality in its connection to the problem of continuity;

(b) explain the role of the infinitesimal in justifying the continuity of the manifold;

(c) defend the logical concept of the infinitesimal against its critics;

(d) provide an *Erkenntniskritische* demonstration of the necessity of the infinitesimal for the cognition of empirical science.

Cohen's criticism of the limit method begins by echoing Hegel's: the inductive inference from finite approximations to the infinite determination has no (determinate) explanation. Indeed, "as long as one still seeks to transition inductively from the finite to the infinite," as takes place through the method limits, "so long will there be a "miracle", which however is explained as soon as one has learned to go about it the other way around, from the infinite to the finite" (PIM; CW 4:39). However, whereas Hegel's solution consists in the idealization of the finite into the infinite, Cohen proposes just the opposite. The infinite from which all mathematical cognition will originate is the infinite of the infinitesimal, the "differential" or the "intensive reality" of the mathematical object.

Now, just how one is to go about explaining the transition from the infinite (or the infinitely small) to the finite magnitude is the second reason for the untimeliness of Cohen's *Principle of the Infinitesimal Method*. For, Cohen aims to discover the "logical secret" that is "intensive reality" not by means of formal logic (thereby anticipating the logicism of Russell and Couturat), but rather in what he calls "Erkenntniskritik". Moreover, this inquiry focuses not on the mere form of the representation of knowledge — as Peano's logic does — but on the mere form of knowledge itself, in the general character of consciousness. Accordingly,"**the concept of the differential**," which is the concept through which the "general law of thought" that is reality is to be discovered, "**springs in the last instance** from the **source and principle of mechanical problems**" (PIM; CW 4:22). That is, it is not from the consideration of mathematics as such, but from the specific conditions of mechanical problems in their unity for thought. This condition, it turns out, is continuity: "Continuity designates a **general character of consciousness**, similar to **identity**. It is therefore a **special** expression of the general law of the **unity of consciousness**" (PIM; CW 4:35). In other words, continuity — as a general principle of knowledge — will not be grounded in the Dedekind-Cantor philosophy, in which continuity is replaced by the concept of order, and magnitude replaced with the concept of ordering relation. (Russell 1903, Part V) Rather, Cohen insists that continuity is a "special presupposition of thought" without which the unity of thought in general — and thus formal logic

and mathematics specifically — *would be unintelligible*. This, indeed, is the heart of Kant's legacy for *Erkenntniskritik*: "**reality** does not consist of raw sensible **feeling** and also not in the pure sensible **intuition**, but must be made valid as a **special** presupposition of **thought**, just like subtance and causality" (PIM; CW 4:14).

Logistics and Erkenntnislogik

Whereas mathematics in the late nineteenth century tends to eliminate the concept of magnitude, and tends toward the formalization of a logico-mathematics that is indifferent to the specifically cognitive conditions of mathematical reflection, Cohen's intention is to take philosophy in just the opposite direction. The reason for this is that, as Cohen objects, "equality and magnitude already presuppose **intuition**."[20] Indeed, this is the comment that provoked the most ire from Russell in his *Principles of Mathematics*. There, Russell claims that equality just is the formal identification of two quantities, and in no way invokes intuition. Thus, according to Russell, Cohen is simply engaged in a regressive — or worse, uninformed — speculation about the nature of mathematical magnitudes, thereby advocating precisely the kind of intuitionist mathematics that Russell's formalism rejects.

If Russell's reading of Cohen's *Principle of the Infinitesimal Method* were justified, then Cohen would indeed be engaged in an absurdly anachronistic project. However, here Russell has misinterpreted Cohen's proposal. For, Cohen is not making the claim that mathematics is *founded* in the concept of magnitude. Rather, he is making the claim that if the concept of magnitude is to be intelligible at all (and most interpretations of mathematics at the time supposed some concept of magnitude), then it must be because of an antecedent connection between the "formal" or "logical" definition of order and continuity. The concept of continuity, Cohen contends, implicates mathematics (or, more appropriately, the comparison of magnitudes)

[20](PIM CW IV:2) Superficially, at least, Cohen appears to be endorsing the intuitionist view of mathematics. For, he appears to be claiming that intuition is indispensible for the comparison of magnitudes, and thus for the possibility of arithmetical calculation. This, of course, is a view common among mathematicians of the late 19th century. For example, Otto Stolz in his textbook, *Allgemeine Arithmetik* claims, "calculation, i.e., the connection of magnitudes (*Grössen*), is the foundation of pure mathematics" (Stolz 1885, 1). Indeed, the most basic operation of arithmetic, counting, "presupposes the comparison of pluralities" (Stolz 1885, 9).

in metaphysics: the metaphysics of continuity, or the "reality" of the manifold. Furthermore, the "reality" of mathematics (as the mathematics of natural science) must be able to *explain* how thought about reality — and especially mechanical reality — is possible with respect to the other "functions of judgment" that constitute epistemic representations: i.e., judgments of natural science. The *Principle of the Infinitesimal Method and its History* is thus an ambitious attempt to isolate two distinct problems for mathematics. The first is the implicit perplexity of mathematics and metaphysics. The second is the entanglement of a mathematics of "reality" with natural physics. Both of these projects may seem to be at odds with the trends in contemporary mathematics, or even mathematics as it was practiced in the late 19th century. However, a foundation of mathematics in intuition or even natural science is certainly not the ambition of Cohen's *Principle of the Infintesimal Method*.

First, the perplexity of mathematics and metaphysics is exemplified by the concept of a magnitude. It is difficult to provide a mathematical definition of magnitude, since the concept implies not only the concept of an interval (i.e., a relation between members of an ordered series), but also that of a continuous extension, which implies (a) an intuitive cohesion between members and (b) an intuitive amplitude of the ordering. The intelligibility of mathematics rests on our ability to dislodge the fundamental logical presuppositions underlying the intuitive intelligibility of "cohesion" and "amplitude". Thus, when Cohen claims that there is an entanglement between mathematics and intuition it is not so that he might restore the primacy of intuition against formalism, but rather to discover the form inherent in intuition. This form — the first "stage" of the a priori — is what Cohen calls the metaphysical. This, indeed, is just the extension of Cohen's original interpretation of the a priori form of sensibility in Kant's *Critique of Pure Reason*.

Second, the entanglement of mathematics and natural physics is implied by the entanglement of mathematics and intuition. For Kant too, intuition is the foundation of the physical because it is the intuitive intelligibility (i.e., the a priori form) of nature. In addition to the intelligibility of the (abstract) magnitude, the concept of a real magnitude implies the reality of a

cohesive magnitude, that is, its connection to a system of possible magnitudes (a manifold of intuition). The concept of the physical, therefore, is just the concept of the systematic connection of the concept of magnitude, i.e., the logical (metaphysical) foundation of number. Cohen's aim, therefore, is not to show that mathematics is grounded in (pre-critical) metaphysics of intuitive or physical reality, but rather to show that mathematics (i.e., the concept of magnitude) is *entangled* with metaphysics, where metaphysics is understood to be the systematic formal connection of order.

The perplexity of mathematics and natural physics may be understood in the following ways. First, from an *Erkenntniskritisches* viewpoint, the origin of mathematics in geometry and arithemetic may be discovered through the scientific investigation of the domain of reality, i.e., in natural physics. Accordingly, the epistemic validity of mathematics derives originally from its application to natural physics, and this is the domain of its validity as an instrument of human knowledge. This does not mean that only constructions of physical reality are possible. Rather, it means that only constructions of physical reality are validated as epistemic constructions, i.e., judgments. This is the approach that Cohen attempts in the *Principle of the Infinitesimal Method*. Second, from an *Erkenntnislogisches* viewpoint, mathematics must deliver the formal logical foundations of the concept of the real magnitude in order to account for the possibility of knowledge of nature; for, only in this way can knowledge of nature be designated as "pure", i.e., derived entirely from a priori structures. This is the approach that Cohen will ultimately pursue in his account of the production of the manifold in the *Logic of Pure Knowledge*.[21] We may not, however, understand logic or mathematics to be *grounded* in intuition or physical reality. This is precisely the opposite of what Cohen intends. Rather, the concepts of intuition and

[21] So construed, the project of *The Principle of the Infintesimal Method* and the project of the *Logic of Pure Knowledge* address the same problem: what is the logical foundation of the determinability of reality, understood as an infinte continuum. *The Principle of the Infinitesimal Method*, however, approaches the problem through the fact of science, and therefore as a "transcendental investigation" as Cohen understands this in the context of *Kant's Theory of Experience*. In the *Logic of Pure Knowledge*, however, Cohen addresses the problem as a pure problem of construction, that is as a problem of the intentional logic of cognition: how are the representations of reality constructed in general? Accordingly, the *Principle of the Infinitesimal Method* is more explicitly entangled with the metaphysics of intuition and natural science than the investigation in the *Logic of Pure Knowledge*. Great care is required when interpreting the distinct results of each investigation. For more on the distinction between *Erkenntniskritik* and *Erkenntnislogik*, see (Edel 1989).

physical reality are precisely those which must be explained by the project of *Erkenntniskritik* and *Erkenntnislogik*.

On the one hand, the pure foundations of mathematics are indeed ordering functions: adjacency and succession; this just is the consequence of the formalist interpretation of sensibility. However, the concept of the mathematical — as Cohen understands this — must provide the concept of magnitude. This concept, however, implies (a) the cohesion of numbers (what Cohen calls continuity, and Russell will call connexity) and (b) the amplitude of intervals, or the systematic determination of a number as a relation to other numbers. To this extent, Russell and Cohen in fact have a common goal: they wish to provide clear and rigorous formal foundations to a system of logico-mathematics. The difference between the two projects is that Cohen construes the problems of logic and mathematics from within the systematic framework of post-Kantian Critical Philosophy, and specifically within the confines of the critique of knowledge. That is, the investigation of logic and mathematics is taken to originate in the human mind; knowledge of logic is therefore possible through critical reflection on the origin of knowledge in the mathematical knowledge of nature. Accordingly, the perplexity of logic and mathematics with the metaphysics of intuition and physical reality is intrinsic to the project of a system of knowledge derived from the critique of cognition.

Indeed, to say that magnitudes presuppose intuition is to say that they *cannot* be appealed to in mathematics without a rigorous foundation through *Erkenntniskritik*. The product of this critique is not — as it is for the "Old Formalists" — a further definition in terms of axiomatics, but a genuinely "logical" — as distinct from a "logistical" — foundation: one that grounds the possibility of knowledge in the a priori possibility of the intentional object of logical representation. Thus, the connection between intuition and magnitude is not the problem just of the formal or intelligible grounds of mathematical concepts (i.e., an axiomatic definition of continuity), but rather the production of a set of relations that correlates with the intentional object of thought. This too, requires a formal foundation in "logic". But this is evidently not provided by the kind of logic that Russell and Couturat have in mind, which construes the

problem of logic and mathematics in terms of constructability, not in terms of the ends of the systematic object of logic. Rather, it is — to repeat the point — Kant's important contribution to have shown that "**reality** does not consist of raw sensible **feeling** and also not in the pure sensible **intuition**, but must be made valid as a **special** presupposition of **thought**, just like subtance and causality" (PIM CW IV:14). Whereas the realization of Russell's epistemology requires the doctrine of *givenness* or "knowledge by acquantaince", Cohen contends that the possibility of givenness — reality — is already a problem for thought, and a fortiori a problem for a foundational logic. Thus, before logic can begin to claim an answer to the problems of human knowledge — and thus also of natural physics — the problem of reality must be given a clear foundation in an *Erkenntnislogik*.

Cohen's intention is thus not to contest the emerging formal logical foundation of the theory of number, but rather to find the connection between formal mathematics and mathematical practice: the mathematical natural sciences. This logic — ultimately an *Erkenntnislogik* — will, however, turn out to be more fundamental than the set-theoretic logic of the "old-formalists"; for, the problem of the objectivity of logic, and the objectivity of the logical object is, so Cohen argues, prior to the question of the epistemic relations that it can enter into. This is the priority of "logic" over "logistics", of intelligibility over form, in the reconstruction of reality. Accordingly, the orientation towards continuity begins not with the *finite* ordering of countable integers, as it does for Dedekind, and even Brouwer, but rather with the infinite. For Cohen, "continuous unity must be thought of as **origin**. The **infinite must be abstracted from the finite, in order to be able to produce the finite from itself**" (PIM; CW 4:32). Unlike intuitionist mathematicians, Cohen asserts the explanatory primacy of the infinite, and seeks to find in it the common ground between intuition and thinking. This is not a condition of the possibility of pure mathematics, but rather a condition of the intelligibility of mathematics for natural physics. For, so Cohen argues, certain concepts — reality, magnitude, continuity — have a physical significance that may be formally interpreted, but is not thereby explained in its full "logical" significance. This explanatory lacuna is most obviously manifest in me-

chanical physics, where concepts of continuity, infinitesimal change and magnitude are not only of explanatory significance, but have special significance for the possibility of physical knowledge.

Infinitesimal Reality

The explanation of the metaphysical and natural-physical foundations of mathematical method implies an *Erkenntniskritische* explication of the concept of continuity and its rational predicate: reality. This begins, for Cohen, in the abstraction from intuition as a *merely psychological* condition of representation, and its (formal) interpretation as an (a priori) source of knowledge: knowledge of the serial manifold of reality. Thus, as Cohen claims, "intuition like thought is an **abbreviation** for a scientific method" (PIM; CW 4:3). What this means is that science proceeds through two methods, which may be described as (a) empirical and (b) rational. The rational methods are those of conceptual construction and judgment: the logic of the syllogism. However, the possibility of the empirical does not rest on intuition as a passive faculty, but, as Cohen argued in *Kant's Theory of Experience*, on the a priori form of sensibility: relations of adjacency and succession. These are the a priori relations of the free construction of the manifold through which alone the empirical is posssible. Thus, mathematics is the "method" of empirical science as a logic of reality.

However, mathematics, for Cohen, is and remains a purely rational pursuit of free construction. He is not an intuitionist. Nevertheless, that upon which mathematics operates is mere "thought-material" (PIM; CW 4:25). It is not reality itself. The question of the (transcendent) reality of mathematics is a further question, one that must be answered by explaining the relation, if any, between empirical modes of cognition (the measurement and comparison of "magnitudes"), and the formal manipulation of continuous intervals of the "real line". For Cohen, the "method" of intuitive knowledge and the "method" of formal knowledge converge in two competing definitions of the differential. For, on the one hand, the point where the tangent meets the curve is a point of null extension. It is merely the intersection of two lines: the

curve and its tangent. Intuitively, however, the point in question is a point on a curve with a determinate position and a determinate relation to its neighbours on the curve. As Cohen argues, "the point of the tangent and the point of the curve ... cannot count as two different points that fall together" (PIM; CW 4:34). The solution is to think of the point on the curve neither as the intersection of two extensional lines, nor as the extensional point (finite infinitesimal) that constitutes the curve. Rather, we must think of the tangent-point and the curve-point as "one point, in regard to the production of the curve" (PIM; CW 4:34). That is, there is *one* point that is the "thought-material" of mathematics and the "real" material of intuition: the differential.

Cohen's interpretation of the infinitesimal is thus indebted to Leibniz's understanding of the differential as a non-extended (inextended) point, which nevertheless has an infinite plurality of (merely possible) determinations of its velocity.[22] That is, the mathematical point is not defined in terms of magnitudes of extension (for it has no such extension), but rather in terms of the determinable relations into which it may enter, i.e., the various lines (functions) through which it may be apprehended as part of a totality. Accordingly, Cohen understands the infinitesimal to be a *rational* condition of the possibility of the construction of the curve: the function, like the line, consists of the infinite set of points that constitute it, i.e., it is *extensionally defined*. However, the function is not extensionally defined through a set of coordinate points (e.g., $\{< 1, 1 >, < 2, 3 >, < 3, 5 >, ...\}$) in intuition. Rather, the extension of the function is the "ideal" set of *differences* between merely possible points. Thus, the extensional conception of the function does not consist of a *determinate* curve in an absolute space (or time), but rather of a universal relation of differences (i.e., the derivative) through which an infinite number of possible curves may be produced (i.e., the integral of the derivative).

But now, since the determinability of the point is logically prior to (a) the possible curve represented by the function and (b) the manifold of intuition in which the perception of the curve is actual, this implies that the determinability of the point (i.e., its possible differentials) is the a priori condition of (a) any possible intelligible continuous modification of the manifold and (b)

[22] See the *New Essays*, where Leibniz discusses his view of inextended points at length.

the determinate manifold of intuition in which curves are perceived as intelligible unities. That is, the "infinitesimal" is not a property of any extension (i.e., of a possible figure or a possible manifold of space or time); rather, it is the pure idea of determinability as the *condition* of magnitude — whether intelligible or merely intuitive. As for Maimon, Cohen supposes that an a priori order of differentials of extension (ideas of the understanding) is the condition not only of determinate figures, but also of the manifold of differences within which such figures are determinate as extensions.

It is precisely the non-extended point — infinite "infinitesimal" or differential — that is the point of contact between that which is a purely rational (or formal) construction, and the rational deployment of intuition as "magnitudes" that may stand in relations of "equality". It is therefore through the critical grounding of calculus in *differentials* (infinitesimals), not in limit magnitudes, that the unification of intuition — the real comparison of immediately given magnitudes — and thought — the abstract manipulation of "thought-material" —- can take place. It is through a critical conception of "reality" as the differential of actuality — both intuitive and formal — that it is possible to think of formal mathematics as a branch of science equal to, and profoundly related to, empirical experimentation.

5.2.2 Origin, Reality and the Rational Manifold

The Principle of the Infinitesimal Method and its History begins the project of interpreting the significance of the rational manifold and explicating the mechanisms of its construction. In this sense, the project may be considered to be related to the project undertaken in Hegel's Doctrine of Being in the *Objective Logic*. However, the project of the *Principle of the Infinitesimal Method* is, in many ways, a failure. The primary failure is not the attempt to found continuity in the concept of the infinitesimal (as Russell would claim), but rather that the *Principle of the Infinitesimal Method* has misconstrued the object of epistemic reconstruction. As we noted, Cohen takes the task of the *Principle of the Infintesimal Method* to be the reconstruction of the (rational, logical) conditions of *intuition*. This, of course, is just the natural extension of the

project of *Kant's Theory of Experience*, which aims to ground the a priori forms of sensibility in the logic of mathematics. However, the difficulty with this approach (and also, as we will see, the difficulty in Cohen's interpretation of the Transcendental) is that the implication of intuition in the foundations of mathematics rests on a psychological prejudice: that science must reconstruct the forms and objects of intuition.

However, the history of science has shown not only that Euclidean geometry is a contingent form of representation, but also that three dimensional geometries and perhaps even time itself are not necessarily adequate for the purposes of the rational construction of scientific representations. Indeed, the connection between our intuitive forms of representation and the representational forms of our best scientific theories appears to be entirely contingent, and the description of "intuition" as a (perhaps abbreviated) scientific method seems entirely unjustified. Accordingly, the mature expression of Cohen's doctrine of the rational manifold is only to be found in his systematic work, *Logic of Pure Knowledge*. There, Cohen dispenses with the project of providing a rational reconstruction of intuition, and therefore abandons the thesis of the (psychological) perplexity of the rational foundation of mathematics. Nevertheless, the problem of the infinitesimal — and its reality — remains the central problem for a logic of production.

Whereas the Kantian doctrine of the faculties proposes two "sources" of knowledge (the equivalent of "two scientific methods"), only one of these is "original" or "pure": "I call all representations **pure** (in the transcendental sense) in which nothing is to be encountered that belongs to sensation" (KRV; AA A20/B34). Taking his lead from Kant's distinction between "empirical" intuition and "pure" intuition, Cohen argues that logic (alone) must be able to provide *all* of the logical materials of construction in a purely a priori (analytical) way, that is, in a way that does not require the empirical demonstration of the possibility of the finite construction of the object in intuition. A pure logic of production, therefore, is not a truncated version of Kant's transcendental synthesis, but rather an idealization of the constructive activity of thought, abstracting from the synthetic conditions of finite construction.

And so, whereas in the *Principle of the Infinitesimal Method*, Cohen is concerned with thought *and* intuition as "sources" of knowledge, in the *Logic of Pure Knowledge*, only one of these sources is taken to be of significance for the project of scientific knowledge: the rational. Thus, for Cohen, "scientific thought begins its history with the concept of the origin" (LRE CW 7:79), and thus with the problem of the free construction of reality. Accordingly, the table of judgments begins with a *judgment* of origin. Kant's classification of the system of judgments begins with judgments of quantity, which serve to *unify* a plurality under the intelligible unity of a rational quantification (all, some, one). Taken in its categorical form, the judgment of *unity* serves to unify the factical plurality (of sense or of objects) under the unity of intelligible form of quantification. As such, it is an operation that synthesizes *the given* into a unity. For Cohen, however, this aspect of Kant's transcendental logic is, as we have seen, problematic. Instead, Cohen claims that thinking must precede the given in the logic of thought. Thus, "this must become the **first** concern of thinking: **to found** (*legen*) the origin of all content that it may produce **in thinking itself**" (LRE CW 7:82). In other words, the first "act" of thinking must be to establish the origin of the intelligible: as that which is in thinking itself.

We can understand Cohen's emphasis on the priority of thinking with respect to the given as an extension of Kant's thesis, "namely that we can cognize of things *a priori* only what we ourselves have put in them" (KRV; AA Bxviii). There, the claim is that the form of the intelligible is grounded in the activity of the subject: the forms of judgment. However, as we saw, Cohen extends the domain of the activity of the subject to include not just the relations between objects, but the relationality of objects themselves, the pure form of *sensibility*.[23] Accordingly, the "transcendental method" can be extended such that it demands a rational grounding for the forms of the objectivity of the thing itself, i.e., the a priority of mathematics. Now, since the content of sensibility may only be apprehended through a system of a priori relations, the first act of thought must be to establish a system of relations through which the given may become

[23] This is not an entirely un-Kantian way of thinking. As we have repeatedly seen, Kant at times claims that all possible *combination* of plurality implies the unifying functions of thought (synthesis), without which they would not be possible.

intelligible in the first instance.

The claim, then, is that the given is not even something that can be apprehended unless it is already the case that something can be thought. This "something", however, remains an indeterminate *relation*, at least at this stage of the development of thought, for it is merely the thought of the determinability of sensibility as a system of relations. What kind of relation this may consist in is yet to be determined. And so, as Cohen insists, "individual things first enter into a connection with each other in the question of the origin. This **connection** is the being (*Seiende*), in distinction to the thing" (LRE CW 7:79). That is to say, the question of the *origin* of thought precedes the question of the *content* of thought.

Prima facie, Cohen's strategy suggests a close kinship with Hegel's method in the *Science of Logic*. There, Hegel begins his reflection on logic with "**being, pure being** — without any further determination" (WL1; HW 21:59). Unlike Kant, Hegel posits being as an order of determinable relations — whose determination is the topic of the first section of the objective logic. In positing being — *pure being* — Hegel does not posit any determinate form of thought, but rather its pure determinability. There is, however, a significant difference between Hegel's conception of being and Cohen's conception of origin. For, Hegel takes unity to be an intrinsic feature of being. That is, pure being is given as determinability, and, more importantly, as determinability with respect to itself. What this means is that logic, for Hegel, consists in the determination of being through being itself. Or, to put this another way, it is the determination of the totality of being that characterizes the determinacy of both being and its beings, and thereby also the determinacy of the logical operation of determination.

For Cohen, however, the judgment of origin posits a determinable relation, but not yet a *system* of relations. For, unlike Hegel, Cohen requires that the manifold of determinables be constructed by thought itself. Being — or the manifold of possible determinations — is not simply the "matter" of thinking, but is itself a constructed unity. It is not given, intelligible, or form in any absolute sense. And so, whereas Maimon and Hegel both assert the infinite givenness of the manifold (i.e., the *totum reale*), the judgment of reality merely asserts the

connection of a relation of composition: the *compositum ideale*. For Cohen, "thought produces **unity** in the **relation of unities**. Thus thought, as the thought of unity, is **conditioned by relation**." (LRE CW VII:91) In other words, it produces the pure possibility of determining a manifold — a priori or otherwise — through a pure relation that, as a condition of the reality, intelligibility or form of being, must be produced. However, as a pure relation, it is, as yet, *nothing*. And yet, as a relation between indeterminates it is the determinable, and insofar it is something: the determinability of the determinable, a purely relative or relational nothing that is still something, or what he calls an *Ichts*: "the relative nothing is wholly fixed, and without other connotation, on its aught (*Ichts*" (LRE; CW 6:93). We may think of this "nothing-something" as the rational ground of the possibility of determinability. It is thus that which *conditions form* of being: being as purely determinable relation, an "**origin-something**" (LRE; CW 6:105). Nevertheless, it is, as activity of thought, a *pure condition*, which it to say, the condition of being is grounded in the *formality* of thinking, or the thought-act of production whereby individuals may be posited *through their determining relations*.

5.3 Problematic Determinacy

5.3.1 The Principle of Production

The major interpretive difficulty in parsing Cohen's logic of origin lies in the fact that in avoiding the paradoxes of *intuitive magnitude*, he has turned to a Leibnizian conception of the *infinitesimal*. What Kant calls an "intensive magnitude", and which appears to be an order of *difference* that is quantifiable but not "extended". This is taken to be the common ground between *pure knowledge* and *pure representation*, that is, between thinking and mathematical representation. Accordingly, the "infinitesimal" may be thought of as foundational for *thinking* as such, and thereby for any possible answer to the question of being. Thinking, as it were, is the foundation of the individual as such, and the ground of the possibility of the concrete (intelligible) representation of differences and similarities. The problem that Cohen's logic highlights

is this: while it is possible to construct mathematical representations by, for example, drawing functions on a graph, or perhaps merely by representing these functions algebraically, these functions are not fully determinate except through their intuitive interpretation as relations of magnitudes. Whereas the *Principle of the Infintesimal Method* attempted to resolve this perplexity into one constitutive of the possibility of mathematics, the *Logic of Pure Knowledge* argues that the implicit connection between mathematics and intuition is not to be explained, but rather to be eliminated from the "pure" rational foundation of the a priori determinacy (reality) of the particular.

Cohen's claim in the *Logic of Pure Knowledge* amounts to the claim that the rational determination of the concept of continuity is necessary to the rational purity of any functional representation, and thus to the possibility of knowledge as such. For, "**continuity is a law of thought**." (LRE CW VI:91) And, "**By way of continuity, all elements of thought, insofar as they are to count as valid elements of thought, are produced out of the origin**" (LRE; CW 6:92). Accordingly, "**infinitesimal analysis is the legitimate instrument of mathematical natural sciences**," and is the "**triumph of pure thought**" (LRE; CW 6:33). Of course, the view that all natural-scientific thought is expressed in the mathematics of differentiable functions is a 19th century prejudice that did not survive long into the 20th century. However, Cohen's conception of the logic of origin is sufficiently flexible to accomodate a wide range of representational systems, provided that they are fundamentally systems of relational order; after all, the concept of "continuity" is, for Cohen, not a concept of intuitive continuity; nor is it the (formal, mathematical), concept of continuity as a concept of reality, understood in a very particular way. What is decisive for the Cohenian position, however, is the priority of origin and continuity in the construction of reality (or whatever representational system is in the end required). In the absence of a rational definition of continuity, functions may still be expressed as graphs or as functions. However, these representations remain entangled in empirical psychology (intuition) until such time as a rational foundation can be provided which finally severs the connection between the concept of the function and the intuitive concept of a magnitude.

Accordingly, Cohen's project — especially in the *Logic of Pure Knowledge* is not opposed to the project of the "old-formalists" (Russell, Couturat). Although Cohen's determination of the rational concept of continuity is quite different to the one proposed by Dedekind and embraced by the formalist school, it is not essentially different in spirit: serial order is the foundation of continuity, and it is an order that is essentially a *product* of the free creation of reason.

If we understand the order of infinitesimals not as an order of *already* ordered elements — a determinable totality, either given or posited — but rather as an order of free creation, we come much closer to grasping the essence of Cohen's proposal. As Gawronsky emphasizes, "**continuity** (*Continuität*) as the process of production is distinguished from **uniformity** (*Stetigkeit*) as the result of this process" (Gawronsky 1910, 48). Gawronsky's point here is that the "continuity" of a function (e.g. the real numbers) is distinct from the "continuity" of the production of a serial order through the production of infinitesimals. Indeed, the former presupposes the latter. Thus, the infinitesimal is not to be considered to be already an *element* of a function, line, manifold, or field, but rather as the condition of its possibility *überhaupt*. Accordingly, "**there is no other means to formulate the laws of natural science; no, not merely to formulate, but rather even no other means to ground them, to lay the foundations and to work on these, than that which is secured and described in the infinitesimal**" (LRE; CW 6:134). That is, the infinitesimal is the ultimate intelligible ground — the *hyperbelon* — of every possible determination of thought. Cohen's proposal relates not to the *product* of thought as an order of continuity, but to thinking itself as essentially the production of continuous forms out of the indeterminate and (a priori) indeterminable compossibility of thought-elements.

Indeed, if we think of the infinitesimal not as an element of a pre-existing order, but rather as the condition of *any* order, whatever its structure, we may be able to understand Cohen's proposal as analogous to the method of construction proposed by Dedekind in his '*Was sind und sollen Zahlen*' (Dedekind 1909). For example, Dedekind takes it to be the case that numbers are products of thought, and, "my own realm of thoughts, i.e., the totality S of all things, which can be objects of thought, is infinite" (Dedekind 1909, 45). Here, however,

Dedekind does not mean to imply a totality in the Maimonian or Hegelian sense — a *totum reale* — but rather an unrestricted domain of composition: the whole of composition. For Dedekind, the domain of numbers is not intuitive, or even connected to a domain of reality, rather consits of "creative acts" (Dedekind 1909, 4), which are independent of any facticity of consciousness: "numbers are free creations of the human mind." (Dedekind 1909, 4) Indeed, Dedekind interprets numbers explicitly in terms of a system of (constructed) differences, which serves as the basis for the apprehension of objects:

> numbers are free creations of the human mind; they serve as a means of apprehending more easily and more sharply the *difference of things*. (Dedekind 1909, 5)

For Dedekind, it is the unrestricted positive character of thought — i.e., its creativity — that makes it possible to generate the irrational numbers from the integers and the rationals. Indeed, the positive fiat of Dedekind's "cuts" appears, in principle, to be no different to the rational positing of the infinitesimal.

However, we still might find Cohen's formulations suspicious. For example, Cohen describes the continuity of the point in terms of curves, which suggests that they are already part of an intelligible system of relations, perhaps the topology of the real manifold: "**this absoluteness of the point**, insofar as in it the **direction** lies, and the **production** of the curve uninterruptedly starts from it, **we will designate as reality**" (LRE; CW 6:130). In order to make sense of these (admittedly oracular) formulations, we must take note that Cohen's claim is that the "curve uninterruptedly starts" from the infinitesimal; the curve is not the infinitesimal, nor is it presupposed by it. That is, the affirmation of reality does not affirm (a) an infinitesimal portion (limit magnitude) of an a priori manifold or even (b) a manifold constituted of infinitesimals, nor even (c) a set of curves through which the determinability of the point is determinate. Indeed, the "absoluteness" of the point in which direction "lies" does not refer to an already existent order of curves, or a topological space that is generated out of them. Rather, the point — as the pure, absolute possibility of relation — just is, like Dedekind's "numbers", the positing of a member of a purely possible ordering scheme. The characteristic of these posited "points"

is that they are the condition of the possibility of order insofar as an order consists of a *system* of relations. Thus, the positing of a successor (as in counting) does not posit the number absolutely, but rather posits the element as one in a myriad of possible relations. This is why, once posited, the "point" becomes successively *more* determinate, even though it has not *an sich* been changed. Rather, its determinacy arises from the increasing number of relations that arise from the (continuous) production of systems of relations.

For example, we may think of the integers as a determinate system of "points". This system of points becomes *more determinate* when, through the positing of rational composites — i.e., the rational numbers — the integers becomes interpretable as a sub-set of rational numbers. They are more determinate not with respect to the original system of ordering (the integers), but rather now because the same symbol (e.g., 4) can represent a point or element within the rational numbers. Its place in each order is different, although it remains the same element considered from the point of different systems of ordering. The original points become still more determinate when they are related through a system of irrationals, such as those produced by Dedekind 'cuts'. This process of relating the original "infinitesimal" or "absolute" points can continue indefinitely, producing multi-dimensional manifolds of real numbers, imaginary numbers, hyper-real numbers and beyond. And so, when Gawronsky distinguishes between *Stetigkeit* as the "characteristic" of a product (i.e., a system of rational posits) and *Continuität* as the production of a system, it is this distinction — between the continuity of the reals, imaginaries, and hyperreals — and the continuity of the production of number systems *as such* to which he is referring. Gawronsky makes the point this way:

> both mathematicians [Dedekind and Cantor] are fully correct when they claim that it is impossible to ground the concept of the continuum in intuition. Furthermore, a purely abstract, purely logical formulation of this concept is absolutely required. However, the criterion described there go no further than this: only this conceptual formulation of the essence of the continuum may be created out of them, only a "characteristic" through which it may be determined, not however the way that one comes to it, not the logical means through which it is produced. (Gawronsky 1910, 43)

And so, whereas Dedekind takes the integers to be the fundamental and "intuitive" (i.e., in need

of no further explanation) relation through which all others are to be understood, Cohen argues that the determinate concept of the real number is *prior* to the determinate concept of integers. That is, within the total (Platonic) domain of number across which we can quantify (without, perhaps knowing how or even what is thereby quantified), any particular element of an ordering system is only as determinate as its place in the most complex ordering system yet developed. Granted, the continuum of the real numbers cannot be developed in thought except through the positing of simple relations of order (that is, it is not a simple compositing of limit magnitudes, as Hegel proposed). However, what is thereby thought is the concept of order, not the concept of the integer. The integers, so Cohen argues, are only determinate as "points of rest" in the continuum of real numbers (which, charitably, we may interpret as points contained in simpler sets of relation, such as the rationals, the integers, the naturals and the positive even numbers); that is, their determinacy as numbers depends on the determinate concept of number, which is not achieved through simple ordering, but rather through the idea of the intelligible unity of the continuum of production of possible systems of ordering. This continuous production is not the continuity of a point-set topology (Dedekind-completeness + compactness) but the intellectual power to represent (or formally construct) systems of ordering relations of arbitrary complexity.

5.3.2 Production and Determinacy

For Kant, the totality of the manifold of space and time is "given" in the a priori form of intuition: space and time. Space and time, however, are not empirical intuitions, but rather "pure" intuitions. As Cohen argues in *Kant's Theory of Experience*, however, pure intuitions are not *extensional* modifications of the sensible manifold, but a priori *methods* for the ordering of sensibility, i.e., *intensional* structures. Accordingly, pure forms of intuition are just "concepts" with a different logical form: serial order. The hypothesis of faculty dualism, of course, asserts that these forms have their origin in different faculties of the mind. However, if we abstract from the psychological conditions of particular representations and focus only on the problem of the production of universally valid judgments, then both syllogistic structures and serial orderings

appear to be products of "subjective" activity.

As we have seen, Maimon and Hegel both take the intellectual manifold to be a priori determinable through a logic of the determinable totality; this requires (a) that the totality of the determinable (the manifold, being) be represented in consciousness prior to its conceptual determination (*the principle of reality*) and, (b) that every affirmative determination of the subject, object and truth of a judgment implies the reciprocal and relational determination (as determinate, determination, or truth) with respect to the determinable, or the (*the principle of determinability*). The first condition requires that the manifold be given *extensionally* (i.e., as the whole and its (implicit) parts, not necessarily as "in space"), and not just *intensionally* (i.e., as the (mere) principle of the whole through constructable parts) as it is for Kant. The second condition requires that the predicate of reality be analytically (or through an intellectual synthesis) determinate in all of its possible applications, even if this determinacy is only implicit; this is Maimon's criterion of "material truth", and Hegel's claim about the reality of the concept.

Accordingly, I have characterized the logics of Maimon and Hegel as logics of totality; for, determinacy — or, as for Hegel, ideal determinability — consists of the explication of the rational parts that are immanent in a factical whole. Thus, the whole is given, and the totality — as the whole composed through its parts — is implied by the facticity of the whole, even if its rational reconstruction is temporally modalized. However, Cohen rejects both central theses of the logic of totality: the *principle of reality* and the *principle of determinability*. Rather, he endorses what I will call the *principle of production*.[24] The principle of production holds that determinability is the result of the production (rational positing) of (a) an unrestricted system of determinables and (b) determinable relations between determinables. The positing of determinables is, as we have seen, the function of the *judgment of origin*. The positing of determinable relations (order) is the function of the *judgment of reality*. For Maimon and Hegel, the manifold is an *extensional* totality. For Cohen, on the other hand, the manifold is a "pure" method for the construction of a manifold. That is, the manifold is not a *totum reale*, whose

[24]Here, for example, is a sample formulation: "**Thought produces the foundations of being**. The ideas are these foundations, these self-produced **laying-of-foundations**" (LRE; CW 6:20).

parts must be rationally reconstructed, but a *compositum reale* that must be produced in order for anything to be thought at all.

For example, the individual member of the manifold, the "infinitesimal" (the aught, *das Ichts*) is a pure posit, but is also absolutely indeterminate and (*de facto*) indeterminable. That is, *qua* individual, it is not even determinable. The determinability of the rational product of the jdugment of origin only acquires determinability insofar as it can be related to another posited origin. It is the possibility of the relation between the first posit and the second that establishes (a) the determinability of the first (and second) rational product and (b) the relation that expresses the determinacy of the rational posit. Accordingly, indeterminacy is a constitutive part of any rational construction; for, the individuals of the production of thought (infinitesimals, origin) have no determiancy *an sich*. Rather, they first acquire determin*ability* through the "continuous" production of a *system* of infinitesimals, i.e., the through production of a system of possible relations of determination.

Prima facie, Cohen, like Maimon, has restored the rational significance of the predicate 'reality', and in particular its Leibnizian significance. For, (phenomenal) reality and determinacy both consist of systems of differences, i.e., reciprocally determinate relations between rational posits (monads). However, while Cohen endorses the thesis that reality is determinate through a rational manifold, the nature of this manifold is radically different from the one proposed by Maimon or Hegel. For, Maimon and Hegel take the manifold to be explicitly (Maimon) or implicitly (Hegel) determinate in its totality in the fact of sensibility. For Cohen, on the other hand, the concept of the manifold is not bound to sensibility, intuition, or to the fact of any qualitative or psychological experience. Rather, Cohen understands the determinability of the manifold to be the result of a cumulative process of "production" with no analytically determinate boundary. The constructions of logic are therefore not analogous to the factical nature of sensibility, and need not satisfy its conditions. Instead, the pure manifold is bound only by the rational conditions of knowledge.

Thus, the determinability of the origin (i.e., of the rational posit) consists in a *system*

of posited relations. Maimon too understands reality to consist of a system of posited relations (i.e., the ideas of reason, which organize the ideas of the understanding); however, Maimon understands the determination of the totality to be the condition of the determination of the particular. For Cohen, however, the condition is reversed: the determination of particular relations constitutes the manifold. This manifold increases in complexity as more infinitesimals are posited, and more relations become determinable. Cohen, therefore, understands the determinability of the origin to increase as more systems of posited relations are added through which the origin may be determined.

As we saw in the last chapter, the determinacy of number — 'plurality' — arises from the identification of a number with itself, i.e., the identification of a thought-element (perhaps an ordinal) as itself, and not something else. However, the intelligible determinacy of this self-identity does not derive from the intrinsic nature ("being") of a number, but rather from the system of differences (i.e., other rational posits), i.e., through the judgment of 'totality'. The judgment of totality, however, is an indeterminate judgment. For, as we have seen, it is the cancellation (not the "*Aufhebung*") of privation: not *me*-determinate. Accordingly, the determinacy of the judgment of plurality (i.e., the identification of reality) is the judgment of an individual with respect to an indeterminate totality. For example, the antique concept of number is largely indeterminate, for it posits a limited number determinable relations: the (positive, non-zero) integers. Rational numbers provide further determinacy to the concept of number by positing a system of relations (ratios) through which numbers may be related. However, these numbers are — at least initially — thought of as ratios, not numbers, and do not make the integers more determinate; integers are, as we saw, thought of as determinate in themselves as individuals, at least on the Aristotelian, physicalist interpretation of number. Thus, the number 2 is a 'plurality' insofar as it is not 4.

Reality, therefore, is not an absolute predicate; we cannot say that a number — 2, 2.5 or $\sqrt{2}$ — is *absolutely* determinate. For, it is only determinate with respect to the system of number that is its *limitation*. Thus, for example, 2 is not determinate with respect to the integers,

but rather with respect to the rationals. For, it is through the system of rational numbers that the systematic relation of 2 to 3, or 3 to 4 can be fixed.[25] Still, however, this only means that the integers are determinate *relatively*, that is, with respect to the system of rationals. The rationals, however, are only determinate when their limitation — the real numbers — can be rigorously established. That is, the method of Dedekind 'cuts' (for example), delimits the rational numbers with respect to a continuous system of real numbers.

Accordingly, Cohen emphasizes that the reality of number — its self-identity as "plurality" — is not in itself determinate (as x, or as a determinate number), but rather through the system of *differences* that make its determinacy possible:

> As a law of thought, continuity would mean for the infinitesimal the relation of x to its dx. For reality, on the other hand, x is not of foremost importance. **Continuity means the relation of the** dx**s**, the relation **of the infinitesimal elements**. (LRE; CW 6:137)

We may, of course, generalize the claim to avoid the appearance of presupposing the real continuum. For the concept 'number', continuity means the relation of the determinables. While the real numbers are determinate through the infinitesimals, the integers are determinate through the rationals, and the rationals through the reals. As Gawronsky emphasizes (see above), "continuity" here does not mean "*Stetigkeit*" as the product of thought (the reals), but rather the continuity with which number is produced, or better, producable. The determinacy of the individual, therefore, is a regulative ideal, and takes the place of the idea of the "world-system". The determination of the determinability of the origin is therefore what Cohen calls a "task" for thought, through which the "pure" determinacy of the individual (in this case number) becomes possible through an unlimited regression of systems of determination. That is, number is not an absolute determination, but a method of determination: the anticipation of the reality of the actual. This, of course, just is the interpretation of the *principle of pure individuality* as it is applied to the manifold.

[25] We might object that 2 is indeed determinate, since it is determinate with respect to the order of successors, i.e., it is the successor of the successor of zero. The problem with this understanding of the integers is that here the relations of the integers is thought of as absolutely determinate: with respect to zero, which is not an integer at all (see **Section 5.3.3** below for more on this problem).

And so, whereas Maimon understands the totality of the manifold to be determinable through a (Leibnizian) logic of difference and Hegel understands being to be the pure determinable through a (dialectical) logic of distinguishable limitations, Cohen denies that the totality of the manifold is qualitatively or symbolically intelligible prior to its systematic rational construction. This construction, however, is positive: it requires the positing of infinitesimals and the construction of increasingly complex systems of reciprocal relations. Accordingly, Cohen must deny the thesis of reciprocal determination. If reality is the determinability of a system of relations, this system is a pure construction with no logical complement. For, in order to determine the real complement of a real system we would need to suppose that the manifold is a totality in which every positive determination corresponds to a determinate negation. This, however, Cohen denies. As a result, Cohen may also reintroduce limitative judgment (privative judgment), since the negation of a positive system of relations consists of the unlimited set of heterogeneous constructions. The extension of this set is, of course, not logically determinable.

5.3.3 NSA and the Infinitesimal

As I noted above, the concept of the infinitesimal was, for all intents and purposes eliminated from mathematical analysis by the limit method and the concept of an infinite calculation. This remained the case until the early 1960's when Abraham Robinson rehabilitated the concept of the infinitesimal in his *Non-Standard Analysis* (Robinson 1996). Non-Standard Analysis (NSA) proposes a system of numbers — the hyperreals — that lie "in the gaps" between the real numbers, much as the real numbers lies "in the gaps" between the rational numbers.[26] The hyperreal numbers may be thought of as 'infinitesimals', that is, as the non-extended boundaries

[26]The topological metaphor here is suggestive, but perhaps somewhat misleading. Just as Dedekind proposed that we 'cut' the rational number line — which is everywhere dense — to produce the irrational numbers, the hyperreals may be produced by 'cutting' the real number line — which is everywhere continuous — to produce the hyperreals. The only problem with the analogy is that our intuitive understanding of topological spaces is that of real space: thus, there are no "gaps" in the real series as there are in the rational series. Nevertheless, we may think of a hyperreal topology as an extended topology in which the real line is wholly covered by the hyperreal line, and contains other "infinitesimal" points besides.

of intervals of the real line.[27] These infinitesimals separate real intervals of the real line (real magnitudes, or , in less loaded language, the actual determinacy of limits) through an actual infinitesimal, which at the same time has no *real* value. The question now is this: to what extent can Cohen's infinitesimals be compared with Robinson's hyperreal infinitesimals?

There are a number of ways in which we might approach the question. First, we may note that Cohen did not have the mathematical sophistication to produce a formal definition of the infinitesimals in terms of an extension of the reals, nor of a hyperreal manifold as the intelligible condition of the real manifold. Furthermore, there is no evidence that he attempted any such thing. However, we may want to argue that Cohen *anticipated* the concept of the infinitesimal. Indeed, one of the main advantages of NSA is that it allows us to provide an intuitive introduction to the calculus, one that avoids the complex constructions of delta-epsilon convergence.[28] As we have seen, it is precisely the (perhaps merely apparent) arbitrarity of the delta-epsilon limit method that drew criticism from both Hegel and Cohen.[29] And, whereas Hegel proposed the idealized completion of the limit method in the concept of "the ones" as the limitation of magnitude (and thus, as we have seen, as dependent on the intuitive manifold and its intervals as real magnitudes), Cohen proposes a system of rational posits: the infinitesimals. *Prima facie*, these rational posits — which have no magnitude but are rather that out of which

[27]The real line contains one hyperreal number: 0. Zero is not a real number because it is not a determinate real, rational or integer number, but is rather the *limitation* of this determinacy. For example, Frege's method of constructing the integers begins by positing zero as the null set, i.e., as an integer that is not an integer, since it is not the successor of anything. Similarly, 0, as the boundary between positive and negative real numbers cannot be produced through a Dedekind 'cut', since between the least rational positive number and the greatest rational negative number, there is not only zero, but also least and greatest irrational numbers. That is, the method of 'cuts' does not uniquely produce irrational numbers at the boundary of positive and negative real numbers unless zero — a hyperreal boundary — is already admitted.

[28]It is significantly easier to provide an axiomatic treatment of real analysis using infinitesimals than it is to do the same through the Bolzano-Cauchy-Weierstrass limit method; however, whereas the method of limits can begin with our intuitive grasp of the line as a real continuum, infinitesimal analysis must begin with some notions from set theory. See, for example, Jerome Keisler's *Elementary Calculus: an Infinitesimal Approach* (Keisler 1976) or James Henle and Eugene Kleinberg's *Infinitesimal Calculus* (Henle 1979). An axiomatic treatment of elementary analysis can be provided relatively simply, as in Alain Robert's *Nonstandard Analysis* (Robert 2003). Nevertheless, there is still resistance to teaching the non-classical approach. (I had to learn analysis through limit methods, and there was, as a non-mathematician, nothing intuitive about it.)

[29]Both Hegel and Cohen criticize the apparent "Aristotelianism" of the limit method, i.e., that the approach towards the limit is always finite. However, these criticisms are somewhat off the mark, as a formalist interpretation of the limit method allows us to infer (ideally) the existence of the limit without having to carry out the calculation.

magnitude — or better, a quantifiable system of ordered relations — can first be produced, anticipates the function, if not the formal execution, of Robinson's infinitesimals.

Richard Arthur, for example, has argued that Leibniz's concept of the infinitesimal in fact proposes infinitesimals not as the limit of magnitude — i.e., as real numbers — but rather as pure fictions that may be thought (posited) as the actual limitation of the real (Arthur 2007). Thus, when Leibniz's proposes that $x + dx = x$, he is not — as is often thought to be the case — that we dx is a magnitude that may simply be disregarded (Newton). Rather, he is claiming that dx is no magnitude at all, but rather a "fiction", which is a purely rational posit that serves as the actual limitation of magnitude: an infinitesimal in Robinson's sense.[30]

For Cohen, it is clearly the case that the products of the judgment of origin — the infinitesimal "*Ichts*" — function as the *positive* limitation of reality. For, the rational construction of reality — i.e., the concept 'number' as the determinacy of of a system of relations — arises out of a system of productive judgments, or more specifically, out of a determinate quantification (what Cohen calls a "*Ruhepunkt*") as the 'reality' of judgment. In other words, reality — as the determinacy of a system of posits, and thus as the determinability of order, number or "real magnitude" — is already a specific determination of the general determinability of thinking as such.

And so, even if Cohen nowhere makes the case that the judment of origin produces hyperreals, it is also not a possibility that can be excluded by the *Logic of Origin*. The reason for this is that, whereas the *Principle of the Infinitesimal Method* construes the problem of reconstructing the manifold from within the psychological boundaries of intuition, the *Erkenntnislogisches* project of a system of pure posits explicitly avoids limiting the system of determinables to an intuitive conception of number or the manifold (e.g., $\mathbb{R}^4$).[31] Indeed, for Cohen, the concept

[30] Robinson makes a similar suggestion in (Robinson 1969) and (Robinson 1996). In particular, Robinson wants to distinguish between Leibniz' fictionalist view of actual infinitesimals, and de L'Hôpital's realist view of infinitesimals as least magnitudes (De L'Hôpital 1716).

[31] Indeed, insofar as Cohen believes that thought is an act of creation *ab nihilo* (rather than *ex nihilo*), he may quitebe accused of allowing intellectual intuition. By this I do not mean the ability to sense the super-sensible, nor even the ability to be conscious of our own mental activity. Rather, I mean by 'intellectual intuition' the capacity for thought to produce plurality out of its unity. To the extent that thought produces something that is other than itself (rather than merely a modification of its unity), one may believe that Cohen is appealing to a form

'number' is a *category*, and as such, it is a historically contingent construct, necessary for the purposes of scientific and logical representation at a certain stage in its development. And, just as the concept 'number' replaced 'magnitude', so too the concept 'order' replaced 'number' and was in turn replaced by the concept 'calculation' (Sinaceur 1994). Moreover, both 'space' and 'time' are categories; they are particular determinations of the functions of judgment (substance and reality, respectively) that are necessary to the representation of scientific knowledge at a particular stage in its (historical) development.[32]

Whereas Leibniz can be said to anticipate NSA and Robinson's infinitesimals, Cohen's account can only be said to anticipate the general determinability of an infintesimal. Now, the attempt to ground the infinitesimal in the concept of the 'intensive magnitude', and to interpret these as a rational condition of the intuition of determinate magnitudes suggests that Cohen wanted to separate the concept of magnitude from the concept of number as such; indeed, this is the task of the *Principle of the Infinitesimal Method*. However, Cohen's *Erkenntniskritische* account in the *Principle of the Infinitesimal Method* remains entangled in the conception of "intuition" and its boundaries as a "scientific method". The *Logic of Pure Knowledge* dispenses with the problematic identification of method and intuition, but also does not provide an explicit substitute for the intensive as the intelligible limitation of magnitude. Rather, there

of intellectual intuition. However, I do not think that the objection will be successful. Since everything begins (and ends) in thought, the judgment of origin merely amounts to a denial of the Hegelian claim that one must think of every determinate object of thought always as systematically related to the totality of thinking as such. Cohen denies this, and instead insists that one may simply posit an object as a determinable, whose sole charactersitic is its self-identity. We cannot, of course, get far by doing this, since no two objects that are characterized through nothing more than self-identity could be thought at the same time; they would (by the principle of the identity of indiscernibles) be indiscriminable, and thus identical. Accordingly, the judgment of reality — that of a relation between two posits — serves to posit a relational difference of discriminability. This may be — but evidently need not be — an ordering function. In any event, Cohen could not, as Frege does, define the origin extensionally (as the null set), but would have to do so intensionally. From this qualification, we can see whence Cohen's concern with the indeterminacy of number systems arises: there is no absolute origin, but only an origin of intensional differences. An origin of any relational system of differences may be stipulated, but this is only an arbitrary definition. One might get more traction out of a consideration of zero as a special number (*the* origin) with respect to which all other relations can be determined, but which is not in itself determinable.

[32]The Marburg School is celebrated for its early and active embrace of special relativity, which was otherwise met with widespread resistance from the philosophical community, especially in those quarters most indebted to the Kantian (and Hegelian) paradigm of "real" cognition. As we can see from this brief consideration of the positive character of the judgment of origin, however, the functions of judgment do not prescribe their eventual objects — as they do for both Maimon and Hegel.

Cohen understands number as arising from the moment of identification of the mathematical judgments, i.e., the judgment of plurality. Of course, a number (or magnitude) only is what it is because of the (implicit) limitation of its boundaries. However, once again, Cohen does not formulate the concept of an explicit hyper-real boundary, but only requires that the limit method (construed as an "Aristotelian" progressive approximation) be complemented by the ("Platonic") idea of the limit as objectively valid prior to its construction —- that is, he insists on a formalist conception of the existence of well-defined objects.

Cohen's infinitesimals, therefore, are not strictly speaking analogous to Robinson's infinitesimals or Leibniz' 'fictions'. For, Cohen's infinitesimals do not stand in an explicit relation to the real numbers or their determination. Rather, Cohen's concept of the origin as the pure, positive activity of thinking suggests that he understands the 'infinitesimal' to be a generic concept of the (purely intelligible) product of thought, and the individual element within possible systems of relations. More explicity, Cohen's 'infinitesimal' is the generic concept of the element of a "mathematical" system, i.e., a system of ordered relations. Accordingly, Robinson's hyperreals are just a special case of Cohen's 'infinitesimal'. For, Cohen's understanding of the unlimited (or, "continuous") productivity of the judgment of origin could not be limited to the production of the hyperreal, but must already include the possibility of extensions to the hyperreal — the hyper-hyperreal, and so on.

Chapter 6

Logic, Truth and Revelation

6.1 Logic and System

As we have seen, there are two ways of establishing the objectivity of our empirical representations: first, through a claim about empirical psychology (affection) (see page 107); second, through an a priori claim about the nature of sensibility as a faculty governed by implicit relations, i.e., a system of reciprocally determining "parts". Here, we have not examined the first possibility in much detail — above all because it appears to commit Kant to a number of premises that are incompatible with the project of transcendental idealism and the critique of metaphysical logic. The second possibility has, thus far, appeared under two different guises: first, as the problem of affinity (see page 107) or the problem of the intrinsic intelligibility of the manifold (see page 110); second, as the problem of matter (see page 125) as the correlate of the Analogies of Experience. These problems point, first, to the implicit physicalism of Kant's understanding of the manifold of reality, and, second, to the crucial role played by this physicalism — and especially the continuity of appearances — in the legitimation of (a) the unity of the individual object of experience and (b) the necessity of relations (persistence, succession, and simultaneity).[1]

On the other hand the elements of Cohen's formalist interpretation of the *Critique of Pure Reason*, and in particular the interpretation of the synthesis of inner sense, make it impossible to interpret sensibility as either (a) a source of knowledge about intrinsic particulars, or (b) a system of reciprocally conditioning parts. As we saw (see **Chapter 2**), Cohen understands sensibility to be grounded in a formal system of ordered relations — the mathematics of adjacency and succession — rather than in a "real" system of reciprocally conditioned parts. From

[1](KRV; AA A177/B219) Friedman, rather charitably, I think, interprets Kant's conception of the manifold as inherently kinematic, or at least kinematic insofar as synthetic reconstruction takes place (Friedman 1992b). While it may be true that the synthesis of figures necessarily takes place in time, for Kant, and is therefore a kinematic reality, it seems unlikely that this could be the case for the objects of perception. In what sense are extended but stationary spatial objects of perception brought to apperception as moving not just in time, but also in space? It seems that alteration is a property of time (and, accordingly, a condition of perception of space in general), but not a property of extension as such. In what sense does the Metaphysical (or Transcendental) exposition assert or defend such a claim? Nevertheless, it seems that kinematic laws are taken to govern possible objects of experience a priori. Thus, what can be represented mathematically is already, so to speak, a part of (real) analysis. However, on Friedman's construal of Kant's philosophy of mathematical construction, to be a part of analysis is *already to be a part of physics* (i.e, a physics of fluxions and their fluents).

the point of view of systematic idealism, this formalism leads to two important consequences. First, Cohen owes us a more extensive account of transcendental logic in order to explain the unity of the object as a logical construct of the totality of the manifold, and not merely as a fact of experience (see **Chapter 3**). That is, Cohen must account for the unity of the object — and thus the continuity of appearances — without appealing to facts about empirical experience (or more specifically "facts" about sensibility) in order to justify the claim that transcendental idealism is an objective idealism and not merely a subjective idealism. Second, the requirements of an extended transcendental logic imply the rational determinability of difference. That is, the manifold — the characteristic determinable of Kant's new epistemology — cannot be subordinate to a sensible condition; rather, it must be possible to construct an intellectual representation of (ordered) difference if objective cognition — in the sense Kant intends — is to be possible at all.

As we saw, the same demands arise for both Maimon and Hegel in their attempt to validate the methods of transcendental idealism in view of the apparent failure of the Transcendental Deduction. However, unlike Maimon and Hegel, Cohen does not commit to the *principle of determinability*. Rather, Cohen takes the a priori manifold ('origin') to be an open-ended method of rational construction, endorsing instead the *principle of continuity*. Indeed, not even the manifold of space-time ($\mathbb{R}^4$) is absolute, since both space and time (and even number!) are merely "categories", and thus (historically contingent) concepts, not fundamental functions of judgment. Higher dimensional (or imaginary) extensions or other transformations of numbers are (in accordance with the indeterminacy of origin) always possible. Thus, the manifold is potential in two respects: first, because it is not actually represented in any single representation (but only *in potens* in a judgment of 'contradiction'), and second, because the manifold, as the fundamental determinable object of knowledge, is and remains a *task*, and thus an anticipation: it is always possible to expand the domain of knowledge through new theoretical constructions within new domains of relational construction. Before we examine the systematic implications of Cohen's proposal, however, it will be helpful to briefly revisit the axis on

which the systematic idealism of Kant's immediate successors turns: Spinozism.

6.1.1 Skepticism and Spinozism

One of the central claims of transcendental philosophy is that it enables us to overcome, or at least to mitigate, some varieties of skepticism. In order for the transcendental claim to succeed in responding to the skeptic, however, it is not sufficient to say that subjective forms are necessary to the possibility of the empirical object (transcendental judgment). Rather, it must also be shown that there actually are objects of experience (the empirical claim). If this last claim cannot be established, then the transcendental solution does not succeed in responding to the empirical skeptic, but rather only extends the commitment to subjective idealism, and terminates in what Maimon calls "rational dogmatism". For, what the transcendental idealist has shown is only that the objectivity of objects (i.e., their intelligible structure), if it is to be experienced at all, must be subjective in origin. The claim that our epistemic claims about such objects are objective (i.e., that they are mind-independent) can only be established through a further claim about the nature of our sensible faculty and its relation to objective being, or, as we have seen, through an extension of the project of a transcendental logic.

Now, the question of the objectivity of the subjective appearance of unity is at the centre of one of the most influential early attacks on Kant's transcendental philosophy, that of Friedrich Henrich Jacobi.[2] Jacobi's claim against Kant is that the transcendental philosophy only succeeds in overcoming subjective idealism if we accept the substantial unity of the "thing in itself" as the causal antecedent of subjective states to which objective unity is ascribed in objective judgment (i.e., if we commit to Transcendental Realism). If we deny that there is such a

[2]Jacobi raises the claim against Kant in his *David Hume on Faith, or Realism and Idealism* (Jacobi 1787), but reiterates some of his criticisms in the second and third editions of his *On Spinoza's Doctrine in Letters to Mendelssohn* (Jacobi 1787). Jacobi was neither the first nor the only writer to raise the claim of nihilism against rationalist idealism. However, Jacobi succeeded in framing the problem (through Lessing's alleged kabbalistic Spinozism) in a popular intellectual context, and presented the problem as a radical — and existential — decision between nihilism and faith. No doubt, Jacobi missed the point of Kant's program of mitigated idealism, but Jacobi's polemic did succeed in convincing many that the radical contingency of Empirical Realism (of which the thing-in-itself is merely the intelligible spectre) was a deeply inadequate solution to the "crisis" of reason.

unity, then Transcendental Idealism collapses into subjective idealism, as either dogmatic rationalism (Leibniz, Maimon) or skeptical idealism (Berkeley). If, however, we accept the claim that subjective states are determined by (transcendent) objective unities, then our subjective states are causally determined, a claim that, Jacobi argues, leads the Kantian philosophy back to Spinozism, fatalism and ontological nihilism.

However, as we saw in **Chapter 3**, Kant's solution to the Mathematical Antinomies rests not on a formal distinction between the structure of empirical and intelligible unity. Rather, it rests on the claim that the infinite series of conditions that constitute unity in the serial manifold cannot be determinately reconstructed. That is, what prevents the empirical *totum* from being thought as a *compositum* is not a formal difference between the a priori structures of the serial manifold and the a priori structures of intelligibility (after all, the schematism must show that these are interpretable), but rather the distinction between determinate and indeterminate modes of the serial manifold. Indeed, the claim that finitely determinate intelligible structures (concepts) and their infinite counterparts (ideas) rests on a distinction between finite and infinite modes of a single domain of intelligible structures is strengthened by an addition to the B edition of the Analytic of Concepts. Here, Kant considers the antique position on the transcendentals: "*quodlibet ens est unum, verum, bonum*." That is, everything is one, true and good. In the context of the *Critique of Pure Reason*, Kant reinterprets these "transcendentals" as features of the three moments of quantitative judgments (and thus of judgment as such): unity, plurality and totality. Accordingly, all judgments represent

(a) **unity** of the concept, which one can call **qualitative unity** insofar as by that only the unity of the comprehension of the manifold of cognition is thought;

(b) **truth** in respect of the consequences. The more true consequences from a given concept, the more of its objective reality. One could call this the **qualitative plurality** of the marks that belong to a concept as a common ground (not thought of in it as a magnitude).

(c) **perfection**, which consists in this plurality conversely being traced back to the unity of

> the concept, and agreeing completely with this one and no other one, which one can call **qualitative completeness** (totality). (KRV; AA B114)

These transcendentals, however, just correspond to the three moments of quantity: universality, generality and singularity (i.e., the negation of a general predicate). In claiming that the classical transcendentals may be thought of as implied in the structure of judgment itself, Kant suggests that unity, truth and perfection are implied by conceptual construction. That is, the rational construction of conceptual representations on the basis of the categories alone anticipates the individuality of the object as "agreeing completely with this [concept] and no other one", which anticipates the "**qualitative completeness**" of the object, or its "totality". Indeed it appears to be the case that rational construction anticipates — without realizing — the totality of relations of the manifold of appearance; this just is a condition of the interpretation of judgment as standing under "regulative" ideals.

However, the failure of the solution to the Mathematical Antinomies seems to demand an infinite solution within transcendental logic itself. As we saw, both Hegel and Maimon reject Kant's claim that finite representation is a condition of intelligibility. This, in turn, suggests that the Kantian system of transcendental ideals — and thus 'unity', 'truth' and 'perfection' — are attainable within the critical paradigm through a system of transcendental concepts. If, like Maimon, Fichte and Hegel, we understand the negation of the general predicate as (necessarily) at the same time the determination of a correlative singular predicate (totality), then the moment of perfection (i.e., the predicate of totality, or the complete concept) is part of the set of determinate predicates. The "idea" of an individual object is thus not a different kind of predicate (i.e., a singular term), but rather just a concept with a narrow extension: it denotes a single object through all of its determinations. So understand, however, concepts are not really distinct from ideas (i.e., they are not a distinct form of predication), but are merely finite approximations of the infinitely determinate idea. Moreover, the transcendental ideal — the *ens realisimum*, or the real determination of God as the most real individual — appears to be nothing more than the complete determination of the manifold as a *compisitum reale*, the "**sum total of all possi-**

bility" (KRV; AA A573/B601), or "everything that can ever be given in it, taken together as an **absolute whole**" (KRV; AA 511/B583). And, as the resolution to the Mathematical Antinomies suggests, what distinguishes a *compositum ideale* from the *compositum reale* is not that they are *heterogeneous*, but rather that the former can be thought by a finite cognizer whereas the latter can only be thought by an infinite cognizer. Following Maimon and Hegel, then, the *ens realisimum* is not only immanent in the structure of transcendental logic, but is, as a qualitative totality (i.e., sensibility interpreted as the *totum reale*), also a condition of the determinability of *any* unity as a determinate object of thought. Totality is not only immanent to the structure of thought for itself, but is at the same time the determinable content of thought (the whole of space and time) in itself. All that remains to be determined is the (teleological) convergence of that which is in itself (intuition) and that which is for itself (thought) in judgments of the universal (predicate) in the particular (object), or judgments of the absolute.

Indeed, if Kant's commitment to the classical transcendentals is interpreted as a commitment to the continuity of (ideal) unity, truth and perfection with the constructive activity of judgment, and if a distinction in this continuum rests solely on the determinacy of sensible intuition, then the idealist doctrine of symbolic difference or "intellectual intuition" implies the intelligible identity of (a) real grounds as these are given in sensibility and (b) intelligible grounds as these are represented ideally through symbolic construction or "intellectual intuition." That is, if we were (perhaps counterfactually) able to provide infinitely determinate concepts, then the ends of thought (unity, truth and perfection) would *in fact* realize the identity between intuition and understanding, or the identity of thought and being, whence the epithet "philosophy of identity". For Maimon, this identification of sensible (subjective) and symbolic (objective) orders manifests as the identification of appearances and "ideas of reason", which are merely infinite composites of the "ideas of the understanding" or "symbolic differentials". For Hegel, on the other hand, this identification of qualitative totality (appearance) and quantitative totality (form) culminates in the doctrine of the "idea" as the perfection of the concept. Indeed, Jacobi's accusation that Kantianism is ultimately Spinozism appears to be not only ac-

curate, but prophetic. For, if Kant's transcendental logic is to be rescued from the paradoxes of finite intelligibility, while at the same time retaining a commitment to empirical unities, then the only rationalist solution to the problem appears to be to simply bite the bullet and identify the order of external causes (extension) and the order of internal causes (mind) just as Spinoza proposed. Accordingly, the idealist commitment to the intellectual synthesis of the totality of determinables (the *compositum reale* as the *ens realisimum*) as the intelligible structure of the absolute appears to be unavoidable. For, it is only if thought is capable of infinitely determining the finite as it is given "absolutely" in sensibility — thus transcending the finitude of the human standpoint — that knowledge of individuals is possible at all.

Accordingly, the interpretation of "absolute truth" as the perfection of the concept as the idea is taken up explicitly in the characterization of logic, characteristically in Hegel's logic (here in the *Encyclopedia*):

> As the unity of the subjective and the objective Idea, the Idea is the Concept of the Idea, for which the Idea as such is the ob-ject, and for which the object is itself — an object in which all determinations have come together. This unity, therefore, is the **absolute truth and all truth**, it is the Idea that thinks itself, and at this stage, moreover, it is [present] **as** thinking, i.e., as **logical** Idea." (Hegel 1992, §236), translated in (Hegel 1991)

The implicit teleology of Kant's reinterpretation of the classical transcendentals provides the outline for a logic in which the idea is immanent to the manifold itself. Thus, unity, truth and the good (perfection) are immanent to the structure of logic as the determination of the manifold. That is, the rational construction of the infinite logical totality — the *compositum reale* — is necessarily one, true and good; accordingly, every finite rational construction just is a finite realization of this *compositum*, and the limit of this construction — the absolute — just is the one, unique *ens realisimum* that realizes the classic transcendentals in the unity of a complete cognitive act.

Now, the Spinozist system — *The Ethics* — is, like the *Critique of Pure Reason*, a doctrine of the determinability of the manifold taken as a total system of determinables: modifications of the one substance. However, whereas Kant's manifold is the pure manifold of

nature (perhaps "matter"), the object of determination in Spinoza's *Ethics* is not just nature, but also God (*deus sive natura*), which includes the nature of Man (ethics) as the instantiation of God's will. Moreover, the Spinozist system is essentially preoccupied with the problem of individuating — or differentiating — modifications of the manifold (i.e., modes of substance) as intrinsically determinate individuals that may be determined through the concept of their "striving" to be what they are; that is, one may interpret Spinoza's conception of *conatus* to be precisely that through which modes are individuated.[3] Indeed, it's unclear how modes can even be individuated in extension through monadic predicates. Without the concept of the *conatus*, there do not appear to be individual modes of extension in Spinoza's system. As we have seen, however, it is precisely this problem — the individuality of the empirical individual — that Kant's transcendental logic fails to resolve, at least to the satisfaction of his successors. Insofar as the objectivity of Kant's new method of epistemology is to secure the claim of objectivity, the problem of the unity of the individual within the serial manifold must be resolved, or a concession to the empirical skeptic (empirical unity is a brute fact) is inevitable. The systematic determination of organic life (*conatus*) in Spinoza's *Ethics* therefore seems to provide just the tool to resolve the problem: The empirical unity of the individual is to be explained not in terms of the serial properties of the manifold (i.e., as a mathematical function), but rather through the organic relation between the individual and the whole — a logic of the infinite that looks back to Aristotle, not forward to Boole, Peano and Frege.

Indeed, the Spinozist system provides a number of theoretical proposals that were taken up enthusiastically by post-Kantian systematic idealists:

(a) It is possible to individuate modifications of substance (and therefore of extension) through monadic predicates, i.e., through concepts of *conatus*, thereby preserving the primacy of traditional (Aristotelian) logic and its ontology;

[3]Despite Spinoza's frequent attacks on final causation, the *conatus* lends itself to a teleological interpretation, or at least suggests an alternative reconstruction of the project of the *Ethics*. Certainly, the post-Kantian systematic idealists grasped the project of an *Ethics* of freedom as the expression of a system of final causes, and made it central to their strategy in the rehabilitation of an ontologically committed logic.

(b) It is possible to realize *both* theoretical philosophy (nature) and practical philosophy (ethics) through the systematic determination of the manifold through a total set of monadic predicates: a system of nature *and* a system of freedom;

(c) The two systems of determinations of the totality — necessity and freedom — may consist of the determination of a *single* manifold through a single set of predicates construed in two different ways: considered now (a) from the (finite) point of view of sensibility, i.e., as a serial manifold of causes, and now (b) from the (infinite) point of view of reason, i.e., as a totality of final causes.

While the return to Spinozism may seem to represent a regression to a (pre-critical) mode of metaphysical speculation, it nevertheless offers significant theoretical resources *within* the structure of Kant's Critical Philosophy, assuming that the problem of a "Spinozistic" mathematics can be adequately resolved, i.e., if the principle of determinability can be shown to apply to the manifold as a reciprocally determining set of determinables. This can readily be seen if we reconsider these central aspects of Spinoza's system in terms of the epistemological uncertainty faced by post-Kantian systematic idealists (see page 34):

(a) The new epistemological structure — **transcendental logic** — needs to be expanded not by adding a mathematical annex (transcendental logic as synthesis) to Aristotelian logic, but by rehabilitating the system of teleological concepts through which Aristotle's logic originally functioned, i.e., systematic organicism, through which serial order is intelligible as the finite expression of a totality;

(b) The new epistemological method — **the determination of the absolute manifold** — consists of the unification of the serial determinations of the manifold (empirical properties) through the rehabilitated system of teleological concepts, i.e., the doctrine of the "concept";

(c) The new epistemological system — **the system of human knowledge** — implies a new object for the *system of sciences*: the totality of nature as a system of "freedom" or "will';

The Spinozist system seems to offer solutions to precisely those problems that arise from the Kantian proposal and its perceived failures. The project of a "system" of monadic predicates is thus grounded in the idea of extending transcendental logic — the system of a priori principles of the intelligibility of the individual — into a total system of teleological concepts through which the "whole" of reality becomes intelligible as the systematic inflection of "freedom"; this project is realized in Hegel's logic through the principle of determinability and the principle of intelligible totality.

6.1.2 The Logic of Purity (Unity)

The Kantian interpretation of the classic transcendentals so enthusiastically embraced by post-Kantian systematic idealism is not, however, available to Cohen. The reason for this is straightforward: there is no determinable relation between the a priori form of intuition (i.e., consciousness as a determinable) and the a priori form of thought (i.e., the totality of the determinations of the determinable). Certainly, in *Kant's Theory of Experience*, Cohen is committed to the view that the manifold consists of the serial structures of intuitive space-time, or at the very least the real manifold ($\mathbb{R}^4$) of Newtonian mechanics. The very same formal commitments underlie Cohen's position in *The Principle of the Infinitesimal Method and its History*. In both of these works, Cohen's reconstruction of the Kantian position appears to lead inevitably to logicism and ultimately to the position of other systematic idealists.

However, Cohen's mature view rejects the claim that the manifold is either (a) limited to the specific form of intuition or (b) limited to the historically contingent structure the manifold of "nature" in the science of any particular era. In generalizing the concept 'reality' to a schema for the (continuous) construction of serial relations (i.e., in generalizing the concept 'number'), Cohen has, in effect, denied the claim that the structure of the "manifold" can be determined as a priori (i.e., as universal, generic or necessary) either (i) through the structure of logic, or (ii) through the analysis of the structure of the "fact" of consciousness or science. That is, there is no a priori determinable relation between the formal structures of "logic" — including the

serial structures of 'reality' — and a *unique* system (set or manifold) of serial relations. This does not, of course, mean that the manifold is heterogeneous to the structures of logic; Cohen is certainly a logicist in the sense that he believes that all of the structures of mathematical representation derive from the structures of "logic". However, he is also committed to the view that we cannot determine — a priori — what mathematical structures are most appropriate to the ends of knowledge in any particular case or at any particular point in the history of science (past or future). In other words, what structure of mathematical representation is to be applied in any particular case of epistemic representation is not a necessary consequence of the formal structure of logic, but of the historically contingent fact of science and its theoretical grounding at a particular point in the history of thought.

We can take note of two consequences that follow from Cohen's mature position. First, it is not possible to claim that all possible structures of knowledge are modifications of the *totum reale*, construed as the sensible manifold. For, in denying Kant's intuitionism, Cohen also denies that the form of intuition — a priori or a posteriori — places any constraint on epistemic representation. That is, non-Euclidean geometries, imaginary dimensions, higher dimensions and hyper-real "infinitesimals" are all candidates for epistemic representation, even if they can have no correlate in intuition; 'reality' is a property of a system of mathematical relations, not a property of a system of sensible relations. Second, it is not possible to claim that the formal structure of space and time (adjacency and sucession, or serial order) are logical consequences of the formal structure of thought, as Hegel does. That is, the actual form of the manifold employed in any particular cultural form of knowledge is not determined by logic; the Principle of Sufficient Reason does not apply to our choice of underlying mathematical systems.

Now, because there is no unique relation between (a) the structure of logic (*Erkenntnislogik*) and (b) the mathematical construction of 'reality', it no longer follows that there is a unique intelligible object — the *ens realisimum* as the unique *compositum reale* — towards which *all* rational construction necessarily converges. That is, "everything that can ever be given in it, taken together as an absolute whole" (KRV; AA 511/B483) is no longer a necessary,

unique end of rational construction. Accordingly, "perfection" can no longer be considered to be implicit in the structure of judgment. For, it is no longer the case that the infinite determination of a concept implies that appearances agree "completely with this [concept] and no other one" (KRV; AA B114); the *Logic of Origin* allows for a plurality of distinct totalities, none of which may claim to totalize "reality", insofar as — by the principle of continuity — every system of mathematical relations may be subsumed under a different, and perhaps more expansive, system of relations.

As a result, "truth" can no longer be considered to be implicit in the structure of judgment, insofar as there is no determinate relation between the determination of the determinable (the object of experience) and the determinability of the determinable (the possibility of experience). That is, not every possible formal construction is *eo ipso* a possible fact of science. For, the objects of judgment (for Cohen, *Erkenntnisse*) that are the product of rational construction are merely formal constructions; they are formally valid constructions from rational principles, not modifications of a unique reality with *de facto* metaphysical import. For Kant, however, the domain of rational construction is subordinate to the sensible manifold; accordingly, all valid synthetic constructions are *eo ipso* real possibilities. That is, what can be asserted as a valid modification of the manifold is "true", since it just is a modification of the manifold that is actual at some time; "reality", therefore, entails metaphysical possibility just in virtue of being a possible modification of the *totum reale*. Thus, Kant may claim, "the more true consequences from a given concept, the more of its objective reality" (KRV; AA B114).

For Cohen, on the other hand, the domain of possibility described by rational construction is that of logical possibility, or those constructions that are formally valid. But since valid constructions of the manifold are not conditioned either by the "affinity" or "materiality" of a reciprocally determining sensible manifold, particular instantiations of 'reality' only have epistemic validity with respect to the (contingent) fact of science. Cohen's *Logic of Origin* is, accordingly, a logic of validity; it provides "purity", construed as the derivability (i.e., a priori

possibility) of a particular form of epistemic representation from a set of a priori principles.[4]

In order to see why, and in what sense, the *Logic of Pure Knowledge* is the anticipation of unity, we must return to our earlier discussion of the Cohenian table of judgments (see page 189), now to the final class of judgments: the judgments of method (modality). Now, recall that the first moment of each class of judgments is a determination of the judgment of origin. If the original (indeterminate) unity of thought is 'origin', if its mathematical determinability is 'reality', if its natural physical interpretation is 'substance', then the judgment of methodology is possibility, or 'hypothesis'. Cohen traces the word to its Platonic origin (see, for example, LRE CW 6:430), but we also recognize it as a Kantian term, specifically from the Discipline of Pure Reason (KRV; AA A770/B798). There, 'hypothesis' is the status accorded to those a priori 'objects' whose hypostasis is necessary to the judgments of experience. Here too — in Cohen's judgments of 'method' — the empirical realist moment of thought must be taken into account. However, as we have seen, Cohen's formalist interpretation of the *Critique of Pure Reason* eliminates the sensible moment not just in turning away from "empirical" cognition, but insofar as the fact of science itself is an intelligible structures, not a fact related (a) to any special features of our sensible nature of (b) any special features of the manifold of nature inferred from the limitations of intuition.[5]

However, as Cohen recognizes, *unity* is the highest principle of the *Critique of Pure Reason*: it is (a) that through which the Transcendental Deduction is possible as an argumentative strategy, and (b) the fundamental principle of all of the synthetic principles insofar as consciousness is grounded not simply in a particular consciousness (the synthetic unity of apperception) but in a particular cognition as well. Accordingly, Cohen understands the unity of consciousness to be the highest *principle* of consciousness, the ground of the synthetic unity of

[4]See, for example, (Edel 1994). Just as, for Kant, the objectivity of metaphysics becomes a hypothetical science warranted by the requirements of the epistemology of natural science (KRV; AA A 769-782/B797-810), for Cohen, the epistemic forms of the "categories" (as the particular forms of logico-mathematics) becomes a hypothetical science warranted by the formal requirements of the representation of natural scientific "facts of experience."

[5]So, for example, Kant's assumption that Euclidean geometry is proved by the Transcendental Aesthetic (even if the warrant for the claim is somewhat obscure) is not one that Cohen can endorse. Our intuitive faculty does not determine what type of geometry is necessary; only the requirements of science are able to dictate (or better, recommend) one type over another.

apperception: "Kant brought together his **categories** and his **principles** under the expression '**the unity of consciousness**'. He made the unity of consciousness into the **highest principle**" (LRE CW 6:417).

However, Kant's "unity of consciousness" contains a number of important ambiguities. For the purposes of the Transcendental Deduction, the unity of consciousness is purely formal; it is the dependence of the forms of space and time on the functions of judgment (the principles), and the dependence of both on the unity of apperception (i.e., consciousness). However, in the Analogies of Experience — and so also in the Mathematical Antinomies — Kant's interpretation demands more than the merely formal unity of the manifold as the correlate of consciousness of the object; rather, it posits "**nature in general**, as lawfullness of appearances in space and time" (KRV; AA B165), through which the object is grounded in *matter* rather than merely in the concept of *alteration*, or motion. Indeed, the categories of modality — and in particular the category of actuality, where Kant attempts a Refutation of (Cartesian) Idealism — implies a relation to conscious awareness: "that which is connected with the material conditions of experience (of sensation) is **actual**" (KRV; AA A217/B266). This "condition", however, cannot be interpreted as the (merely formal) condition of adjacency or succession, but must rather introduce an empirical condition ("something persistent", "*etwas Beharrliches*") that leads necessarily to a phyicalist interpretation of the correlate of experience, or a phenomenological ontology.[6] Moreover, the unity of consciounsess (of the synthetic object) is possible only through the unity of nature (the determinability of a quasi-physical manifold of relations), which in turn appears to be related to an immanent unity ("something persistent"), or what may be nothing other than the empirical unity of the object, that fact of experience which is at most presumed by the possibility of a transcendental analysis of the (subjective) conditions

[6](KRV; AA B275) There are, of course, many ways to interpret Kant's argument in the Refutation of Idealism. My only contention here is that however the argument may run, the objective interpretation of the correlate of intuition (actuality) is physical. This is already implied by the function of unity in a judgment of relation, i.e., of substance as persistence. In appealing to persistence as phenomenally rather than conceptually grounded, Kant is thus claiming that conceptually ungrounded phenomena can legitimately be treated as substances, i.e., as instantiations of opaque concepts. This is a phenomenological claim (since it rests on subjective grounds), rather than an objective claim. Accordingly, the last chapter of the *Metaphysical Foundations of Natural Science* is a 'phenomenology'.

of the unity of an empirical object.

But now, since Cohen has rejected every explicit dependence of the formal functions of thought on structural or material features of sensibility, the possibility of the unity of consciousness as (a) the formal unity of consciousness, as (b) the unity of a system of laws (nature), as (c) the unity of the fact of consciousness (i.e., as the fact of experience) and finally, as (d) the unity of the empirical individual that is a possible object of knowledge, must all be grounded in logic itself. That is, whereas Kant appears to accept that some features of the unity of the fact of consciousness are empirical, and thus intelligibly ungrounded, Cohen's Platonist reading of the *Critique of Pure Reason* requires that *all* unity arise solely from the subject and its intellectual faculty: thought. Thus, the general determinable hypothesis of thought is the pure unity of scientific consciousness, i.e., the indeterminate system of mathematical laws (of nature) that may be rationally produced, starting from the judgment of origin as the pure possibility of the construction of a system of determinables. As for Kant, the 'hypothesis' is the condition of possibility of a judgment of empirical knowledge. But whereas Kant construes 'possibility' as a metaphysical domain (those modifications of the totality of space and time that are made possible by the categories and the principles of synthesis), Cohen understands 'possibility' as an epistemic domain: as that which can be thought in accordance with the principles of an *Erkenntnislogik*: "**it is possible means: it makes possible new knowledge** [*Erkenntnisse*]" (LRE; CW 6:432). This condition, of course, cannot be sensibility itself, but rather an 'original' hypothesis, i.e., *system of hypotheses*. Accordingly, "**the unity of consciousness is defined through the unity of scientific consciousness**" (LRE; CW 6:16). More precisely, we should say that judgments of possibility (as the original, indeterminate form of hypothesis), "**must produce the category of consciousness**" (LRE; CW 6:424). That is, consciousness is not the original unity through which synthesis first becomes possible (i.e., spontaneity), but rather a formal unity: that which is the product of the continuous production of reality and its determination in a *merely possible* "substance" (i.e., determinable manifold ("reality") with posited dynamical properties) that first makes possible the cognition of a determinate object of

consciousness — without determining that object in any way. It is, so to speak, the equivalent of the manifold of space and time, which, for Kant, is "given" in its totality in intuition. For Cohen, this "manifold" of determinable relations is first and foremost a construction of thought, and always only one among an indeterminate domain of possible manifolds.

Whereas for Kant, the possible is always a possible object of intuition, possibility, for Cohen, is never more than a domain of logical relations. To be sure, this domain may be thought of as "law", i.e., as the necessary intelligible basis for a domain of natural relations. This, indeed, was the case with the real domain ($\mathbb{R}^4$) during the reign of Newtonian mechanics. In order that we might think of an actual object of consciousness (i.e., a particular fact of science), we must inevitably appeal to the category of plurality, or that through which a determinate quantification of the manifold may first be thought. Thus any particular (be it *law* or *concept*), may only be thought of as a magnitude: "**Actuality is a category, but a critical category. It is dependent for the particular on magnitude**" (LRE; CW 6:478). Thus, actuality is not (a) related to sensibility, (b) to the empirical, or (c) to the present. Rather, judgments of actuality refer not to a metaphysical modality, but to an epistemic modality, or "method": this (substance, law, concept) is a determinate something within a particular domain of intelligibles. The *fact* of consciousness, accordingly, is neither formally nor intelligibly related to the fact of sensibility, or to the temporalized structure of some thing in itself. Rather, the fact — while always "given" through culture — remains an actuality *sub specie aeternitatis*: it is not a modality of awareness, but a determination of a particular domain of intelligibility. Similarly, 'necessity' as a methodological judgment of natural science is not a judgment about the metaphysical character of an object or even a law (which, in the end, is just a function of substance), but rather the 'contradiction' of any particular individual. Accordingly, the judgment of necessity is a methodological judgment of the generality of the hypothesis, or better, of the system of hypotheses within the hypothetical domain of a particular system of relations ('reality') construed as a physical domain ('substance'). It is the judgment that substance, law or concept must be thought as a characteristic feature of that particular structure of hypothetical

representation. So, for example, the law of gravitation is not only a law, but a necessary law of Newtonian mechanics, and thus applies to every possible representation of substance (the physical domain) within the paradigm of Newtonian mechanics.

Taken together, the judgments of method show that the system of judgments is an open-ended construction. We have already seen this in the anticipatory structure of, for example, time (anticipation) in the judgment of plurality (see page 184) and also in construction of the real manifold through the successive determination of systems of number. However, through the judgments of method, we now see that the ultimate task of the system of judgments is the generation of the intelligibility of the fact of science as an individual fact that is grounded within the unity of a system of laws of nature. However, whereas Kant takes the unity of consciousness (the synthetic unity of apperception) to be the spontaneity that grounds the possibility of any synthetic combination (and therefore requires a deduction of some kind), Cohen assumes the possibility of synthetic combination (indeed, it will be shown in the *Ethics of Pure Will*), and takes the unity of consciousness to be the system of natural laws within which the singularity of the fact of consciousness is grounded as a particular (i.e., a determinate limitation of the totality of possible laws within a given paradigm).

We may think of this (categorially) as the unity of (scientific) consciousness. As we know, however, this "categorial" interpreation of possibility is historically contingent. The true structure of knowledge need not be represented in any particular consciousness, but is rather the unity of the system of scientific hypotheses. Thus, if the concept of number is itself an "anticipation" (and thus the "scientific" concept of time as infinite series), the concept of the unity of science (consciousness, or the system of hypotheses) is an anticipation of a different kind. This anticipation is not the anticipation of metric time (i.e., the series that is intelligible through the rule of its generation), but historical time: the history of scientific knowledge. Like Hegel's history, the history of scientific knowledge has an "end" contained in its principles; unlike Hegel's history, however, this end cannot be deduced from the original principles of thought, because these original principles are organized around a judgment of privation (ori-

gin), not affirmation (being) and its determinate negations. 'Unity', for Cohen, is thus the anticipated horizon of a system of natural laws that accomodates the singularity of every fact of science. It is that manifold (i.e., 'reality', the system of relations) within which substance (or the system of physical laws) and its necessity grounds every fact of science within a unique system of representations. The "end" of science is the realization of that most comprehensive representational system within which all facts of science are grounded. It is, then, the realization of the transcendental ideal — an *ens realisimum* — within a particular paradigm. However, if consciousness — or empirical science — is the anticipation of the transcendental ideal, the transcendental ideal is not a priori determinable on the basis of the mere form of thought; only the completion of "history" will determine the ultimate relational structure of the totality of actual being.

As we have seen, the fact of science is historically contingent. It cannot be decided a priori whether the present scientific paradigm is adequate to the representation of every determinate representation of scientific fact.[7] The construction of new systems of relational representation (new mathematics) inevitably opens the possibility of new, more comprehensive representations of scientific fact. In effect, the advance of mathematics brings with it the possibility of new facts of science, and therewith a new, perhaps wholly disjoint transcendental ideal now anticipated by a new system of scientific representation. Thus, if unity is anticipated by science, it is not a unique unity that is thereby implied. The uniqueness of the transcendental ideal would be the truth of science, demonstrated through its unique adequacy as the representational structure containing all possible facts of science. However, such a structure (a) is not given, even implicitly, in the form of intuition, nor (b) can it be known from the standpoint of any point in the development of science, but only from its completion. Indeed, the revolution in physics implied by general and special relativity were not anticipated from the standpoint of

[7] The Marburg programme anticipates the point later emphasized by Thomas Kuhn in *The Structure of Scientific Revolutions* (Kuhn 1962). While the Marburg account does not highlight the disruptive effect of changes in major research programmes, it does highlight the dependence of a research programme on an adequate formal basis. Developments in mathematics may be necessary to the comprehensive treatment of new forms of scientific knowledge (and this priority need not be chronological; sometimes scientific fact demands new mathematics for its adequate interperetation).

Newtonian mechanics.

Thus, while, for Kant, the progress of knowledge tends towards not only unity, but also *truth*, and ultimately *perfection*, there is, for Cohen, no unique system of relations (reality) or laws (substance) within which either truth or perfection might be realized. Rather, *Erkenntnis-logik* provides, at most, intelligibility. This intelligibility is manifest as the *unity* of a system of scientific representations, through which individual facts of science are interpreted not as singular facts, but as particular instances of the generic system of possible (formal) representations. Logic provides the materials for a system of hypotheses. It does not — and cannot — provide a warrant for belief (i.e., an objective ground of truth) on the basis of merely formal grounds.

6.1.3 The Character of System (Truth)

We can locate the break between Cohen's systematic project and the holism of the more familiar systems of post-Kantian idealism in the delineation between logic (and its correlate, Nature) and ethics (and its correlate, Man) by looking back to Cohen's second major work, his interpretation of Kant's ethics in *Kant's Foundation of Ethics* (Cohen 1910). For, Maimon, Fichte, Schelling and Hegel all turned to a Spinozistic *identification* of thought and will, of logic and ethics through an Aristotelian conception of Nature and an (unconvincing) account of the reciprocal determination of the infinite and the finite. Cohen, on the other hand, appears to have grasped the problem of the polyadic nature of the manifold, and instead interprets the categories as mere epithets for combinations of mathematical functions. However, because of this interpretation of the nature of the predicates of judgments of theoretical reason, it is impossible for Cohen to identify the (monadic) predicates of ethics ('Agent X does y') with the functions of mathematics ('$\forall x \forall y, f(x) = y$'). The attribution of action presupposes objects that are not (*pace* Fichte and Hegel) functions of the manifold of theoretical reality. Necessarily, then, Cohen retains Kant's distinction between the logic of theoretical judgment and the rational "predication" of (pure) practical reason.

This break is decisively marked by Cohen's study of Kant's practical philosophy in *Kant's Foundation of Ethics*. There, Cohen clearly formulates the dualism that he attributes to Kant, and which will decisively demarcate his own approach to systematic idealism from the Spinozism of other systematic idealists:

> This alone is the question of ethics: the possibility of **another type of reality** than that which can be advocated by nature through its scientific validity.
>
> In this conception of the problem, the founder of the transcendental ideal stands next to the creator of the doctrine of ideas: since Plato, Kant is the first to formulate the task of ethics.
>
> Ethics, according to Kant, does not teach what is, but **what should be**. (KBE; CW 2:5)

Cohen's formulation is striking in many respects. For, Cohen formulates the question of ethics not, as Kant surely did, as the characterization of the good will, or even as that which is an end in itself. Rather, Cohen formulates the problem of ethics in terms of *reality*. As we have already seen, the question of reality is, for Cohen, deeply connected with the problem of the intelligibility of being (and in particular mathematical determination), not with the affective power or transcendent being of the thing in itself. Accordingly, Kant's ethics, Cohen claims, is fundamentally concerned not with action, but with the *intelligibility* of action: it's being or its reality. In this formulation, however, Cohen appears to break not only with Kant's conception of the *Critique of Practical Reason* (which aims to prove the possibility of pure practical reason through *merely practical* reason!) but even to revise the conception of idealism advanced in *Kant's Theory of Experience*.

This reinterpretation of the systematic character of Transcendental Philosophy begins with Cohen's interpretation of the Third Antinomy. There, Cohen advances what appears to be a familiar criticism of the resolution of the Third Antinomy:

> In fact, the achievement of this constructed unification of freedom and natural necessity is for the moment only negative; freedom too has a merely negative meaning. If the noumenon can lay claim to freedom, then according to the solution to the Antinomy, this means only independence from causal laws, but not some arbitrary control over the causal laws. It means only the opposition between the concept noumenon and causality, which opposition is given in itself through the distinction between appearances and the thing in itself. (ERW; CW 2:124)

Prima facie, Cohen is merely pointing to the familiar distinction between transcendental freedom (which is required for the possibility of an active synthesis of apperception) and practical freedom, or will. However, from the distinction between appearances and the thing in itself, Cohen draws an astonishing parallel:

> Thereby the orientation is given for the correct understanding of every contradiction and every unification. Just as the thing in itself only means to postulate something above reality, which law determines, a cognitive value in order to cover the abyss of intelligible contingency though an unconditioned being, so also can the free noumenon have no other meaning than to cross the yawning abyss of intelligible contingency in the infinite series of natural conditions of human action. (ERW; CW 2:124)

That is, the solution of the Third Antinomy — far from being a failure — provides the unique solution to the unification of the contradictions of the Transcendental Dialectic, which is at the heart of Kant's argument for Transcendental Idealism. Indeed, as Cohen reads the argument of the Third Antinomy, it is not in fact the argument of the Transcendental Aesthetic that secures the ideality of space and time (for, on Cohen's interpretation, it only grounds the objectivity of experience in the intelligible structures of mathematics), but rather the resolution of the Antinomies — and thus the Transcendental Ideal — that secures the ideality of knowledge as pure (i.e., positive) with respect to its sensible objects.[8]

The free noumenon is not an object of experience (and thus it is no substance) whose "freedom" (transcendental or otherwise) requires a defense; to understand the Third Antinomy as a defense of freedom is already to hypostasize the idea of the noumenon as a substance to which causal properties can be ascribed and must be defended, and this would be nothing more than Transcendental Realism. According to Cohen, however, this is not the task of the Third Antinomy. Rather, the free noumenon is *posited* in order to explain not the intelligibility of

[8]It is often asserted that the "Neo-Kantians" disregard most of the Transcendental Dialectic, and focus instead on the Transcendental Analytic. This is true to the extent that, at least for the Marburg School, the mathematical interpretation of the form of intuition obviates the categories and renders the Dialectic as an object of metaphysical speculation superfluous. Nevertheless, Cohen was attentive to the problems highlighted by the Transcendental Dialectic, even if he did not believe that theoretical reason was capable of justifying metaphysical claims of the kind that Kant had hoped for. If the complaint is that the Marburg reading of Kant's theoretical philosophy is revisionary, then it is certainly justified. If the complaint is that the Marburg reading ignores the metaphysics of Kant's system, then the claim is simply false, and the first one hundred pages of *Kant's Foundation of Ethics* is all the proof that one need adduce.

freedom, but the intelligibility of an infinite series of conditions that lead to human action. Accordingly, the thing in itself is likewise not posited as the real or intelligible *ground* of the object of experience. Rather, the thing in itself is only the concept of an unconditioned being that makes it possible to bridge the gap between that in experience which can be intelligibly grounded (i.e., in the law of its mathematical production) and that which is as yet ungrounded: the anticipated being of partially intelligible empirical observation.

Whereas Kant appears to characterize the thing in itself as a rational inference to a causal antecedent of sensibility, or perhaps as a real correlate to the intelligible unity of the object, perhaps the real (but unintelligible) ground of empirical unity, Cohen understands the thing in itself not as a placeholder for the *object*, but as a the intelligible sign of the as-yet undetermined *intelligibility of the object* (i.e., the law of its reality). Thus, if reality is that which is determined by the law of natural physics (i.e., the mathematical manifold that is determinate through mathematical functions), then the thing in itself is the assumption of reality in the absence of intelligible determination. It is ideal not in an ontological sense, but in the teleological sense that Cohen identified in the passage above: the thing in itself is *what ought to be* determined as real through an intelligible law, but remains (for the time being) unintelligible. Accordingly, the thing in itself does not even have an ontological interpretation; it is at most an epistemic or symbolic posit.

This is not just a curious reading of the thing in itself, but a tectonic shift in the structure of Kant's idealism. For, immediately, the thing in itself no longer has the properties of an object, but like the construal of "experience" as the fact of science, the thing in itself is just a law of natural science that is not yet determined; it is perhaps best thought of as the bare concept of a functional relation. It is the anticipation of intelligibility in the face of the "abyss" of what remains — with respect to current science — contingent. The thing in itself is thus not a special noumenal structure, nor does it have any possible metaphysical valence. Rather, it is simply part of the theoretical apparatus of a sytem of intelligible laws. It is a placeholder; nothing more. What had been interpreted by nearly all of Kant's interpreters (and perhaps by Kant himself)

as an inevitable metaphysical postulate arise from Empirical Realism has become, for Cohen, merely the idea of a "variable" that represents gaps in our knowledge. The thing-in-itself is accordingly the promise (or better, prophecy) of intelligibility, and thus it is contiguous with the concept of reality (as the determinability of law) and necessity (as objective laws of nature). It may thus also be understood as in some sense the "origin" of the intelligibile, since implicit in its ideality as that which ought be determined is the postulate of its determinability: the ideality of the thing in itself just is the "laying of foundations" for the possibility of constructing a reality that can be determined through a relational manifold, but is not yet. Accordingly, the thing in itself is nothing more than the indeterminate idea of the determinable, an unintelligible stop-gap that is, while indeterminate, that which must be thought of as prior to any determinability. We think the thing-in-itself not as an unconditioned ontological being, but rather, Platonically, as the ultimate intelligibility of being in its most generic form: ultimate intelligibility.

The same transformation takes place in the concept of the noumenon. Any potentially ontological or metaphysical dimension in the concept of the noumenon is immediately sublated into the pure intelligibility of law:

> **Thus it is not the freedom of the noumenon that is saved, but rather a noumenon of freedom that is assumed**. That abyss, which freedom covers over, must correspond to a noumenon, which represents an intelligible law, a new individual character of causality, an unknown kind, not merely action, but rather to capture the original effectivity that brings the sensible to appearance. (KBE; CW 2:125)

However, the noumenon that is assumed in the resolution of the Third Antinomy is not characterized by the *Critique of Pure Reason*. Rather, it represents a kind of causality (or better, a kind of law) that falls outside of the apparatus of theoretical reason; it is not another (competing) law of nature, but a law of a different kind altogether.[9] Cohen's claim, then, is that Kant does not

[9] That is, Cohen is not interpreting the solution of the Third Antinomy as a weak form of compatibilism. Instead, the very idea of determination (law of nature) rests on the antecedent system of law: the laws of the free production of thought, or the lawfullness of laws (*Gesetzlichkeit*). Insofar as this reading is compatabilist, it is the laws of nature that need to be shown to be compatible with the (unrestricted) free production of thought, not the other way around. If any deflationism is proposed, it is deflation in the direction of the freedom of thought. This approach — deriving the theoretical from the practical — has a superficially Fichtean flavour. However, Cohen insists that only science can teach us the nature of the principles of construction, and thus the ultimate structure of thought as *Erkenntnislogik*. Even if, in the *Religion of Reason*, Cohen ascribes the original discovery of the concept of law to

intend to defend the effective will, nor even secure the transcendental freedom of the noumenal agent that is presupposed by the synthesis of apperception. Rather, Cohen understands Kant's strategy to be *positive*: we must posit a noumenal freedom — an intelligible law — that falls *outside of the infinite series of conditions* in order to make sense of the fact that an infinite series of conditions is intelligible to us *at all*. It is thus not freedom that is thereby secured, but rather intelligibility that is assumed. The Third Antinomy is therefore to be understood both (a) as the independence of ethics from logic and (b) as the priority of the law of free production (the lawfulness of law), or ethics, with respect to the system of natural laws.

In at least one sense, Cohen's claim is not revolutionary. It may generally be taken to be the case that the *Critique of Pure Reason* requires a more robust defense of the effective noumenal agent than is offered by the *Critique of Pure Reason* itself. Even if we set aside the problem of affection and any organicist interpretation of 'function' as a (psychological or metaphysical) characterization of thought, we still need to account for the sense in which the synthetic unity of apperception may be described as "spontaneous" and independent of the series of conditions that is posited as both necessary and objective. It is generally admitted that an adequate notion of freedom is not secured by the resolution of the Third Antinomy, and a more robust defense of the idea of autonomy, such as that provided by the *Groundwork of the Metaphysics of Morals* must be undertaken to complete the project. One of the features of Cohen's interpretation of the *Critique of Pure Reason* is that the fact of science is in itself proof of transcendental freedom. No argument need be supplied. Kant could have applied the same argument to the object of experience (and, indeed, the spontaneity of synthetic unity of apperception is a necessary condition of the possibility of objects of experience). However, Kant also aims to be ontologically committed, which is not a concern for Cohen. Whereas Kant sees the defense of (at least transcendental) freedom, and thus the possibility of ethics, to be necessary to the metaphysical commitments proposed by a critical metaphysics, Cohen

monotheism (and thus to Judaism), this can only be construed as the implicit self-limitation of free production for the sake of universality (law) and not the discovery of the logic through which generally universal claims could be constructed, which is alone the domain of the investigation of real relations, i.e., of logic, broadly construed.

sees ethics not as the defense of the possibility of science, but rather as the vindication of its methods. Just as, for Cohen, mathematics grounds intuition, it is ethics that grounds logic.

Cohen's strategy in interpreting Kant's Idealism can be summarized in four key claims.

(a) *Kant's Theory of Experience* assimilates the conditions of intelligibility (the a priori, metaphysics) to the system of theoretical principles, i.e., the concept of substance is grounded in intelligible (mathematical) modifications of the manifold, and does not provide the intelligible ground (unity) of the latter.

(b) *Kant's Foundation of Ethics* assimilates the postulates of intelligibility to the system of theoretical principles by implicitly modalizing the system of intelligible laws in historical time. The noumenal agent and the thing in itself are anticipations of law; they are not possible laws, but the possibility of law, the determinability of the determinable as raw intelligibility, or thinking as such.

(c) The formal unity of thought is thereby grounded not in the cognitive activity of a noumenal agent, but in the anticipative structure through which the intelligibility of contingency is postulated. The positive activity whereby the intelligibility of contingency is assumed is ascribed to a noumenal agent that is not a metaphysical principle, but a formal law or principle of anticipation that is heterogeneous to the laws of natural science.

(d) The possibility of knowledge is thus expressly ideal in the sense that it is the anticipation of *what ought to be*: the reality of Nature ought to be determined through a system of laws. However, insofar as theoretical knowledge is a doctrine of *what ought to be*, it is subordinate to the science of the laws of what ought to be: ethics.

Through the these four claims, Cohen's *System of Critical Idealism* can be contrasted with the project of *Spinozistic Systematic Idealism*:

(a) Whereas the logic of identity attempts to subordinate mathematics to Aristotelian logic by interpreting the continuum through the idea of the whole and its limitations, Cohen grounds

Kant's categories (and thus the conceptualized forms of Aristotelian logic) in functions of the manifold of serial relations of space and time.

(b) Whereas the logic of identity interprets 'history' to be the realization of a logically necessary actualization (*Verwirklichung*) of the logical structure of being, Cohen understands 'history' to mean the deferral of complete intelligibility (i.e., the positing of the thing in itself) as the reality of the contingent in the future . 'History', for Cohen, is thus not the actualization of reality (and thus rationality), but the realization (and thus rationalization) of actuality.[10]

(c) Whereas the logic of identity grounds the unity of thought *and* action in the concept of a single determinable totality (subjective, as for Maimon and Fichte, or absolute, as for Hegel), Cohen understands the unity of thought to be a theoretical postulate. There is no determinable whole from which individuals may be determined by (monadic) limitation, but rather a null-point (origin) from which forms both theoretical and practical may be constructed, but according to distinct principles of construction. There can be no absolute totality of thought and action, and thus the "pantheism" or "romanticism" of systematic idealism must be rejected.

(d) Whereas the logic of identity assimilates ethics to logic by grounding both in an organicist

[10]This formulation may be somewhat obscure. To put it in more graphic terms, we may understand actuality, in Hegel's logic, to be the determination of an index between a necessary starting point and a necessary end. Thus, for Hegel, actuality is only contingent as a mode of an underlying necessary order, or 'history'. For Cohen, however, philosophy begins neither from the beginning, nor the end, but starts in the middle, with the fact of culture. *Erkenntniskritik* uncovers the positive aspect of thought in grounding actuality; Ethics, however, uncovers the teleological orientation of thinking, or the positive aspect of thought in the anticipation of the intelligibility of what remains contingent in the fact of culture. For Cohen, both the past and the future are reconstructed from the standpoint of the present. 'History' is therefore the intelligibility of actuality suspended over the abyss of contingency: an uncertain past and an uncertain future. This just is the inversion of the principle of intelligible totality into the principle of pure individuality. The retrospective aspect of Cohen's understanding of intelligible contingency is eloquently captured in Benjamin's image of the "Angel of History", who looks back upon the wreckage of the past, while flying blindly into a future devoid of any dialectical determinacy (**?**). This affinity between Cohen's epistemology of the "abyss" of intelligible contingency and Benjamin's "dialectics at a standstill" has often been noted. (See, for example, (Deuber-Mankowsky 2004), (Deuber-Mankowsky 2005).) *A fortiori*, we should understand Cohen's fracturing of dialectical logic (i.e., rejecting the principle of the excluded middle but also rejecting the principle of determinability, a.k.a. the judgment of origin) anticipates Adorno's *Negative Dialectics* (**?**).

logic modelled after an Aristotelian system of teleological concepts, Cohen sharply distinguishes between the logic of nature (mathematics) and the logic of ethics (teleological judgment, or idealism) in which the logic of nature is ultimately grounded.

Spinozistic Systematic Idealism assimilates the Kantian interpretation of the classical transcendentals into a system of unity: unity, truth and the good are the (necessary) end of *every* rational determination insofar as these transcendentals are inscribed in the logic of determination itself. For Cohen, on the other hand, "unity" is the normative modality of logic as a system of representations. Such representations are neither (merely) subjective nor do they represent objective truth. For, they are merely valid (*gültig*) insofar as they satisfy the norm of unity (i.e., of formal consistency). In order for claims of *objective truth* to be raised, however, it must be possible to represent the facticity of experience as something other than a given: the intelligibility of objective truth rests in the idealization of experience. The ideality of representations, however, is secured by ethics, not by a (metaphysical) theory of theoretical representations. As Cohen's interpretation of the resolution of the Third Antinomy shows, "truth" is the normative modality of ethics as a system of idealism. Thus, ethics is true only insofar as it is ideal: that its representations conform to the inner lawfulness of thought as a positive system.

However, ethics cannot determine from the principles of thinking (i.e., logic) the content of ethics. For, the content of ethics is the human, which is not produced by the form of logic (or even of ethics) as a mere construction. The ultimate orientation of ethics — the human — cannot be thought, but can, at most, be given. Insofar as it is given, however, the end of ethics is located in feeling, not in thought or will. Aesthetics, therefore, is the anticipation of the content of ethics not in its substance but only in its form: the lawfullness of law, or *Gesetzlichkeit*. Although aesthetics does not provide the objective content of the human as the end of ethics, it does provide the intelligible correlate of any adequate concept of the end of ethics: the idea of the good beyond being. Aesthetics is the intelligibility of a content of which we can at most be conscious, but which cannot be derived from the principles of logic and ethics: it is the

apophatic revelation of objective lawfullness through subjective feeling. Thus, the good — as the lawfullness of law — is the normative modality of aesthetics as the revelation of the intelligibility of facticity insofar as it can be present in consciousness at all.

Accordingly, Cohen does not reject the classical transcendentals, but rather recovers them within the systematic structure of a system of knowledge. Each application of thought anticipates one of the "transcendental" moments, and may be interpreted as a system of anticipations.[11]

(a) The *Logic of Pure Cognition* provides a method (mathematical construction) through which the unity of thought may be anticipated in its principles. Of course, the totality of thought is not given in the principles of thought, since the structures of reality and intelligibility are constructed by human cognizers, and thus are finitely realized. Their progressive completition is the measure of the progress of a scientific culture.

(b) The *Ethics of Pure Will* provides a method (jurisprudence) through which the truth of thought may be anticipated in the idealization of anticipation itself: the idea of eternity as the end of all positive law. The Kingdom of Ends is not, of course, realized in any extant system of laws; rather, it is through the idea of God, or the perfect attributes of action, that the constitutive teleological principles of ethical thought can be interpreted

(c) The *Aesthetics of Pure Feeling* provides a method of the analysis of art through which the *goodness* of thought may be anticipated in the immament actuality of eternity in feeling.

Cohen's method of logic is thus not a logic of totality (i.e., a logic of determination), but rather a logic of origin (i.e., a logic of determinability). The completeness of the system of principles of thought is not contained as a principle in the foundations of the logic, but is at most anticipated by its structure. This is so for two different reasons. First, although the principles

[11] More explicitly, whereas the *Spinozistic Systematic Idealism* is constructed on the basis of a teleological logic (i.e., a logic that is necessarily related to the totality of the determinable), Cohen's logic is constructed on the basis of the anticipated form of a system of laws: as unified, true, and good. None of the ends of thought, however, can be derived from the mere form of representational determination however.

of thought provide the constructive foundation for all possible valid constructions (i.e., unity), the boundary of the infinite totality of what may be constructed is not determined a priori. The construction of the complete system of epistemic representations is therefore what Cohen calls a 'task': it is a priori determinable in form, but historically contingent in its truth. Second, the logic of thought is, as Cohen emphasizes, a product of the analysis of the fact of culture; it is, therefore, historically contingent, and it is only through the critical verification of the principles of thought in ethics (i.e., the law of truth), that the principles of thought may be taken to be secured. And so, while the method of mathematical construction anticipates the totality of the manifold, this anticipation provides only the unity of validity of the construction. The demonstration of the aptness of the principles of thought is only achieved through the ethics.

6.2 Ethics, Revelation and Religion

6.2.1 Ethics and Eternity

The *Ethics of Pure Will* is perhaps Hermann Cohen's most important philosophical work. While Cohen's 1871 interpretation of Kant's *Critique of Pure Reason* has probably had a more lasting impact in epistemological circles, and while Cohen's *Logic of Pure Knowledge* is a dense and carefully wrought elaboration of the consequences of the idealization of mathematics proposed by *Kant's Theory of Experience*, it is in the *Ethics of Pure Will* that Cohen gives full expression to his new and unique form of idealism: *Messianic Idealism*.

For Cohen, ethics is that through which philosophy, as a system, is able to secure its special character as a *positive* science. Thus, for Cohen, it is with Socrates that "**ethics, as the doctrine of man, became the centre of philosophy**. And it is first in this centre that philosophy acheives its independence and its particularity, and thus also its unity" (ERW; CW 7:1). The positive character of Cohen's programme is not immediately apparent in the doctrine of knowledge — where representations may be taken as referring to transcendental objects, or at least as signs of some "fact of science" that transcends the specifically positive activity of thought. However, in ethics, the positive activity of thought is manifest in the (arbitrary) capacity of thought to create possible forms that are not correlates of the reality (or being) of Nature.

Prima facie, Cohen has returned to the Fichtean view — that the activity of self-determining thought is the original positing of a domain of action (law), which is subsequently limited through the idea of reciprocity (right), and ultimately through the idea of fact (self-limitation) as the production of empirical reality: the apotheosis of the subject.[12] However, there remains a significant distinction between Cohen's approach and that of Fichte and Schelling. In particular, Cohen denies that it is the self that posits itself (as free individual) in the fundamental act of consciousness; the facticity of the individual remains, on Cohen's account, essentially

[12] That is, the self-conscious subject is the author of both its own actions and the sensibly given. In this sense, the subject is modelled after the *imago dei*. Schelling, for example, explicitly invokes the tetragrammaton in identifying the "final real ground of knowledge" as the claim "**I am, because I am!**" (Schelling 1856, 167).

ungrounded, and is not connected with the metaphysics of the absolute subject or the principle of the identity of thought and being. Rather, for Cohen, the positive creation of law is only possible through thought (or what we might also call consciousness). As we know, however, this consciousness is neither phenomenal nor necessarily individual in character. Indeed, it is consciousness only insofar as it is universal in character. The error of the Fichtean method is to begin with the concept of the individual and to attempt to derive from this concept the concept of the good. As Cohen claims, "**the individual is and remains the core concept of the Man of psychology**" (ERW; CW 7:11). And, furthermore, "the individual of psychology would become for philosophy the foundation of individualism, egoism and solipsism", or subjective idealism (ERW; CW 7:12). This general characteristic of logic *qua* liberal ethics is rejected by Cohen as tainted (a) by an erroneous metaphysics of the subject and (b) by an original solipsism from which it cannot escape.

Cohen's criticism not only distances his position from that of Fichte and Schelling (or even Hegel), but also, and decisively, from that of Kant and Plato. For, both Kant (*The Critique of Practical Reason*) and Plato (*The Republic*) begin their examination of the good with the concept of the individual: what is it for an agent (person, or will) to be a "good" or "just" agent? To this extent, Plato and Kant provide merely psychological accounts of moral action — at most a moral psychology. Indeed, Plato comes closer to Kant in understanding that the problem of justice (or goodness) must first be resolved as a social problem before the problem of the individual can be addressed. For Plato, however, even the pseudo-politics of *The Republic* remain a kind of psychology, grounded, as they are, in the functional, organic concepts of the practical agent: sensibility, will and intellect.

In contrast, Cohen proposes an ethics that is primarily concerned with a different object. It is not the individual that is the object of knowledge, but the universal concept of the agent: "**The object of ethics is man**" (ERW; CW VI:2). Indeed, the object of ethics is not (factical) man of biology, sociology, or moral statistics, but rather the *idea* of humanity as a moral concept. Thus, as Cohen insists, "**without beginning with the concept of totality, the**

concept of Man not only cannot be perfect, but rather cannot even be developed or properly framed" (ERW; CW 7:7). Ethics, for Cohen, is not grounded in the original form of the "well-governed" subject, but in the idea of a well-ordered community. Despite the Aristotelian aspect of Cohen's proposal — that we discover the nature of ethics through the political — Cohen takes the political to be subordinate to the anthropological: humanity as such. For Cohen, humanity is not, as it is for Kant, the immanent end of a psychology of action, but rather the original determinability of action through the ideal possibility of lawfulness (i.e., of a universal maxim). Thus the concept 'man' or 'humanity' is not biological, psychological or empirical. Rather, it is the idea of a universal community that is produced through a positive system of law: the kingdom of ends.

Cohen's resistance to the idealist program of a "Spinozism of freedom" must therefore be understood not as a reaction against the (romantic) pantheistic or atheistic consequences (ultimately, apotheosis) of the philosophy of nature, but rather as a reaction against the strictly liberal conception of the nature of logic, which inevitably assimilates the will to natural causation. That is, it is because human freedom is of the same type as natural determination that monism, and the liberal ethico-logic that it is grounded in, fails to provide an adequate, teleological conception of human agency, and thereby misconstrues the concept of the human as the apotheosis of the free will. As Cohen insists, "**the danger of pantheism** is not originally in the threat to the idea of God; this is merely its consequence. The mistake in the pantheistic foundation and principle relates to the concept of Man, and thus to the problem of ethics. If God and Nature are the same, then Nature and Man are at least the same" (ERW; CW 7:16). Within the *System of Critical Idealism*, it is not the transcendence of God that must be preserved, but rather the transcendence of the ideal of humanity, since only this ideal can provide the ultimate orientation of idealism: "[the Idea] must have no thing in itself standing in the background. The idea is the ought. The idea means nothing other than the preface to the practical use of reason, which is summarized in the ought. **The value of the being of ethics lies in this ought**" (ERW; CW 7:27). Thus, whereas the model for Spinozistic systematic idealism is the single

ultimate principle (i.e., the freedom of the self-determining subject or absolute thought) from which all reality may be derived, Cohen's variety of idealism begins with a teleological ideal (humanity) *that it does not produce for itself*, which is the boundary condition (or asolute end) of the principles of logic and the productivity of ethics.

Accordingly, what pure ethics secures is not the practical efficacy of the psychological will (i.e., affect), but rather the idea of the pure will (i.e., the form of action) insofar as this can be determined with respect to the ideal of humanity as the end consciousness itself. That is, ethics shows the originality of pure thought as the capacity to think the good beyond being, without thereby also determining the freedom of the will as a metaphysical problem. In thinking the end of thought itself, thinking provides *for itself* the foundation upon which the constructions of logic are (a) intelligible as anticipations of (pure) being and (b) grounded in the "hypothesis" as the production of intelligible grounds. What is thereby thought is not the absolute, but rather *the anticipation of the absolute*. For, the good beyond being (i.e., the kingdom of ends) is not thought in its absolute determinacy (as *factum*), but only as the absolutely intelligible (as pure being), or that which is the pure negation of givenness and facticity and the indefinite affirmation of intelligibility.

If idealism consists of the view that the content of thought is in principle intelligibly grounded — not merely given or revealed — then its consequence is a teleological conception of thought: thinking is oriented towards the transparency of that which is necessarily thought, but which is not transparent to thinking itself in the fact of knowledge (i.e., in empirical science). However, Cohen's reinterpretation of the correlation between the manifold of being and the unity of the object makes it impossible to assume that there is a uniquely determinate, absolutely intelligible ground of knowledge. Theoretical representation is pure, but it cannot be true, because it is merely formal, and therefore can and must tolerate a plurality of empirically adequate intelligible grounds: theoretical pluralism. This is the consequence familiar from more contemporary forms of scientific idealism, as, for example, in the work of Carnap, Reichenbach, Quine and — in his idiosyncratic way — Rorty. Cohen, however, does not accept

the epistemic relativism that seems to follow inevitably from the view. For, Cohen does not think that truth can or should be grounded in knowledge. Rather, the idea of truth rests in the *idea* of the thing-in-itself. As we saw, however, this "idea" for Cohen is not ontological, nor is it a candidate for hypostasis. Rather, it is the idea of the perfectability of knowledge, i.e., it is the *verum* of the traditional transcendentals, which we call the 'thing-in-itself'. As the perfectability of knowledge, however, the thing-in-itself is not a thing at all, but the anticipation of anticipation. That is, it is the end beyond knowledge, but which grounds the intelligibility of the claim of truth — if not directly the form (or logic) of theoretical claims. Thus, Cohen identifies the problem of truth explicitly with with the concept of the good beyond being from Plato's *Republic*.

Accordingly, for Cohen, it is ethics that must provide the intelligible ground of the claim to truth raised by a knowledge claim, insofar as such a claim is to be more than a mere form of representation. From this follows Cohen's definition of truth: "**Truth means the unity and the harmony of the theoretical and the ethical problematic**" (ERW; CW 7:89). If empirical theory appears to be threatened by relativism and empty formalism, it is secured (if only as hypothesis, i.e., as a historically contingent truth) by ethics, which determines the domain of the intelligible as that which can legitimately be thought of as intelligibly grounded: law. Thus, for Cohen, "truth means the climax of being (*oùsia kaì àlétheia*). **Truth depends on this elevation to the idea of the good**" (ERW; CW 7:86). This is — emphatically — not the "primacy" of the practical, or the grounding of theoretical reason (the self-determination of the determinable) in practical reason (the determinability of the determinable). Cohen distances himself both from the explicit identification of logic and ethics (Fichte, Schelling and Hegel) and the implicit (ontological) grounding of theoretical reason in an effective subject (Kant). Rather, the primacy of ethics, for Cohen, is *epistemic*. It is because thought is able to think something purely for itself — something that cannot even be a concept, and can have no reality as an object of experience — that thinking is legitimated in providing its own intelligible foundations, even if such a pure idealism appears to be a vicious circle that leads to relativism and nihilism:

> The logic of pure cognition has above all rediscovered the fundamental Platonic insight, that every ground of being is not to be assumed or sought in foundations that are given in themselves. **The idea is hypothesis** ... The originality of Plato consists solely in the characterization of the idea as hypothesis. (ERW; CW 7:97)

That is, thinking is adequate to itself because of and insofar as it is able to think of its absolute self-sufficiency as the intelligibility of being. The content of ethics, therefore, is not the idea of (organic) "flourishing", the "good life", or "becoming what you are" (actualizing your reality, etc.). Rather, the pure content of ethics is the pure idea of the will: thinking in and for itself, as the groundless unity that is itself the adequate origin of the grounds of thought only insofar as it is originally oriented towards its intrinsic, if implicit, end: humanity. That is, thinking just is the laying of foundations (*zu Grund liegen*) for the sake of the ends of thought.

The specificity of Cohen's idealism comes into sharp focus when we consider just what the pure end of thought must be. In Kantian terms, we might think that the end that thought gives to itself is the teleological orientation of the moral law: autonomy, or perhaps the good will as the end of all rational activity (including the theoretical). For Cohen, however, the former is inadequate because of its metaphysical implications, while the latter is more properly thought of as the content of (applied) ethics, not the pure ground of intelligibility. Rather, that which is the pure end of thinking is the idea of the perfection of perfectability, or the idea of the good-in-itself: the idea of God. In the idea of God, thought — and thus science — provides itself at once with the horizon of its determinability (i.e., the perfection of pure thinking) and the legitimacy of its formal method (logic) when faced with the skeptical challenge. Science lays claim to truth in and through the idea of God. And, it is the character of "God" that determines the special character of Cohen's idealism:

> Ethics alone could not secure the expression of truth. Rather, it was championed by religion. One can neither blame nor praise the Greek religion for having made pregnant use of the word for their gods; indeed, it was **monotheism** that seized upon this word for the one God. God is true, and God is the truth; these are the profound expressions through which the **prophets** conceive of the one God. (ERW; CW 7:87)

Prima facie, Cohen's position appears to be little different than the familiar liberal ethics of

systematic idealism, not only that of Fichte, Schelling and Hegel, but also the ontotheology of Leibniz and Descartes: a tradition with a continuous origin, one might argue, beyond Plato and Pythagoras. However, the specificity of Cohen's idealist project — and its very special limitations — can be made explicit by a more precise determination of the ideal of perfection that can be called 'messianic'.

Cohen carefully distinguishes between the concept of the totality that applies to the temporal structure of natural physics (knowledge) and that which applies to the anticipation of the determinability of intelligible structure. For, while "space, as infinite space, means the totality of space" (ERW; CW 7:400), there is no totality of time, for time represents, for Cohen, not merely a dimension of reality, but the anticipation of reality. Physics is true of space *at* all times, but is essentially *in* time as the realization of *motion*. In order to think the totality of time, therefore, we can only think of what is true of all time. Thus, we require an "analogue" of infinity that can be thought of *in time* but as true *outside of time*: this concept is eternity, an ideal or "messianic" temporality.[13]

The concept of a time beyond natural time is not provided by a natural physics that has its foundation in natural religion, since "myth has no representation of the future; it misplaces the peace of humanity and nature in the past, in the golden age" (ERW; CW 7:405). "Religion" — or at least the monotheistic religion of Judaism, Christiantiy and Islam — locates ethics not in the time of nature, but in a time that transcends the time of nature: the eternity of the ideal. For Cohen, however, "eternity" is not a temporal structure that derives from an extra-human agency (i.e., the time of God's redemption). Rather, from the human standpoint, "eternity" is the anticipation of the actuality (or facticity) of ethical perfection, i.e., the realization of the kingdom of ends on earth. Thus, eternity is, like natural time, an anticipation. However, unlike

[13]That is, the laws of natural physics are the anticipation of the relational spatial determination of objects at some point in the future; the nature of natural time is this anticipation. However, in order to think of the *true* physics, one must think of a set of laws that do not anticipate the future of the manifold, but *are* the intelligible ground of the whole of the manifold itself, i.e. a science that is beyond the history of science and its progress. Kant calls this the 'transcendental ideal', and later idealists call it 'consciousness as such' or 'the absolute'. For Cohen, however, it is just the idea of a completed physics, but, as an *idea* it can only be grounded in something that is transcendent with respect to nature, not in nature itself (at pains of returning to romanticism and pantheism).

natural time it does not anticipate natural being, but the perfection of intelligibility through the perfection of an ethical system, and, by extension, the harmony of the theoretical and the ethical problematic in the intelligibility of a system of natural law that is grounded in the pure intelligibility of the positive laws of the pure will.[14] The theological temporal-concept 'eternity' is in reality the intelligibility of the pure productivity of thought insofar as we believe that a single, unified system of ethics is the necessary end of ethical "progress". And so, "eternity, abstracted from time and related to the pure will, means only the **eternal progress of moral work** [*sittlichen Arbeit*]" (ERW; CW 7:410). The pure will, therefore, is not the efficacy of the subject, but rather the idea of consciousness as the intelligible ground of a system of purely positive laws (ethics), through which the ideality of science in general becomes intelligible.

Accordingly, just as logic does not produce nature, but only its intelligibility as "reality", ethics does not produce (metaphysical) freedom, but only its intelligibility as will: "**Willing is the being of the ideal**" (ERW; CW 7:425). That is, the will is not, for Cohen, as it is for Kant, a reflective act, the purified faculty of desire, through which an intervention in the causal order becomes intelligible. Rather, will is the pure production of a being (moral reality, *Sittlichkeit*) that is orthogonal to the being of the natural sciences. There is no causal collision between the will and nature (which is rather a problem for a psychology of affects); rather, there is a methodological distinction between the logic of knowledge (mathematics) and the calculus of the will (jurisprudence). Whereas liberal idealism construes freedom as the basic form of causation or determination from which all other laws may be derived (from some implicit inner structure in freedom *qua* logic), Cohen's ethics is grounded solely in the idea of a unique system of universal laws that are valid at all times, even if they cannot be known, a priori, from the mere principles of production. Ethics is not given in nature, but is a task whose end in no way derives from the structures of theoretical or ideal (ethical) intelligibility.[15]

[14]To say that the system of science is grounded in the pure will does not mean that the pure will is the real ground of science, but only that the lawfullness of the pure will is the ideal ground of intelligibility. This is, of course, still thought in the image of God, but in the absence of any metaphysical hypostasis. This hypostasis can only be attributed to a divine will that is also creative, not merely ideal.

[15]Cohen emphatically distances himself from Kant's interpretation of the human form as the Ideal of Beauty: "**let us remove the ideal from the inventory of the aesthetic, and reserve it for Ethics**" (ERW; CW 7:421). The

Cohen's ethics, therefore, is not — or at least not immediately — a system of morality (*Sittlichkeit*), but rather a defense of the intelligibility of a pure idealism (i.e., a system of purely intelligible laws that are determined by no empirically antecedent fact) as the foundational activity of thought in the construction of a system of ideal representations, i.e., systematic idealism. Systematic idealism, therefore is the theory of the ideal, which necessarily leads to a theory of purely intelligible laws, or the laws of what *ought to be*: ethics. Thus, "**these Three moments are contained in the ethical ideal: perfection; the perfecting of perfection** [*die Vervollkommung*]; **the imperfection of the perfecting of perfection**" (ERW; CW 7:424). These three moments may be explicated as follows:

(a) The idea of the will is grounded in the idea of God, or the idea of truth as the perfection of being;

(b) The creative activity of the will is the creation of morally intelligible being, i.e., "action" as the exercise of the purely creative mode of thought;

(c) The anticipation of the perfectability of the will in the idea of an eternity — of peace and justice — which is not yet.

In other words, the essential concept of ethics is the concept of the holy will *of all humanity* (i.e., the law of the kingdom of ends) as the perfection of a system of laws that provide for the reciprocity of a community of holy wills. However, like logic, ethics is an abstract method for which the essential concept of ethics is in fact an idea, or a teleological orientation. Logic is oriented towards unity; ethics is oriented towards truth. Finally, ethics, anticipates its completion in a real order of perfect wills. The ideality of ethics does not — and cannot — ground the actuality of a community of pure wills. Rather, it can, at most, provide the idea of such

problem with Kant's admission of an ideal of beauty is that it opens the door to a reflective re-animation of nature. That is, the Aristotelian conception of intrinsic causes that is expelled from the logic of the determination manifold in the *Critique of Pure Reason* is restored in the *Critique of the Power of Judgment*, where we are permitted to think of nature "as if" it were so determined. While there are significant Aristotelian aspects to Kant's "system", Cohen's idealizing interpretation must reject even the analogy of natural animism.

a community: the idea of the good. The determination of such a community *as actual* is not within the domain of the productivity of ethics.[16]

The place of Cohen's *Ethics* within the system of classical transcendentals can now be made explicit. As we saw above, logic anticipates (a) its own completion (and thus unity, *unum*) and is therefore oriented towards (b) a truth that it cannot determine from within its own framework. Similarly, ethics anticipates (b) its own completion (and thus truth, *verum* and is therefore oriented towards (c) a good that it cannot determine from within its own framework. Thus, whereas Kant attempted to locate the transcendentals within the structure of logic — indeed of quantity — in the Analytic of Concepts, Cohen argues that the transcendentals — unity, truth, and the good — are three different modes of teleological orientation. Unity is the end of logical determination, and truth is the end of ideal production (thought). The good, however, is only assumed by ethics as the anticipated end of human existence (i.e., of factical being). Ethics itself cannot produce the idea of the good, as the end of the completion of ethics. The validation of the system, however, rests on the intelligibility of the idea of the good. This intelligibility, however, cannot be provided by ethics, not even through the idea of a perfected system of laws, since this would only provide the ground of ethical truth insofar as it is oriented towards an end that it does not itself determine, but can at most satisfy. That is, just as Cohen does not ground the principles of construction in the fundamental logic of thought but rather understands them to be determinable only with respect to an idea (truth) that transcends logic itself, ethics must construct the idea of the pure will as a will that is oriented towards an idea — the good — that is not a product of the principles of ethics (i.e., is not the culmination of a necessary system of maxims of action) but is rather the determining condition of a system of universal law.

The ultimate horizon of ethics as the foundation of a systematic idealism is the idea of humanity as the end of the pure lawfullness of thought. This idea, however, transcends

[16]Whereas the liberal logics of Fichte and Hegel propose that ethical community (*Sittlichkeit* can be derived from logic itself, Cohen argues that a rational ethics can only produce the idea of such a community. The facticity of humanity as actually instantiating such a community transcends the productive power of thought, as we will see below.

systematic idealism and the pure production of the ideal in the system of ethics, since it is the end of ethics: that towards which it can only be determined, but cannot itself determine.

> Thought alone does not create the doctrinal concept of idealism. Being is no less the being of the ideal than it is the being of nature, no less the being of the will than the being of thought. The will is an equally powerful factor of idealism: the will of the ideal. The will does not fantasize, for which thought gives it wings, and it does not merely dally in the whirl of a restless dance to which it is driven by desire; rather, it operates in the production of a genuine being. A Promethean creation is its image: humanity, as eternity, as the ideal of moral self-consciousness. (ERW; CW 7:426)

Cohen's doctrine of the pure will thus leaves us with the following picture. The possibility of knowledge rests in the idea of truth. Truth, in turn, is the idea of the perfection of the idea, that is, it is the idea of the will of God as the perfection of humanity. However, the idea of humanity is not determined insofar as it is necessarily assumed by the possibility of ethics.[17]

6.2.2 Aesthetics and the infinite (The Good)

For Cohen, Kant's foundation of aesthetics in the *Critique of the Power of Judgment* is inevitably a piece of psychology. For, an aesthetics founded on the feeling of *life* — as the feeling of intra-subjective effectivity — is at most subjectively grounded, since it does not and cannot claim any objective correlate. Reflective judgments grounded in the feeling of life are not merely subjective, but exclusively subjective, since they are not even — like ideas or concepts — candidates for objectivating hypostasis.[18] Thus, as Cohen understands Kant, "**pleasure and**

[17] While Cohen's *Ethics of Pure Will* appeals to the concept of freedom and the idea of the free will, the purpose of the ethics is not the maximization of freedom as the realization of good. Indeed, freedom is only possible *insofar as it brings about an ethical community*, or *Sittlichkeit*. Accordingly, Cohen's ethics, unlike Kant's, is not libertarian. The perfection of pure practical reason is not an end in itself. Only humanity — as the concept of a community of agents — is a permissible end of reason. Cohen's ethics is therefore best described as communitarian rather than liberal, insofar freedom is not intrinsic to the agent, but attributed to a (legal) subject just insofar as the law of that action (whether intentionally adopted or not) is oriented towards the realization of ethical community. Cohen's ethical communitarianism bestows freedom on individuals through the law, not the other way around.

[18] Kant introduces the "feeling of life" in a footnote at the beginning of the *Critique of Practical Reason*, and arises as a species of the feeling of coordination between the faculties, and specifically of the effectivity of the faculty of desire with respect to a form provided by the understanding. The feeling of pleasure and displeasure in the *Critique of Pure Reason* arise from the (spontaneous) coordination of the imagination and the understanding in the absence of a determining concept, i.e., in the free play of the faculties. As such, pleasure and displeasure are related to the feeling of life as the subjective awareness of cognitive coordination.

displeasure are expressions for the question of awareness [*Bewußtheit*]" (ARG; CW 8:120), that is, aesthetic concepts are reflective concepts of phenomenological states, and cannot be candidates for objective validity, intelligibility, or intelligible hypostasis.

Accordingly, aesthetic judgment cannot be grounded in "mere" feeling, but must be grounded in the intelligibility of the concept of feeling. That is, feeling cannot be a mere state (such as pleasure and displeasure), but must originally be grounded in the idea of some sort of functional relation. Cohen proposes, as we have already anticipated, a theory of affects to replace the Kantian doctrine of pleasure and displeasure. Whereas the feeling of life — and its subtypes, pleasure and displeasure — is purported to arise from the intrasubjective coordination of the functions of the faculties (in their free play or through the idea of the good), affect arises always in correlation with an object; the objectivity of the theory of affect arises from this correlation with the object of affection. Accordingly, Cohen's theory of aesthetic feeling is grounded in a theory of affect construed as the feeling of movement — of communication between subject and object: "**The production of feeling must return to the question of the feeling of movement**" (ARG; CW 8:146).

Cohen's theory of affect, however, does not refer back to the Aristotelian conception of active and passive natures, of causal or motor powers of the organism. If anything, Cohen's understanding of affection is closer to the Cartesian theory of animal spirits, of afferent and efferent motions that are equally part of the activity of the organism. However, whereas Descartes posits physical motions through the nervous system (cite), Cohen's theory of affection is idealized. That is, the feeling of movement is not grounded in physics, but in the idea of an objective correlation as the direction of effectiveness between subject and object. We might, therefore, say that Cohen's theory of aesthetic feeling is a theory of the intelligibility of impression and expression, or the intelligible component of feeling. This is not to say that we do not have feelings of pleasure and displeasure. Rather, it is only to say that the only component of feeling that does not belong, properly speaking, to the domain of the pscyhology of awareness (phenomenology) is the ideally grounded theory of afferent and efferent feeling: the theory of

affection.

Now, for Kant, the *Critique of Judgment* shores about the account of the *Critique of Pure Reason* and the *Critique of Practical Reason* by attempting to prove that a certain kind of given intelligibility is apt for theoretical or teleological judgment. That is, the subjective (a priori) structures of reason and the understanding may be subjectively validated through reflective judgment in order to mitigate the threat of empirical and moral chaos (cite). For Cohen, however, the intelligibility of Nature and Humanity already provide objective intelligibility (which is not to say objective reality). Accordingly, the purpose of Cohen's *Aesthetics* is not to show that we are warranted in assuming that the transcendental a priori escapes from subjective idealism; rather, he intends to explain the intelligible grounds of the idea of correlation. For, if correlation is assumed (a) in theoretical judgment, insofar as our judgments are about objects and (b) in practical judgment, insofar as our judgments are about other subjects, then the idea of a correlation — as a relation to a being that is transcendent with respect to the pure production of thought. The correlate of the correlation is, therefore, not the "intelligible" thing-in-itself (which, as we saw above, Cohen interprets as merely an anticipated lawfullness), but the objective correlate of subjective experience: feeling. It is, in other words, what knowledge is ultimately about.

Superficially, then, the problem of correlation is analogous to the problem of the thing-in-itself. For, in both cases, we are concerned with the intelligible ground of the object of judgment. However, whereas Kant asks a potentially ontological question, "what is the intelligible ground of the object?", Cohen asks an epistemic question, "what is the intelligible ground of the judgment of objectivity?" That is, Cohen wants to know what part of feeling may be grounded a priori as a condition of the possibility of objectivity. The answer to this question is the idea of the correlation, of a possibly objective (but not hypostasized) relation between the subject and object, which is grounded in the theory of communicative (afferent and efferent) feeling. Accordingly, whereas Kant's aesthetics are preoccupied with a form that may be intelligible independently of determining judgment. Cohen's theory of aesthetic judgment is

concerned not with form, but with content; specifically, it is preoccupied with the human (*der Mensch*) as the special content of feeling.

As we anticipated in the previous section, the coordinating idea of Cohen's system is the idea of humanity as the three-fold anticipation of being. That is, humanity — as consciousness in general — represents, (a) the perfection of being (unity), (b) the perfectability of being (truth) and (c) the perfecting of perfection (the good). Accordingly, Cohen's system may dispense (i) with the sublime, which explicitly refers to the super-human, and (ii) with the teleological, which is also grounded in the idea of an intelligible cause. Rather, Cohen aims to understand the artistic representation of both Nature and the Human through the coordinating idea of the system: humanity as the correlation between human and human. Thus, in both its afferent and efferent modes, aesthetic feeling is feeling for the human; it is love of the human not as form, but as idea of moral perfectability, made manifest through the artist's representations. Thus, for Cohen, pure (or intelligible) feeling, as the basis of art, "is the **love of the nature of man**, that is a part of nature ... it is the original force [*Urkraft*] of art. **It alone is love**" (ARG; CW 8:182). That is, art, insofar as it is possible, is the representation of the correlation with either Nature or Humanity as intrinsically grounded in the idea of humanity.

Cohen's position leads him to a radical rejection of Kant's proposals in the *Critique of the Power of Judgment*. For, both judgments of the sublime and judgments of natural teleology are grounded in mythical and religious feeling respectively. These forms of judgment do not propose an objective relation to anything intelligible, but are rather only a reflective hypostasis of the subjective as warrant for objectivating determining judgments. Cohen, however, wants to make sense of the ways in which we may take ourselves to be in relation to intelligible transcendence, insofar as theoretical and practical judgments propose such a correlation. The duality of Cohen's aesthetics results in two modes of aesthetic reflection. The first mode, the afferent, reflects the intelligibility of a subjective projection of our sensible response to the object onto the object itself. This mode, Cohen calls 'desire'.[19] The second mode, the efferent,

[19]Unlike Kant, Cohen does not take the will to be a modification of the faculty of desire. Indeed, Cohen posits no such psychological faculties. Instead, 'desire' refers merely to the intelligibility of the correlation as a projection

reflects the intelligibility of our reflection of response to an object onto ourselves. This mode, Cohen calls 'humour', an epithet that will require some explanation. However, what remains crucial in both modes is that they must be grounded in the idea of humanity, as a belief in the perfectability of humanity, or what Cohen calls 'love'.

Now, the aesthetics of desire are intrinsically related to sensuality and sexuality, insofar as they comingle with — but are intelligibly distinct from — the animal drive for satisfaction and reproduction. However, the pure feeling of desire is not grounded in animal impulse (*Triebe*), but, as I have emphasized, in the pure idea of the correlation. Accordingly, the aesthetics of desire are not confined to the aesthetics of the human form. Rather, the archetype of the aesthetics of desire is landscape painting. For, it is in landscape painting that the yearning for beauty and love is most palpable.[20] Indeed, if the intelligible ground of pure feeling is the idea of humanity, then it is necessarily the case that the aesthetic representation of nature is implicitly humanized. And so, whereas Kant's interpretation of the reflective judgment of nature privileges the supernatural, Cohen's emphasis is squarely on the human, since "**there is no aesthetic Nature without the immanence of man**" (ARG; CW 8:216). Desire is not merely the flattering of the senses or the titilation of animal drives, but an intelligible desire for the perfection of the nature of humanity.

The aesthetics of humour, on the other hand, is essentially oriented toward the way in which we reflect the correlation with the object upon ourselves. For Cohen, the paradigmatic art-form of humour is portraiture, which always retains some element of the caricature. For, whereas the representation of abject suffering provokes the pathos of sympathy or compassion, the aesthetic or reflective representation of the correlation to another turns not on an ethical correlation, but a merely intelligible correlation. That is, the portrait manifests at once the moral perfectability of the human (i.e., the human (*den Mensch*) as the intelligible correlate of aesthetic reflection) and its imperfection (or caricature) as burdened by its natural being.

of the subject onto that which is distinct from it.

[20]This aspect of landscape painting is not limited to Western art forms. One need only study the landscape painting of China and Japan to see an evocative example of the aestheticization of nature.

Whereas desire is the manifestation of the human within the natural, humour is the manifestation of the natural within the human. The human portrait is not — as both Kant and Plato propose — oriented towards the perfection of the human form (i.e., the "ideal of beauty"), but towards the *perfectability* of the human. It is insofar as we are and remain natural creatures — "warts and all", as the saying goes — that we become more than we are: mere animals. Humour, or what we might call 'the irony of the portrait', gives rise to the efferent intelligibility of humanity as the perfectability of the intelligible agent burdened by its animality. This irony is, for Cohen, represented in the smile, whereby animality and its transcendence are manifest in their suspended opposition.

Finally, although Cohen rejects Kant's account of the sublime, he does understand the proximity of art and religion. Since, "religion, in its ultimate ground, wants to be a **feeling of the infinite**" (ARG; CW 8:197).[21] Indeed, in the *Aesthetics*, Cohen claims that the religious — insofar as it is a concept of the infinite — and thus, "**the infinite can be thought and can be willed; where it is felt, there rules aesthetic consciousness**" (ARG; CW 8:198). The aesthetic appreciation of absolute intelligibility, however, is not the (totemic) figuration of the Godhead, but the anticipation of the infinite through the finite correlation with the human other. As Cohen insists, "the aesthetic ascent to the infinite is nothing other than pure **love of the nature of man**, which is itself **infinite in its perfectability**" (ARG; CW 8:267). Thus, the figuration of the divine is not the incarnation of the Godhead, but the apotheosis of the human. It is the anticipation of the perfection of humanity (i.e., the suspension of its natural condition), that permits the aesthetic intimation of the perfection of humanity, the mystical aesthetic of eternity.

Cohen's *Aesthetics* represents the completion of his extant *System of Critical Idealism*.

[21] Cohen will revise this view in the *Religion of Reason out of the Sources of Judaism*. There is clearly a danger in understanding the religious as the aesthetic feeling of the infinite, for this suggests that religion is "ultimately grounded" (and these are Cohen's words!) in an mystical (infinite) aesthetics of desire. However, in the *Religion*, Cohen resists the aestheticization of religion, and, in distinguishing between mythical religion and the religion of reason, he creates a path whereby ethics can purify religious of the ecstatic moment of feeling. This, as we will see, takes place in the transformation of aesthetic contemplation of the infinite, to the interpretation of feeling as infinite law: divine commandment, or *mitzvah*. The ancient Judaic prohibition against graven images is, in Cohen's account, taken as a decisive resistance against the aestheticization of God and the ethics of humanity. This point is extremely important for any who would compare Cohen and Rosenzweig, for the moment of revelation in Rosenzweig is undeniably aesthetized.

However, it is far from providing a satisfactory answer to the questions raised in the aftermath of Kant's Copernican revolution. To be sure, Cohen provides a robust methodology for investigation of the fact of culture *Kulturfaktum*, whereby we may understand culture — and its history — to reveal the underlying intelligibility of thought (or idealism) as the systematic intelligibility of human consciousness (and not awareness) as the "system of science" through which humanity perfects itself. We might even be prepared to accept the (historical) contingency of such an interpretation, and the fact that the concept of "progress" implied by any historical deduction is already normative — surely we cannot understand history as the uninterrupted progress of human reason. However, even if we accepted the "intelligible contingency" of an analytic system of science, Cohen's project would still — and necessarily — fail. For, the orienting idea of the system of sciences is the idea of humanity. Indeed, it is only on the basis of the idea of humanity that we are able to assess progress at all, since it is only through the idea of humanity that unity, truth and the good are intelligible at all. However, without a more substantive content to such an idea, it is unclear precisely what the content of the *System of Critical Idealism* could be, or how it could ever hope to adjudicate between competing conceptions of the necessary principles of the intelligibility of any particular fact of culture.

The problem is that Cohen presents the *System of Critical Idealism* as a system of laws: the unity of law, the system of law, and the perfectability of a system of laws. However, this provides a criterion of validity for any possible system of idealism. At least in its broad conception, this characterization of the task of systematic philosophy would not be objectionable for Kant, Fichte, Schelling or Hegel. In distinguishing himself from his idealist predecessors, however, Cohen orients the teleological indeterminacy of his formal framework around the idea of humanity. This idea, however, is not equivalent — as the *Aesthetics* at times suggests — to the mere idea of the lawfullness of law, or the hypostasis of an intelligible system. More explicitly, the intelligibility of ethics as transcendent with respect to a system of nature demands a conception of a system of ends that supercedes every system of laws that could ever be provided by a merely intelligible system of laws. In order for Cohen to defend the dualism of

Critical Idealism, he requires a concept of that which is transcendent with respect to mere law: humanity. That is, he must be able to show that the "kingdom of ends" as a community of human generosity is the intelligible orientation of any legitimated system of idealism. However, the distinction between a community of law and a community of generosity is not provided by a mere community of law. No more is it provided by the reflective interpretation of feeling through the idea of such a community.

In order for Cohen to secure the *System of Critical Idealism* as an idealist system that remains decisively independent from the liberal-ethical systems of his antecedents, he requires a robust idea of humanity. Such an idea of humanity, however, cannot be provided by the formalism of the *System of Critical Idealism* itself. The possibility of aesthetic, moral and theoretical determinacy, indeed, even the idea of progress in culture, depends decisively on the anticipation of humanity. However, what such an anticipation entails is, at most, the content of revelation.

6.2.3 Revelation and Transcendental Idealism

The problem of the content of revelation, of course, has its roots in Kant. Jacobi's claim that Kantianism is essentially Spinozism is unwarranted, or perhaps merely polemical. For, the problem of singularity is one that Kant takes quite seriously. This can be seen, first, in his (seemingly half-hearted) claim that the system of pure categories is "as it were a system of the **epigenesis** of pure reason" rather than "a kind of **preformation-system** of pure reason" (KRV; AA B167), since the latter might (justifiably) be thought of as requiring an innate (and thus subjective) system of concepts, rather than one derived from the necessary functions of objective judgment. Indeed, Kant emphatically rejects dogmatic rationalism, not just in the well-known passages from the Introduction, but also, and, I think, more decisively, in the Doctrine of Method, especially in the Discipline of Pure Reason. There, Kant explicitly rejects the possibility of a dogmatic philosophy (i.e., philosophy on the basis of pure principles, or systematic idealism), since "in philosophy one must not imitate mathematics in putting the definitions

first" (KRV; AA A730/B759). Indeed, the only permissible use of reason is heuristic: as the hypothetical conditions necessary to the interpretation of empirical experience (and not even experience a priori!):

> Since we cannot construct *a priori* the least concept of the possibility of dynamical connection, and the category of the pure understanding does not serve for thinking up such a thing but only for understanding it where it is encountered in experience, we cannot originally cook up, in accordance with the categories, a single object with any new and not empirically given property and ground a permissibly hypothesis on it ... In a word: it is only possible for our reason to use the conditions of possible experience as conditions of the possibility of things; but it is by no means possible for it as it were to create new ones, independent of these conditions, for concepts of this sort, although free from contradiction, would nevertheless also be without any object. (KRV; AA A770-1/B798-9)

At least with respect to the objects of experience, Kant seems prepared to accept that the unity of the object could be "revealed" in experience, even if this fact (a) is not intelligibly grounded in its *actuality* and thereby (b) destroys the possibility of "rational philosophy" as a system of (pure) concepts, leaving only "transcendental philosophy" and a system of a priori concepts. This is hardly Spinoza's rationalism.

Although Kant says remarkably little on the subject of our actual empirical experience (the *Anthropology* excepted), the empirical nonetheless figures prominently throughout the three *Critiques*, and often at decisive moments. Consider, for example, the role played by moral feeling as testament to the actuality of the moral law in the *Critique of Practical Reason* (KPrV: AA 5:75). However, perhaps the most striking example of revelatory moment of the empirical is in the *Critique of the Power of Judgment*, where Kant appears to concede the possibility of (aesthetically) assessing the actuality of an ideal. For Kant, the "**human being** alone is capable of an ideal of **beauty**, just as the humanity in his person, as intelligence, is alone among all the objects in the world capable of the ideal of **perfection**" (KU; AA 5:233). That we might have access — even reflective — to an empirical ideal is surprising in the light of Kant's otherwise pessimistic view of the possibility of cognizing the ideal, "by which I understand the idea not merely *in concreto* but *in individuo*, i.e., as an individual thing which is determinable, or even determined, through the idea alone" (KRV; AA A568/B596). However, that the empirical

instantiation of the idea of the human as an end in itself is at least thinkable (even if only reflectively) appears to be a condition of the possibility of ethics in general. Recall, for instance, the "humanity" formulation of the moral law from the *Groundwork of the Metaphysics of Morals*: "*So act that you use humanity, whether in your own person or in the person of any other, always at the same time as an end, never merely as a means*" (GMS; AA 4:429). *Prima facie*, we must posess a concept (or rather idea) of the possibility of moral perfection (i.e., 'humanity') without which pure practical judgment is inconceivable.[22]

However, what the idea 'humanity' actually entails only comes into focus when we consider that the Doctrine of Virtue of the *Metaphysics of Morals* imposes upon us a (supererogatory) duty to benevolence, as a matter of exercising and perfecting our moral capacity.[23] However, the ends of virtue — unlike those of mere right (*Recht*) — do not derive from the universalizability criterion ("*act only in accordance with that maxim through which you can at the same time will that it become a universal law*" (GMS; AA 4:421)), but rather only from the idea of humanity as an end in itself.[24] However, there are only two duties of virtue: "They are *one's own perfection* and *the happiness of others*" (MS; 6:385). These consist, respectively, in

[22]Some recent commentators, and especially Allen Wood (Wood 2008), have argued that there is an oft-overlooked practical dimension to Kant's philosophy — most especially involving the cultivation of our natural sociability through moral and aesthetic education. One might, at a stretch, say that the content of 'humanity' is a practical wisdom (*phronesis*) that is not (a) a form of theoretical knowledge nor (b) a form of moral knowledge, but rather (c) the exercise of a practiced heuristic of judgment. While it is undeniably the case that Kant believes that we must develop our natural sociability as a condition of the effective exercise of judgment, it seems to me unlikely that this *aesthetic* model of the refinement of judgment is sufficient to secure the objective validity that Kant takes to be necessary not just to the possibility of virtue, but to the possibility of a doctrine of right, and *a fortiori* the objectivity of theoretical judgment. Whatever the case may be, the content of 'humanity' is neither subjectively nor objectively grounded in principles of construction. Whether the "revelation" of 'humanity' takes place through deliberate practice or through extatic disclosure, the epistemic status of such "wisdom" remains equally problematic. Cohen is not unaware of the practical dimension of Kant's moral doctrine, but distinguishes clearly between the theoretical possibility of moral knowledge (law) and the application of law in empirical individuals, which is more properly a problem for (moral) psychology.

[23]This is already the case in the *Groundwork of the Metaphysics of Morals*: "to be beneficent where one can is a duty" (GMS; AA 4:398).

[24]On this point, Kant is explicit:

> The doctrine of right dealt only with the *formal* condition of outer freedom (the consistency of outer freedom with itself if its maxim were made universal law), that is, with **right**. But ethics goes beyond this and provides a *matter* (an object of free choice) an **end** of pure reason which it represents as an end that is also objectively necessary, that is, an end that, as far as human beings are concerned, it is a duty to have. (MS; AA 6:380)

the perfection of the rational faculty (i.e., of moral judgment) and concern for the well-being of others. Moreover, our duty to others consists of three components: benificence, gratitude and sympathy. Cultivating these duties requires not only that we take an active interest in the empirical welfare of another, but that we act with the empirical well-being (happiness) of others *as an end*:

> Benevolence is satsifaction in the happiness (well-being) of others; but benificence *is the maxim of making other's happiness one's end*, and the duty to it consists in the subject's being constrained by his reason to adopt this maxim as a universal law. (MS; AA 6:452 — my emphasis)

To do so, however, requires that we have at least some access to the empirical states of others, as a condition of being able to determine what actions might affect their well-being. For, it is only if we can be conscious of the empirical needs of others that we are able to act with these in mind as the end of our (virtuous) actions. This, in turn, imposes upon us a duty of sympathy, since,

> while it is not in itself a duty to share the sufferings (as well the joys) of others, it is a duty to sympathize in their fate; and to this end it is therefore an indirect duty to cultivate the compassionate natural (aesthetic) feelings in us, and to make use of them as so many means to sympathy based on moral principles and the feeling appropriate to them. (MS; AA 6:457)

While it is not possible to enter into the details of Kant's conception of ethical virtues here, it is important to note that while Kant acknowledges that we are bound to respect principles of freedom, the bulk of the content of ethics — the theory of virtue — rests on a content that cannot be intelligibly grounded. For the idea of humanity cannot simply be an innate idea, but is, at best, a feature of our natural sociability; indeed, it is in precisely these terms that Kant describes the concept of humanity in both the *Critique of the Power of Judgment*[25] and the

[25]For example:

> The beautiful interests empirically only in **society**; and if the drive to society is admitted to be natural to human beings, while the suitability and tendency toward it, i.e., **sociability**, are admitted to be necessary for human beings as creatures destined for society, and thus also as a property belonging to **humanity**, then it cannot fail to be the case that taste should also be regarded as a faculty for judging everything by means of which one can communicate even his **feeling** to everyone else, and hence as a means for promoting what is demanded by an inclination natural to everyone. (KU AA 5:297)

Anthropology.[26] Although Kant calls our sympathy for the empirical states of others "aesthetic", this refers to the cognitive mode in which they are available (i.e., as reflective, not determining), which does not allow us to interpret them as objectively grounded. At least according to the taxonomy of epistemic claims in the *Critique of Pure Reason*, "taking something to be true is only subjectively sufficient and is at the same time held to be objectively insufficient, then it is called **believing** [*glauben*]" (KRV; AA A822/B850). We could, without doing violence to Kant's intent, translate '*glauben*' as 'having faith', thereby more closely capturing the sense both (a) of the kind of content that is revealed in sympathetic emotions of humanity and (b) our attitude towards this content. While we are still far from the claim that Kant is a fideist, 'humanity' clearly shows the extent to which Kant's account — and especially the ethics — depends on ungrounded content.

We can see aspects of the revelation of the content of ethics in Kant's idealist successors. This is particularly evident in Fichte's attempts to derive ethics from mere experience in the *Foundations of Natural Right* and the *System of Ethics*. Fichte argues that we become aware of our own autonomy (and thus, the content of the moral law) through "the subject's being-determined as **its being-determined to be self-determining**, i.e. as a summons [*Aufforderung*] to the subject, calling upon it to resolve to exercise its efficacy."[27] Even if we could

[26] Again: "The way of thinking characteristic of the union of good living with virtue in *social intercourse* is *humanity*" (An.; AA xx:277).

[27] (Fichte 2000, 31) Fichte's appeal to the "summons" is perhaps no more perplexing than his appeal to the "check" (*Anstoß*). For, in both cases, Fichte requires that we *posit* the cause (or, better, origin), of what we take to be revealed. For example:

> The external being that is posited as the cause of the summons must at the very least presuppose the possibility that the subject is capable of understanding and comprehending; otherwise its summons to the subject would have no purpose at all. The purposiveness of the summons is conditional on the understanding and freedom of the being to whom it is addressed. Therefore the casue of the summons must itself necessarily possess the concept of reason and freedom; thus it must itself necessarily possess the concept of reason and freedom; it must be an intelligence, and — since this is not possible without freedom, as has just been shown — it must also be a free, and thus a rational, being, and must be posited as such. (Fichte 2000, 35)

But it seems paradoxical to claim, on the one hand, that the summons is our "being-determined" (i.e., passivity), and on the other, to claim that it is the intelligible product of a cause that we must posit (i.e., activity). Fichte provides a notoriously vague account of how one is supposed to understand either the check or the summons as actually revealed, if indeed they are only intelligible as products of our own activity.

successfully interpret Fichte's claim about the way in which freedom is revealed to the subject, there would still be a stark distinction between Kant's conception of what is revealed to us through sympathetic emotion and what is revealed in the summons. Indeed, for Fichte, the content of ethics is exhausted by the revelation of autonomy. For Fichte, ethics — indeed reality — is freedom all the way down. Indeed, we see the same emphasis on liberal ethics in Hegel's account both of the emergence of the subject and the emergence of the general will in the *Phenomenology of Spirit*, the *Encyclopedia* and the *Outline of the Doctrine of Right*. While we may have affective, purely sensible bonds to one another as a feature of our animality, the emergence of the subject — indeed, its defining characteristic — is its independence from its affective nature. It is precisely through the negation of his animal dependence that the slave overcomes the fear of death to realize the freedom of self-consciousness.[28] What has gone missing from Kant's account of ethics in the idealist reception is the duality of ethics as both a doctrine of right *and* a doctrine of virtue. Of course, it is hardly surprising that this should be the case. In turning to an Aristotelian conception of humanity as the immanent genus of the individual, ethics, as the logical explication of the concept of the human, just is both (a) the universal principle of right action and (b) the particular affect of the individual.[29] On this

[28]Despite the (deserved) celebrity of the master-slave dialectic in the *Phenomenology of Spirit*, Hegel is not interested in the recognition of the other as a particular other in this context. Indeed, this cannot be the case, since it is through the master-slave dialectic that one first becomes capable of recognizing one's own subjectivity for what it is: the capacity to separate oneself from the material conditions of existence; this is not inter-subjective recognition but self-recognition. If one were to point to an instance of inter-subjectivity in the *Phenomenology of Spirit*, it would be in the conflict between subjective and objective *Sittlichkeit* represented by Antigone and Creon. There, at least, there is the possibility of reciprocal recognition (the intelligibility of the moral principle of another) and rational (if irreconcilable) disagreement. Here too, however, what we recognize is not the other in his particularity, but only the principle of his will insofar as it represent a principle of intelligible action (subjective, objective, or absolute). Indeed, this is the structure that Hegel develops in his *Outline of a Philosophy of Right* (Hegel 1995): intersubjectivity consists not in the recognition of alterity but in the recognition of objective principles of freedom, symbolically realized in the contract. Indeed, despite Hegel's prominence as a thinker of inter-subjectivity, Hegel is remarkably disinterested in singularity of the other, or even the empirical conditions of this other.

[29]It is, of course, possible to interpret Kant as grounding humanity — as the end of moral judgment — in the *praxis* of moral deliberation. Hence, Kant's emphasis on aesthetic education, socialization, normalization, casuistics and the cultivation of the habits of good judgment generally may be seen as the underpinning of a naturalized conception of reason. That towards which reason tends, as the end in itself, just is the product of a practically acquired *phronesis* in the exercise of virtue (appropriate to a time and place), and not the transcendental ideal of a "kingdom of ends" that orients our moral judgment (and thus also our theoretical judgment) towards an a priori ideal of human community. This is clearly not Cohen's interpretation, nor is it the one that I favour, but I will concede that it is not *prima facie* incoherent to claim that Kant has a naturalized (or naturalizable, within an Aristotelian nature) conception of the ends of reason.

interpretation, our duty to be the best instance of what we are (i.e., human) just entails that we act toward the common good in the realization of our personal end. Kant's distinction between duties of respect (i.e., of universality) and duties of love (i.e., of the end in itself) is both unnecessary and untenable. Moreover, on the liberal account, the content of revelation — as the moral law, and faith therein — is exhausted by (a) the recognition of one's own autonomy and (b) the generality of this autonomy as the common good of the genus 'human'. Revelation of a pure religion — even as Kant conceives of this — that is anything other than the natural end of the subject itself is not only unnecessary, but impossible.

Cohen's Platonizing reading of Kant's *Critical Project*, however, cannot follow the same route. Indeed, Cohen's account not only abolishes the possibility of feeling as an epistemic source generally — it is not subject to laws, not even in aesthetic contemplation — but also eliminates the affective dimension of the discipline of moral, theoretical and aesthetic judgments. Indeed, precisely those aspects of Kant's view which most readily lend themselves to an Aristotelian interpretation are deliberately rejected by Cohen's idealizing account.[30] This places a tremendous burden on the concept of humanity, however. For, as the keystone of the teleology of ethics, it is also thereby the central arch of the teleological system of reason. If reason can be said to have any interest, or any end in itself, it must be that of humanity: the focal point of Kant's *Critical System* is none other than the concept of humanity as an end in itself. However, since this end in itself implies duties to others that are ungrounded — which, indeed, may be incapable of grounding within the Kantian paradigm, at least on a Platonizing

[30] As Kant points out in the Introduction to the *Critique of Practical Reason*,

> we shall not have to do a critique of **pure practical** reason, but only of **practical** reason as such. For, pure reason, once it is shown to exist, needs no critique. (KPrV:;AA 5:15)

The methodology of the *Critique of Practical Reason* thus employs a theory of affection (and thus a moral psychology) as the basis for a defense of the possibility of pure practical reason through merely practical reason, i.e., as this is manifest through affect (and thus through the faculty of desire, or the effective will). The account of the *Critique of Practical Reason* is thus a mixed exposition of pure principles (maxims) and psychological evidence (affections) through which reason is practically effective. Cohen, however, aims to provide a pure exposition of pure practical reason in the form of practical knowledge: law. While a moral psychology and a doctrine of affection (or something similar) are necessary for consideration of the application of the pure laws of practical reason, they are not necessary if we take the factical basis of the moral law to be jurisprudence rather than the first-personal moral feelings of the effective will (e.g., respect).

interpretation — it appears to be the case that the entire architecture of Critical Reason rests on a commitment to the welfare of others that cannot be justified, but can at most be found and embraced.

However, the doctrine of morality (including the *Critique of Pure Reason* and the *Metaphysics of Morals*) does not prove that morality is in itself desirable. It follows only that it is rationally necessary. For, as Kant points out, "there is not the least ground in the moral law for a necessary connection between the morality and the proportionate happiness of a being belonging to the world as part of it and hence dependent on it" (KPrV; AA 5:124). The well-known solution to the problem, in the *Critique of Pure Reason*, is the postulate of the highest good: God. Since, "in the practical task of pure reason, that is, in the necessary pursuit of the highest good, such a connection is postulated as necessary: we **ought** to strive to promote the highest good (which must therefore be possible)" (KPrV; 5:125). But if Kant has pointed to the limitation of the defence of practical reason, the solution is only partial. For even if the postulate of the possibility of a perfectly effective will is admitted, this in no way resolves the original problem, since what the postulate of the highest good shows is at most that we may believe that our moral actions will bring about the happiness *of others*. It does not follow from this that our own well-being is thereby secured. The pursuit of our own happiness is, recall, not even a morally permissable end.

That towards which we aspire in moral action is the realization of a holy will, i.e., perfection of our own rational nature and the (effective) capacity to realize the empirical well-being of others. However, this is only the objective end of a subjective principle. The objective intelligibility of ethics as the realization of a moral order — a *mundus intelligibilis* that includes our own happiness as a possibility — is not secured by the possibility of a holy will, nor through the subjective necessity of the categorical imperative (i.e., through the testimony of moral feeling). The objectivity of the moral law must aim towards (a) an objective end (the humanity formulation of the categorical imperative) but must also rest on (b) an objective (and not merely subjective) formal principle (the kingdom of ends formulation of the categorical imperative).

For, it is only through the possibility of a kingdom of ends, where, "by a **kingdom** [we] understand a systematic union of various rational beings through common laws" (GMS; AA 4:433) that our own happiness as commensurate to our virtue is intelligible at all. The objectivity of the categorical imperative — indeed, even only insofar as it secures the doctrine of right, not even the doctrine of virtue — rests in the formulation of the kingdom of ends and the supposition of a community of rational agents (i.e., humanity), which itself is not secured by the subjective principle of moral action, the objectivity of the maxim as oriented towards humanity, the testimony of moral feeling, or even the postulate of a holy will capable of realizing the empirical happiness of the other. Without the concept of the kingdom of ends, ethics is unintelligible. Pure reason, however, cannot supply the concept of the kingdom of ends. For a defense of this concept, we must turn to the *Religion within the Boundaries of Mere Reason*.

In turning to the *Religion within the Boundaries of Mere Reason*, one might expect to find a philosophical account of revelation, and perhaps therein an account of the idea of humanity and the welfare of the community as the intrinsic content of religious faith. However, any such expectation is disappointed. Indeed, it's not clear how Kant could accept the possibility of revelation, if, as the *Critique of Practical Reason* insists, God is a necessary *postulate* of pure practical reason (KPrV; AA 5:124). How could a postulate possibly be the origin revelation? And, indeed, the only possible source of revelation, for Kant, is reason itself, since only through the revelation of reason can heteronomy be avoided. Our faith must be based on faith in pure reason alone. Nevertheless, revelation is necessary, but not as a demand of reason. Revelation is a necessary condition of the establishment of a church due to a defect in human nature:

> The only faith that can found a universal church is *pure religious faith*, for it is a plain rational faith which can be convincingly communicated to everyone, whereas a historical faith, merely based on facts, can extend its influence no further than the tidings relevant to a judgment on its credibility can reach. Yet, due to a peculiar weakness of human nature, pure faith can never be relied on as much as it deserves, that is, [enough] to found a Church on it alone. (Rel.; AA 6:103

However, even if we accept the *fact* of revelation as the (scriptural) basis for a particular religious institution, this revelation itself is defective. For, it can never reveal the true religion —

which arises from pure reason alone — but only a *heteronomous* religion based on fact whose legitimacy is at most historical:

> If, however, we assume statutory laws of [God's] will, and put our religion in observing them, then cognition of these laws is possible not through our own mere reason but only through revelation. And, whether given to each individual secretely or publicly — that it may be propagated among human beings through tradition or scripture — this revelation would be *historical* and not a *purely rational faith*. (Rel.; AA 6:104)

Kant's hostility to revelation extends in particular to Judaism, which, while it is precisely that historical religion which is the condition of the pure religion, must, because of its very historicity, be negated:

> We cannot, therefore, begin the universal history of the Church (inasmuch as this history is to constitute a system) anywhere but from the origin of Christianity, which, as a total abandonment of the Judaism in which it originated, grounded an entirely new principle, effected a total revolution in doctrines of faith. (Rel.; AA 6:127)

Judaism thus finds itself a paradoxical situation: it is both the necessary condition of the true (pure) religion grounded solely in reason, and at the same time that which must be negated — even annihilated — as a condition of the liberation of reason from a superstitious adherence to a heteronomous statutory law, the emancipation of faith from history and its contingency.[31]

6.2.4 Revelation and Critical Idealism

If the problem of revelation is acute for Kant's idealism — at least insofar as we take its foundation to be ideal — the question is all the more decisive for Cohen's explicitly idealist foundation of the *System of Critical Idealism*. For, if the concept of humanity cannot be given any determinate content, then the system cannot be given any content: it remains a formal *method* whereby

[31] Kant's strategy in the *Religion within the Boundaries of Mere Reason* comes remarkably close to Spinoza's rejection of religious parochialism in the *Theological-Political Treatise* (Elwes 1951). The anti-Judaic consequences of both views have been widely noted. No doubt the concatentation of Kant's hostility to Judaism and Spinoza's blithe erasure of any religious plurality contributed to the generally anti-Judaic tenor of post-Kantian German Idealism.

the intelligibility of science and law may be explained. That is, the *System of Critical Idealism* is not a philosophical system at all, no more can it provide an adequate justification of the transcendental ideals of a system of thought (unity, truth, and the good), since the ultimate transcendental — the good — is no more than a supposition from the point of view of the system. The system on its own can provide *hope*, insofar as we believe that human history will, in some sense, prove to be the history of the realization of the good (i.e., ethical community).[32] However, it cannot even provide faith (*Glauben*, in Kant's sense), since the idea of the good is not even subjectively grounded, but remains merely anticipated as the indeterminate (and indeterminable) idea of the fact of ethical community.

That this is so should come as no surprise, however. For, Cohen's project in the *Sytem of Critical Idealism* is to provide a system of the critique of culture. The *Kulturfaktum* must be given; from this, critical analysis may provide its intelligible (ideal) grounds. Its facticity, however, remains ungrounded from the perspective of such an idealism, because it is concerned with culture only insofar as it provides the universal intelligibility of cultural consciousness, or the historically contingent realization of systems of lawfullness. That a system of lawfullness can be provided, and that such a system of lawfullness could be unique or objectively valid (rather than merely formally or subjectively valid) cannot be established by the system itself.

However, the problem for Cohen's *System of Critical Idealism* is that in the absence of any determinacy, the idea of humanity cannot serve as a determining orientation for ethics. That is, if ethics is just the free production of a system of laws, there is no way to adjudicate between differing systems of laws insofar as they are all universal; the possibility of rational disagreement — and thus some form of relativism or skepticism — remains, even within the constraints of critical idealism.

As we saw, for Kant the doctrine of right does not provide the doctrine of virtue. For Cohen, the criterion of universality — and even a criterion of universal reciprocity — will not

[32] In Kant's taxonomy from the *Critique of Pure Reason*, such a hope would be no more than "opinion" insofar as it is not even subjectively valid, since subjective validity is reserved for aesthetic judgment. And, indeed, there is no way to understand human history as the history of moral progress except through a normative concept of history that is disjoint from the order of events.

provide for the mutual concern for the welfare of the community that is demanded by the idea of the realization of a kingdom of ends.

Throughout the *Ethics*, however, Cohen points to the specifically Judaic character of ethics. He characterizes this moment as "messianic", and, as we have seen, this messianic character is determined as the anticipation of the idea of the good, which is not itself provided by the *Ethics*, but only assumed as the end of the actualization of thought. Ethics therefore has a necessarily historical component, insofar as it can only be realized in history, and by actual cognizers. The reason for this is that only individual cognizers may have a determinate idea of an actualized concept of humanity: the idea of humanity realized within the individual as a contingent (subjective, given) concept. Characteristically, these individuals are represented as *critics* of the injustice inherent within a particular system of laws. That is, the individual cognizer appeals to a subjective concept of humanity — as the end of thought itself — in order to criticize an objective system of laws. This historical, critical moment of ethics, Cohen refers to as 'prophetic', drawing the designation from the social prophets of the Hebrew Bible.[33] The prophetic moment of ethics is therefore the intervention of history within the order of the ideal, or better, the intervention of the individual — and a subjective concept of humanity — within the eternal and objective order of ethics as the domain of truth.

The paradox of Cohen's ethics is therefore that a system of pure idealism can only have its foundation in the subjective concept of 'humanity', which is deployed by individuals cognizers in order to serve as a "critical revelation" of the content of ethics, and thus as the content of the *System of Critical Idealism*. The content of the prophetic critique of law must be considered to be a revelation from the point of view of the system, since its content cannot be derived from either logic or ethics, but is presupposed by it as a condition of its completion. Furthermore, this revelation can only be critical, since the claim of the concept of humanity against the system of laws can only be negative: the prophet can at most claim that the existing system of laws falls short of realizing the ethical ideal of the kingdom of ends insofar as it

[33] 'Social prophets' generally designates the prophetic texts from the period subsequent to the destruction of the first Temple and the Babylonian captivity: Jeremiah, Isaiah, Ezekiel and Nehemiah.

does not satisfy the prophet's subjective concept of humanity. That is, the prophet points to social injustice and demands the revision of a system of laws, but does not — indeed cannot — provide a *merely subjectively grounded* system of laws, which, as we know, can only be the product of an ethics, or a universal system of law.

As we saw, a content of Kant's doctrine of virtue is only intelligible insofar as we can provide a determinate concept of humanity, however this may be grounded. The merely reflective "ideal of beauty" is inadequate for such a purpose because it provides no grounds from which a maxim of action could ever be determined. However, a determinate idea of humanity would not be a universal idea at all, but only a subjective concept, since it would necessarily be grounded in our receptivity to the empirical states of another. That is, the claims of the doctrine of virtue may only be made from a subjective standpoint, which nonetheless raises a (problematic) claim to universality insofar as virtuous actions are norms (oughts) that ought to be, but which are not themselves objectively grounded in the concept of freedom. Similarly, Cohen cannot appeal to the reflective or aesthetic mode of contemplation in order to justify prophetic critique within the *System of Critical Idealism*. For, aesthetic reflection provides at most the intelligibility of feeling as adequate to the idea of lawfullness. The play of our emotions in a reflective state, on the other hand, may be the origin of a subjective concept of humanity (i.e., we may learn about the real conditions of moral judgment through aesthetic contemplation), but the determining — if subjective — ground of a concept of humanity is not the idea of lawfullness in general, but the determinate empirical content of the underlying sympathetic feeling. That is, the idea of humanity derives its content from moral feeling, not from the idea of the lawfullness of that feeling with respect to the ideas of unity and truth. While the intelligibility of the aesthetic — and thus the idea of a normative claim to objectivity (this ought to be an ought) — provides the *form* of a prophetic claim, the actuality of any prophetic, or moral critical claim can only be grounded in the underlying sympathetic feeling of the prophetic critic.

Thus, insofar as any claims of the *System of Critical Idealism* may be taken to be binding, or *normative*, this normativity derives from the original normative claim of the subjective,

sympathetic ground of moral feeling. We may, in science, demand of others that they agree to the (historically contingent) representations of natural science only insofar as that truth is grounded in the legitimacy of a purely positive system of laws. The legitimacy of a purely positive system of law, in turn, is not grounded in the form of the act whereby the law is produced (i.e., the form of freedom), is grounded in the good as the possibility of prophetic critique.[34] The good, however, is not provided or legitimated by any system of principles or objective criterion. It rests on a subjective concept of humanity, which in turn derives from sympathetic moral feeling. It is only the combination of a determinate concept of humanity with the idea of lawfullness as such that ethics, and thus also logic, can raise legitimate normative claims. While the *System of Critical Idealism* remains a method — and not a system of content — the legitimacy of its critical claims, indeed, of any critical claim raised in the name of rational consistency through such a system, rests in the end on the subjective grounds of the concept of humanity.[35] Cohen's system cannot claim legitimacy — even as a system of merely formal validity — unless the normative claim of critique is justified. And this justification is only possible through an account of the legitimacy of the subjective concept of humanity, that is, through an account of revelation.

Revelation, however, cannot be a part of the *System of Critical Idealism*. It is a necessary companion to the philosophical system of critique, insofar as it provides the legitimacy of the normativity of critique, but it cannot itself be provided within the system of philosophical critique. For, the subjective concept 'humanity' can only be legitimated from the point of view of an individual consciousness. What is more, insofar as that consciousness is individual it is

[34]That is, the prophet need not criticize the existing system of laws in order for it to be ethically valid; the prophet need only be able to (legitimately) contest the system of laws in order for it to claim universality. Thus, the form of every valid system of ethical laws is that it is *not yet invalid*, insofar as it is merely the anticipation of a true system of ethics in its perfectability. The "good" system of ethics lies in the future.

[35]Superficially, Cohen's approach to the legitimation resembles Habermas' theory of rational consensus: truth as that to which everyone in an ideal speech community could agree. However, whereas Habermas grounds the idea of moral truth in the reciprocal bonds of a community of practice (and thereby invites cultural relativism), Cohen allows each individual an immanent — if negative — access to the good in the idea of the perfectability of law, which is a *negative* conception of truth: truth is not grounded in a single, ultimate consensus, but rather is grounded in the intelligibility of the absence of dissent. Habermas can only avoid cultural relativism if he accepts a robust doctrine of human nature, i.e., a contingent (empirical) basis.

no longer *Bewußtsein* as the universality of a necessary form, but rather *Bewußtheit* as the individuality of phenomenological immanence. Thus, the original distinction between objective and subjective consciousness returns, and locates the claims of humanity outside of the system of idealism, and within a realist dimension. For Cohen, this realism is actualized as *religion*. Thus, we must turn to Cohen's final (and posthumous) work in order to articulate the legitimacy of the normative claims of critique on a subjective, phenomenological basis: the doctrine of moral passivity presented in the *Religion of Reason out of the Sources of Judaism*.

Now, Cohen's theory of revelation in the *Religion of Reason* can be summarized in a simple claim: revelation is the revelation of reason. For, Cohen describes revelation in its most general terms as follows: "**that God comes into relation with man**" (RR 71). Revelation, as the relation between man and God "**is the creation of reason**" (RR 72). And, furthermore, "it is only by virtue of revelation that the **rational creature**, man, comes to be" (RR 71). *Prima facie*, Cohen has returned us to the Kantian position: the content of revelation is disclosed by reason, or it is not intelligible content at all. This, however, is not an adequate interpretation of Cohen's approach in the *Religion of Reason*. This aspect of Cohen's philosophy of religion has, indeed, been widely discussed, and frequently criticized. However, we risk misunderstanding the importance of the *Religion of Reason* if we take it to be the case that *reason* (*Vernunft*) is merely a faculty for rules. Rather, Cohen, like Kant, understands reason to be the capacity of determining rules in accordance with ends. But, as I have argued, the ends of reason are transparent within the *System of Critical Idealism*. If these ends are to be incorporated into an account of thought — or at least one based on the critical methods described in Cohen's system — then this content must be *revealed*. That is, the content of revelation is not the rule, nor the faculty for rules, whereby rationality is achieved. Rather, the revelation of reason can only be the revelation of the end of thinking itself: the idea of humanity as a subjective concept.[36]

[36]Kant's *Critique of Pure Reason* is also a critique of the faculty of reason as the ability to construct rules (provided by the understanding) in accordance with the interest of reason (truth). However, whereas Kant appears to take the ends of reason to be unproblematic, Cohen's system — as we have seen — is oriented around the problem of the transcendental ideals (unity, truth, the good) insofar as the ends themselves provide the intelligible ground of the determination of a system of laws. But since the ends of reason are not constructed by the system itself, it can *only* be the case that the ultimate end of reason — the good — must be revealed. Thus, what is revealed

If humanity is, as the end of rational (and thus also ethical) judgment, the content of reason as such, it is not a content that may be thought of as a pure judgment (*reinem Erkenntniss*). To identify the intellectual synthesis from formal principles proposed by Cohen's *Logic of Origin* with "rational construction" loosely construed (as, for example, construction from rules) is to misunderstand not only the function of reason in ethical judgment, but also the content that is presupposed by the very possibility of such a judgment. Rather, humanity, as the content of reason, provides the indispensible orientation and outline without which even ethics itself is a mere method of formal construction. As Cohen insists, "In ethics **The I of man becomes the I of humanity**" (RR 13). Thereby, however, 'humanity' becomes an empty concept. It is not the *concept* of psychology — or of individual self-consciousness — but rather the *idea* of *what ought to be*, that is, of eternity or the kingdom of ends. Indeed, "the method of ethics fails and must fail in regard to the new problem of the Thou, while at the same time the concept of the individual demands the Thou" (RR 16). What religion provides to ethics is the possibility of the determination of the content of 'humanity': an ultimate orientation of the *System of Critical Idealism* that transcends the methodological critique that it propounds.

Judaism, for Cohen, is a "religion of reason" precisely insofar as it represents the systematic connection between the intelligible moments of ethics and aesthetics (law and lawfullness) in their real actualization. This implies a *realist phenomenology* of the concept of humanity, or the idea of humanity insofar as it is subjectively realized as the concrete consciousness of another individual. Thus,

> the share of religion in reason is the share of religion in morality, and no problem of morality takes precedence over this problem of the fellowman. The possibility of ethics is tied to this problem. If the fellowman is leveled down to the next man, it would still be questionable whether sociology could arise; but there is no question that in that case ethics would be impossible. And since the share of religion in reason consists in its share in morality, if ethics is impossible religion also becomes untenable; for then the correlation disintegrates: man then is no longer fellowman, the link in the correlation with God, and no other concept of man but the moral concept of the fellowman can be established in this correlation. Hence,

is not form, nor any property thereof, but rather only the end to which all thought must be oriented if it is to be called 'rational thinking'.

> ethics and religion depend, with regard to the concept of man, on the concept of the fellowman. (RR 115)

And, if ethics depends on the concept of the fellowman, then so too does the completion of logic. For, as we saw, the completion of logic is the teleological orientation that derives from idealization, and this idealization is grounded only in ethics.

Thus, "revelation is the sign of reason, which is not animal sensuality but comes from God and connects with God" (RR 84). However, if revelation comes from God and connects with God, it does so only through the fellowman. It is the suffering of the other — the manifest empirical suffering that provokes the passive affect, compassion or *mitleid* — that is the *content* of revelation as the revelation of reason. For, it is through this revelation that the correlation of man and God, of man and the absolute ideal, becomes possible:

> the concept of the fellowman conceals a correlation of its own, namely that of man and man, but in this narrower correlation there is merely an initial unfolding of the content of the more universal one. For the correlation of man and God cannot be actualized [*in Vollzug treten*] if the correlation of man and man is not first included. The correlation of man and God is in the first place that of man, as fellowman, to God. (RR 114)

Thus, revelation — and, indeed, God's revelation — does not take place at Sinai, but everyday, in the street, in the suffering of another. This revelation, however, can only take place insofar as the phenomenology of sympathy (and its psychological correlate) is interpreted as objectively grounded. This ground is not theoretical Nature, but rather the idea of God as the ground of law. This interpretive stance, however, is distinctively religious; it is a claim raised on the basis of "faith", not "knowledge".

And so, whereas Kant insists that moral sympathy must be aesthetic, Cohen rejects the implication that we are *spectators* to the suffering of the other. Rather, compassion, as a passive affect, is already taken to be effective. That is, it is already the beginnings of action — i.e., of the psychological effectivity that is presupposed by an effective will — even prior to the formulation of any maxim, or intelligible principle of action. It is possible for Cohen to give passive affect a positive interpretation, since the passivity of the psychological or effective

subject does not threaten the independence of thought; for Cohen, the theory of moral feeling is not psychological and metaphysical: affection is not in this context a causal concept. Rather, as we saw, affection is the intelligibility of feeling lawfullness; it is the idea of sensibility — or the given — as intelligible even though it does not have its origin in that intelligibility. And so, whereas aesthetic contemplation gives rise to desire or to humour, moral contemplation is already compassion. That is, the realist attitude of the "religious" stance takes the psychological affect of compassion to have systematic significance:

> The observation of another man's suffering is not an inert affect to which I surrender myself, particularly not when I observe it not as a natural or empirical phenomenon, but when I make of it a question mark for my whole orientation in the moral world (RR 18).

This attitude — the realist interpretation of affect as an objective motivation — is not and cannot be provided by the critical stance of philosophy. Rather, it depends on a commitment to subjective conditions as testifying to objective conditions: that the meaning of the suffering of the other is not in the first instance an empirical *fact* but a normative claim — commandment.

For Cohen, the specifically systematic content expounded in the *Religion of Reason* thus reaches its climax with the development of the concept of the fellowman as a singular instance of the genus 'humanity', but whose manifest suffering is the ground of the normativity of the concept 'humanity' as a subjective instance of the idea of the lawfullness of the political, which ought to be free of this — and any — suffering. Accordingly, "if the meaning of physical suffering, of physical ill in the human world, has always been a question for theodicy, then one could perhaps state this meaning in a paradox: the suffering, the **passion** [*Leiden*] is for the sake of **compassion** [*Mitleid*]" (RR 17). It is insofar as the suffering of the other can be interpreted (religiously) as demanding a critical revaluation of social and political structure that the subjective affect of compassion grounds the claim of the prophet to speak in the name of God for the sake of Man. Thus, prophetism, or social critique on the basis of the objective interpretation of subjective affect, is the essential achievement of a religion of reason as that which alone can provide to a system of philosophy the grounds of its intelligibility.

> Monotheism completes its development in the prophetic teachings; from the social-moral point of view, one may even say it develops toward the prophets, for the **peculiar** characteristic of the prophetic teaching consists in the connection of the alleged independence of evil with the alleged independence of morals. The prophet does not know this isolation. He knows only the correlation of God and Man, of man and God. He is therefore as much interested in **politics** as in the divine rule of the world. And politics for him certainly includes foreign, international politics, but it is, in the first place, social politics. (RR 132)

This transition — from the subjective affect of compassion to the prophetic claim against the political — is the special contribution of religion not just to ethics, but to a systematic conception of reason. For, it is only the religious standpoint that makes possible the interpretation of compassion as an objective motivation, thus a law *that is also a commandment*.

Cohen's understanding of revelation is peculiar in many ways. First, the content of revelation is not the revelation of law (as either the decalog or the Torah). Second, revelation does not come from a "face-to-face" encounter with the transcendent itself. Rather, revelation is the revelation of an absolute end, which is given through the face-to-face encounter with another human being and their particular suffering. Moreover, this revelation, since it takes place in feeling, cannot have any a priori determinable content, nor can it be thought of as an intelligible event. Rather, 'humanity', as the content of revelation, is (a) a *subjective concept* on the basis of which we may raise normative claims against existing systems of *objective law* and (b) an immediate motivation (i.e., a psychological effectivity) to raise this claim.

Cohen's concept of revelation is unusual in that it is neither purely theological, nor purely ideal. If the fundamental commandment of monotheism is to love God, this command manifests itself first and perhaps only as the immanent affect — *Mitleid* — through which we *already* love the fellowman. Thus, revelation may be described as providing immanent evidence that we are already implicated in a community of ends. However, it is precisely the facticity of this evidence — that it is *already there* — that gives rise, for Cohen, to the feeling of shame. For, the fact that of the community of ends and the fact of the suffering of the other can point to a temporality that cannot be anticipated. While logic anticipates unity and ethics anticipates eternity, the revelation of affective community with humanity reveals the *time of*

responsibility: that we are bound to a community of affective ends that *precedes* our ability to make a system of ends intelligible.

The apparent paradox of Cohen's proposal is that subjectively grounded concepts (our individual concept of 'humanity') are supposed to provide sufficient grounds for raising a claim against objective systems of law. In Kant's taxonomy, this would mean that mere "faith" can raise a legitimate claim against knowledge. Worse still, on the Kantian account, the aesthetic feeling that is the ground of the concept 'humanity' implies that the concept is not even subjectively valid, i.e., it is mere "opinion". For Cohen, however, the distinction between 'subjective' and 'objective' does not rest on a the distinction between that which is hypostasized as the intelligible grounds of empirical scientific claims and that which is rationally necessary as the condition of mere thought. Rather, for Cohen, both types of structure (transcendental and rational conditions) are subjective in origin. Since Cohen does not allow for a "metaphysical" hypostasis of intelligible structures, objectivity derives not from a (transcendental) necessity, but rather from the (immanent) necessity of the idea of lawfullness (*Gesetzlichkeit*); it is the system of science, not the structure of objective claims about intuitive representations, that grounds the distinction between subjective and objective claims.

To be sure, the ground of the critical claim against an objective system of laws is not the feeling that gives rise to compassionate motivation. Rather, compassionate motivation has objectivity only through the idea of lawfullness to which it testifies. The prophetic claim, therefore, contains no trace of the subjective or particular conditions that gave rise to the claim. Prophetic critique is an intelligible critique, even if its motivating ground is contingent:

> The moral excellence of the prophets consists in their refusal to measure and weigh the differences between well-being and ill according to subjective differences, to which disease and death itself belong. Instead, they measure them according to the objective social contradictions that upset the balance of society. (RR 133)

The critical or "prophetic" claim against the existing system of laws does not rest on the subjective concept of humanity as such; that is, it does not appeal to the particularity of my representation of the suffering of another or any representation of a subjective feeling or affect.

Morever, it does not make a claim about the origins of the suffering of the other. Regardless of whether the suffering of the other has its grounds in natural or political phenomena, its remedy is political: the ends of the political are the elimination of suffering and injustice. The kingdom of ends is thus not a *kingdom of respect* in which mutual recognition is the grounding norm, but rather a *kingdom of love* in which mutual benificence is the grounding norm.

Accordingly, the objectivity of ethics (and thus the objectivity of knowledge) rests on a claim about the 'messianic' conditions of eternity: 'If the kingdom of ends is realized, then there will be no suffering.' The prophetic claims does not (and cannot) affirm the antecedent — a metaphysical claim about eternity. Rather, the prophetic claim negates the consequent: there is suffering, therefore the kingdom of ends is not realized. At the same time, however, the prophet must also insist on the normative dimension of the kingdom of ends. That is, it is not sufficient for the prophet to point out that eternity "is not yet." Instead, the prophetic claim can only be an intelligible claim: that lawfullness is necessary to the intelligibility of the political. Accordingly, prophetic critique has two distinct moments:

The Weak Messianic Claim is a privative claim, which consists of the denial of the actuality of the kingdom of ends: the kingdom of ends is yet to come. We may formulate this claim as follows: 'Since the kingdom of ends is not yet realized, the political is not completely intelligible.' As such, it is a merely empirical claim.[37]

The Strong Messianic Claim is a positive, normative claim about the intelligibility of the political: politics must be a politics of redemption. We may formulate this claim as follows. 'If politics is intelligible, then it must aim to realize the kingdom of ends.' As such, it is a substantive thesis about the relation between politics and the theological.

The weak messianic claim is the revelation of the suffering of the other. However, the prophetic

[37]The labels 'strong' and 'weak' messianism derive from Walter Benjamin's distinction between retrospective and prospective messianism. For Benjamin, the "weak" messsianic power of historiography provides a limited form of redemption insofar as it testifies to suffering — and thus injustice — in the past. The historian, therefore, can redeem a fragment of the past by bearing witness to suffering and injustice. On the other hand, "strong" messianism implies the redemption of the future itself, or that through which the kingdom of ends (and thus the end of time, or eternity) can be brought into being.

appropriation of the revelation of the suffering of the other implies a broader, normative claim that is not justified by the appearance (or the affect) of compassionate suffering. Rather, the strong messianic claim rests on the intelligibility of a normative community; it is only through the *idea* of ethical community (*Sittlichkeit*) that the revelation of the other can be interpreted as a normative, political claim.

For Cohen, the reality of revelation is the suffering of the other: in the sympathy of *Mitleid* the affective community of humanity is revealed as the real content of the law (i.e., Torah). However, the intelligible content of revelation is not the law itself, but only the negation of its actualization. That is, the law is not revealed, but only the insufficiency of the current system of law. This is a negative revelation, or *apophasis*.[38] It is precisely in this formulation of revelation — as the fact of a responsibility that precedes every act of thought — that we can see the proximity of Cohen's religious thought to the contemporary consideration of the problem of ethical responsibility. The paradigmatic formulation of the problem of the otherness of the other — i.e., the empirical suffering of the other as a fact that transcends the domain of that which may be intelligibly constructed by the subject — derives from Levinas' phenomenological criticism of existential decision, particularly in *Totality and Infinity* (Levinas 1971) and *Otherwise than Being* (Levinas 1978).

Like Levinas, Cohen understands the reality of revelation to be an unintelligible consciousness of the suffering of the other — what Levinas calls the 'saying'. The saying — or the responsibility to others that precedes the intelligible act of liberal ethics — may be understood to be analogous to the revelation of *mitleid*: compassion reveals a pre-intelligible commitment to an affective community. Like Levinas, Cohen also understands the saying as a claim against the existing ethical order — or any voluntaristic conception of ethical grounding — which Levinas calls the 'said'. The revelation of responsibility, for Levinas, implies a critique of the adequacy of any conception of ethics grounded in the concept of the autonomy of the will, or a

[38] As is well known, Cohen returns to the medieval tradition of negative theology, and particularly the negative theology of Maimonides. The peculiar character of Cohen's negative theology is most clearly expressed in his 'Ethics of Maimonides' (translated and annotated in (Cohen 2004)).

liberal ethics. For Levinas — as for Cohen — the revelation of responsibility is apophatic: we learn (too late) of a responsibility that was not discharged.[39]

However, whereas Levinas' exposition of the saying rests on a phenomenological investigation of appearance as grounded in the productivity of an autonomous (or self-determining) subject, Cohen understands the revelation of responsibility to be the completion of an intelligible system. That is, the system of philosophy is actual only insofar as its intelligibility is at the same time *real*. Thus, for Cohen, the revelation of responsibility is immediately connected to the political, since the "saying" of revelation implies — through the psychological effectivity of compassion — the prophetic critique of existing systems of law in the name of lawfullness. That is, whereas Levinas' exposition of responsibility provides a justification for the weak messianic claim (i.e., the testimony of the suffering of the other), the phenomenological exposition of responsibility does not provide any explicit connection to the strong messianic claim. And so, while Levinas clearly shows how a voluntaristic structure (i.e., autonomy) is inadequate to the grounding of ethical responsibility (and thus fails to ground the wide duties of the doctrine of virtue), the connection between ethics and politics in Levinas' phenomenology remains opaque.[40] For Cohen, on the other hand, that connection is already implicit in the formulation of the prophetic claim against the political *status quo*.[41]

As we have seen, Cohen's *System of Critical Idealism* is a system of formal validity. Here, 'formal validity' may be understood not only as the validity of logical construction, but

[39]The implied claim here is that Cohen's doctrine of revelation is much closer to the tradition of "dialogical ethics" than is often thought to be the case. Dialogical ethics implies not only (as in Buber (**?**)), an immanent mode of consciousness in which the other is ethically transparent (i.e., the revelation of the "saying" or compassion), but also the consciousness of a factical state (paradigmatically, love) through which consciousness is itself individuated (as, for example, in Rosenzweig (**?**)).

[40]The subject of the connection between ethics and politics in Levinas is the most contested aspect of his work. For, Levinas provides few explicit indications of the relation between ethics and politics, even though the saying appears to imply a transformation not just in our conception of the subject, but also in our conception of the political through which peace and justice are to be realized.

[41]The claim could be expanded as follows: while both Cohen and Levinas provide an account of the universality of revelation through the suffering of the other (and Levinas' account of this is certainly more robust than Cohen's), Levinas is not able to provide an account of the prophetic appropriation of revelation as the basis for political critique. Cohen's understanding of prophetic critique implies an understanding of the political that is distinct from the liberal politics that are the target of Levinas' criticism. Moreover, it's not entirely clear how one can formulate the intelligibility of eternity (i.e., the normativity of the strong messianic claim) in the absence of a more robust conception of a philosophical system.

also as the adequacy of a system of principles (whether determining or positive) with respect to a particular end of human thought. However, such a system remains formal and "empty" (i.e., without positive content) insofar as it is merely a system of *critique*, or a system whereby normative claims become possible, but not actual. The legitimation of the normativity claimed by a system of critique — the claim of what Rosenzweig calls "philosophy" upon the subject — is not grounded in a system of critical reason itself, but rather in a doctrine of passive affect. If such a doctrine of passive affect were taken to be *self-sufficient*, then we would, no doubt, approach the Nietzschean proposal of a pure "psychology" of drives: the will to power. However, for Cohen — and for Rosenzweig as well — the passive affection of "religion" is not self-sufficient. On its own, it is merely subjugation, or at most what Levinas calls "subjectification". In order that the passivity of moral feeling may not collapse into *Schwämerei*, or moral sensualism, philosophy — or the universality of critical claims — must be attached to every instance in which affect is claimed as a motivation for action.

It is for this reason that every "prophetic" claim to justice is both (a) religious and (b) philosophical. For, on the one hand, it is always a claim to restore or renew the law. On the other hand, it is always a claim to restore or renew the law *for the sake of lawfullness*. The former is provided by religion, and a realist commitment to the subjective concept of humanity, while the latter is provided by philosophy — and the system of critique — insofar as every prophetic critique raises a normative claim that ought to become universal and binding for all. Whatever claims may have been made about the independence of Cohen's *Religion of Reason* from the body of his philosophical system, reports of Cohen's late conversion to an existentialist perspective have been greatly exaggerated. The religious dimension of Cohen's thought is not independent of the system of philosophical critique. Rather, religion — as a messianic religion — cannot, for Cohen, be grounded in a theory of affect at all. If religion is to avoid being merely myth and superstition, it requires a concept of history that is only provided by a philosophical conception of time (the future) and grounded in a system of laws (intelligibility).[42]

[42] Indeed, even if Nietzsche professes that the *Übermensch* is a futural concept (as in *Zarathustra*, it seems rather that Nietzsche's understanding of the overcoming of morality is therapeutic, and therefore represents a return to a

(mythical) original state of "health" (which is quite explicit in *The Anti-Christ*. For Cohen, Nietzsche's psychology is, in the end, merely a mythology of the human: apotheosis as the restoration of a lost state of human perfection in which the will is both (a) natural and (b) perfect. This too appears to be the logical consequence of *Fichte's* liberal state. Cohen, on the other hand, takes guilt (*Schuld*) to be the original state of the subject. Like Levinas, Cohen believes that recognition of moral autonomy always comes "too late", in the "said" that comes after the "saying", and which necessarily falls short of its objective. And so, like Levinas, Cohen is concerned with an idea of humanity that *ought to be*, which can only be grounded in a concept of humanity that *is not*, and has *never been*.

6.3 Conclusions

6.3.1 Post-Cohenian Idealism

It is not easy to assess the importance of Hermann Cohen's philosophical legacy. Almost as soon as it was published, Cohen's *System of Critical Idealism* was already outdated. Russell's *Principles of Mathematics*, first published in 1903, just one year after the first edition of Cohen's *Logik der reinen Erkenntnis*, provided a new and formidable synthesis of logic and mathematics that went far beyond what Cohen had imagined. Moreover, systematic idealism, as a philosophical project, came under attack both on the Continent and in the Anglo-American philosophical traditions.

Perhaps Cohen's most significant philosophical contribution is his innovative, if ultimately inaccurate, interpretation of Kant's *Critique of Pure Reason*. What Cohen's formalist interpretation showed was that it is possible to recover the original, epistemological revolution announced by the *Critique of Pure Reason* without resorting to the speculative logic of the philosophy of the subject. Cohen not only correctly assessed the epistemological significance of the Copernican revolution, but also correctly diagnosed (if accidentaly) the lingering metaphysical commitments of the Critical Project. In other words, Cohen provided a direct path from Kant's Copernican revolution to a contemporary view of the place of logic and metaphysics in scientific explanation.

Granted, this interpretation does extreme violence to the nature of transcendental philosophy and its proposed solutions, and Cohen has been justly criticized . However, from a contemporary perspective, this may not be a bad thing. For, the Kantian proposal has a problematic connection to intuitionist mathematics, and — even more troubling — its supposed anti-skeptical force (as a "transcendental argument" of sorts) derives from a highly suspect hypostasis (substantialization, even) of a mechanical conception of "matter" , without which neither the transcendental claim of objectivity, nor the resolution of the Mathematical Antinomies can succeed. Cohen provides a way of retaining some of the key insights of the transcendental

program, while also avoiding the paradoxes — and possibly also the dogmatism — of Kant's solution.

Over the years, many have objected that Cohen's reading of Kant is misguided, inaccurate, or otherwise harmful to Kant scholarship. To such objections we can only reply that Cohen was aware of, and embraced the revisionary character of his interpretation of the critique. This is explicit even in the first line of the Preface to the first edition of *Kant's Theory of Experience*, and it is a reading whose methodology he was often at pains to defend.[43] Indeed, Cohen presents his own system explicitly as an alternative to the "romantic" philosophies of post-Kantian idealism — Fichte, Schelling and Hegel.

We may ultimately disagree with Cohen's criticisms of Fichte and Hegel,[44] but, as I have tried to show, we must still take into account (a) the requirements of the programme of systematic idealism and (b) the content, object and structure implied by such a philosophical programme if we wish to show that earlier, more ambitious versions of the project of systematic idealism are to be preferred over Cohen's more modest attempt.

---

[43]Cohen's most sustained effort to justify his revisionary method of philosophical interpretation is found in his introduction to F.A. Lange's *Geschichte des Materialismus*. (**?**) However, the historical method that he proposed is deployed widely after his work in *The Principle of the Infinitesimal Method and its History*, including long sections of the *Logic of Pure Cognition*, which are devoted to the task of conceptual history, not the explicit development of the logical judgments themselves. This has prompted some to complain that Cohen does not provide arguments for his claim. This is, in a sense, justifed. For, Cohen does not aim to start from some accepted premises or axioms (as Fichte does), nor does he aim to start without presuppositions at all (as Hegel does). Rather, Cohen understand the task of *Erkenntniskritik* — of which the *System of Critical Idealism* is a part — to be engaged in the project of the critique of culture. The starting point of such an enterprise is contingent (the fact of culture), and can provide only a relative legitimation of the objectivity of its claims, conditioned as they are by the contingency of history itself. However, the critical historical analyses that are found throughout Cohen's systematic works are intended to demonstrate the (historical) necessity of particular concepts and categories, and thereby serve as "historical arguments" for the positions adopted in the *Logic* and elsewhere. These are not deductive arguments, but historico-critical deductions, which, much like Hegel's *Phenomenology of Spirit*, serve as (historically) immanent proofs of the concepts they are intended to defend.

[44]As I have already pointed out, Cohen's interpretation of Hegel is indefensibly metaphysical. However, his criticism of the general suppositions of the logic — the principle of determinability, the principle of totality — are much harder to dismiss. For those who wish to rehabilitate the Hegelian program, the challenge is not so much to defend Hegel against metaphysical interpretations of the *Science of Logic*, but to show that Hegel's key premises — determinability and totality — are defensible. Although I have not been able to show this in any detail here, I believe that justifying these claims — especially in the light of contemporary logic and mathematics and their applications to natural science — would be a difficult enterprise indeed.

Abbreviations

Immanuel Kant

I have primarily consulted the Cambridge Edition of the Works of Kant, and all citations are from this edition. (Kant, I. (1996-). *Cambridge Edition of the Works of Kant* (ed. Paul Guyer & Allen Wood). Cambridge University Press.) In German, I have used the Suhrkamp edition, based on the Insel Verlag edition. (Kant, I. (1958). *Werke*. Insel Verlag.) I have also used the translation conventions of the Cambridge Edition when rendering my own translations of Hegel, Maimon and Cohen to maintain consistency. For example, '*Begriff*' is translated as 'concept' (not 'notion') and '*Erkenntnis*' is translated as 'cognition', except where 'knowledge' is implied (as is often the case in Cohen). I have also used bold-face to indicate *Sperrdruck* and reserved italics for foreign words. Cohen, for example, was particularly fond of *Sperrdruck*, even though his texts were generally published in roman type, not Fraktur.

Anth.	AA	*Anthropology from a Pragamtic Point of View* (Kant 2007)
KRV	AA A/B	*Critique of Pure Reason* (Kant 1998)
KPrV	AA	*Critique of Practical Reason* (Kant 1996)
KU	AA	*Critique of the Power of Judgment* (Kant 2000)
JL	AA	*Jäschke Logic* (Kant 2004)
MGS	AA	*Metaphysical Foundations of Morals* (Kant 1996)
MS	AA	*Metaphysics of Morals* (Kant 1996)
Pro.	AA	*Prolegomena to Any Future Metaphysics* (Kant 1992)

Hermann Cohen

Hermann Cohen's published works have been reprinted by Georg Olms Verlag. (Cohen, H. (19-). *Werke* (ed. Helmut Holzhey). Olms.) At present, only the *Religion of Reason* (Cohen 1995) has been translated into English as well as some of the Jewish writings.

KTE1	CW 1.2	*Kants Theorie der Erfahrung, 1. Ausgabe* (Cohen 1885)
KTE2	CW 1.1	*Kants Theorie der Erfahrung, 2. Ausgabe* (Cohen 1871a)
KBE	CW 2	*Kants Begründung der Ethik* (Cohen 1910)
LRE	CW 6	*Logik der reinen Erkenttnis* (Cohen 1902a)
ERW	CW 7	*Ethik der reinen Willen* (Cohen 1907)
ÄRG	CW 8 & 9	*Ästhetik des reinen Gefühls* (Cohen 1912)
RV	CW	*Religion der Vernunft* (Cohen 1919) (Cohen 1995)

Georg Friedrich Wilhelm Hegel

Most extant translations of Hegel's works are problematic in one way or another. (I have not yet had occasion to consult Di Giovanni's new translation of the *Wissenschaft der Logik*.) Instead, I have provided my own translations based on the Meiner critical edition of Hegel's works. (Hegel, G.W.F. (1978-). *Werke* (ed. Otto Poggeler & Walter Jaeschke). Meiner.)

EN	HW 20	*Encyclopaedia* (Hegel 1991)
WL1	HW 21	*Wissenschaft der Logik: Objective Logik* (Hegel 1978)
WL2	HW 22	*Wissenschaft der Logik: Subjective Logik* (Hegel 1981)
PhG	HW 9	*Phenomenologie des Geistes* (Hegel 1977)

Salomon Maimon

Some of Maimon's works are not extant. Some of his works have been reprinted by Georg Olms Verlag. (Maimon, S. (2000). *Gesamelte Werke* (ed. Valerio Verra). Olms.) However, I have worked primarily from the original edition, since the Olms edition adds no critical apparatus (not even errata). There is a critical edition of Maimon's *Versuch über die transcendentale Philosophie* by Meiner (ed. Florian Ehrensberg), and a translation into English (Maimon 2010) from which my translations are drawn. All other translations are my own.

VTP	MW 2	*Versuch über die transcandentale Philosophie* (Maimon 1790)
KU	MW 7	*Kritische Untersuchung über die Menschlichen Geist* (Maimon 1797)
VNL	MW 5	*Versuch einer neuen Logik* (Maimon 1794)

Bibliography

Adickes, E. (1924). *Kant und das Ding an Sich*. R. Heise.

Allison, H. E. (2004). *Kant's Transcendental Idealism, Revised and Enlarged Edition*. Yale University Press.

Ameriks, K. (2000). *Kant and the Fate of Autonomy*. Cambridge University Press.

Arthur, R. (2007). 'x + dx = x': Leibniz's Archimedean infinitesimals.

Barrow, I. (1735). *Geometrical Lectures*. Stephen Austen.

Beiser, F. (1993). *The Fate of Reason*. Harvard.

Beiser, F. (2002). *German Idealism: The Struggle Against Subjectivism*. Cambridge University Press.

Bennet, J. (1966). *Kant's Analytic*. Oxford University Press.

Bergman, S. H. (1967). *The Philosophy of Solomon Maimon*. Magnes Press.

Berkeley, G. (1754). *The Analyst*. Tonson and Draper.

Bird, G. (2006a). The Neglected Alternative: Trendelenburg, Fischer and Kant. In *A Companion to Kant*. Blackwell.

Bird, G. (2006b). *The Revolutionary Kant*. Open Court.

Bishop, E. (1967). *Foundations of Constructive Analysis*. McGraw-Hill.

Bishop, E. . D. B. (1985). *Constructive Analysis*. Springer.

Bolzano, B. (1950). *Paradoxes of the Infinite*. Yale University Press.

Breazeale, D. (2002). Der Satz der Bestimmbarkeit: Fichte's Appropriation and Transformation of Maimon's Principle of Synthetic Thinking. *Deutscher Idealismus - Ein Internationales Jahrbuch 1*.

Brouwer, L. E. J. (1981). *Brouwer's Cambridge Lectures on Intuitionism*. Cambridge University Press.

Cantor, G. (1915). *Contributions to the Founding of the Theory of Transfinite Numbers*. Open Court.

Carlson, D. (2001). Hegel's Theory of Quantity. *Cardozo Law Review 23*, 2027–2155.

Cassirer, E. (1899). *Descartes' Kritik der mathematischen und naturwissenschaftlichen Erkenntnis*. Marburg.

Cassirer, E. (1910). *Substanzbegriff und Funktionsbegriff: Untersuchung über die Grundfragen der Erkenntniskritik*. Cassirer.

Cassirer, E. (1920). *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit, Bd. 3*. Bruno Cassirer.

Cohen, H. (1871a). *Kants Theorie der Erfahrung*. Ferdinand Dümmlers.

Cohen, H. (1871b). Zur Controverse zwischen Trendelenburg und Kuno Fischer. *Zeitschrift für Völkerpsychologie and Sprachewissenschaft*.

Cohen, H. (1878). *Platons Ideenlehre und die Mathematik*. Marburg.

Cohen, H. (1883). *Princip der infinitesimal Methode und seine Geschichte: Ein Kapitel zur Grundlegung der Erkenntniskritik*. Ferdinand Dümmlers.

Cohen, H. (1885). *Kants Theorie der Erfahrung, 2. Auflage*. Ferdinand Dümmlers.

Cohen, H. (1902a). *Kants Theorie der Erfahrung, 3. Auflage*. Bruno Cassirer.

Cohen, H. (1902b). *Logik der reinen Erkenntnis*. Bruno Cassirer.

Cohen, H. (1907). *Ethik des reinen Willens*. Bruno Cassirer.

Cohen, H. (1910). *Kants Begründung der Ethik, 2. Auflage*. Bruno Cassirer.

Cohen, H. (1912). *Ästhetik des reinen Gefühls*. Bruno Cassirer.

Cohen, H. (1919). *Religion der Vernunft: aus den Quellen des Judentums*. Gustav Fock.

Cohen, H. (1995). *Religion of Reason Out of the Sources of Judaism*. Scholars Press.

Cohen, H. (2004). *Ethics of Maimonides*. University of Wisconsin Press.

Cohen, P. (2008). *Set Theory and the Continuum Hypothesis*. Dover.

Couturat, L. (1914). *The Algebra of Logic*. Open Court.

De L'Hôpital, M. (1716). *Analyse des infinement petits, 2e édition*. François Montalant.

Dedekind, R. (1909). *Essays on the Theory of Number*. Open Court.

Descartes, R. (1984). *The Philosophical Writings of Descartes, Volume II*. Cambridge University Press.

Descartes, R. (1985). *The Philosophical Writings of Descartes, Volume I*. Cambridge University Press.

Deuber-Mankowsky (2004). The Ties Between Walter Benjamin and Hermann Cohen: A Generally Neglected Chapter in the History of the Impact of Cohen's Philosophy. *Journal of Jewish Thought and Philosophy 13*(1), 127–145.

Deuber-Mankowsky, A. (2005). Hanging over the Abyss: On the Relation between Knowledge and Experience in Hermann Cohen and Walter Benjamin. In *Hermann Cohen's Critical Idealism*. Springer.

Dummett, M. (1977). *Elements of Intuitionism*. Oxford, Clarendon Press.

Dussort, H. (1963). *L'école de Marbourg*. Presses Universitaires de France.

Edel, G. (1989). *Von der Vernunftkritik zur Erkenntnislogik*. Alber.

Edel, G. (1994). Die Entkräftung des Absoluten. Ursprung und Hypothesis in der Philosophie Hermann Cohens. In *Neukantianismus: Perspektiven und Probleme*. Königshausen & Neumann.

Elwes, R. (1951). *The Chief Works of Benedict de Spinoza, Volume 1*. Dover.

Feder, J. G. H. (1787). *Über Raum und Causalität: zur Prüfung der kantischen Philosophie*. Johnn Christian Dietrich.

Fichte, J. G. (2005). *The System of Ethics*. Cambridge University Press.

Fichte, J. G. F. (2000). *Foundations of Natural Right*. Cambridge University Press.

Fischer, K. (1870). *Anti-Trendeleburg, 2. Auflage*. Hermann Dabis.

Franks, P. (2005). *All or Nothing: Systematicity, Skepticism and Transcendental Arguments in German Idealism*. Harvard University Press.

Frege, G. (1884). *Die Grundlagen der Arithmetik*. Wilhelm Koebner.

Friedman, M. (1990). Kant on Concepts and Intuitions in Mathematical Sciences. *Synthese 84*(2), 213–257.

Friedman, M. (1992a). *Kant and the Exact Sciences*. Harvard University Press.

Friedman, M. (1992b). Kant's Theory of Geometry. In *Kant's Philosophy of Mathematics: Modern Essays*. Kluwer Academic.

Friedman, M. (2000). *A Parting of the Ways: Carnap, Cassirer and Heidegger*. Open Court.

Fries, J. F. (1828). *Neue oder anthropologische Kritik der Vernunft, 2. Auflage*. Christian Friedrich Winter.

Gawronsky, D. (1910). *Das Urteil der Realität*. Ph. D. thesis, Marburg.

Gibbs, R. (Ed.) (2006). *Hermann Cohen's Ethics*. Brill.

Hegel, G. (1977). *The Phenomenology of Spirit*. Oxford University Press.

Hegel, G. F. W. (1992). *Enzyklopädie der philosophischen Wissenchaften im Grundrisse*. Meiner.

Hegel, G. W. F. (1978). *Wissenschaft der Logik: Die objektive Logik*. Meiner.

Hegel, G. W. F. (1981). *Wissenschaft der Logik: Die subjektive Logik*. Meiner.

Hegel, G. W. F. (1991). *The Encyclopaedia Logic*. Hackett.

Hegel, G. W. F. (1995). *Grundlinien der Philosophie des Rechts*. Meiner.

Heidegger, M. (1985). *Schelling's Treatise on the Essence of Human Freedom*. Ohio University Press.

Henle, James M. and Kleinberg, E. M. (1979). *Infinitesimal Calculus*. MIT Press.

Henrich, D. (2003). *Between Kant and Hegel: Lectures on German Idealism*. Harvard University Press.

Hintikka, J. (1967). Kant on the Mathematical Method. *The Monist 51*, 352–375.

Holzhey, H. (1986a). *Cohen und Natorp*. Schwabe.

Holzhey, H. (1986b). *Ursprung und Einheit : die Geschichte der "Marburger Schule" als Auseinandersetzung um die Logik des Denkens*. Schwabe.

Holzhey, H. (Ed.) (1994). *Hermann Cohen*. P. Lang.

Horstmann, R.-P. (2008). Hermann Cohen on Kant's Transcendental Aesthetic. *Philosophical Forum 39*(2), 127–38.

Jacobi, F. H. (1787). *David Hume über den Glauben: oder Realismus und Idealismus*. Leowe.

Jacobi, F. H. (1789). *Ueber die Lehre des Spinoza in Briefe an den Herrn Moses Mendelssohn, Neue vermehrte Ausgabe*. Goett. Loewe.

Kant, I. (1992). *Theoretical Philosophy, 1755-1770*. Cambridge University Press.

Kant, I. (1996). *Practical Philosophy*. Cambridge University Press.

Kant, I. (1998). *Critique of Pure Reason*. Cambridge University Press.

Kant, I. (2000). *Critique of the Power of Judgment*. Cambridge University Press.

Kant, I. (2004). *Lectures on Logic*. Cambridge University Press.

Kant, I. (2007). *Anthropology, History and Education*. Cambridge University Press.

Keisler, J. (1976). *Elementary Calculus: An Infinitesimal Approach*. Prindle, Weber & Schmidt.

Köhnke, K. C. (1991). *The Rise of Neo-Kantianism: German Academic Philosophy between Idealism and Positivism*. Cambridge University Press.

Kuhn, T. (1962). *The Structure of Scientific Revolutions*. University of Chicago.

Lachterman, D. (1992). Mathematical Construction, Symbolic Cognition and the Infinite Intellect: Reflections on Maimon and Maimonides. *Journal for the History of Philosophy 30*(4), 497–522.

Leibniz, G. (1989). *Philosophical Essays*. Hackett.

Levinas, E. (1971). *Totalité et infini*. Kluwer.

Levinas, E. (1978). *Autrement qu'être ou au delà de l'essence*. Martinus Nijhoff.

Lotze, H. (1884). *Logic*. Clarendon Press.

Macdowell, J. H. (1994). *Mind and World*. Harvard University Press.

Maimon, S. (1790). *Versuch über die Transscendentalphilosophie*. Christian Friedrich Voß und Sohn.

Maimon, S. (1794). *Versuch einer neuen Logik oder Theorie des Denkens*. Ernst Felisch.

Maimon, S. (1797). *Kritische Untersuchungen über den menschlichen Geist*. Gerhard Fleischer dem Jüngern.

Maimon, S. (2010). *Essay on Transcendental Philosophy*. Continuum.

Melamed, Y. (2004). Salomon Maimon and the Rise of Spinozism in German Idealism. *Journal of the History of Philosophy 42*(1), 67–96.

Munk, R. (Ed.) (2005). *Hermann Cohen's Critical Idealism*. Springer.

Natorp, P. (1910). *Die logischen Grundlagen der Exakten Wissenschaften*. B.G. Teubner.

Nelson, E. (2005). Completed versus Incomplete Infinity in Arithmetic. http://www.math.princeton.edu/ nelson/papers.html.

Newton, I. (1803). *Principia*. Knight and Compton.

Nietzsche, F. (1967). *The Genealogy of Morals and Ecce Homo*. Random House.

Orth, E. W. (2001). Trendelenburg und die Wissenschaft als Kulturfaktum. In *Hermann Cohen und die Erkenntnistheorie*. Königshausen & Neumann.

Parsons, C. (1969). Kant's Philosophy of Arithmetic. In *Philosophy, Science and Method: Essays in Honor of Ernest Nagel*. St. Martin's Press.

Parsons, C. (1983). *Kant's Philosophy of Mathematics: Postscript*. Cornell University Press.

Parsons, C. (1984). Arithmetic and the Categories. *Topoi 3*.

Paton, H. (1936). *Kant's Metaphysic of Experience*. Allen Unwin.

Pinkard, T. (1981). Hegel's Philosophy of Mathematics. *Philosophy and Phenomenological Research 41*(4), 452–464.

Pistorius, H. A. (2007). Rezension von J. Schulzes Erläuterung zur Kritik der reinen Vernunft. In B. Gesang (Ed.), *Kants vergessener Rezensent*. Meiner.

Poma, A. (1997). *The Critical Philosophy of Hermann Cohen*. SUNY Press.

Poma, A. (2006). *Yearning for Form and Other Essay's on Hermann Cohen's Thought*. Springer.

Posy, C. J. (Ed.) (1992). *Kant's Philosophy of Mathematics: Modern Essays*. Kluwer.

Prauss, G. (1974). *Kant und das Problem der Dinge an Sich*. Bouvier.

Quine, W. V. O. (1953). *From a Logical Point of View*. Harvard University Press.

Reich, K. (1992). *The Completeness of Kant's Table of Judgments*. Stanford University Press.

Robert, A. M. (2003). Nonstandard Analysis. *Dover*.

Robinson, A. (1969). *The Metaphysics of the Calculus*, Chapter 10, pp. 153–163.

Robinson, A. (1996). *Non-standard Analysis*. Princeton University Press.

Russell, B. (1903). *The Principles of Mathematics*. Norton.

Schelling, F. (1856). *Sämmtliche Werke, 1. Band*. J.G. Gotta'scher Verlag.

Sellars, W. (2000). *Knowledge, Mind and the Given*. Hackett.

Shabel, L. (1998). Kant on the 'Symbolic Construction' of Mathematical Concepts. *Studies in the History and Philosophy of Science 29*(4), 589–621.

Shabel, L. (2003). Reflections on Kant's Concept (and Intuition) of Space. *Studies in History and Philosophy of Science 34*, 45–57.

Shabel, L. (2004). Kant's "Argument from Geometry". *Journal of the History of Philosophy 42*(2), 195–215.

Sinaceur, H. (1994). Calculation, Order and Continuity. In *Real Numbers, Generalizations of the Reals, and Theories of Continua*. Kluwer.

Spinoza, B. (1992). *The Ethics*. Hackett.

Stern, R. (Ed.) (1999). *Transcendental Arguments: Problems and Prospects*. Clarendon Press.

Stolz, O. (1885). *Vorlesung über Allgemeine Arithmetik: nach den neueren Ansichten*. Teubner.

Strawson, P. (1966). *The Bounds of Sense*. Methuen.

Sutherland, D. (2006). Kant on Arithmetic, Algebra and the Theory of Proportions. *Journal of the History of Philosophy 44*(4), 533–558.

Thielke, P. (2001). Getting Maimon's Goad: Discursivity, Skepticism and Fichte's Idealism. *Journal of the History of Philosophy 38*(1), 101–134.

Thielke, P. (2003). Intuition and Diversity: Kant and Maimon on Space and Time. In *Salomon Maimon: Rational Dogmatist, Empirical Skeptic*. Kluwer.

Thompson, M. (2008). *Life and Action*. Harvard University Press.

Thomson, I. (2000). Ontotheology? Understanding Heidegger's Destruktion of Metaphysics. *International Journal of Philosophical Studies 8*(3), 297–327.

Tonelli, G. (1994). *Kant's Critique of Pure Reason within the Tradition of Modern Logic*. Georg Olms.

Trendelenburg, A. (1840). *Logische Untersuchungen, 1. Band*. G. Bethge.

Trendelenburg, A. (1843). *Die logische Frage in Hegels System*. F.A. Brockhaus.

Trendelenburg, A. (1862). *Logische Untersuchungen, 2. ergänzte Auflage*. S. Hirzel.

Trendelenburg, A. (1869). *Kuno Fischer und sein Kant*. S. Hirzel.

Van Cleve, J. (1999). *Problems from Kant*. Oxford University Press.

Vuillemin, J. (1954). *L'héritage kantien et la révolution copernicienne*. Presses Universitaires de France.

Wolff, M. (1995). *Vollständigkeit der kantischen Urteilstafel: mit einem Essay über Freges Begriffsschrift*. Vittorio Klostermann.

Wood, A. W. (2008). *Kantian Ethics*. Cambridge University Press.